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AAMRL-TR-89-047

AD-A231 283

A TUTORIAL ON THE ANGULAR POSITIONS AND VELOCITIES OF GROUND OBJECTS VIEWED FROM AIRCRAFT (U)

Herschel C. Self

ARMSTRONG AEROSPACE MEDICAL RESEARCH LABORATORY

OCTOBER 1989

PERIOD OF PERFORMANCE: JUNE 1988 TO SEPTEMBER 1989

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FOR THE COMMANDER

CHARLES BATES, JR.
Director, Human Engineering Division
Armstrong Aerospace Medical Research Laboratory

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution is Unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AAMRL-TR-89-047	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Harry G. Armstrong Aerospace Medical Research Laboratory 6b. OFFICE SYMBOL (If applicable) AAMRL/HEA	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Wright-Patterson AFB OH 45433-6573	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO: 62202F PROJECT NO.: 7184 TASK NO: 11 WORK UNIT ACCESSION NO.: 58

11. TITLE (Include Security Classification)
A Tutorial on the Angular Positions and Velocities of Ground Objects Viewed from Aircraft (U)

12. PERSONAL AUTHOR(S)
Herschel C. Self

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Summary	FROM Jun 88 TO Sep 89	1989, October	82

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP
05	05	

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Angular Motion, Alpha Velocity, Angular Velocity, Azimuth Velocity, Declination Velocity, Dynamic Acuity, Dynamic Visual Acuity, Flow Diagram, Flow Lines, Relative Velocity,

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report is a primer or tutorial on the angular motion of ground objects relative to level-flying aircraft. Both qualitative and quantitative description is used. Equations are derived in detail for angular relationships and for angular velocities in alpha, azimuth, and declination. Terms are defined and included are tables, graphs and worked-out examples and problems. Problems in low-altitude flight, dynamic visual acuity, apparent motion, and angular velocities at aircraft windows are discussed. 25 *Angular motion,

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr Herschel C. Self	22b. TELEPHONE (Include Area Code): (513) 255-8887 22c OFFICE SYMBOL: AAMRL/HEA

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

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PREFACE

This report was prepared in the Human Engineering Division of the Armstrong Aerospace Medical Research Laboratory (AAMRL), Wright-Patterson Air Force Base, Ohio. The work was performed under Project 7184, "Man-Machine Integration Technology," Task 718411, "Design Parameters for Visually-Coupled and Visual Display Systems." Special thanks are due to Dr H.L. Task of the Crew Systems Effectiveness Branch of AAMRL for encouragement to organize working notes into a technical report and for reading the document. Thanks are due to the Crew Station Integration Branch of AAMRL for supplying the B-52 window plots generated by the COMBIMAN computer program developed for the Human Engineering Division of AAMRL. The assistance of Miss Tanya Ellifritt and Miss Yolanda Crawford of AAMRL in preparing the manuscript is appreciated.

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SUMMARY

In high-speed low-altitude flight, objects on the ground are viewable for only a short time and can have high angular velocities. This impairs navigation, flight safety, and the detection and tracking of targets. The present paper is a tutorial on angular velocity. The apparent motion of ground objects and texture streaming is discussed. Detailed derivations of equations for the angular velocities of ground objects are presented in text book fashion. Universal tables of angular velocity based on the equations are provided. Example problems are presented and solutions are provided. The construction of flow charts is described. How to plot angular velocity at locations on aircraft windows is illustrated with figures of the windows of a B-52 aircraft for different-sized pilots and seat adjustments. The variation in angular velocity with aircraft height is described by examples and graphs. The influence of angular velocity on dynamic visual acuity is reviewed.

Previous literature used ground references to calculate angular velocity. The present paper uses azimuth and declination, which are observer references. Earlier papers concentrated on results or use of equations with little attention to how to derive and apply the equations. The present paper shows how.

PURPOSE

Viewed from an aircraft, ground objects have angular velocities and eventually exit from an observer's field-of-view. In high-speed flight at low altitudes, angular velocities can be high and available time short, making it difficult for aircrews to find ground objects and take effective action. System design, analyses of system performance, and mission planning all require data on angular relationships and angular velocities. Not all of the required equations are in the literature, and some equations are tedious to derive. There is also a shortage of tables and graphs for analysis purposes. To make up for this shortage, the present tutorial report was written as a primer on the angular positions and angular velocities of ground objects viewed from an aircraft in level flight. Both qualitative and quantitative descriptions of angular motion are presented.

1.0 INTRODUCTION - AIRCRAFT MOTION AND ANGULAR VELOCITIES

For an aircraft flying very low and very fast, the angular velocities of ground objects near the aircraft can be very high. Ground objects may be quite close to the aircraft when aircraft motion unmasks them from concealing vegetation, terrain, or man-made objects. Rapid angular motion in both azimuth and declination, both at different angular velocities, combined with little available viewing time, make it difficult for a crewmember to detect, identify and take action against ground objects. At low aircraft altitudes, vibration and turbulence, by degrading visual capabilities, increase the difficulty of carrying out these tasks. Systematic search is difficult or impossible, and aiming at and tracking ground objects are both difficult and inaccurate. Danger, stress and workload are all high in low-altitude high-speed flight. The problems in such flight regimes are indicated by the short list of problems of Appendix 4.

Since vehicle motion may degrade a crewmember's ability to observe, as well as his ability to take action, vehicle motion must be taken into account in the design of weapon systems, in analyses of system performance, and in the planning of missions.

Taking angular motion rates and available time into account requires equations for angular velocity as a function of aircraft speed and height, and the location of a ground object. Location may be specified relative to the aircraft or relative to the ground. For some purposes, such as aiming or tracking with a weapon or a sensor, or just where to look, the observer wants to know how many degrees clockwise from straight ahead, and how many degrees down from level or horizontal. These two angles are azimuth or azimuth angle A, and declination or declination angle (dip angle) D, respectively. (Note that azimuth angle (angle to the side) may also be defined as the angle between a straight-ahead horizontal line (zero azimuth) and a line from the aircraft passing directly over the ground point or object.) (Declination may also be defined as the angle between a horizontal line (zero declination) passing directly over a ground object and a line from the aircraft to the object or ground point.) Figure 13 in Appendix I depicts the various angles and distances.

An equation containing distance ahead R and offset distance S from the flight path is not suitable for pointing or aiming. For straight and level flight, define alpha as the angle between a straight-ahead line along the flight path in the sky, i.e., along the aircraft velocity vector, and a line from the aircraft to a ground point or object. An equation for angular velocity in Alpha of ground objects relative to the observer is derived as Equation 16 in Appendix 1, where d_i/dt is the derivative of the angle with respect to time, i.e., is angular velocity:

$$d\alpha_i/dt = 57.296(V/H)\mathrm{Sin}\ D_i \mathrm{Cos}\ D_i \sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i} \text{ Degrees/Second}$$

For some purposes, specifying object location relative to the terrain is useful. This type of specification was used by Erickson (1965), who used offset distance S of the ground object from the flight path and its distance R_i ahead of the aircraft. With this terrain reference, angular velocity in alpha is given by Equation 15 derived in detail in Appendix 1:

$$d\alpha_i/dt = 57.296V\sqrt{H^2 + S^2}/(H^2 + S^2 + R_i^2)$$

Havron (1962) used yet another specification of object position relative to the terrain. He used X and Y coordinates in an aircraft that is diving or landing.

There are three angular velocities of particular interest. Azimuth velocity or angular rate in azimuth, declination velocity or angular rate in declination, and alpha velocity or angular rate in alpha. Appendix 1 of the present paper derives equations for all three angular velocities, the relationships between them, and equations for ground range, slant range, distance ahead, and offset of objects from the aircraft's ground path. The various equations for angular rates and angular relationships, as well as the other quantities mentioned, are not difficult to derive. However, some of the equations are long and tedious enough to invite errors in derivation. For readers of this report who want to peruse the derivations of the equations, Appendix 1 gives detailed derivations. The equations apply to straight level flight.

Tables are provided for quick determination of angular rates. Equations are given for calculating and graphing flow fields on azimuth-declination plots. The derived equations are applied to find the angular velocity rates in various areas of the windows of an aircraft from the pilot's eye position. Since this paper is tutorial, it uses a textbook approach and derivations of equations are presented in detail. Since low-altitude flight at high-speed provides conditions that pose angular rate problems for crewmembers, this flight regime is emphasized in the examples. However, the equations and tables are not limited to low altitude flight.

1.1 The Apparent Motion of Ground Objects

The motion of an aircraft over the ground changes the azimuth and the declination of ground objects: every object and every ground point is constantly changing position relative to the aircraft and observers on it. To an airborne observer, objects have angular velocities. In perception, the terrain, not the observer, appears to be moving. The apparent angular velocity of a point on the ground has the same magnitude as the angular velocity of the aircraft observed from the ground point. It is readily apparent to passengers in ground and airborne vehicles that nearby objects move rapidly, while distant objects, i.e., objects with low declinations, move slowly or not at all. Sometimes, distant objects appear to move with the observer. It is also readily noticed that the angular velocity of a ground object also varies with its position in the field of view, i.e., varies with its azimuth and declination. As distance decreases, the angle subtended at the eye by any dimension of an object increases, and it appears to grow in size. Angular motion becomes more apparent and, at close range, may be very high. As distance decreases, approach speed appears to increase, and motion downward and outward both increase. At close range, objects appear to fly outward at high speed. Once past an object, it appears to recede with decreasing velocity, while inward and upward motion rates are decreasing and apparent size is shrinking at a decreasing rate.

Except for objects directly in front of the vehicle, the paths in azimuth and declination are curves. Each object follows its own curved path. This path does not vary with aircraft speed, although speed along the path is directly proportional to vehicle speed. Thus, if aircraft speed is multiplied by n, then angular velocity in azimuth and in declination are also multiplied by n. If an observer views the ground with a device that has a magnification M, angular motion rate is magnified by M.

When angular motion rates are high, an observer may notice that ground objects and ground texture appear to flow, forming a flow field. The apparent paths are flow lines or streamers, and they are useful in perception of motion. Gibson (1955) discusses the optical expansion pattern in aerial locomotion. He notes that the flow of optical stimulation due to the motion of an observer provides a continuous feedback of information used in controlling motion. Changing optical stimulation conforms to the principle of motion parallax or motion perspective. Retinal motion provides information about both space and the motion of the observer. A frontal surface perpendicular to the line of locomotion yields a radial pattern of velocities, a streamer pattern, while a longitudinal surface parallel to the line of locomotion yields a unidirectional pattern of velocities. The first is perceived as an expansion when moving toward the surface and a contraction when moving away from the surface. The second is perceived as a flow.

2.0 CONSTRUCTION OF FLOW CHARTS

As noted earlier, ground objects and ground texture appear to an airborne observer to move along paths that are called streamers or flow lines. A flow line may be plotted on an X-Y ground plot or on an azimuth-declination plot. A plot or graph with several flow lines is called a flow graph, flow diagram, or flow chart. It is a picture of streamer paths, the paths of apparent motion of ground points.

An azimuth-declination flow diagram depicts the path of ground objects in the field-of-view. The path's direction at any point in the field is the direction of motion of the object at that point. All flow lines on a graph may start at the same declination below horizontal. In constructing the graph, initial azimuth may be varied in steps, one flow line starting at each step and moving down the graph. Figure 1 is an example of a flow diagram. It contains several flow lines, each with a different initial or starting azimuth A_0, but with the same initial declination D_0 of 1^o. In this figure, time (t) was varied in steps from zero until each line reached the edge of the graph. The graph is for a height of 200 feet and a ground speed of 440 feet/second (300 miles/hour). The paths of flow lines, although plotted for a given speed, are the same for any speed at the same height. However, speed along a flow line is proportional to aircraft speed. Position on a flow line at any time t also depends on aircraft speed.

An example showing how to construct a flow line on a flow chart for straight and level flight at a height above ground of H feet with a ground speed of V feet/second illustrates the procedure for constructing flow charts. All lines of the chart are constructed by the same method. Equation 34 in Appendix 1 gives azimuth A_i after t seconds as A_i= ArcTan $1/(\mathrm{Cot}A_0 - Kt)$, where $K = (V \mathrm{Tan}\, D_0)/H \mathrm{Sin}\, A_0$. In these equations, A_0 and D_0 are initial azimuth and declination, respectively, i.e., azimuth and declination when t = 0. In the present example, let initial declination D_0 be 1^o below horizontal and initial azimuth A_0 be $.5^o$ for a height H of 200 feet and a speed V of 440 feet/second (300 miles/hour). For these conditions, the constant K is K = (440/200)Tan

1^o/Sin $.5^o$ = 4.40050. With this value of K, Equation 34 is then A_i = ArcTan 1/(Cot.5 - 4.4005t) = ArcTan 1/(114.589 - 4.4005t) . Here, t is in seconds. The corresponding declination D_i is given in Appendix 1 by Equation 35 as D_i = ArcTan (Tan D_0/Sin A_0) = ArcTan (Tan 1^o/Sin $.5^o$) Sin A_i) = D_i = ArcTan (2.00023 Sin A_i). Figure 1 is the graph of a flow line based on these equations. The time in seconds since the initial condition is indicated at each plotted point. Using the same procedure, other flow lines starting at other initial locations may be constructed to obtain the flow chart of Fig 2. For example, it is easily shown, by the above equations, that the equations for a flow line starting at a declination D_0 of 1^o and an azimuth A_0 of 10^o are A_i = ArcTan 1/(5.67128 - .221143t) and D_i = ArcTan (.100520 Sin A_i).

When plotting a flow chart or when planning action of some kind, it may be necessary to determine how long it takes for an object at a position of azimuth A_0 and declination D_0 to reach an azimuth A_i. The time required is obtained by solving the A_i equation in the above paragraph for t. For the conditions of the example, Tan A_i = 1/(114.589 - 4.40050t), from which t = 26.0400 - 1/4.40050 Tan A_i. The time to reach 45^o, for example, is t = 26.0400 - 1/4.4005 Tan 45^o = 25.813 seconds. As a second example, what time is required for A_i to reach 90^o? For A_i = 90^o, Tan A_i = infinity, hence t = 26.040 = 1/(infinity) = 26.040 - 0 = t = 26.040 seconds. Alternatively, for A_i = 90^o, from Equation 6 in Appendix 1, R_i = H Cos A_i/Tan D_i, from which R_0 = H Cos A_0/Tan A_0 = (200) Cos.5/Tan 1 = 11,457.6 feet. At a speed of 440 feet/second, time for A_i to reach 90^o is the time for R_i to reach zero, thus t = 11,457.6/440 = 26.040 seconds, as before.

When a flow chart with several lines is to be constructed, much labor will be saved by using a computer plotter. Figure 1 was constructed using a hand calculator and plotting by hand. The equations of Appendix 1 permit labeling selected points on the flow lines with ground range R_i^*, slant range r_i, distance ahead R_i to the ground point or object, elapsed time t, and angular velocity in alpha d $_i$/dt, in azimuth dA_i/dt, or in declination dD_i/dt, or any combination of these. A picture of an aircraft window or the field-of-view of an optical

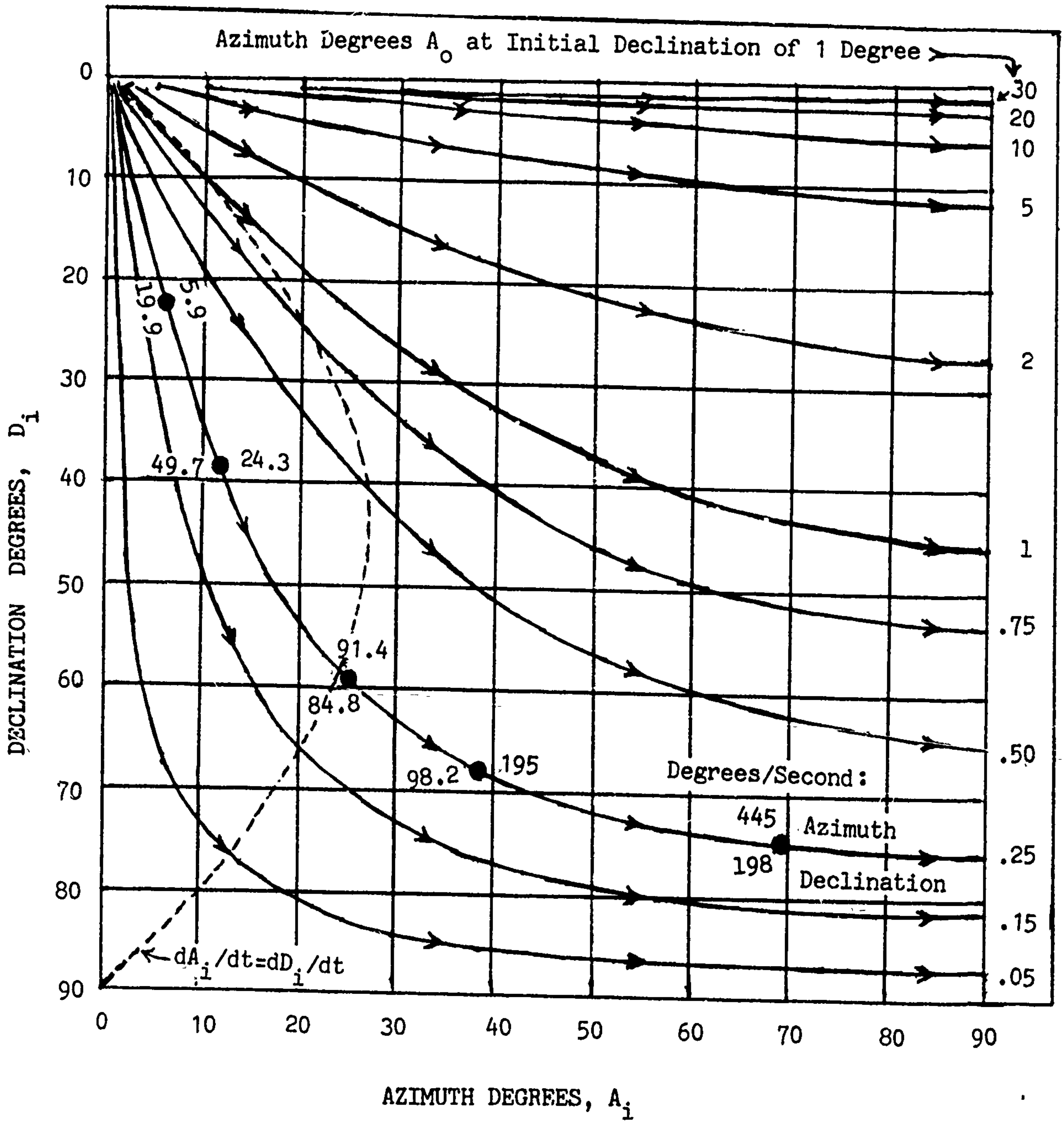


Fig 1. Flow diagram in azimuth and declination for an aircraft height of 200 feet. The curves are motion paths for ground objects and ground texture. Angular velocities are for a speed of 440 ft/sec. The dashed line indicates field locations having the same azimuth and declination velocities at all aircraft speeds.

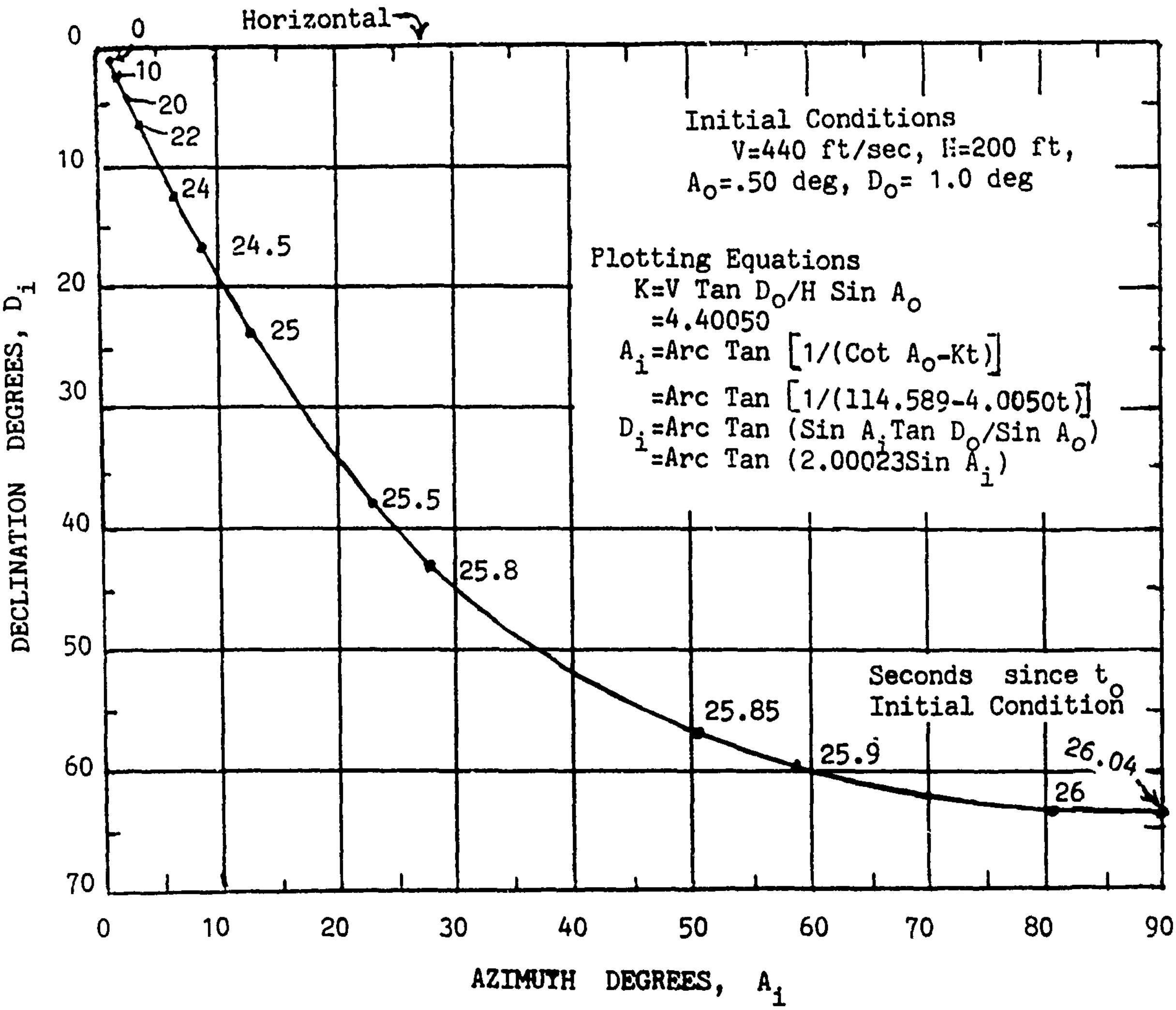


Figure 2. An example flow line for an aircraft height of 200 feet. Time markers are for an aircraft speed of 440 feet/second.

device or display may have a flow chart superimposed on it. For the flow line in Figure 1 that starts with an initial declination of 1^0 and azimuth of $.25^0$, selected points have been labeled with azimuth and declination velocity. Calculation of these velocities will be covered later on.

When display magnification M is not unity, angular rates at positions in the field-of-view will be magnified by M. Thus, if magnification is 2, rates are doubled. Flow charts for diving aircraft are discussed by Havron (1962) for X-Y ground plots, i.e., for flow lines plotted against ground coordinates.

3.0 ANGULAR VELOCITY IN ALPHA BY EQUATIONS

Erickson (1965), in his analysis and review of visual detection of ground targets, examines the angular rate in the angle alpha. Angular rate is also called angular velocity. Alpha is the angle between the flight path in the sky of an aircraft and a line from the aircraft to an object on the ground. This angle is also the angle between a line-of-sight that is level and straight ahead and a line to the ground object. For straight level flight at constant speed, the equation that Erickson provided for angular velocity in the angle alpha is $d\alpha_i/dt = V\sqrt{H^2+S^2}\,/(H^2+S^2+R_i^2)$ radians/second. Here, V is aircraft ground speed, H is aircraft height above the ground, S is offset distance of a ground object from the ground path or track of the aircraft, and R_i is the object's distance ahead of the aircraft measured parallel to the aircraft ground path. To obtain angular velocity in degrees/second, the above equation, which provides radians/second, must be multiplied by 57.296, converting it to Equation 15 in Appendix 1 of the present report. The equation is derived in detail in the appendix. In his report, Erickson (1965) includes a nomograph for angular velocity in alpha prepared by Roy Dale Cole. This nomograph yields angular velocity without having to do any calculation to an accuracy adequate for some purposes. Unfortunately, available copies of the report provide the nomograph in a small size and with poor reproduction quality. It is easier, quicker, and more accurate to use the angular velocity Equation 15.

From the equation, it is apparent that angular velocity in alpha is directly proportional to aircraft ground speed V. It is also clear that the angular velocity increases as distance ahead R_i decreases. The angular velocity situation for height changes is more complex, since increasing height also increases the declination of a ground object. Equation 15 also reveals that angular velocity in alpha is maximum when distance ahead R_i is zero. For a zero R_i, a ground object is directly beneath the aircraft, or is on a line passing beneath the aircraft and perpendicular to the ground path. Here, Equation 15 becomes $d\alpha_i/dt = 57.296\, V/\sqrt{H^2+S^2}$. When distance ahead R_i and offset S are both zero, Equation 15 becomes $d\alpha_i/dt = 57.296\,(V/H)$, the angular velocity for objects directly beneath the aircraft. This position under the aircraft is the location where ground objects have the highest angular velocity.

An equation for angular velocity in alpha for ground objects on the ground path or ground track of the aircraft, where azimuth is zero, is obtained by setting offset distance S to zero, yielding $d\alpha_i/dt = 57.296VH/(H^2 + R_i^2)$. For objects straight ahead at a height H, $\mathrm{Tan}\,D_i = H/R_i$, from which $R_i = H/\mathrm{Tan}\,D_i$. Substituting for R_i in the above equation, $d\alpha_i/dt = 57.296\,VH/(H^2 + R_i^2) = 57.296\,VH/(H^2 + H^2/\mathrm{Tan}^2D_i) = 57.296\,(V/H)/(1 + 1/\mathrm{Tan}^2D_i) = 57.296(V/H)/(\mathrm{Tan}^2D_i + 1)/\mathrm{Tan}^2D_i = 57.296\,(V/H)/(\mathrm{Sec}^2D_i/\mathrm{Tan}^2D_i) = 57.296\,(V/H)\,\mathrm{Tan}^2D_i\mathrm{Cos}^2D_i = 57.296\,(V/H)(\mathrm{Sin}^2D_i/\mathrm{Cos}^2D_i)\mathrm{Cos}^2D_i = d\alpha_i/dt = 57.296\,(V/H)\mathrm{Sin}^2D_i$ degrees/second for ground objects on the aircraft's ground path. Alpha velocity for zero azimuth is also derivable from Equation 16 in Appendix 1. $d\alpha_i/dt = 57.296(V/G)\mathrm{Sin}D_i\mathrm{Cos}D_i\sqrt{\mathrm{Tan}^2D_i+\mathrm{Sin}^2A_i}$. When A_i is zero, $\sqrt{\mathrm{Tan}^2D_i+\mathrm{Sin}^2A_i} = \sqrt{\mathrm{Tan}^2D_i+0} = \mathrm{Tan}D_i = \mathrm{Sin}D_i/\mathrm{Cos}D_i$, so that $d\alpha_i/dt = 57.296\,(V/H)(\mathrm{Sin}\,D_i\mathrm{Cos}\,D_i)(\mathrm{Sin}\,D_i/\mathrm{Cos}\,D_i) = 57.296\,(V/H)\mathrm{Sin}^2D_i$, as above. From this equation, note that, when azimuth is zero, angular velocity in alpha increases as Sin^2D_i, approaching zero as declination D_i approaches zero. Also, for declination $D_i = 90^0$ (straight down), angular velocity in alpha is 57.296(V/H) degrees/second.

As shown earlier, when declination as well as azimuth is zero, the ground object is straight ahead on the level of the aircraft and Sin^2D_i is zero, so that angular velocity in alpha, by the equation, is zero. The aircraft is on a collision course with the ground object and the object is at the center of an expanding pattern of flow lines. The vectors of all angular velocities point radially away from the impact point. Collision can occur in level flight with tall objects, such as buildings, radio and television towers, tall trees, power lines, hills and cliffs, or upward sloping terrain.

From Equation 16, when a ground object is at an azimuth A_i of either 90^0 (straight right) or 270^0 (straight left), $\sqrt{\mathrm{Tan}^2D_i+\mathrm{Sin}^2A_i} = \sqrt{\mathrm{Tan}^2D_i+1} = \sqrt{\mathrm{Sec}^2D_i} = \sqrt{1/\mathrm{Cos}^2D_i} = 1/\mathrm{Cos}\,D_i$. The equation then becomes $d\alpha_i/dt = 57.296(V/H)\mathrm{Sin}\,D_i\,\mathrm{Cos}\,D_i\,(1/\mathrm{Cos}\,D_i) = 57.296\,(V/H)$ Sin

D_i. Thus, for an azimuth of either 90^0 or 270^0, angular velocity in alpha increases as the sine of the declination angle, reaching a maximum, as before, of 57.296 (V/H) for $D_i = 90^0$ (straight down). Also, for objects at either 90^0 or 270^0, alpha velocity varies inversely with height.

When declination is constant, variation of azimuth from 0^0 to 360^0 defines a circle on the ground. From Equation 16 in Appendix 1, $d\alpha_i/dt = 57.296\ (V/H)\mathrm{Sin}\ D_i \mathrm{Cos}\ D_i \sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i}$, it is clear that minimum alpha velocity occurs when azimuth is 90^0 and 270^0, where sin A_i has its maximum of 1. It is also clear that minimum alpha velocity occurs when azimuth is 0^0 and 180^0, where Sin $A_i = 0$. The minimum in both cases, from the equation, is:

$$d\alpha_i/dt = 57.296\ (V/H)\mathrm{Sin}\ D_i \mathrm{Cos}\ D_i \sqrt{\mathrm{Tan}^2 D_i + 0}$$

$$= 57.296\ (V/H)\mathrm{Sin}\ D_i \mathrm{Cos}\ D_i \mathrm{Tan}\ D_i$$

$$= 57.296\ (V/H)\mathrm{Sin}\ D_i \mathrm{Cos}\ D_i (\mathrm{Sin}\ D_i/\mathrm{Cos}\ D_i)$$

$$d\alpha_i/dt = 57.296\ (V/H)\mathrm{Sin}^2 D_i \mathrm{Cos} D_i.$$

Going back to Equation 16, with constant declination, alpha velocity on the ground circle defined by this constant declination increases from 0^0 to 90^0, decreases from 90^0 to 180^0, increases from 180^0 to 270^0, and decreases from 270^0 to 360^0. The minima at 0^0 and 180^0 are equal, and the maxima at 90^0 and 270^0 are equal. The decline from 90^0 to 180^0 mirrors the growth from 0^0 to 90^0.

To determine how alpha velocity varies with change in offset distance S, note that the derivative of angular velocity with respect to offset S is change in angular velocity with change in offset. Similarly, the derivative with respect to height is change in angular velocity with change in height. To calculate these derivatives, alpha velocity Equation 16 in Appendix 1 may be written as $d\alpha_i/dt = 57.296V \sqrt{H^2 + S^2}/(H^2 + S^2 + R_i^2) = KV(H^2 + S^2)^{1/2}\ (H^2 + S^2 + R_i^2)^{-1}$. Note that, when W and Z are functions of S, d(Wz)/ds = W(dZ/ds) + Z(dW/ds). Applying this differentiation rule to the above alpha velocity equation, and differentiating both size of the equation,

$$d(d\alpha_i/dt)/dS = KV(H^2 + S^2)^{1/2}(-1)(H^2 + S^2 + R_i^2)(2S) + KV(H^2 + S^2 + R_i^2)^{-1}(1/2)(H^2 + S^2)^{-1/2}(2s) =$$

$$d(\alpha_i/dt)/dS = \frac{-2SV\sqrt{H^2 + S^2}}{(H^2 + S^2 + R_i^2)^2} + \frac{SV}{(\sqrt{H^2 + S^2})(H^2 + S^2 + R_i^2)}$$

Placing both fractions over a common denominator and combining them,

$$d(d\alpha_i/dt)dS = \frac{[-2SV(H^2 + S^2) + SV(H^2 + S^2 + R_i^2)]}{(\sqrt{H^2 + S^2})(H^2 + S^2 + R_i^2)^2} = \frac{SV[R_i^2 - (H^2 + S^2)]}{(\sqrt{H^2 + S^2})(H^2 + S^2 + R_i^2)^2}$$

From this equation it may be seen that, when $R_i^2 > (H^2 + S^2)$, an increase in offset distance S increases the angular rate, and when $(H^2 + S^2) > R_i^2$, an increase in S decreases the angular rate in alpha. This result was shown in Erickson's paper (1965), with the steps not shown. Erickson noted, fro: examination of the equations, that substituting HV for SV in the numerator gives

$$d(d\alpha_i/dt)dH = HV[R_i^2 - (H^2 + S^2)]/(\sqrt{H^2 + S^2})(H^2 + S^2 + R_i^2)$$

for the rate of change of angular velocity in alpha with change in aircraft height H. Thus, his Figure 13 show a boundary on the ground where the sign of $d(d\alpha_i/dt)$ changes.

Some of the results obtained in this section from examination of the equations for alpha velocity are as follows:

1. Angular velocity in alpha is directly proportional to aircraft ground speed, but varies in a more complex way with height.

2. The closer an object on the ground is to an aircraft, the higher its angular velocity in alpha.

3. The maximum angular velocity in alpha of any ground object is 57.296 (V/H), and is for objects directly beneath the aircraft.

4. For objects on the ground path or track of the aircraft (zero azimuth), alpha velocity is $d\alpha_i/dt=57.296\ (V/H)\mathrm{Sin}^2 d_i$, which has a maximum of 57.296(V/H) when D_i is 90^0 (straight down), and a minimum of 0 for $D_i = 0$ (horizontal). Also, an equation not containing D_i for objects on the flight path, which gives the same results, is $d\alpha_i/dt = 57.296VH/(H^2 + R_i^2)$. This equation has a maximum of 57.296 (V/H) when R_i is zero, straight down in this case, and a minimum of 0 for infinite R_i, which is for horizontal viewing.

5. For a given declination D_i, maximum alpha velocity is for objects with an azimuth of either 90^0 or 270^0. For objects at either of these two azimuths, $d\alpha_i/dt = 57.296(V/H)\mathrm{Sin}\ D_i$ or, equivalently, 57.296(V/H), which are maximum for $D_i = 90^0$ (straight down), or for offset S = 0, respectively, and minimum of zero for $D_i = 0$ (horizontal), or for infinite offset S (also, horizontal).

6. Angular velocity in alpha is zero for an object directly ahead on a level with the aircraft (H=0). In this case, the aircraft is in a collision course with the object, and the object is at the center of a radially expanding flow pattern.

7. For any given declination, variation of azimuth from 0^0 to 360^0 describes a circle of ground points that are all equally distant from the aircraft. For objects on this ground circle, alpha velocity is lowest at zero azimuth (straight ahead), increases as azimuth increases to 90^0 (directly right), then decreases as azimuth further increases to 180^0 (directly to the rear). The decrease from 90^0 to 180^0 mirrors the increase from 0^0 to 90^0. Angular velocity in alpha at 0^0 and 180^0 are equal, but of opposite sign (approaching versus receding).

8. When $R_i^2 > (H^2 + S^2)$, an increase in offset distance S increases alpha velocity, and when $R_i^2 < (H^2 + S^2)$, an increase is S decreases angular rate.

4.0 EQUATIONS, TABLES AND GRAPHS FOR ANGULAR VELOCITY

There are three angular velocities that are of interest for objects on the ground: velocity in alpha, azimuth velocity, and declination velocity. In the following discussion, each will be defined and equations will be given for calculating angular velocity. Universal tables are provided in Appendix 2 for each of the three angular velocity types. The equations and tables are for straight level flight at a constant velocity. The tables are for an aircraft ground speed of 440 feet/second (300 miles/hour) at an altitude of 200 feet above ground, but are usable for any speed or altitude, as explained later. Alpha is the angle between a level line along the aircraft flight path in the sky and a line from the aircraft to the ground point or ground object being observed. An equation for angular velocity in the angle alpha, using aircraft velocity V, height above ground H, azimuth angle A_i and declination angle D_i to the object, is derived in Appendix 1, Equation 16. This equation is $d\alpha_i/dt = 57.296(V/H)\mathrm{Sin}D_i\mathrm{Cos}D_i\sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i}$ degrees/second. Here, $d\alpha_i/dt$ is the derivative with respect to time of the angle alpha, i.e., rate of change of alpha, commonly called velocity in alpha or just alpha velocity. D_i is declination below horizontal, and A_i is azimuth or angle off from straight ahead.

As may be noted from inspection of the equation, it does require some calculation which can be considerable if angular velocity is required for several azimuths and declinations. In this case, a universal angular velocity table may be of value for avoiding much computation. Table A1 in Appendix 1 is such a table. The table values are based on an aircraft speed of 440 feet/second at a height of 200 feet. For different height H in feet and speed V in feet/second, multiply table entries by (V/440)(200/H), i.e., by (5/11)(V/H). This can be done because, in the equation for alpha velocity given above, V and H appear as the multiplier V/H.

The data of table A1 are plotted in Figure 3, with velocity in alpha on the vertical axis, declination on the horizontal axis, and families of curves for various values of azimuth, one curve for each

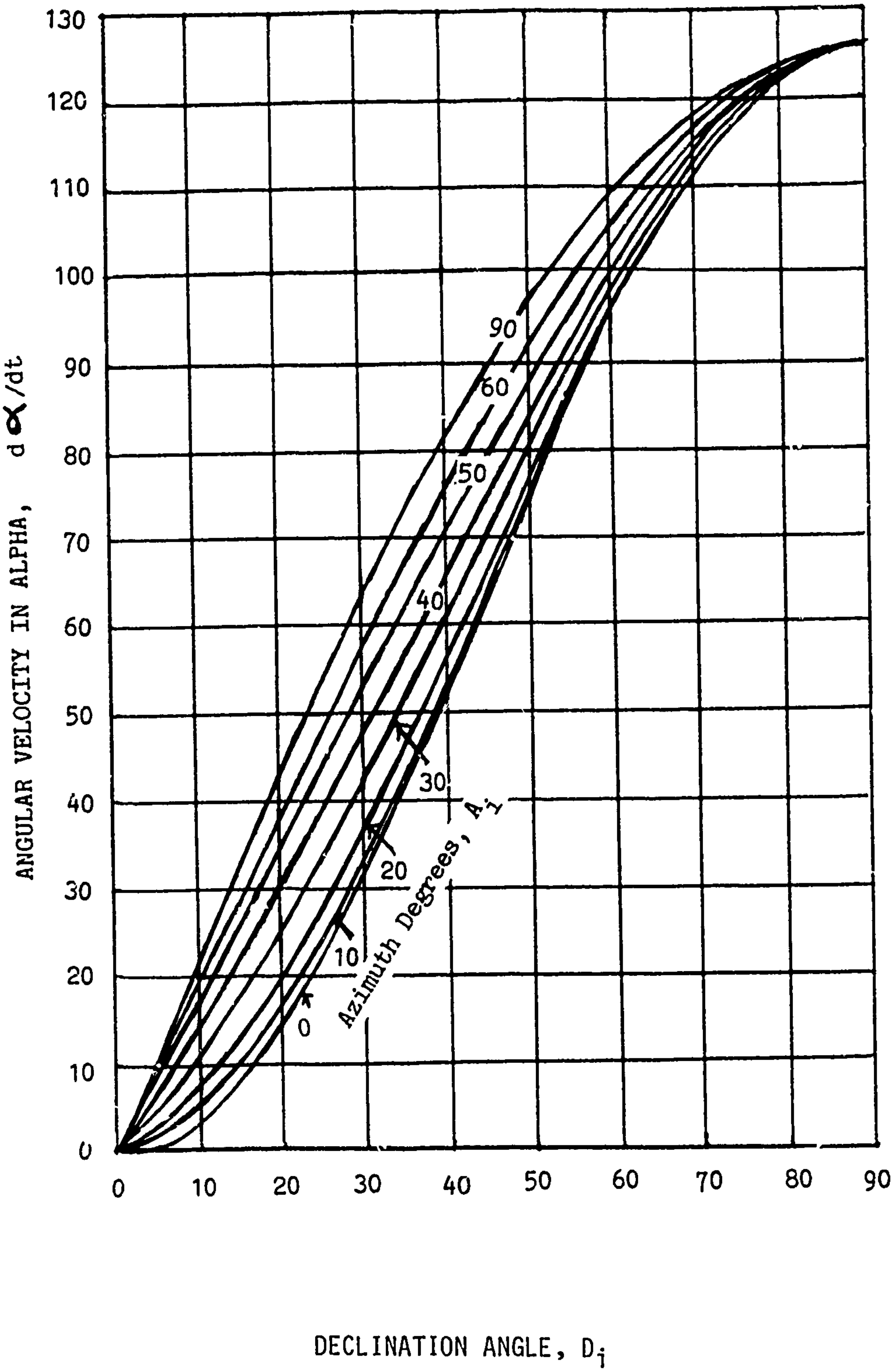


Figure 3. Vel ty in alpha for various azimuths for an aircraft speed of 440 feet/ ond at a height of 200 feet above ground. The graph is a plot of the data in Table A1.

azimuth. The same data could, of course, be plotted with azimuth on the horizontal axis and a series of curves, one for each declination. From the figure, note that the azimuth curves all start at zero declination and converge to a peak at 90^0 declination. Had declination been plotted out to 180^0, a mirror image of the azimuth curves for 0^0 to 90^0 would have been produced. All curves fall to 0^0/second at an azimuth of 180^0. The peak of the curves is at a declination of 90^0 and, as explained elsewhere, is 57.296 (V/H) = 57.296(440/200) = 126.05 degrees/second.

When the distance ahead of the aircraft R_i and offset S from the aircraft ground path are given, azimuth A_i and declination D_i may be calculated from equations in Appendix 1: A_i = Arc Tan(S/R_i), and D_i = Arc Tan$(H/\sqrt{H^2+S^2+R_i^2})$, respectively. These values may then be used in the alpha velocity equation given above. A more direct approach, when S and R_i are given, is to use Equation 15 from Appendix 1: $d\alpha_i/dt = (57.296 \cdot \sqrt{H^2 + S^2})/(H^2 + S^2 + R_i^2)$.
For some purposes, such as aiming, pointing, or tracking, the angular velocities in azimuth and declination are more applicable than velocity in alpha $d\alpha_i/dt$. Equations for angular velocities in azimuth and declination, using A_i and D_i, rather than R_i and S, are derived in Appendix 1.

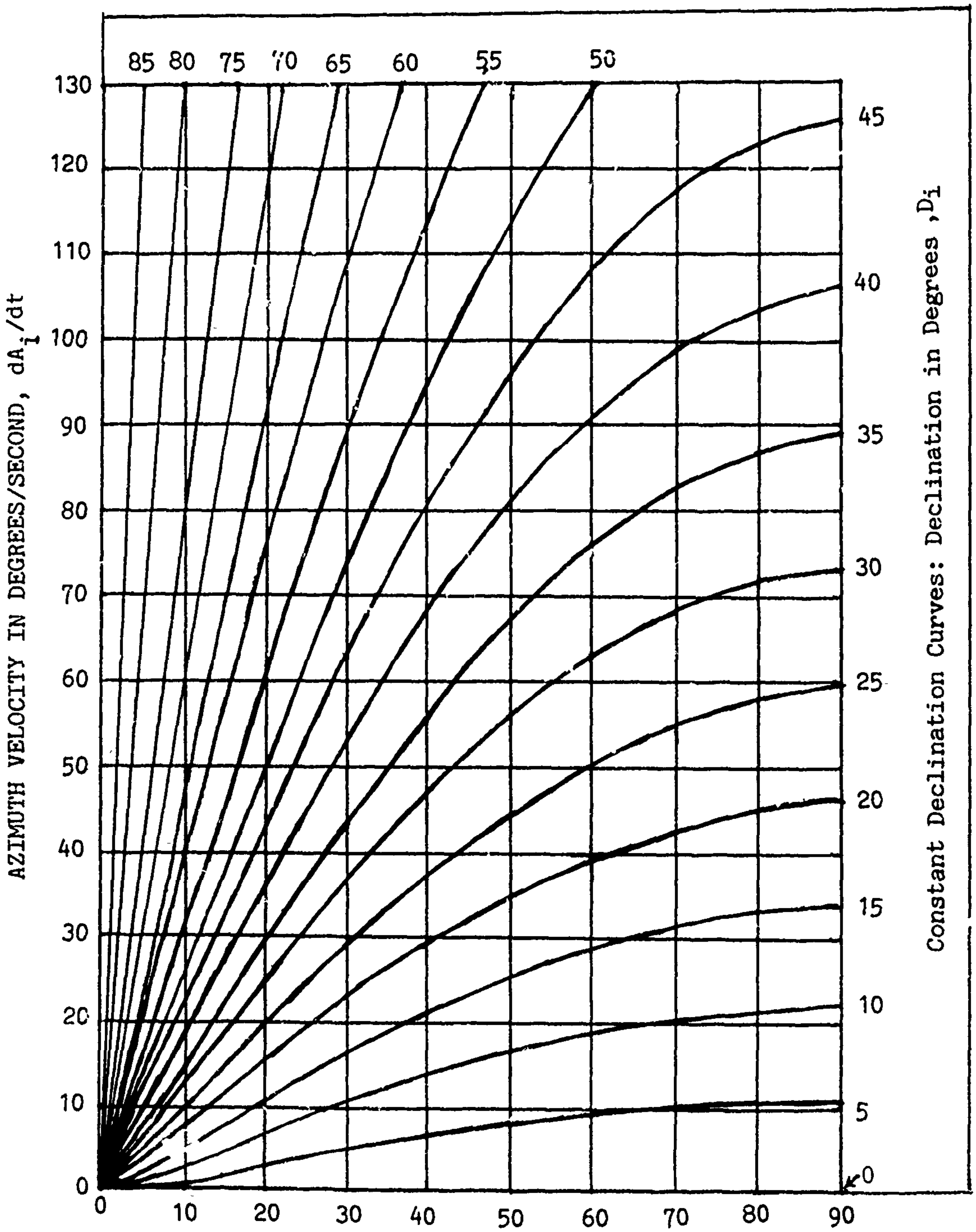
The azimuth or azimuth angle of a ground object is an angular measure of how far to the side of straight ahead it is located. It is the angle between a straight-ahead line level with the aircraft and a level line at aircraft height passing over the ground object. Azimuth velocity dA_i/dt is rapidity of change of azimuth. An equation for azimuth velocity derived in Appendix 1 is Equation 21: dA_i/dt = 57.296 (V/H)$\mathrm{Sin}A_i\mathrm{Tan}D_i$ degrees/second. The constant 57.296 is the number of degrees in one radian, i.e., $180/\pi$.

Equation 21, with A = 440 feet/second, and H = 200 feet, was used to calculate the entries in Table A2 of Appendix 2, a universal azimuth velocity table. For aircraft velocity V other than 440 and height 200, multiply the table entries by (5/11)(V/H). Had the tables not been intended to provide actual angular velocities for examples and graphs, table entries could have been calculated using only 57.296 Sin A_i Tan D_i, and, for velocity V and height H, entries would be multiplied by V/H, rather than by (5/11)(V/H).

The data of Table A2 may be plotted on a graph, with azimuth on the horizontal axis and azimuth velocity on the vertical axis, with one curve for each selected value of declination. This was done to generate Figure 4. The graph is for straight and level flight at a speed of 440 feet/second (300 miles/hours) at a height of 200 feet. Note, from the graph, that, for any declination (for any curve), as azimuth increases, azimuth velocity increases (curves have a positive slope), but at a decreasing rate (curves less steep). Also, note that as declination increases, azimuth velocity increases (higher curves), and increases at a higher rate (steeper slopes) with azimuth increase. Examination of the Sin A_i Tan D_i of the angular velocity equation leads to the same conclusions.

Inspection of either the graph or the table from which it was constructed reveals that azimuth velocity may be quite high when neither azimuth nor declination are very high. For example, suppose that, for an azimuth of 30^0, it is required to determine the declination below which azimuth velocity exceeds 30^0/second. On the graph, trace up along a vertical line at a 30^0 azimuth. Note that the vertical intersects the declination curve for 25^0 just short of 30^0/second. Thus, for any declination greater than just a little more than 25^0 at a 30^0 azimuth, azimuth velocity exceeds 30^0/second. The equation for the table yields an exact value. Here, dA_i/dt = 126.05 Sin A_iTan D_i = 30 = 126.05 Sin 30 Tan D_i, from which Tan D_ii = 30/126.05/Sin 30^0 = .47600, and D_i = ArcTan .47600 = 25.45^0 declination.

Azimuth velocity may also be calculated from offset S and distance ahead R_i, rather than from azimuth and declination. The equation for doing this is derived in Appendix 1 as Equation 20: $dA_i/dt = 57.296VS/(S^2 + R_i^2)$. Offset S is the distance off to the side of the aircraft ground path of the ground object, and distance ahead R_i is the



AZIMUTH IN DEGREES, a_i

Figure 4. Azimuth velocity for various declinations for an aircraft speed of 440 feet/second at a height of 200 feet above ground. The graph is a plot of the data in Table A2.

distance of the ground object ahead of the aircraft measured along the ground parallel to the ground path or ground track of the aircraft.

An equation for angular velocity or angular rate in declination is derived in Appendix 1 as Equation 22: $dD_i/dt = 57.296\ (V/H)\ \mathrm{Cos}\ A_i\ \mathrm{Sin}^2 D_i$ degrees/second. This equation was used to calculate the entries in Table A3 in Appendix 2. The entries are for an aircraft ground speed of 440 feet/second and height above ground of 200 feet. For other values of V and H, multiply the table entries by (5/11) (V/H). Angular velocity in declination is plotted as a graph in Figure 5, with angular velocity in declination on the vertical axis and declination in degrees on the horizontal axis. Curves for azimuth values from o^0 to 90^0 were plotted. Note the flattening out of the angular velocity curves, as would be expected from the Cos A_i term in the equation, reaching zero for a 90^0 azimuth (straight to the side). Also, note how declination velocity increases as declination increases, as expected from the $\mathrm{Sin}^2 D_i$ term in the equation.

Declination velocity may also be calculated using an Equation containing offset S and distance ahead R_i, instead of azimuth and declination. An equation for this is dervied in Appendix 1 as equation 22a: $dD_i/dt = 57.296VHR_i/(H^2 + S^2 + R^2{}_i)\sqrt{S^2 + R_i{}^2}$.

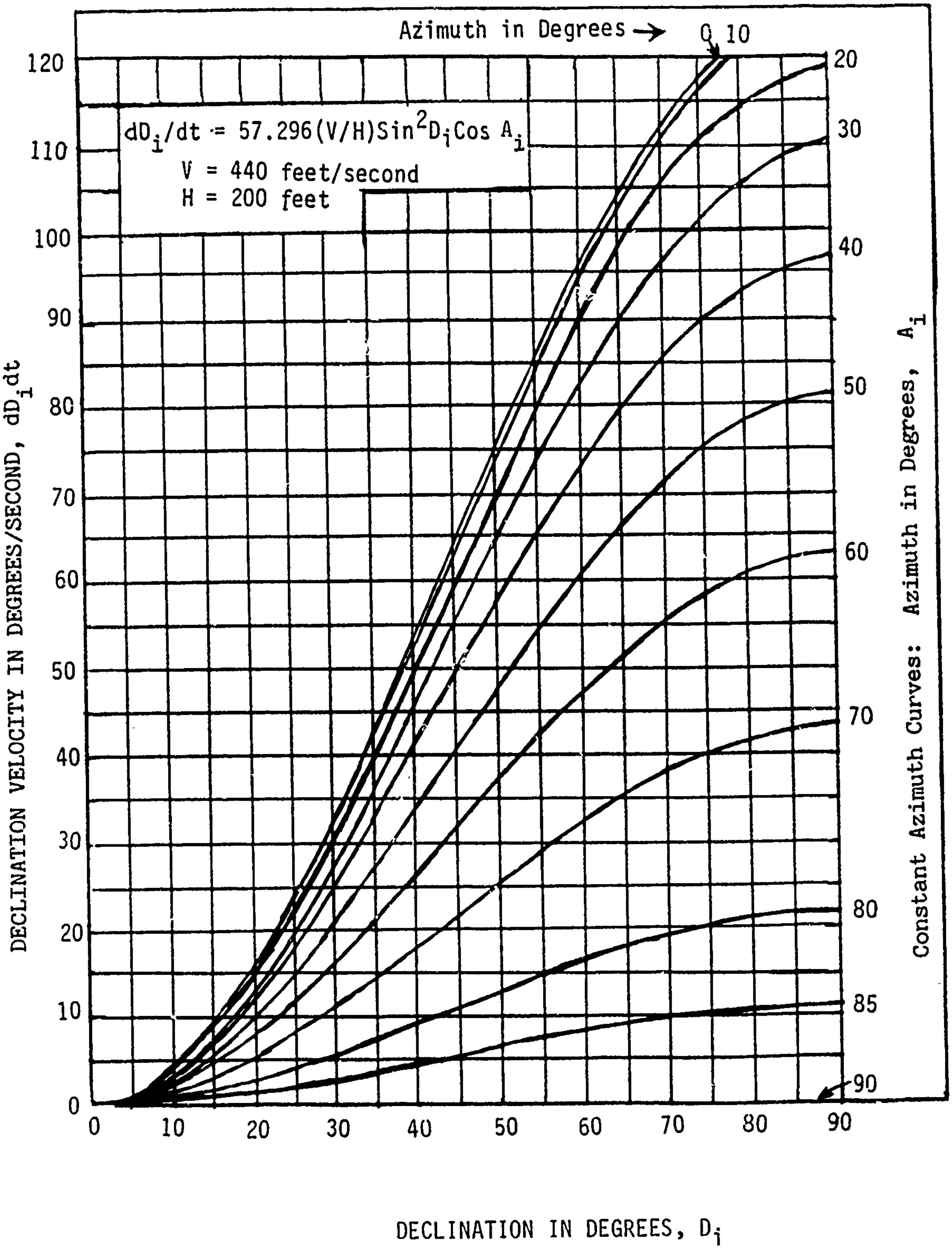


Figure 5. Declination velocity for various azimuths for an aircraft speed of 440 feet/second at a height of 200 feet above ground. The graph is a plot of the data in Table A3.

5.0 VARIATION IN VELOCITY IN ALPHA WITH AIRCRAFT HEIGHT

It is instructive to work out numerical examples in which angular velocity in alpha is compared at different aircraft heights above the terrain. For a first example, suppose that ground speed V, offset distance S from the aircraft ground path, and distance R_i of the ground object ahead of the aircraft are the same for both aircraft. In this case, the same ground object is viewed by observers at different heights above ground. Angular velocity in alpha for aircraft flying straight and level at a constant ground speed V and height H above ground is given by Equation 16 in Appendix 1 as $d\alpha_i/dt = 57.296\ V\sqrt{H^2 + S^2}/(H^2 + S^2 + R_i^2)$. Let H_1 be the height of the lower aircraft, and H_2 be the height of the higher aircraft. To compare angular velocities, use the ratio M of the alpha velocity of the higher aircraft to that of the lower one. Using the above equation, this ratio is $M = (d\ _i/dt)_2/(d\ _i/dt)_1 = \left[\sqrt{(H_2{}^2+S^2)/(H_1{}^2+S^2)}\right]\left[(H_1{}^2 + S^2 + R_i{}^2)/(H_2{}^2 + S^2 + R_i{}^2)\right]$. To illustrate variation of the angular velocity ratio M with height differences, suppose that the lower aircraft is at a height of 200 feet, and that the upper aircraft is directly above it and ten times higher at 2,000 feet. With these heights, the ratio of the angular velocities is $M = \left[\sqrt{(2{,}000^2 + S^2)/(200^2 + S^2)}\right]\left[(200^2 + S^2 + R_i{}^2)/(2000^2 + S^2 + R_i{}^2\right]$. When several values of distance ahead R_i are used with a given value of offset distance S, a curve of the ratio M may be plotted for that offset distance. In Figure 6, offset distances of 0, 200, 400, 800, 1,200, 2,000, and 4,000 feet are used with distances ahead R_i varied from 0 to 12,000 feet. Computation with the M equation yields a family of curves, one for each offset distance S. The family of curves produced this way is shown in Figure 6.

From examination of the curves in the figure, it is clear that, the shorter the standoff distance S, the shorter the distance ahead R_i at which the angular rate for the upper aircraft surpasses that for the lower aircraft, and the more rapidly the velocity ratio M increases with distance ahead R_i, i.e., the steeper the curves. Note that, for a zero distance ahead R_i, the S curves are lower for lower offset distances S, i.e., the lower the angular velocity ratio M. It is also

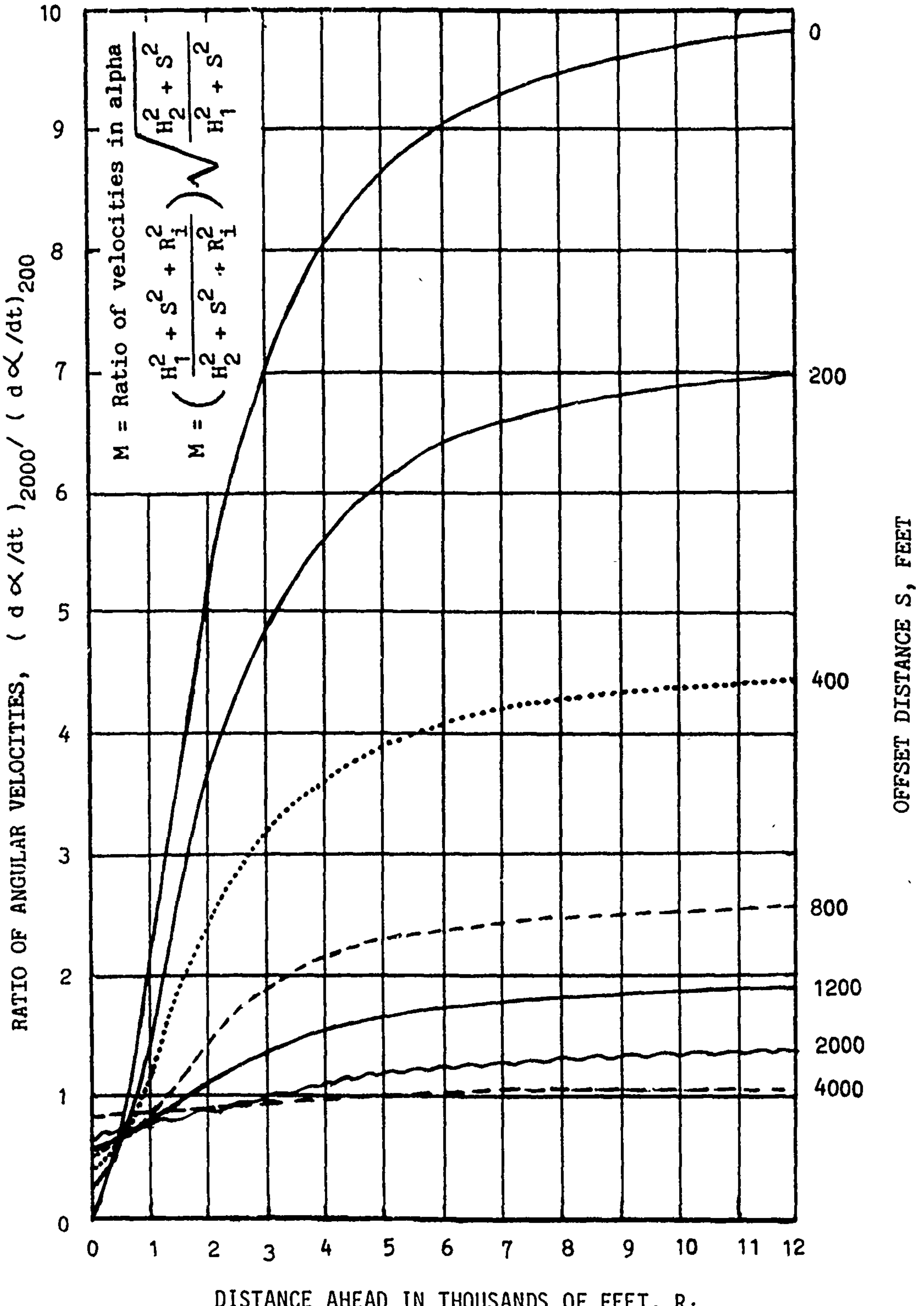


Figure 6. Ratio of angular velocity in alpha at a 2000-foot height to velocity at a 200-foot height when both aircraft speeds are equal and distance ahead R_i is the same for both. One aircraft in directly above the other.

clear that the higher aircraft is presented with lower angular velocities ($M<1$) only when distance ahead R_i is less than about 1,000 feet. Appreciably lower angular velocity for the upper aircraft is present only for R_i less than about 500-1,000 feet. It is clear that, for many combinations of offset and distance ahead, observers in the upper aircraft see angular rates that are higher to much higher than observers in the lower aircraft. Note, from the graph, that the largest S value curve plotted was for an offset S of 4,000 feet, and that this curve fell close to the M = 1, or equal angular velocity line, for all distance ahead. Also note that, for offsets of 0-200 feet at distances of over 2,000 feet, upper aircraft angular velocities are 3.5 to almost 10 times greater than those for the lower aircraft.

In the examples above, both aircraft were above the same ground point, with observers viewing the same object at the same distance ahead. When an aircraft is flying at a higher altitude, objects on the ground may be detected and observed at greater distances, since there may be less masking by vegetation, buildings, and terrain. For a second example comparing angular velocities at different heights, let the lower aircraft be at a 200-foot height and the upper one at 2,000 feet, as before. Now, however, let the upper aircraft view the ground object from a distance ahead R_i that is ten times as great as for the lower aircraft. The same ground object is viewed from both vehicles with the same offset S and aircraft speed V. Before calculating the angular velocity ratio M, a look at the ratio of slant ranges may be of interest. Slant range is the straight line distance from aircraft to ground object. From Equation 2 in Appendix 1, $r_i = \sqrt{H^2 + S^2 + R_i^2}$. The slant range ratio for the two air vehicles in this example is $\sqrt{(10H)^2 + S^2 + (10R_i)^2} / \sqrt{H^2 + S^2 + R_i^2} = \sqrt{100H^2 + S^2 + 100R_i^2} / \sqrt{H^2 + S^2 + R_i^2}$. For ground objects on the ground path of the aircraft, where S is zero, this ratio is $\sqrt{100\,(H^2 + R_i^2)} / \sqrt{H^2 + R_i^2} = 10$. For ground objects, then, slant range for the upper vehicle is 10 times as great for objects directly ahead, and decreases from 10 as offset increases from zero.

In this second example, the angular velocity ratio M is $M = (d\alpha_i/dt)_{2000}/(d\alpha_i/dt)_{200} = M = \sqrt{(2{,}000^2 + S^2)(200^2 + S^2)}\ \left[(200^2 + S^2 + R_i^2)/(2{,}000^2 + S^2 + 100\ R_i^2\right]$. Variation of S and R_i in this equation yields a family of curves shown in Figure 7. From this figure it may be seen that, when observing from a height ten times as great and with the ground object's distance ahead R_i ten times as great, results are very different from those of the first example. For distances ahead of more than about 700 feet for the lower vehicle and 7,000 for the upper one, at offsets up to 1,600 feet, the largest shown in the figure, alpha velocity is less than .4 as high for the upper aircraft. For a zero offset, where slant range is ten times as great, the angular velocity ratio curve is flat at .1.

Flying higher and observing ground objects while they are at a longer distance ahead of the aircraft can, in some situations, be beneficial by considerably reducing angular velocity in alpha, allowing more observing time, and being able to view ground objects masked at lower altitudes by intervening terrain or vegetation and manmade objects. The reduced angular velocity and the smoother ride sometimes present at higher altitude may permit better vision, better aiming and more accurate tracking of ground objects.

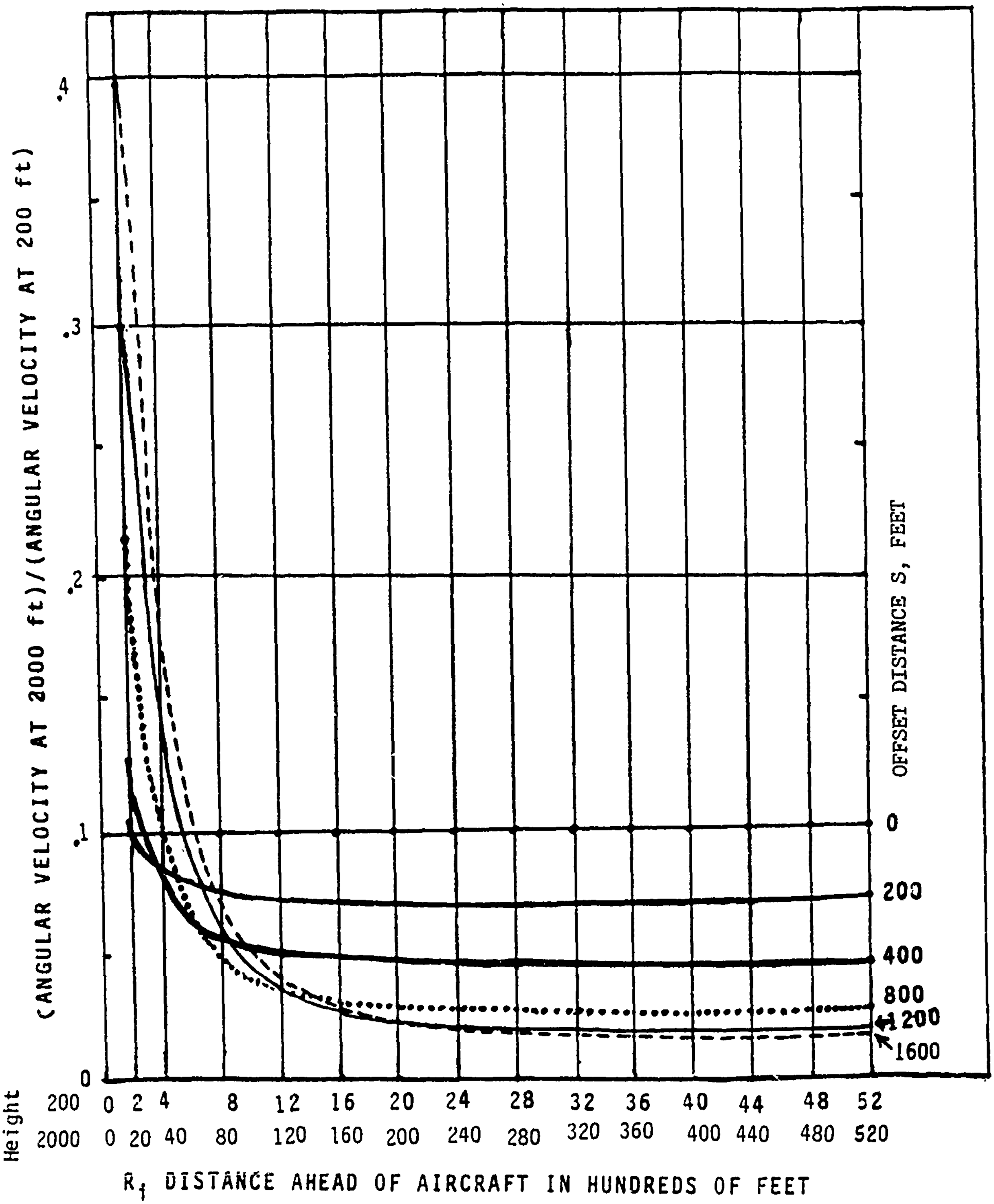


Figure 7. Ratio of angular velocity at 2000 ft to angular velocity at 200 feet when distance ahead is 10 times as great for the higher aircraft. Angular velocity is the rate of change of the angle alpha.

6.0 ANGULAR VELOCITY AT LOCATIONS ON AIRCRAFT WINDOWS

Relative to the eye of an observer in an aircraft, every point and object on the ground has an azimuth angle, or angle to the side of straight ahead, and a declination angle, or angle down from horizontal. Every point on the ground may be regarded as lying on a straight line from the point to the eye. The location or point on the window through which the line from the point passes defines the window location, conveniently expressed as the window azimuth and window declination of the ground point. Both of these window angles will vary with changes of the eye's location relative to the window. An observer with a greater eye-to-seat height can look down at a steeper angle, and given ground object appears at a higher point on the window. When the seat is moved forward, a steeper down look is possible, objects farther to the side appear in the window, and ground objects appear lower on the window and closer to zero window azimuth. Since aircraft windows are close to airborne observers, small changes in eye location produce appreciable changes in the window location of ground objects. However, ground objects are distant, so that changes of eye position within an aircraft have negligible effort upon the azimuth and declination of ground objects. Observer height and seat adjustment, then, change the angular coordinates of points on a window. Also, as fuel is used, in level flight the tilt or pitch of the aircraft may change so that, relative to a horizontal line from the observer's eye, window declinations change.

The azimuth and declinations of points on an aircraft window may be obtained by taking a photograph of the window with a level camera located at the observer's eye position and pointed straight ahead. An azimuth-declination or azimuth-altitude coordinate grid in the camera can be superimposed on the picture or added later. The grid allows every point on the window to be assigned an azimuth and a declination. There are several sources of error in making and using coordinate grids. One error source is to use nominal (marked) camera lens focal length rather than actual focal length. The two may differ by as much as 5% or more. Camera lens image distortions, particularly with wide-angle lenses, can also produce measurement errors. Window distortions of the outside scene can cause significant errors in azimuth and declination: ground objects are not quite where they appear to be.

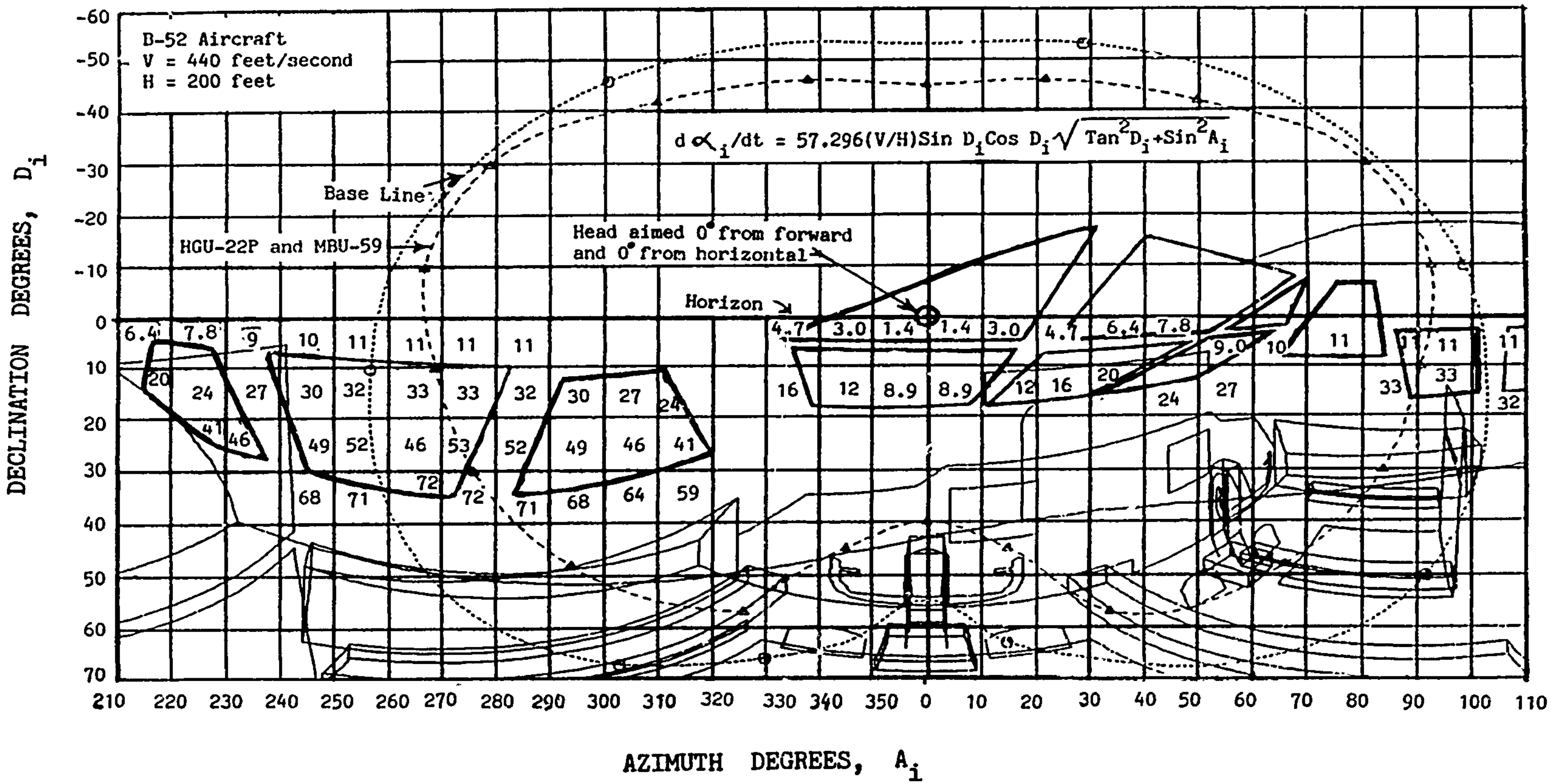
Once angular coordinates have been determined for points on an aircraft window, angular velocities in alpha, in azimuth, and in declination can be worked out. One way to determine angular velocity is to use the angular velocity equations in Appendix I. However, this method can require considerable computation. For example, note that by Equation 16 in Appendix 1, angular velocity in alpha is:

$$d\alpha/dt = 57.296\ (V/H)\mathrm{Sin}D_i\mathrm{Cos}D_i\sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i}$$

An alternate to equations is to use tables listing angular velocity at selected azimuths and declinations, provided that the steps in angle size for azimuth and declination are adequate. If the aircraft speed and height of interest are not 440 feet/second and 200 feet, respectively, table entries must be corrected. Because the tables contain V and H as a multiplier, tables can be corrected for the speed V and height V of interest by multiplying them by $(H_0/V_0)(V/H)$, i.e., by $(200/440)(V/H)=(5/11)(V/H)$. For example, the tables may be used for an aircraft speed of 1550 feet/second at an altitude of 250 feet by multiplying table entries by $(H_0/V_0)(V/H)=(200/440)(1550/250)=2.818$.

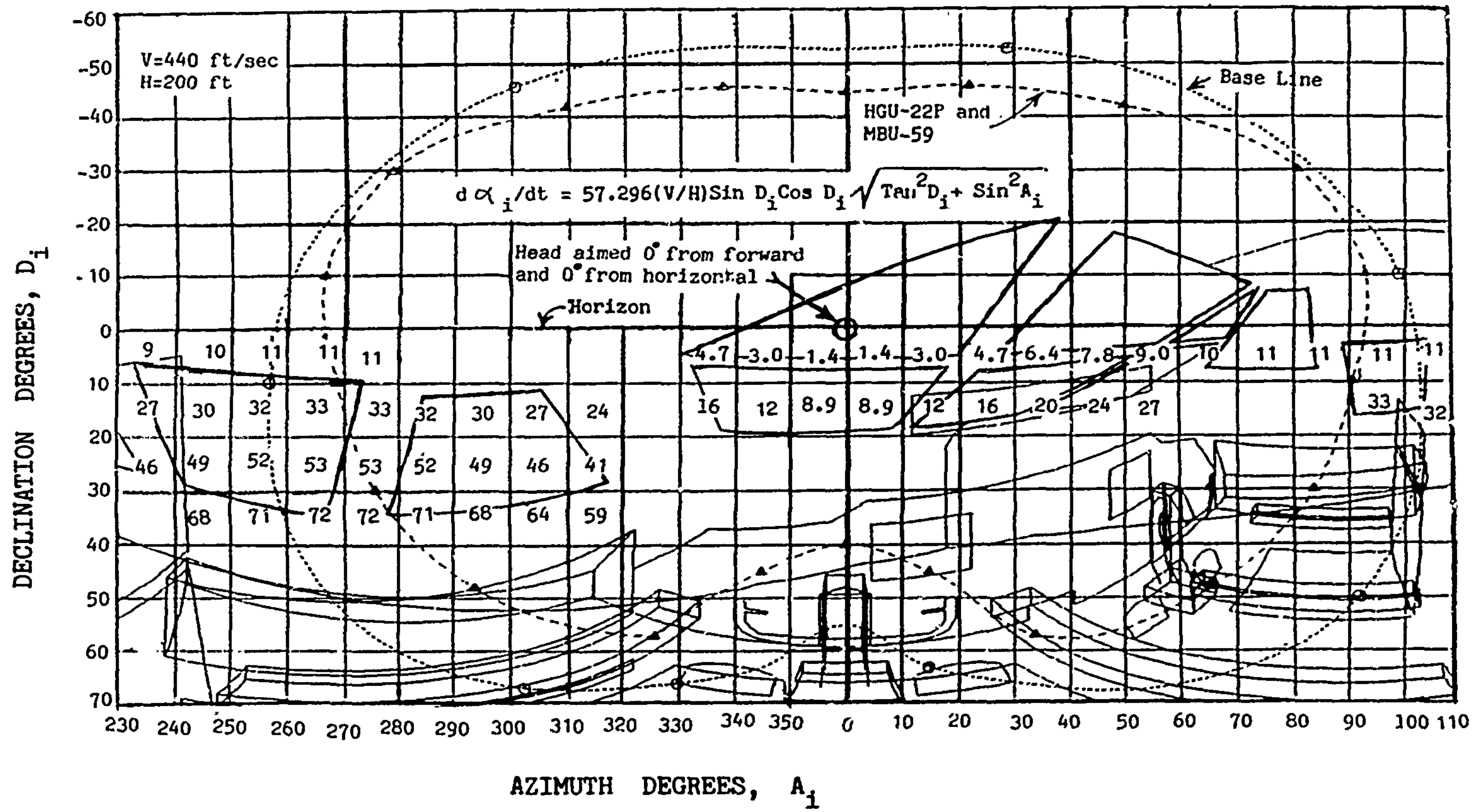
To demonstrate how angular velocities in aircraft windows may be determined, plots were obtained showing the forward windows of a U.S. Air Force B-52 bomber as seen from the pilot's eye position. The plots contained a grid of azimuth (horizontal) versus declination (vertical). The plots were obtained from the AAMRL COMBIMAN computer program, and are shown in Figures 8, 9, and 10. The first two are for a male with an eye-to-seat distance corresponding to a 95th percentile Air Force male. Figure 7 is with the seat full back, and Figure 9 is for the seat full forward. Both seats are full up, i.e., at the maximum seat height position. Figure 10 is for a 5th percentile Air Force male with the sear full back and full up. In the three figures, within the grid squares, are numbers representing angular velocity in degrees/second in the angle alpha, the angle between a level line straight ahead from the pilot's eye and a line to the window position. Each number is for angular velocity at the center of the grid square in which it appears.

The three figures indicate angular velocity in the angle alpha. By using Table A2 for azimuth velocity, the numbers would be for azimuth velocity, or, Table A3 could be used for declination velocity. Examination of the three figures is instructive about how the azimuth and declination of points on windows and the angular velocities in the angle alpha corresponding to these locations vary with pilot eye-to-seat height and seat adjustment. The figures could have a flow diagram superimposed on the picture instead of or with the numbers giving angular velocity.



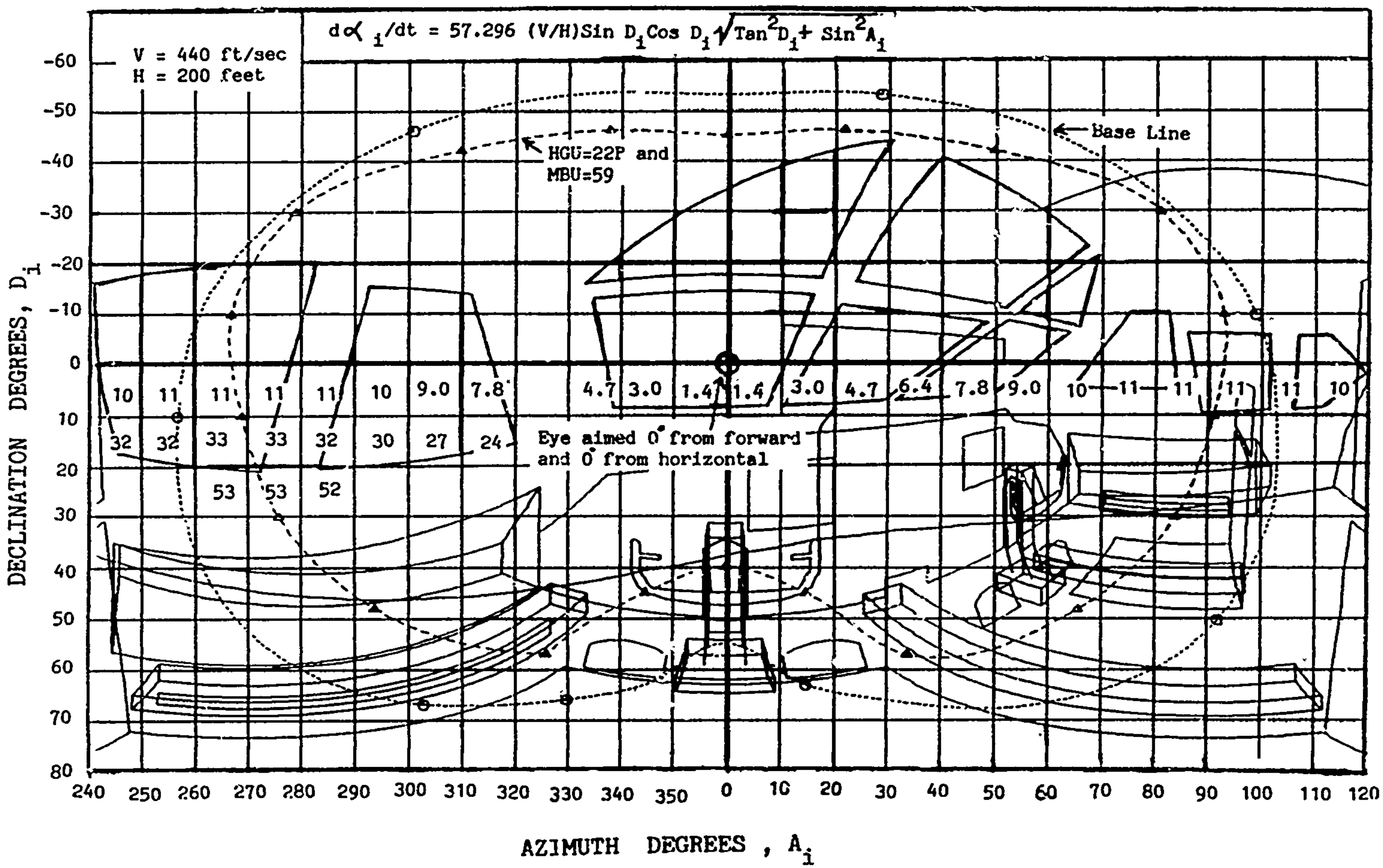
AZIMUTH DEGREES, A_i

Figure 8. Angular velocity in alpha for ground objects for straight level flight. Numbers apply to centers of squares for a 200-foot height at 440 feet/second. For other values of H and V, multiply values shown by 5H/11V. Number 6 window: 95% male, middle eye location, pilot seat full back, full up.



AZIMUTH DEGREES, A_i

Figure 9. Angular velocity in alpha for ground objects for straight level flight. Numbers apply to centers of squares for a 200-foot heigh 440 feet/second. For other values of speed V and hieght H, multiply values by 5H/11V. No. 6 window, 95% male, middle eye location pilot seat full forward, full up.



AZIMUTH DEGREES, A_i

Figure 10. Angular velocity in alpha for ground objects for straight level flight. Numbers apply to centers of squares for a 200-foot height at 440 feet/second. For other values of V and H, multiply values by 5H/11V. No. 2 window, 5% Male, middle eye position, pilot seat full back, full up.

7.0 STUDIES ON DYNAMIC VISUAL ACUITY

The use of angular velocity data in system design and mission planning requires data on the effect of angular velocities on human vision and perception. Havron (1962) reviewed 16 studies from the literature on the ability of observers to perceive angular motion and evaluated the ability of pilots to use angular motion cues from ground objects and ground texture. Erickson (1965), in his extensive analysis and review of the visual detection of targets, examined angular velocity in the angle alpha. He also reviewed results from some studies on threshold contrast and target detection.

Rather than repeating Havron's literature review and Erickson's material on target detection, the present paper reviews several studies on dynamic visual acuity as influenced by angular velocity.

One of the effects on an observer of relative angular velocity is a reduction in the observer's visual capabilities. One visual ability that decreases with angular motion is visual acuity, the ability to discern or resolve fine details in objects. When there is no relative motion, visual acuity is called static visual acuity. When either the observer or the viewed object is moving, i.e., when relative angular motion is present, visual acuity is called dynamic visual acuity. Dynamic visual acuity, then, is the ability to discriminate minute detail in objects with relative motion. Dynamic visual acuity is influenced by all of the factors or variables that influence static visual acuity, including the luminance, contrast and polarity (light on dark or vice versa) of the viewed test object or acuity test pattern, the orientation of the test pattern, type of pattern, length-to-width ratio of bars in grid patterns and the number of bars, size of the field-of-view, viewing or exposure duration, test administrator's criterion of resolution or legibility, and observer confidence level that a pattern is resolved. Some of these factors or variables are discussed in detail by Olzak and Thomas (1986), and others are covered by Stevens (1951) and by Farrell and Booth (1984). Individual observers also often differ greatly in acuity, adding to variability in data. In dynamic visual acuity an additional factor or variable is difference in acuity with different direction of motion of an observed object or pattern. Whether or not an object is tracked by the eyes also makes a difference. Different reports differ in test conditions, i.e., in the values of the variables or factors that influence reported results. Results for acuity as a function of angular velocity are thus expected to differ, often by an appreciable amount, from one report to another. However, acuity-velocity curves have similar shapes in different studies, as will appear in discussion of results. It is apparent, then, that applying numerical values of dynamic acuity from the literature must be done with care.

The technical and scientific literature on dynamic visual acuity and the detection of moving or relatively-moving objects, both in the laboratory and in flight tests is very extensive, and only a few such studies can be discussed in this paper. Foley (1957), using moving digits, found that increasing the separation between digits improved legibility, and that dynamic acuity was better for horizontal motion than for vertical motion and better for upward motion than downward motion. Also, symbol movement from right to left was better than left to right.

Miller (1958) studied visual acuity for a stationary object and a moving observer, finding slightly poorer visual acuity for a moving observer than for a moving object. In the angular velocity range of 20-120 degrees/second, Miller found an acuity loss on the order of one minute or arc or less.

Elkin (1961) examined dynamic visual acuity with monocular tracking of Landolt-C rings at target angular velocities of 30, 60, 90 and 120 degrees/second, anticipatory tracking times of .2 and 1 second, and exposure time of .2 and .5 seconds with 12 subjects. Dynamic visual acuity worsened with increase in angular velocity. Acuity decreased from 1.35 arc minutes to 2.42, and from .79 to 1.10 arc minutes at the shortest and longest exposure-pair conditions, respectively. Acuity was better with 1 second tracking time than with .2 second, and acuity decreased less with increased angular velocity at the longer tracking time and at the longer exposure time. The equation, acuity $Y = A+BX^3$, adequately described acuity as a function of angular velocity.

Miller and Ludvigh (1962) used dark Landolt-C rings as resolution patterns or targets. They found that, for individuals, static and dynamic acuity can be markedly and significantly different. They showed that dynamic visual acuity for a stationary observer viewing a moving object decreases (worsens) with increase in the angular velocity of objects moving at a right angle to the line-of-sight. It was shown that the equation $Y=A+BX^3$ describes dynamic visual acuity Y, where X is angular velocity and A is static visual acuity, i.e., acuity when there is no motion. A and B in the equation are constants whose values depend upon conditions, such as illumination, direction of motion, contrast, etc. The equations held for subject motion, target motion and different directions of motion. In the range of 0 to 40 degrees/second of angular velocity, a visual acuity of about 2 minutes of arc is a representative value for laboratory conditions. When angular velocity reaches 120 degrees/second, visual acuity has decreased to about 10 minutes of arc or worse, depending upon conditions. Similar results were obtained earlier by Ludvigh (1949) who first formulated the semi-empirical equation $Y=A+BX^3$.

Lippert and Lee (1965) used black alphanumeric symbols subtending 39 minutes of arc on a high luminance backgrouna, with vertical top-to-bottom motion. Visibility was compared for 7.5 and 39 arc minute spacings. Mean object velocities for both zero and 100%, legibility was about three times higher for the 7.5 degree symbol spacing. Performance was about twice as good with a 30-degree aperture as with a 3-degree aperture.

Snyder and Greening (1965) examined the effect of direction and velocity of relative motion upon dynamic visual acuity. Acuity was appreciably better (lower acuity threshold) in the horizontal plane (0 degrees) of motion than in the vertical plane (90 degrees). Miller (1958) had found only a slight difference, In the 30 and 60 degree planes, proportional decreases in acuity (higher acuity thresholds) were found. As the plane of motion changes from horizontal to vertical, acuity decreased. As angular velocity perpendicular to the line-of-sight increased from zero to two radians/second (115^{o}/sec), the visual acuity worsened from 1.00 to over 3.00 minutes of arc. Miller and Ludvigh (1962) had found a static acuity of 2 minutes of arc. Snyder and Greening found that visual acuity decreased (worsened) as rate of approach to the observer increased. They also found that the exponent n in the equation $Y=A+Bx^{n}$ was 2.3 rather than the 3 of Miller (1958) and of Miller and Ludvigh (1962). Snyder and Greening noted that studies in the literature that compared static and dynamic visual acuity had used only observers with superior (20/20 or better) static visual acuity. This truncation of the range of static acuity made it statistically unlikely that a significant correlation would be obtained between the two types of acuity. Snyder and Greening's correlations between static and dynamic visual acuity were positive and very small, of the same magnitude as those of other investigators who also used only observers with good static visual acuity.

Levine and Jauer (1972) examined dynamic acuity with TV imagery on a 5-inch TV cockpit display with image motion rates of zero to 10.5 inches/second at an 86 inch viewing distance. Critical detail in the target symbol subtended angles of .7 to 4.4 minutes of arc. The task was to determine the orientation of the symbol. For target-critical detail smaller than 2.2 minutes or arc, visual acuity decreased (worsened) linearly with increasing motion rate. They concluded that, for general design purposes, angular rates of 1.67 and 5 degrees/second require 1.4 and 1.9 minutes or arc visual angle, respectively, to equal visual acuity with a stationary one-minute-of-arc target.

Levine and Youngling (1972) used a cockpit mock-up and a 9 inch 525 line Contrac monitor and a flying spot scanner to display spliced panoramic photographs. Image scale was 12,500 and images were on display for 1, 2, 3, 4, 5 and 6 seconds, simulating aircraft speeds of 675 down to 113 knots. There were 24 targets and 12 subjects. Difficult targets had an image size on the display of 1/5 inch, and easy ones were about 3.4 inch. Each target was briefed along with immediately-adjacent territory. Performance decreased as speed increased for both easy and hard targets at motion rates greater than 1.7 inches/second. Performance did not increase for display time over 3 seconds. Viewing distance was not specified, but if about 28 inches in the cockpit mock-up is assumed, angular velocity corresponding to the 1.7 inches/second was about arcTan (1.7/28) or 57.296 (1.7/28), which is about 3.5 degrees/second. If an 86 inch viewing distance is assumed, a long distance, but one used by Levine and Jauer (1972), the rate is (57.296)(1.7/86), or about 1.13 degrees/second.

Reddy (1975) examined the effect of contrast and linear velocity upon dynamic visual acuity using randomly-oriented black Landolt rings of different sizes with nine different white and grey backgrounds. Illumination was 120 feet candles. An increase in linear velocity markedly decreased dynamic visual acuity, while an increase in viewing time in the range of 1/8 to 1/2 second increased acuity. Dynamic visual acuity, as expected, was appreciably higher with higher contrast. The effect of linear velocity upon acuity is dependent upon viewing distance, for linear velocity and viewing distance determine angular velocity. Reddy, unfortunately, did not specify viewing distance.

Target detection using a TV sensor and a TV monitor when angular motion is present is complicated by TV resolution and the direction of motion across the display of objects relative to the scan lines of the system. Erickson et al (1974) did a series of four studies using, as targets, Landolt-Cs, square-wave grids, and vehicles moving at various velocities. Detection of pattern detail was measured as a function of pattern velocity, direction of motion, pattern size and pattern velocity. Tables and graphs show the loss in resolution as a function of these variables. A section of the report applies experimental results to the design of electro-optical systems. Estimates are made of the number of raster lines, sensor field-of-view, and display size required for satisfactory operator performance. Results and recommendations from this report are too voluminous to report here.

Those readers interested in TV viewing of moving objects will find this report very useful.

7.1 Summary of Dynamic Visual Acuity

Factors that influence static acuity, such as scene illuminance, object contrast, object pattern, and viewing time, also influence dynamic visual acuity. For the same angular velocity dynamic visual acuity is about the same whether the observer or the object is moving. Dynamic visual acuity is better for horizontal motion than vertical motion, and better for upward motion than downward motion. Symbol legibility is better for symbol movement from right to left than from left to right. Visually tracking objects with relative motion results in much less loss of acuity than viewing without tracking.

The data from dynamic visual acuity studies are a good fit to the equation $Y=A+BX^n$, where Y is dynamic acuity in minutes of arc, A is a measure of static visual acuity, and X is angular velocity in degrees/second. The value of the exponent n has varied from as low as 2.34 to 3.00, with 3 being the most often reported value. Since more minutes of arc represents poorer visual acuity, dynamic visual acuity worsens with increase in angular velocity. Incidentally, visual acuity is often specified as the reciprocal of acuity in minutes of arc, so that larger numbers represent better acuity. The correlation between static and dynamic visual acuity is low: an individual's dynamic acuity must be measured, as it can not be predicted with adequate accuracy from his static visual acuity. The low correlations may be partly an artifact caused by using only observers who have good static visual acuity.

Visual acuity has usually been examined and measured using high-contrast test targets. However, acuity varies with the contrast of the test object or target. Targets viewed from aircraft, particularly objects of military interest, are often of low to very low contrast. The dynamic visual acuity of an airborne observer is reduced by vibration, buffeting, lack of clarity of the atmosphere, and the optical defects of the windscreen and its lack of cleanliness.

As ordinarily measured, visual acuity is related to ability to perceive high and intermediate spatial frequencies. Owsley, Sekular and Siemsen (1983) found that acuity, as measured by high contrast test patterns, is not related to ability to perceive low-contrast spatial frequencies.

Visual acuity is only one measure of visual ability and more comprehensive measures of visual ability are required to predict observer vision from aircraft. Two such measures are modulation transfer function (MTF) and contrast sensitivity. For discussion and data on modulation transfer function, the reader is referred to Westheimer (1986). For contrast sensitivity, the reader is referred to Boff, Kaufman and Thomas (1986) and Boff and Lincoln (1986).

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APPENDIX 1

DERIVATION OF EQUATIONS

(A) Derivation

The equations derived in this report are based on the situation geometry depicted in figures 11 and 12. In fig.11, an aircraft at point P_i is in straight level flight at a height H with a constant speed V. An object or point of interest on the ground is at point T which is offset from the aircraft ground path by a distance S. Angular direction to the point on the ground may be designated by the angle alpha or α_i, which is the angle between the flight path in the sky of the aircraft and a line from the aircraft to the ground point. A second angular designation which is sometimes more useful is to use two angles: an azimuth or azimuth angle, A_i which is angle to the side of straight ahead, and declination or deċlination or declination angle, D_i, which is angle down from horizontal. Point Q is an imaginary point directly above the ground point of interest and at the height of the aircraft. Point G_i is a point on the ground directly beneath the aircraft. At the bottom of fig. 11 are five right angle triangles taken from the situation picture above them. Triangles B an E have the same angles and their sides are of the same length.

The following equations are based on Fig. 11.

In triangle A, $\operatorname{Sin} D_i = H/r_i$, from which

(1) $$r_i = H/\operatorname{Sin} D_i$$

In triangle B, $R_i^{*2} = S^2 + R_i^2$.

In triangle A, $R_i^{*2} + H^2 = r_i^2$, from which $R_i^{*2} = r_i^2 - H^2$.

Equating the two values of R_i^{*2},

$r_i^2 - H^2 = S^2 + R_i^2$, from which $r_i^2 = H^2 + S^2 + R_i^2$. Taking square roots,

(2) $$r_i = \sqrt{H^2 + S^2 + R_i^2}$$

Equating r_i values from (1) and (2),

$H/\operatorname{Sin} D_i = \sqrt{H^2 + S^2 + R_i^2}$, from which

(3) $$\operatorname{Sin} D_i = H/\sqrt{H^2 + S^2 + R_i^2}$$

From (3), $\operatorname{Sin} D_i = \sqrt{1 - \operatorname{Cos}^2 D_i} = H/\sqrt{H^2 + S^2 + R_i^2}$.

Squaring both sides,

$$1 - \operatorname{Cos}^2 D_i = H^2/(H^2 + S^2 + R_i^2)$$

$$\operatorname{Cos}^2 D_i = 1 - H^2/(H^2 + S^2 + R_i^2) = (H^2 + S^2 + R_i^2 - H^2)/(H^2 + S^2 + R_i^2)$$

$\operatorname{Cos}^2 D_i \equiv (S^2 + R_i^2)/(H^2 + S^2 + R_i^2)$. Taking square roots,

(4) $$\operatorname{Cos} D_i = \sqrt{S^2 + R_i^2}/\sqrt{H^2 + S^2 + R_i^2}$$

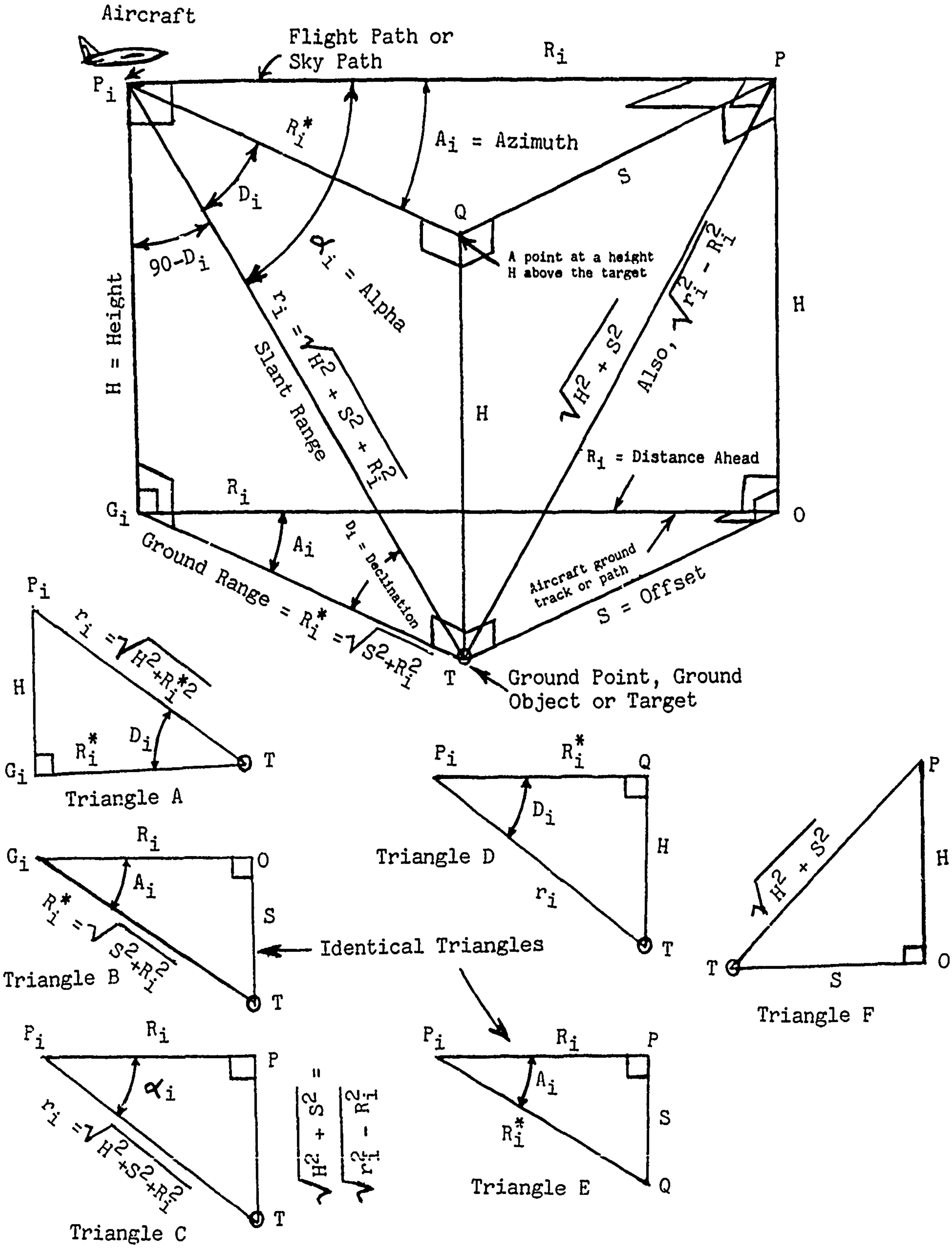

Fig. 11. Geometry of the aircraft-terrain situation.

From triangle A, Tan D_i = H/R_i^*, from which $R_i^* = H/\text{Tan } D_i$.

From triangle B, Sin A_i = S/R_i^*, from which R_i^* - $S/\text{Sin } A_1$.

Equating the two values of R_i^*, $H/\text{Tan } D_i = S/\text{Sin } A_i$, from which

(5) $$S = H \text{ Sin } A_i/\text{Tan } D_i$$

From triangle B, Tan A_i = S/R_i, from which $R_i = S/\text{Tan } A_i$.

From (5), $S = H \text{ Sin } A_i/\text{Tan } D_i$, so that $R_i = S/\text{Tan } A_i$ =

$(H \text{ Sin } A_i/\text{Tan } D_i)/\text{Tan } A_i = (H \text{ Sin } A_i/\text{Tan } D_i)(\text{Cos } A_i/\text{Sin } A_i)$=

(6) $$R_i = H \text{ Cos } A_i/\text{Tan } D_i$$

From triangle A,

(7) $$R_i^* = H/\text{Tan } D_i$$

From triangle B,

(8) $$R_i^* = \sqrt{S^2 + R_i^2}$$

From triangle B, Sin A_i = S/R_i^*, , from which

(9) $$R_i^* = S/\text{Sin } A_i$$

From triangle C, Cos $\alpha_i = R_i/r_i$.

From (1), $r_i = H/\text{Sin } D_i$. Substituting this value of r_i,

Cos $\alpha_i = R_i/r_i = R_i/(H/\text{Sin } D_i)$ =

(10) $$\text{Cos } \alpha_i = (R_i/H) \text{ Sin } D_i$$

From triangle C, Cos $\alpha_i = R_i/r_i$.

From (2), $r_i = \sqrt{H^2 + S^2 + R_i^2}$. Substituting this value of A_i,

Cos $\alpha_i = R_i/r_i$ =

(11) $$\text{Cos } \alpha_i = R_i/\sqrt{H^2 + S^2 + R_i^2}$$

From triangle C, Sin $\alpha i = \sqrt{H^2 + S^2}/ r_i$

From (2) $r_i = \sqrt{H^2 + S^2 + R_i^2}$, so that

(12) $$\text{Sin } \alpha_i = \sqrt{H^2 + S^2} / \sqrt{H^2 + S^2 + R_i^2}$$

From (5), $S = H \sin A_i/\tan D_i$, so that

$$\sqrt{H^2 + S^2} = \sqrt{H^2 + (H^2 \sin^2 A_i/\tan^2 D_i} = \sqrt{(H^2/\tan^2 D_i)(\tan^2 D_i + \sin^2 A_i)}$$

$$\sqrt{H^2 + S^2} = (H/\tan D_i)\sqrt{\tan^2 D_i + \sin^2 A_i}\ .$$

From (3), $\sin D_i = H/\sqrt{H^2 + S^2 + R_i^2}$, from which

$$\sqrt{H^2 + S^2 + R_i^2} = H/\sin D_i .$$

Inserting these values of $\sqrt{H^2 + S^2}$ and $\sqrt{H^2 + S^2 + R_i^2}$ into (12),

$$\sin \alpha_i = \sqrt{H^2 + S^2} \,/\sqrt{H^2 + S^2 + R_i^2} =$$

$$\sin \alpha_i = (H/\tan D_i)\sqrt{\tan^2 D_i + \sin^2 A_i} / (H/\sin D_i)$$

$$\sin \alpha_i = (\cos D_i/\sin D_i)\sin D_i \sqrt{\tan^2 D_i + \sin^2 D_i} =$$

(13) $$\sin \alpha_i = \cos D_i \sqrt{\tan^2 D_i + \sin^2 D_i}$$

From (10), $\cos \alpha_i = (R_i/H)\sin D_i$.

From triangle B, $\tan A_i = S/R_i$, from which $R_i = S/\tan A_i$.

Substituting this into the above equation,

$$\cos \alpha_i = (R_i/H)\sin D_i = (S/\tan A_i)(1/H)\sin D_i$$

$\cos \alpha_i = (S/H)\sin D_i/\tan A_i$. From (5), $S = H \sin A_i/\tan D_i$, thus

$$\cos \alpha_i = (H \sin A_i/\tan D_i)(1/H)(\sin D_i/\tan A_i) =$$

$$\cos \alpha_i = (\sin A_i)(\cos D_i/\sin D_i)(\sin D_i)(\cos A_i/\sin A_i) =$$

(14) $$\cos \alpha_i = \cos A_i \cos D_i$$

Equation (12), $\sin \alpha_i = \sqrt{H^2 + S^2} / \sqrt{H^2 + S^2 + R_i^2}$ may be written as

$\sin \alpha_i = (\sqrt{H^2 + S^2})(H^2 + S^2 + R_i^2)^{-1/2}$. Note that $\sqrt{H^2 + S^2}$ is a constant. Differentiating both sides with respect to time,

$$(\cos \alpha_i)(d\alpha_i/dt) = (\sqrt{H^2+S^2})(-1/2)(2R_i)(dR_i/dt)(H^2+S^2+R_i^2)^{-3/2}$$

$$(\cos \alpha_i)(d\alpha_i/dt) = (dR_i/dt)R_i\sqrt{H^2+S^2}/(H^2+S^2+R_i^2)\sqrt{H^2+S^2+R_i^2}$$

Now $dR_i/dt = -V$, relative aircraft velocity.

From (11), $\cos\alpha_i = R_i/\sqrt{H^2 + S^2 + R_i^2}$. The equation for $d\alpha_i/dt$ then becomes

$$(R_i/\sqrt{H^2 + S^2 + R_i^2})(d\alpha_i/dt) = VR_i\sqrt{H^2+S^2}\,/(H^2+S^2+R_i^2)\sqrt{H^2+S^2+R_i^2}\ .$$

$d\alpha_i/dt = V\sqrt{H^2 + S^2}\,/(H^2 + S^2 + R_i^2)$ radians/second. To convert to degrees/second, multiply by 57.296.

$$d\alpha_i/dt = 57.296\ V\sqrt{H^2 + S^2}\,/(H^2 + S^2 + R_i^2)\ \text{deg/sec.} \tag{15}$$

From (5), $S = H \sin A_i/\tan D_i$. Replacing the S in H^2+S^2 with this S,

$$H^2 + S^2 = H^2 + (H^2\sin^2 A_i/\tan^2 D_i) = H^2(1 + \sin^2 A_i/\tan^2 D_i) =$$
$$= (H^2/\tan^2 D_i)(\tan^2 D_i + \sin^2 A_i).$$

Using this value of H^2+S^2, and, from (6), $R_i = H \cos A_i/\tan D_i$,

$$(H^2+S^2)+R_i^2 = (H^2/\tan^2 D_i)(\tan^2 D_i+\sin^2 A_i) + H^2\cos^2 A_i/\tan^2 D_i =$$

$(H^2+S^2)+R_i^2 = (H^2/\tan^2 D_i)(\tan^2 D_i+\sin^2 A_i+\cos^2 A_i)$. Now, in general,

$\sin^2 A_i+\cos^2 A_i=1$, and, $\tan^2 D_i+1= \sec^2 D_i$. Using these as substitutions,

$$H^2 + S^2 + R_i^2 = (H^2/\tan^2 D_i)\sec^2 D_i = H^2/\cos^2 D_i\tan^2 D_i =$$
$$= (H^2/\cos^2 D_i)(\cos^2 D_i/\sin^2 D_i) = H^2/\sin^2 D_i.$$

Substituting this value of $H^2+S^2+R_i^2$ and the value of H^2+S^2 worked out above into equation (15),

$$\begin{aligned}
d\alpha_i/dt &= 57.296\ V\sqrt{H^2+S^2}\ /(H^2 + S^2 + R_i^2)\\
&= 57.296V\left(\sqrt{(H^2/\tan^2 D_i)(\tan^2 D_i+\sin^2 A_i)}\right) / (H^2/\sin^2 D_i)\\
&= 57.296\ V\ (H/\tan D_i)(\sin^2 D_i/H^2)\sqrt{\tan^2 D_i+ \sin^2 A_i}\\
&= 57.296(V/H)(\cos D_i/\sin D_i)(\sin^2 D_i)\sqrt{\tan^2 D_i+\sin^2 A_i} =
\end{aligned}$$

$$d\alpha_i/dt = 57.296(V/H)\ \sin D_i\cos D_i\sqrt{\tan^2 D_i+ \sin^2 A_i} \tag{16}$$

From (13), $\cos D_i\sqrt{\tan^2 D_i + \sin^2 A_i} = \sin\alpha_i$. Substituting this value of $\cos D_i\sqrt{\tan^2 D_i + \sin^2 A_i}$ into (16),

$$d\alpha_i/dt = 57.296(V/H)\ \sin D_i\cos D_i \tag{17}$$

From (14), $\cos \alpha_i = \cos A_i \cos D_i$. Using the differentiation rule $d(UW)/dt = U(dW/dt) + W(dU/dt)$, and differentiating both sides,

$$(-\sin \alpha_i)(d\alpha_i/dt) = (\cos A_i)(-\sin D_i)(dD_i/dt) + (\cos D_i)(-\sin A_i)(dA_i/dt)$$

$$(\sin \alpha_i)(d\alpha_i/dt) = \cos A_i \sin D_i)(dD_i/dt) + (\sin A_i \cos D_i)(dA_i/dt),$$
from which

(18) $$d\alpha_i/dt = (1/\sin\alpha_i)\left[(\cos A_i \sin D_i)(dD_i/dt) + (\sin A_i \cos D_i)(dA_i/dt)\right].$$

From (13), $\sin \alpha_i = \cos D_i \sqrt{\tan^2 D_i + \sin^2 A_i}$, from which $1/\sin \alpha_i = 1/\cos D_i \sqrt{\tan^2 D_i + \sin^2 A_i}$. Substituting this value of $1/\sin \alpha_i$ into (18),

$$d\alpha_i/dt = (1/\cos D_i \sqrt{\tan^2 D_i + \sin^2 A_i})\left[\cos A_i \sin D_i (dD_i/dt) + (\sin A_i \cos D_i)(dA_i/dt)\right]$$

$$= (1/\sqrt{\tan^2 D_i + \sin^2 A_i})\left[(\cos A_i \sin D_i/\cos D_i)(dD_i/dt) + (1/\cos D_i)(\sin A_i \cos D_i)(dA_i/dt)\right]$$

(19) $$d\alpha_i/dt = (1/\sqrt{\tan^2 D_i + \sin^2 A_i})\left[(\cos A_i \tan D_i)(dD_i/dt) + (\sin A_i)(dA_i/dt)\right]$$

From triangle B, $\tan A_i = S/R_i = SR_i^{-1}$. Differentiating both sides with respect to time,

$d(\tan A_i)/dt = (\sec^2 A_i)(dA_i/dt) = -(S/R_i^2)(dR_i/dt)$. Now $dR_i/dt = -v$, relative aircraft velocity, so that $\sec^2 A_i (dA_i/dt) = SV/R_i^2$, from which $dA_i/dt = SV/R_i^2 \sec^2 A_i = (SV/R_i^2)\cos^2 A_i$.

From Triangle B, $\cos A_i = R_i/R_i^* = R_i/\sqrt{S^2 + R_i^2}$, so that

$dA_i/dt = (VS/R_i^2)\left[R_i^2/(S^2+R_i^2)\right] = VS/(S^2+R_i^2)$ radians/second. To convert to degrees/second, multiply by 57.296.

(20) $$dA_i/dt = 57.296\ VS/(S^2 + R_i^2)$$

In triangle B, $\text{Sin } A_i = S/\sqrt{S^2 + R_i^2}$, so that $\sqrt{S^2+R_i^2} = S/\text{Sin}A_i$, and , squaring both sides, $(S^2 + R_i^2) = S^2/\text{Sin}^2 A_i$.

From (5), $S = H \text{ Sin } A_i/\text{Tan } D_i$, so that

$S^2 + R_i^2 = S^2/\text{Sin}^2 A_i = (H^2 \text{Sin}^2 A_i/\text{Tan}^2 D_i)/\text{Sin}^2 A_i$ =

$S^2 + R_i^2 = H^2/\text{Tan}^2 D_i$. Substituting in (20) for S and for $S^2+R_i^2$,

$dA_i/dt = 57.296VS/(S^2 + R_i^2)$ =

$dA_i/dt = 57.296(V/H)(H \text{ Sin } A_i/\text{Tan } D_i)/(H^2/\text{Tan}^2 D_i)$ =

(21) $dA_i/dt = 57.296(V/H)\text{Sin } A_i \text{Tan } D_i$

From (3), $\text{Sin } D_i = H/\sqrt{H^2 + S^2 + R_i^2} = H(H^2+S^2+R_i^2)^{-1/2}$.

Diff·:entiating both sides with respect to time,

$$(\text{Cos } D_i)(dD_i/dt) = (H)(-1/2)(2R_i)(H^2+S^2+R_i^2)^{-3/2}(dR_i/dt)$$
$$= -\left[HR_i/(H^2+S^2+R_i^2)\sqrt{H^2+S^2+R_i^2}\right](dR_i/dt)$$

From (4), $\text{Cos } D_i = \sqrt{H^2+S^2}/\sqrt{H^2 + S^2 + R_i^2}$. Inserting this value of $\text{Cos } D_i$ into the above equation,

$$(\sqrt{S^2+R_i^2}/\sqrt{H^2+S^2+R_i^2})(dD_i/dt) = HR_i/(H^2+S^2+R_i^2)\sqrt{H^2+S^2+R_i^2}\ (dR_i/dt)$$
$$dD_i/dt = -\left[HR_i/(H^2+S^2+R_i^2)\sqrt{S^2+R_i^2}\right](dR_i/dt).$$

dR_i/dt is -V, relative aircraft velocity, and, since the equation gives radians/second, it must be multiplied by 57.296 to provide degrees/second. The equation then becomes

(22) $dD_i/dt = 57.296\ HR_iV/(H^2 + S^2 + R_i^2)\sqrt{S^2 + R_i^2}$

From (6), $R_i = H \text{ Cos } A_i/\text{Tan } D_i$.

From (8), $R_i^* = \sqrt{S^2 + R_i^2}$.

From (7) $R_i^* = H/\text{Tan } D_i$. Equating R_i^* values,

$\sqrt{S^2 + R_i^2} = H/\text{Tan } D_i$

From (2), $r_i = \sqrt{H^2 + S^2 + R_i^2}$. Squaring both sides,

$H^2 + S^2 + R_i^2 = r_i^2$. However, from (1), $r_i = H/\text{Sin } D_i$, so that $r_i^2 = H^2/\text{Sin}^2 D_i$, and $H^2+S^2+R_i^2 = H^2/\text{Sin}^2 D_i$.

Substituting the above values of R_i, $\sqrt{S^2 + R_i^2}$ and $H^2+S^2+R_i^2$ into (22),

$$dD_i/dt = 57.296\ HVR_i/(H^2 + S^2 + R_i^2)\sqrt{S^2 + R_i^2}$$

$$= 57.296\ HV(H \text{ Cos } A_i/\text{Tan } D_i)/(H^2/\text{Sin}^2 D_i)(H/\text{Tan } D_i)$$

$$= 57.296(V/H)(\text{Cos } A_i/\text{Tan } D_i)(\text{Sin}^2 D_i \text{Tan } D_i) =$$

(23) $dD_i/dt = 57.296\ (V/H)\text{Cos } A_i \text{Sin}^2 D_i$

From (16), $d\alpha_i/dt = 57.296(V/H)\text{Sin } D_i \text{Cos } D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i}$,

from which $57.296(V/H) = (d\alpha_i/dt)/\text{Sin } D_i \text{Cos} D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i}$.

From (23), $dA_i/dt = 57.296(V/H)\text{Sin } A_i \text{Tan } D_i$, from which

$57.296(V/H) = (dA_i/dt)/\text{Sin } A_i \text{Tan } D_i$.

Equating the above two values of 57.296(V/H),

$$(d\alpha_i/dt)/\text{Sin } D_i \text{Cos } D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i} = (dA_i/dt)/\text{Sin } A_i \text{Tan } D_i$$

$$d\alpha_i/dt = [(\text{Sin } D_i \text{Cos } D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i})/\text{Sin } A_i \text{Tan } D_i](dA_i/dt)$$

$$d\alpha_i/dt = (\text{Sin } D_i \text{Cos } D_i)(\text{Cos } D_i/\text{Sin } D_i \text{Sin } A_i)(\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i})(dA_i/dt)$$

(24) $d\alpha_i/dt = (\text{Cos}^2 D_i/\text{Sin } A_i)\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i}\ (dA_i/dt)$

From (13), $\text{Sin } \alpha_i = \text{Cos } D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i}$, so that

$\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i} = \text{Sin } \alpha_i/\text{Cos } D_i$. Substituting into (24) for $\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i}$,

$$d\alpha_i/dt = (\text{Cos}^2 D_i/\text{Sin } A_i)(\text{Sin } \alpha_i/\text{Cos } D_i)(d\alpha_i/dt) =$$

(25) $d\alpha_i/dt = (\text{Cos } D_i \text{Sin } \alpha_i/\text{Sin } A_i)(dA_i/dt)$

Solving (24) for dA_i/dt,

(26) $dA_i/dt = (\text{Sin } A_i/\text{Cos}^2 D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i})(d\alpha_i/dt)$

Solving (25) for dA_i/dt,

(27) $dA_i/dt = (\text{Sin } A_i/\text{Cos } D_i \text{Sin } \alpha_i)(d\alpha_i/dt)$

From (21), $dA_i/dt = 57.296(V/H) \sin A_i \tan D_i$, from which

$57.296(V/H) = (dA_i/dt)/\sin A_i \tan D_i$.

From (23), $dD_i/dt = 57.296(V/H) \cos A_i \sin^2 D_i$, from which

$57.296(V/H) = (dD_i/dt)/\cos A_i \sin^2 D_i$.

Equating the above two expressions for 57.296(V/H),

$(dA_i/dt)/\sin A_i \tan D_i = (dD_i/dt)/\cos A_i \sin^2 D_i$, from which

$$dA_i/dt = (dD_i/dt)\sin A_i \tan D_i/\cos A_i \sin^2 D_i$$

$$= (dD_i/dt)\sin A_i (\sin D_i/\cos D_i)/\cos A_i \sin^2 D_i$$

$$= (dD_i/dt)\tan A_i/\sin D_i \cos D_i =$$

(28) $$dA_i/dt = (\tan A_i/\sin D_i \cos D_i)(dD_i/dt)$$

To graph the motion of a ground object or ground point relative to an airborne observer, a starting point or initial set of conditions at t=0 is required. The initial conditions may be H and V plus offset distance S and initial distance ahead R_o, or they may be H and V plus initial azimuth A_o and initial declination D_o. Initial conditions could even be H, V, S, and initial slant range r_o or initial ground range R_o. In the following derivations only the first two sets of initial conditions will be addressed.

After t seconds of flight, the aircraft has traveled a distance Vt from its initial position above point G_o of fig.12, and is then above point G_i. Distance ahead of the ground object is then

(29) $$R_i = R_o - Vt$$

From fig.13, $\tan A_i = S/R_i = S/(R_o - Vt)$, from which

(30) $$A_i = \text{Arc Tan}\left[S/(R_o - Vt)\right]$$

From (5), $S = H \sin A_i/\tan D_i$, from which

$\tan D_i = (H/S)\sin A_i$. Taking the inverse,

(31) $$D_i = \text{Arc Tan}\left[(H/S)\sin A_i\right]$$

Equation (31), contains $\sin A_i$. To calculate D_i independently of A_i, note that, in general,

$$\tan A_i = \sin A_i/\cos A_i = \sin A_i/\sqrt{1-\sin^2 A_i}.$$

From (30), $\tan A_i = S/(R_o - Vt)$. Equating these two values of $\tan A_i$,

$\sin A_i/\sqrt{1-\sin^2 A_i} = S/(R_o - Vt)$. Squaring both sides,

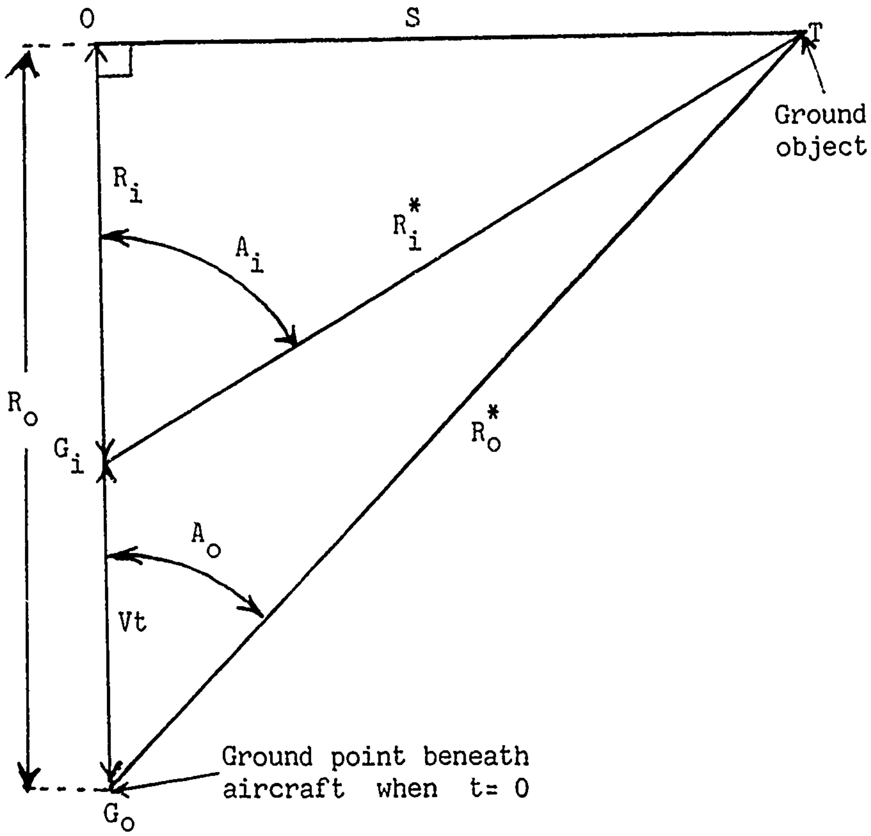

Fig. 12. The ground situation at time zero and after t seconds.

$\mathrm{Sin}^2 A_i/(1-\mathrm{Sin}^2 A_i) = S^2/(R_o-Vt)^2$. Cross multiplying ,

$$\mathrm{Sin}^2 A_i = [S^2/(R_o-Vt)^2](1-\mathrm{Sin}^2 A_i)$$

$$= S^2/(R_o-Vt)^2 - S^2\mathrm{Sin}^2 A_i/(R_o-Vt)^2, \text{ and}$$

$$\mathrm{Sin}^2 A_i + [S^2/(R_o-Vt)^2]\ \mathrm{Sin}^2 A_i = S^2/(R_o-Vt)^2$$

$$\mathrm{Sin}^2 A_i [1 + S^2/(R_o-Vt)^2] = S^2/(R_o-Vt)^2$$

$$\mathrm{Sin}^2 A_i [(R_o-Vt)^2 + S^2]/(R_o-Vt)^2 = S^2/(R_o-Vt)^2$$

$$\mathrm{Sin}^2 A_i [(R_o-Vt)^2 + S^2] = S^2$$

$\mathrm{Sin}^2 A_i = S^2/[(R_o-Vt)^2 + S^2]$. Taking square roots of both sides,

$\mathrm{Sin}\ A_i = S/\sqrt{(R_o-Vt)^2 + S^2}$. Inserting this value of Sin A_i into (31),

$$D_i = \mathrm{Arc\ Tan}[(H/S)\mathrm{Sin}\ A_i] = \mathrm{Arc\ Tan}\left[(H/S)\ (S/\sqrt{S^2+(R_o-Vt)^2}\right] =$$

(32) $$D_i = \mathrm{Arc\ Tan}\left[H/\sqrt{S^2 + (R_o - Vt)^2}\right]$$

When initial conditions are H, V, S, and R_o,equations (30), (31), and (32) provide azimuth and declinatio angles after t seconds of flight. When initial conditions are H, V, A_o, and D_o, the S and R_o of these equations must be replaced.

From (30), $A_i = \mathrm{Arc\ Tan}[S/(R_o - Vt)]$,from which Tan A_i=$S/(R_o-Vt)$.

From (5), S = H Sin A_i/Tan D_i, so that S = H Sin A_o/Tan D_o.

From (6), R_i = H Cos A_i/Tan D_i, so that R_o=H Cos A_o/Tan D_o.

Replacing S and R_o with these values, Tan $A_i = S/(R_o-Vt)$ =

$$\mathrm{Tan}\ A_i = (H\ \mathrm{Sin}\ A_o/\mathrm{Tan}\ D_o)/[(H\ \mathrm{Cos}\ A_o/\mathrm{Tan}\ D_o) - Vt] =$$

$$\mathrm{Tan}\ A_i = (H\ \mathrm{Sin}\ A_o/\mathrm{Tan}\ D_o)/[(H\ \mathrm{Cos}\ A_o - Vt\ \mathrm{Tan}\ D_o)/\mathrm{Tan}\ D_o]$$

Tan A_i = (H Sin A_o/H Cos A_o- Vt Tan D_o). Taking the inverse,

(33) $$A_i = \mathrm{Arc\ Tan}[H\ \mathrm{Sin}\ A_o/(H\ \mathrm{Cos}\ A_o - Vt\ \mathrm{Tan}\ D_o]$$

Dividing both numerator and denominator by H Sin A_o,

$$A_i = \mathrm{Arc\ Tan}[1/(\mathrm{Cos}\ A_o/\mathrm{Sin}\ A_o)-Vt\ \mathrm{Tan}\ D_o/H\ \mathrm{Sin}\ A_o)]$$

$$A_i = \mathrm{Arc\ Tan}[1/(\mathrm{Cot}\ A_o - Vt\ \mathrm{Tan}\ D_o/H\ \mathrm{Sin}\ A_o)]$$

This may be written as

(34) $A_i = \text{Arc Tan}\left[1/(\text{Cot } A_o - Ct)\right]$, where

$C = V \text{ Tan } D_o/H \text{ Sin } A_o$.

From (31), $D_i = \text{Arc Tan}\left[(H/S) \text{ Sin } A_i\right]$, and,

From (5), $S = H \text{ Sin } A_o/\text{Tan } D_o$, so that

$D_i = \text{Arc Tan}\left\{\left[H/(H \text{ Sin } A_o/\text{Tan } D_o)\right] \text{Sin } A_i\right\}$

(35) $D_i = \text{Arc Tan}\left[(\text{Tan } D_o/\text{Sin } A_o)\text{Sin } A_i\right]$

From (5), $S = H \text{ Sin } A_i/\text{Tan } D_i$, so that $S = H \text{ Sin } A_o/\text{Tan } D_o$.

From (6), $R_i = H \text{ Cos } A_i/\text{Tan } D_i$, so that $R_o = H \text{ Cos } A_o/\text{Tan } D_o$.

Replacing the S and R_o of (32) with these values,

$D_i = \text{Arc Tan } H/\sqrt{S^2 + (R_o - Vt)^2} =$

$D_i = \text{Arc Tan}\left[H/\sqrt{(H^2\text{Sin}^2A_o/\text{Tan}^2D_o) + (\{H\text{Cos } A_o/\text{Tan } D_o\} - Vt)^2}\right]$

$D_i = \text{Arc Tan}\left[H/\sqrt{(H^2\text{Sin}^2A_o/\text{Tan}^2D_o)+(H \text{ Cos } A_o - Vt \text{ Tan } D_o)/\text{Tan}^2D_o}\right]$

(36) $D_i = \text{Arc Tan}\left[(H/\text{Tan } D_o)\sqrt{H^2\text{Sin}^2A_o + (H \text{ Cos } A_o - Vt \text{ Tan } D_o)^2}\right]$

When values of both A_i and D_i have to be calculated, it may be more covenient to use equation (35), rather than (36) to calculate D_i.

Note. By varying time t in equations (33)-(36), azimuth and declination at any time t may be calculated, thus permitting flow lines to be plotted on an azimuth-declination plot. The calculated azimuth and declination permit calculation of the azimuth and declination velocities, velocity in alpha, ground range, slant range, distance ahead, and the angle alpha.

B. COLLECTED SYMBOL DEFINITIONS AND EQUATIONS

Alpha, or α_i. The angle between the path of the aircraft in the sky and a line from the aircraft to the ground point or ground object.

$\text{Cos } \alpha_i = \text{Cos } A_i \text{Cos } D_i \qquad \text{Cos } \alpha_i = R_i/\sqrt{H^2+S^2+R_i^2}$

Secondary Equations

$\text{Sin } \alpha_i = \sqrt{H^2+S^2}/\sqrt{H^2 + S^2 + R_i^2} \qquad \text{Cos } \alpha_i = R_i/r_i$

$\text{Sin } \alpha_i = \sqrt{r_i^2 - R_i^2}\ /r_i \qquad \text{Cos } \alpha_i = (S/H)\text{Sin } D_i/\text{Tan } D_i$

$\text{Sin } \alpha_i = \sqrt{H^2 + S^2}/\ r_i \qquad \text{Cos } \alpha_i = (R_i/H) \text{ Sin } D_i$

$\text{Sin } \alpha_i = (\text{Cos } D_i)\sqrt{\text{Sin}^2 A_i + \text{Tan}^2 D_i}$

$\text{Cos } \alpha_i = R_i/\sqrt{H^2+S^2+(R_o-Vt)^2} = (R_o-Vt)/\sqrt{H^2+S^2+(R_o-Vt)^2}$

Azimuth, or A_i. The angle measured horizontally from a level-straight-ahead line to a point at aircraft height directly above a ground point or object. Angular deviation from straight ahead.

$\text{Cos } A_i = R_i/\sqrt{S^2+ R_i^2} \qquad \text{Tan } A_i = S/R_i$

$\text{Sin } A_i = S/R_i^* \qquad \text{Tan } A_i = S/(R_o-Vt)$

$\text{Sin } A_i = S/\sqrt{S^2+ R_i^2} \qquad \text{Tan } A_i = 1/(\text{Cot } A_o-Ct)$, where

$C = V \text{ Tan } D_o/H \text{ Sin } A_o$

Secondary Equations

$\text{Sin } A_i = S/R_i^* \qquad \text{Cos } A_i = R_i/R_i^*$

$\text{Sin } A_i = (S/H) \text{ Tan } D_i \qquad \text{Cos } A_i = (\sqrt{S^2 + R_i^2})/r_i$

Declination, declination angle, or D_i. The angle measured down from horizontal to the ground point or ground object. Angle down.

$\text{Sin } D_i = H/\sqrt{H^2 + S^2 + R_i^2}$

Secondary Equations

$\text{Sin } D_i = H/r_i \qquad \text{Tan } D_i = H/\sqrt{S^2 + (R_o-Vt)^2}$

$\text{Cos } D_i = \sqrt{(S^2+R_i^2)/(H^2+S^2+R_i^2)} \qquad \text{Tan } D_i = (H/S) \text{ Sin } A_i$

$\text{Tan } D_i = H/R_i^* \qquad \text{Tan } D_i = (\text{Tan } D_o/\text{Sin } A_o)\text{Sin } A_i$

$\text{Tan } D_i = H/\sqrt{S^2+ R_i^2} \qquad \text{Tan } D_i = (H/R_i) \text{ Cos } A_i$

$\text{Tan } D_i = (H/\text{Tan } D_o)/\sqrt{H^2 \text{ Sin}^2 A_o - (H \text{ Cos } A_o - Vt \text{ Tan } D_o)^2}$

Distance Ahead, or R_i. The distance ahead of the aircraft of a ground point or ground object measured along the ground parallel to the ground path of the aircraft.

$$R_i = H \text{ Cos } A_i / \text{Tan } D_i$$

$$R_i = \sqrt{H^2 + S^2} / \text{Tan } \alpha_i$$

Secondary equations

$$R_i = S / \text{ Tan } A_i$$

$$R_i = R_i^* \text{Cos } A_i$$

$$R_i = r_i \text{ Cos } \alpha_i$$

$$R_i = \sqrt{r_i^2 - R_i^2} / \text{ Tan } \alpha_i$$

$$R_i = \sqrt{H^2 + S^2} / \text{ Tan } \alpha_i$$

$$R_i = R_o - Vt$$

Ground Range, or R_i^*. The distance along the ground from a point directly beneath the aircraft to the ground point or ground object of interest.

$$R_i^* = H / \text{ Tan } D_i$$

$$R_i^* = \sqrt{S^2 + R_i^2}$$

Secondary equations

$$R_i^* = R_i / \text{ Cos } A_i$$

$$R_1^* = S / \text{ Sin } A_i$$

$$R_i^* = r_i \text{ Cos } D_i$$

$$R_i^* = \sqrt{r_i^2 - H^2}$$

Slant Range, or r_i. The distance from the viewpoint in the aircraft to the ground point or ground object. Object distance.

$$r_i = H / \text{Sin } D_i$$

$$r_i = \sqrt{H^2 + S^2 + R_i^2}$$

Secondary equations.

$$r_i = \sqrt{H^2 + R_i^{*2}}$$

$$r_i = R_i^* / \text{ Cos } D_i$$

$$r_i = R_i / \text{ Cos } \alpha_i$$

$$r_i = \sqrt{H^2 + S^2} / \text{ Sin } \alpha_i$$

Offset, offset distance, or S. Also called target offset. The perpendicular distance of a ground point or ground object from the ground path of the aircraft measured along the ground. Distance to the side.

$$S = H \text{ Sin } A_i / \text{Tan } D_i$$

Secondary equations

$$S = R_i \text{ Tan } A_i$$

$$S = R_i^* \text{ Sin } A_i$$

$$S = r_i \text{ Sin } A_i \text{ Cos } D_i$$

$$S = \sqrt{R_i^{*2} - R_i^2}$$

Alpha velocity, or $d\alpha_i/dt$. Also called velocity in alpha or angular velocity in alpha. rate of change of the angle between the aircraft flight path in the sky and a line from the aircraft to the ground point or ground object of interest.
The following equations yield degrees/second.

$$d\alpha_i/dt = 57.296\ (V/H)\ \mathrm{Sin}\ D_i \mathrm{Cos}\ D_1 \sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i}$$

$$d\alpha_i/dt = 57.296\ V \sqrt{H^2 + S^2}\ /\ (H^2 + S^2 + R_i^2)$$

Secondary equations

$$d\alpha_i/dt = 57.296\ (V/H)\ \mathrm{Sin}\ D_i\ \mathrm{Sin}\ \alpha_i$$

$$d\alpha_i/dt = (\mathrm{Cos}^2 D_i/\mathrm{Sin}\ A_i)(\sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i})(dA_i/dt)$$

$$d\alpha_i/dt = (1/\sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i}\ )\ \left[(\mathrm{Cos}\ A_i \mathrm{Tan}\ D_i)(dD_i/dt + (\mathrm{Sin}\ A_i)(dA_i/dt)\right].$$ dD_i/dt and dA_i/dt in deg./sec.

$$d\alpha_i/dt = (1/\mathrm{Sin}\ \alpha_i)\ \left[(\mathrm{Cos}\ A_i \mathrm{Sin} D_i)(dD_i/dt) + (\mathrm{Sin}\ A_i \mathrm{Cos}\ D_i)(dA_i/dt)\right].$$ dD_i/dt and dA_i/dt in degrees/second.

$$d\alpha_i/dt = (1/\sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i})\left[(\mathrm{Cos}\ A_i \mathrm{Tan}\ D_i)(dD_i/dt) + (\mathrm{Sin}\ A_i)(dA_i/dt)\right].$$ dD_i/dt and dA_i/dt in deg./sec.

Azimuth velocity, or dA_i/dt. Also called angular velocity in azimuth. Rate of change of azimuth.

$$dA_i/dt = 57.296\ (V/H)\ \mathrm{Sin}\ A_i \mathrm{Tan}\ D_i$$

$$dA_i/dt = 57.296\ VS/(S^2 + R_i^2)$$

Secondary equations

$$dA_i/dt = (\mathrm{Sin}\ A_i/\ \mathrm{Cos}^2 D_i)(\sqrt{\mathrm{Tan}^2 D_i + \mathrm{Sin}^2 A_i}\ )(d\alpha_i/dt$$

$$dA_i/dt = (\mathrm{Tan}\ A_i/\ \mathrm{Sin}\ A_i \mathrm{Cos}\ D_i)(dD_i/dt)$$

In the above two equations, $d\alpha_i/dt$ and dD_i/dt, respectively, are in degrees/second.

Declination velocity, or dD_i/dt. Also called angular velocity in declination. Angular rate of up or down motion of points or objects in the field of view.

APPENDIX 2

ANGULAR VELOCITY TABLES

TABLE A1

ANGULAR VELOCITY IN ALPHA, dA_i/dt

Alpha Velocity in Degrees/Second

Azimuth in Degrees

Declination D_i in Degrees	0°	1°	5°	10°	15°	20°	25°	30°	35°	40°	45°
1°	0 038	0 054	0 196	0.384	0.571	0 753	0.930	1 100	1 262	1 414	1 556
5°	0 957	0 976	1 352	2.128	2.990	3.864	4 723	5 555	6 350	7 100	7 798
10°	3 801	3 819	4 240	5 335	6 751	8.295	9 871	11 429	12.935	14 368	15.709
15°	8 444	8 462	8.879	10.062	11 740	13.692	15.769	17 876	19 950	21 945	23.829
20°	14 745	14.762	15 162	16.337	18.093	20.234	22.595	25.054	27 520	29.925	32 218
25°	22 513	22.529	22.903	24.024	25.749	27.920	30.384	33.009	35.689	36.340	40.894
30°	31 513	31.527	31 870	32.907	34 534	36.627	39.053	41 687	44.420	47.159	49.826
35°	41 469	41.482	41.789	42.726	44.212	46.152	48.437	50 957	53.606	56 293	58.936
40°	52 081	52.092	52.361	53.185	54.502	56.241	58 314	60 626	63.086	65.606	68.108
45°	63 025	63.035	63.264	63.969	65.102	66.610	68.423	70.464	72.657	74.923	77.190
50°	73 970	73.978	74 167	74.751	75.694	76.955	78.483	80.216	82.091	84 043	86 010
55°	84 581	84.588	84 739	85.204	85 959	86.973	88.207	89.615	91 148	92.754	94.381
60°	94 538	94.543	94.658	95.012	95.588	96.364	97.312	98.398	99.587	100.838	102.113
65°	103 537	103 541	103 623	103.876	104 289	104 846	105 529	106 314	107 177	108 088	109 020
70°	111 306	111 308	111 362	111 528	111 798	112.165	112.615	113 134	113 705	114 311	114 933
75°	117 607	117 608	117 639	117 734	117 889	118.100	118 359	118 658	118 988	119 339	119 699
80°	122 250	122 250	122 264	122 307	122 377	122 472	122 589	122 724	122 873	123 033	123 196
85°	125 093	125 093	125 097	125 108	125 125	125.149	125 179	125 213	125 251	125 291	125 332
89°	126 012	126 012	126 012	126.013	126 014	126.015	126 016	126 017	126 019	126 020	126 022

Declination D_i in Degrees	50°	55°	60°	65°	70°	75°	80°	85°	90°	95°	100°
1°	1 685	1.802	1 905	1 994	2.067	2.125	2.166	2 192	2.200	2.192	2 166
5°	8 438	9 016	9 526	9 965	10.329	10.615	10.820	10 945	10 986	10.945	10.820
10°	16 945	18 062	19.051	19.903	20.609	21.166	21.566	21 808	21 888	21 808	21 566
15°	25 574	27 160	28.567	29.782	30.793	31.588	32.162	32.508	32.624	32.508	32.162
20°	34 359	36 314	38.057	39.566	40 825	41.817	42 534	42 967	43 112	42 967	42 534
25°	43 298	45.508	47 488	49.209	50 647	51.785	52.607	53 105	53 271	53 105	52 607
30°	52 357	54.700	56.810	58.652	60.197	61 422	62.309	62.846	63.025	62 846	62 309
35°	61 466	63 822	65 957	67.829	69.404	70.656	71.565	72 115	72 300	72 115	71 565
40°	70 520	72.784	74 845	76.660	78 193	79.415	80 304	80 843	81 024	80 843	80 304
45°	79 393	81 471	03.375	85.058	86.485	87.626	88.457	88.962	89.131	88 962	88.457
50°	87 933	89.758	91.437	92.929	94 198	95.215	95 957	96 409	96 560	96 409	95 957
55°	95 981	97.507	98 917	100.175	101 248	102 111	102 741	103 126	103 255	103 126	102 741
60°	103 372	104 578	105 697	106 698	107 555	108 245	108 751	109 059	109 163	109 059	108 751
65°	109 945	110.834	111 661	112 404	113 041	113.555	113 933	114 163	114 241	114 163	113 933
70°	115 551	116.147	116 704	117 205	117 636	117.984	118 240	118 396	118 449	118 396	118 240
75°	120 059	120 407	120 732	121.025	121.278	121.482	121 633	121 725	121 756	121 725	121 633
80°	123 360	123 518	123 667	123 801	123 917	124 010	124 079	124 121	124 136	124 121	124 079
85°	125 374	125.414	125 452	125 486	125 515	125 539	125 557	125 567	125 571	125 567	125 557
89°	126 024	126 025	126 027	126 028	126 029	126 030	126 031	126 031	126 032	126 031	126 031
* 90°	$(180/\pi)(V/H)$ = 126.051 for all values of azimuth A_i										

$d\alpha_i/dt = (180/\pi)(V/H)\sin D_i \cos D_i \sqrt{\tan^2 D_i + \sin^2 A_i}$. The table is for a height H of 200 feet and a speed V of 440 feet/second. Here, $180V/\pi H$ is 126.05071. For other values of H and V, muitiply table entries by $5V/11H$.

* Since $d\alpha_i/dt = (180/\pi)(V/H)\sqrt{\sin^4 D_i + \sin^2 A_i \sin^2 D_i \cos^2 D_i}$, for $D_i = 90°$, $\sin^4 D_i = 1$, and $\cos^2 D_i = 0$, so that $d\alpha_i/dt = (180/\pi)(V/H) = 126.051$ deg./sec.

TABLE A2

ANGULAR VELOCITY IN AZIMUTH, dA_i/dt

Declination in Degrees, D_i	DEGREES/SECOND IN AZIMUTH, dA_i/dt — Azimuth in Degrees, A_i: 1	5	10	15	20	25	30	35	40	45	50
1	.0384	.1918	.3821	.5695	.7525	.9298	1.1001	1.2620	1.4143	1.5558	1.6855
5	.1925	.9611	1.9150	2.8542	3.7718	4.6606	5.5140	6.3254	7.0886	7.7979	8.4479
10	.3879	1.9371	3.8595	5.7525	7.6017	9.3931	11.1131	12.7484	14.2867	15.7163	17.0262
15	.5894	2.9436	5.8650	8.7416	11.5518	14.2740	16.8976	19.3727	21.7103	23.8827	25.8733
20	.8007	3.9986	7.9667	11.8743	15.6914	19.3892	22.9394	25.3149	29.4903	32.4411	35.1451
25	1.0258	5.1228	10.2067	15.2130	20.1034	24.8408	29.3892	33.7139	37.7823	41.5626	45.1451
30	1.2701	6.3428	12.6373	18.8357	24.8907	30.7562	36.3877	44.7423	46.7791	51.4597	55.7492
35	1.5404	7.6925	15.3265	22.8438	30.1873	37.3010	44.1308	50.6248	56.7335	62.4104	67.6124
40	1.8459	9.2183	18.3666	27.3751	36.1752	47.7000	52.8846	60.6667	67.9871	74.7901	81.0238
45	2.1999	10.9660	21.8885	32.6243	43.1119	53.2713	63.0254	72.2997	81.0238	89.1313	96.5605
50	2.6217	13.0924	26.0857	38.8802	51.3787	62.4863	75.1107	86.1635	96.5605	106.2226	115.0763
55	3.1417	15.6897	31.2600	46.5924	61.5702	76.0794	90.0095	103.2541	115.7140	127.2927	137.9026
60	3.8103	19.0284	37.9120	56.5070	74.6720	92.2687	109.1631	125.2268	140.3374	154.3800	167.2476
65	4.7176	23.5596	46.9400	69.9331	92.4537	114.2407	135.1583	155.0472	172.7562	191.1427	207.0746
70	6.0441	30.1839	60.1381	89.6346	118.4489	146.3618	173.1607	198.6418	222.6112	244.8863	265.2977
75	8.2101	41.0005	81.6889	121.7556	160.8957	198.8113	235.2138	269.8262	302.3851	332.6426	360.3685
80	12.4762	62.3049	124.1357	185.0217	244.4996	302.1167	357.4346	410.0321	459.5090	505.4888	547.6215
85	25.1448	125.5711	250.1864	372.8978	492.7711	608.8941	720.3832	826.3896	926.1067	1018.78	1103.69
89	126.0315	629.3900	1253.99	1869.05	2469.88	3051.91	3610.72	4142.05	464[illegible].85	5106.33	5531.94

Angular velocity in azimuth is $dA_i/dt = (180/\pi)(V/H)\sin A_i \tan D_i$. For the above table, V = 440 feet/second and H = 200 feet, so that $dA_i/dt = (396/\pi)(V/H)\sin A_i \tan D_i$. For other values of V and H, multiply table values by 5V/11H.

TABLE A2 (Continued)

ANGULAR VELOCITH IN AZIMUTH, dA_i/dt

DECLINATION IN DEGREES, D_i	DEGREES/SECOND IN AZIMUTH, dA_i/dt										
	Azimuth in Degrees, A_i										
	55	60	65	70	75	80	85	90	95	100	105
1	1.8023	1.9054	1.9941	2.0675	2.1252	2.1668	2.1918	2.2002	2.1918	2.1668	2.1252
5	9.0336	9.5505	9.9947	10.3629	10.6522	10.8605	10.9860	11.0280	10.9860	10.8605	10.6552
10	18.2056	19.2484	20.1437	20.8857	21.4688	21.8885	22.1416	22.2261	22.1416	21.8885	21.4688
15	27.6670	29.2502	30.6107	31.7383	32.6243	33.2621	33.6467	33.7752	33.6467	33.2621	32.6243
20	37.5816	39.7321	41.5802	43.1119	44.3154	45.1817	45.7044	45.8787	45.7041	45.1817	44.3154
25	48.1435	50.9046	53.2713	55.2336	56.7756	57.8854	58.5547	58.7784	58.5547	57.8854	56.7756
30	59.6141	63.0254	65.9568	68.3685	70.2956	71.6698	72.4985	72.7754	72.4985	71.6698	70.2956
35	72.2997	76.4368	79.9923	82.4388	85.2542	86.9208	87.9258	88.2617	87.9258	86.4208	85.2542
40	86.6410	91.5987	95.8594	99.3905	102.1651	104.1622	105.3666	105.7691	105.3666	104.1622	102.1651
45	103.2547	109.1631	114.2407	118.4489	121.7556	124.1357	125.5710	126.0507	125.5710	124.1357	121.7556
50	123.0542	130.0948	136.1468	141.1619	145.1027	147.9392	149.6498	150.2214	149.6498	147.9392	145.1027
55	147.4630	155.9011	163.1527	169.1626	173.8851	177.2842	179.3341	180.0191	179.3341	177.2842	173.8851
60	178.8424	189.0761	197.8708	205.1596	210.8870	215.0094	217.4955	218.3263	217.4955	215.0094	210.8870
65	221.4304	234.1011	244.9901	254.0146	266.1058	266.2099	269.2880	270.3166	269.2880	266.2099	266.1058
70	283.6900	299.9232	313.8739	325.4357	334.5309	341.0601	345.0036	346.3215	345.0036	341.0601	334.5209
75	385.3518	407.4023	426.3522	442.0574	454.3982	463.2808	468.6375	470.4277	468.6375	463.2808	454.1982
80	585.5865	619.0948	647.8915	671.7572	690.5106	704.0087	712.1488	714.8691	712.1488	704.0087	690.5106
85	1180.21	1247.74	1305.78	1352.88	1391.67	1418.88	1435.28	1440.77	1435.28	1418.88	1391.67
89	5915.46	6253.95	6544.85	6785.93	6975.38	7111.73	7193.96	7221.44	7193.96	7111.73	6975.38

When velocity V is not 440 feet/second, and height H is not 200 feet, multiply table entries by 5V/11H.

TABLE A3

ANGULAR VELOCITY IN DECLINATION, dD_i/dt

Degrees/Second in Declination, dD_i/dt

Azimuth in Degrees, A_i

Declination in Degrees, D_i	0	1	5	10	15	20	25	30	35	40	45	
1	0 038	0 038	0 038	0 038	0 037	0.036	0 035	0 033	0 031	0 029	0 027	
5	0 957	0 957	0 954	0 943	0 925	0 900	0 868	0 829	0 784	0 733	0 677	
10	3 801	3 800	3 786	3 743	3 671	3 572	3 445	3.292	3 114	2 912	2 688	
15	8 444	8 443	8 412	8 316	8 156	7 935	7 653	7.313	6 917	6 468	5 971	
20	14 745	14 743	14.689	14 521	14 243	13 856	13 364	12 770	12 079	11 295	10 426	
25	22 513	22 510	22 428	22 171	21 746	21 156	20 404	19.497	18 442	17 246	15.919	
30	31 513	31 508	31 393	31.034	30 439	29 612	28 560	27.291	25.814	24.140	22.283	
35	41 469	41 463	41 312	40 839	40 056	38 968	37 584	35.914	33 970	31.767	29 323	
40	52.081	52.073	51.883	51 290	50 306	48 940	47 202	45 104	42.662	39.896	36.827	
45	63 025	63 016	62 786	62 068	60 878	59 224	57 120	54 582	51.627	48.280	44.566	
50	73.970	73 958	73 688	72 846	71 449	69 509	67 039	64.060	60 592	56.664	52.304	
55	84 581	84 568	84 259	83 296	81 699	79.480	76 657	73.250	69 285	64.793	59.808	
60	94 538	94 524	94 178	93 102	91 317	88 837	85 681	81 872	77.441	72 420	66 848	
65	103 537	103 521	103 143	101.964	100 009	97 293	93 837	89 666	84 813	79 314	73 212	
70	111 306	111 289	110 882	109 615	107 513	104 593	100 877	96.393	91 176	85 265	78 705	
75	117 607	117 589	117.159	115 820	113 600	110 514	106 588	101.851	96 338	90.092	83 161	
80	122 250	122 231	121.785	120 393	118 084	114.877	110 796	105.871	100 141	93 649	86 444	
85	125 093	125 074	124.617	123 193	120 831	117 549	113 373	108 334	102 470	95 827	88 454	
89	126 051	126 032	125 571	124 136	121 756	118 449	114 241	109 163	103 255	96.560	89 131	

Declination in Degrees, D_i	50	55	60	65	70	75	80	85	90	95	100	105
1	0 025	0.022	0 019	0 016	0 013	0 010	0 007	0.003	0.000	-0.003	-0.007	-0.010
5	0 615	0 549	0 479	0 405	0 327	0 248	0 166	0 083	0 000	-0.083	-0.166	-0.248
10	2 443	2 180	1 900	1 606	1 300	0 984	0 660	0 331	0 000	-0.331	-0.660	-0.984
15	5 428	4 843	4 222	3 569	2 888	2 185	1 466	0.736	0 000	-0.736	-1.466	-2.185
20	9 478	8 457	7 373	6 232	5 043	3 816	2 560	1.285	0 000	-1.285	-2.560	-3.816
25	14 471	12 913	11 257	9 515	7 700	5 827	3 909	1.962	0 000	-1.962	-3.909	-5.827
30	20.256	18 075	15 756	13 318	10 778	8 156	5 472	2 747	0 000	-2 747	-5.472	-8.156
35	26 656	23 786	20 735	17 526	14 183	10 733	7 201	3 614	0 000	-3.614	-7 201	-10.733
40	33 477	29 873	26 041	22 010	17 813	13 480	9 044	4 539	0 000	-4 539	-9.044	-13.480
45	40 512	36 150	31 513	26 636	21.556	16 312	10 944	5.493	0 000	-5.493	-10 944	-16.312
50	47 547	42 427	36 985	31 261	25 299	19 145	12 845	6 447	0 000	-6 447	-12 845	-19.145
55	54 368	48 514	42 291	35 746	28 929	21 891	14 687	7.372	0 000	-7.372	-14 687	-21.891
60	60 768	54 225	47 269	39 954	32 334	24 468	16 416	8 240	0 000	-8.240	-16 416	-24.468
65	66 552	59 387	51 769	43 757	35 412	26 797	17 979	9 024	0 000	-9.024	-17 979	-26.797
70	71 546	63 842	55 653	47.040	38 069	28 808	19 328	9 701	0.000	-9 701	-19 328	-28 808
75	75 596	67 457	58 803	49 703	40 224	30 439	20 422	10 250	0 000	-10 250	-20 422	-30.439
80	78 581	70 120	61 125	51 665	41 812	31.641	21 228	10 655	0 000	-10 655	-21.228	-31.641
85	80 408	71 751	62 547	52 867	42 784	32 377	21 722	10 903	0 000	-10 903	-21.722	-32.377
89	81 024	72 300	63 025	53 271	43 112	32 624	21 888	10 986	0 000	-10.986	-21.888	-32.624

Table entries are for H = 200 feet and V = 440 feet/second. For other entries, multiply entries by (5/11)(V/H). Entries calculated from $dD_i/dt = (150/\pi)(V/H) \cos A_i \sin^2 D_i = 126.0507 \cos A_i \sin^2 D_i$.

APPENDIX 3

EXAMPLE PROBLEMS

Problem 1

An object on the ground is 2,000 feet ahead of an aircraft and is offset from the flight path by 265 feet. The aircraft is flying straight and level at 1,000 feet/second(681.8 miles/hour) at an altitude of 200 feet above ground. Using the equations in Appendix 1, calculate the angle alpha and the object's azimuth and declination. Use the azimuth and declination to calculate alpha again. Then calculate the object's ground range and slant range, its angular velocity in alpha, in azimuth, and in declination. For velocity in alpha, use both Eqn. 15 and Eqn. 16. For velocity in azimuth, use both Eqn. 21 and Eqn. 20. For velocity in declination, use both Eqn. 23 and Eqn. 22. Obtain declination velocity from azimuth velocity by using Eqn. 28. Check angular velocity computations by using velocity values in Eqn. 18, which contains velocities in alpha, azimuth and declination.

In working out the answers, note that what is given is distance ahead R_i = 2,000 feet, offset distance S = 265 feet, aircraft speed V = 1,000 feet/second, and aircraft height H = 200 feet.

Solution

The angle alpha α_i is obtained from Eqn. 12, from Appendix 1,

$$\alpha_i = \text{ArcSin} \sqrt{(H^2 + S^2)/(H^2 + S^2 + R_1^2)}$$

$$= \text{ArcSin} \sqrt{(200^2 + 265^2)/(200^2 + 265^2 + 2{,}000^2)} = 9.4250 \text{ degrees.}$$

Azimuth angle A_i, from triangle B, Fig. 12, Tan $A_i = S/R_i$, so that

$$A_i = \text{ArcTan}\ (S/R_i) = \text{ArcTan}\ (265/2000) = 7.54773 \text{ degrees.}$$

Declination angle D_i, from Eqn. 5, S = H Sin A_i/Tan D_i, so that

$$D_i = \text{ArcTan}\left[(H/S)\text{Sin } A_i\right] = \text{ArcTan}\left[(200/265) \text{ Sin } 7.54773\right] = 5.66144 \text{ degrees}$$

To obtain the angle alpha from azimuth and declination, use Eqn. 14,

$$\alpha_i = \text{ArcCos}\ (\text{Cos } A_i \text{Cos } D_i) = \text{ArcCos}\ (\text{Cos } 7.54773 \text{ Cos } 5.66144) =$$

$$= 9.4252 \text{ degrees, as before.}$$

$4.62806 = (1/\sin 9.42520)\left[\cos 7.54773 \sin 5.66144)(2.76382) + (\sin 7.54773 \cos 5.66144)(3.73037)\right] = 4.62807$. This checks out, considering the number of digits (places) used in the calculations.

Summary of worked

Alpha α_i = 9.42520 degrees.

Azimuth A_i = 7.54773 degrees.

Declination D_i = 5.66144 degrees.

Ground range R_i^* = 2,017.48 feet.

Slant Range r_i = 2,027.37 feet.

Angular velocity in alpha $d\alpha_i/dt$ = 4.62806 degrees/second.

Azimuth velocity dA_i/dt = 3.73037 degrees/second..

Declination velocity dD_i/dt = 2.76382 degrees/secon ,

Problem 2

An aircraft is flying straight and level at a speed of 500 feet/second at an altitude of 190 feet. The pilot detects a missile launcher at an azimuth of 20 degrees and declination of 15 degrees. What is the launcher's offset distance S, ground range R_i^*, slant range r_i, and distance ahead R_i, and what is the launcher's angular velocity in azimuth, in declination, and in the angle alpha? Find angular rates by using equations from Appendix 1 and also by using tables A_1-A_3 in Appendix 2.

Solution

By Eqn. 5 , offset distance $S = H \sin A_i/\tan D_i =$
$= (190 \sin 20)/\tan 15 = 242.52$ feet.

By Eqn. 7 , ground range $R_i^* = H/\tan D_i = 190/\tan 15 = 709.1$ feet.

By Eqn. 1 , slant range $r_i = H/\sin D_i = 190/\sin 15 = 734.1$ feet.

By Eqn. 6 , Distance ahead $R_i = H \cos A_i/\tan D_i =$
$= 190 \cos 20/\tan 15 = 666..3$ feet.

By Eqn. 21 , azimuth velocity $dA_i/dt = 57.296\,(V/H)\sin A_i \tan D_i =$
$= 57.296\,(500/190)\sin 20 \tan 15 = 13.82$ degrees/second.

Ground Range R_i^*, from Eqn. 9, $R_i^* = S/\text{Sin } A_i = 265/\text{Sin } 7.54773 = 2{,}017.48$ feet.

Slant Range r_i from Eqn. 1, $r_i = H/\text{Sin } D_i = 200/\text{Sin } 5.66144 = 2{,}027.37$ feet.

Angular velocity in alpha, from Eqn. 15,

$$d\alpha_i/dt = 57.296\, V\sqrt{H^2 + S^2}/(H^2 + S^2 + R_i^2) =$$

$$57.296(1{,}000)\sqrt{200^2 + 265^2}/(200^2 + 265^2 + 2{,}000^2) =$$

$$= 4.62806 \text{ degrees/second.}$$

Angular velocity in alpha, from Eqn. 16,

$$d\alpha_i/dt = 57.296(V/H)\,\text{Sin } D_i \text{Cos } D_i\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i} =$$

$$= 57.296(1{,}000/200)\,\text{Sin } 5.66144 \text{ Cos } 5.66144\sqrt{\text{Tan}^2 5.66144 + \text{Sin}^2 7.54773}$$

$$= 4.62806 \text{ degrees, as above.}$$

Angular velocity in azimuth dA_i/dt, from Eqn. 21,

$$dA_i/dt = 57.296\,(V/H)\,\text{Sin } A_i \text{Tan } D_i =$$

$$= 57.296\,(1{,}000/200)\,\text{Sin } 7.54773 \text{ Tan } 5.66144 = 3.73037 \text{ degrees/second.}$$

Angular velocity in azimuth, from Eqn. 20, $dA_i/dt = 57.296VS/(S^2 + R_i^2) =$

$$dA_i/dt = (57.296)(1{,}000)(265))/(265^2 + 2{,}000^2) = 3.73037 \text{ degrees/second.}$$

Angular velocity in declination dD_i/dt, from Eqn. 23,

$$dD_i/dt = 57.296\,(V/H)\,\text{Cos } A_i \text{Sin}^2 D_i =$$

$$= 57.296\,(1{,}000/200)\text{Cos } 7.54773 \text{ Sin}^2\, 5.66144 = 2.76382 \text{ degrees/second.}$$

Angular velocity in declination dD_i/dt, from Eqn. 22,

$$dD_i/dt = 57.296\, VHR/(H^2 + S^2 + R_i^2)\sqrt{S^2 + R_i^2} =$$

$$=(57.296)(1{,}000)(200)(2{,}000)/(200^2 + 265^2 + 2{,}000^2)\sqrt{265^2 + 2{,}000^2} =$$

$$= 2.76382 \text{ degrees/second, as above.}$$

To obtain azimuth velocity from declination velocity by using Eqn. 28,

$$dA_i/dt = (\text{Tan } A_i/\text{Cos } D_i \text{Sin } D_i)(dD_i/dt) =$$

$$= (\text{Tan } 7.54773/\text{Cos } 5.66144 \text{ Sin } 5.66144)(2.76382) = 3.73037 \text{ degrees/second, as before.}$$

To check computation of angular velocities by using Eqn. 18,

$$d\alpha_i/dt = (1/\text{Sin}\alpha_i)\left[(\text{Cos } A_i \text{Sin } D_i)(dD_i/dt) + (\text{Sin } A_i \text{Cos } D_i)(dA_i/dt)\right] =$$

By Eqn. 23, declination velocity $dD_i/dt = 57.296\ (V/H)\text{Cos }A_i\text{Sin}^2D_i =$
$= 57.296\ (500/190)\text{Cos } 20\ \text{Sin}^2 15 = 9.491$ degrees/second.

By Eqn. 16, Angular velocity in alpha is

$$d\alpha_i/dt = 57.296\ (V/H)\text{Sin }D_i\text{Cos }D_i\sqrt{\text{Tan}^2D_i+\text{Sin}^2A_i} =$$
$$= 57.296(500/190)\text{Sin } 15\ \text{Cos } 15\sqrt{\text{Tan}^2 15 + \text{Sin}^2 20} = 16.38 \text{ deg/sec.}$$

Alternatively, by Eqn. 15, $d\alpha_i/dt = 57.296V\sqrt{H^2 + S^2})/H^2+S^2+R_i^2) =$

$$57.296(500)\sqrt{190^2+242.54^2}/(190^2+242.5^2+666.3^2) =$$
$$d\alpha_i/dt = 16.38 \text{ degrees/second, as above by Eqn. 16.}$$

To find angular velocities by using Tables A1-A3, note that the correction factor for speed and height is (5/11)(500/190)= 1.1962.

From Table A2, dA_i/dt = (11.552 from table)(correction factor) =
= 11.552(1.1962) = 13.82 degrees,second.

From Table A3, dD_i/dt = (7.9345)(1.1962) = 9.491 degrees/second.

From Table A1, $d\alpha_i\ dt$ = 13.69(1.1962) = 16.38 degrees/second.

Note that these angular velocities using the tables are the same as those above calculated from the velocity tables.

Problem 3

For what azimuth A_i is angular velocity in alpha 36 degrees/second when declination D_i is 30 degrees for an aircraft flying straight and level at a height H of 200 feet with a ground speed of 440 feet/second? Check the answer by inserting it back into the equation for angular velocity in alpha, $d\alpha_i/dt$.

Solution

Eqn. 16 in Appendix 1 for angular velocity in alpha is
$d\alpha_i/dt = 57.296(V/H)\text{Sin }D_i\text{Cos }D_i\sqrt{\text{Tan}^2D_i + \text{Sin}^2A_i}$. Dividing both sides of the equation by 57.296(V/H)Sin D_iCos D_i yields
$H(d\alpha_i/dt)/(57.296\ V\text{Sin}D_i\text{Cos }D_i) = \sqrt{\text{Tan}^2D_i + \text{Sin}^2A_i}$. Squaring both sides of the equation,

$$\left[H(d\alpha_i/dt)/57.296V\ \text{Sin }D_i\text{Cos }D_i\right]^2 = \text{Tan}^2D_i + \text{Sin}^2A_i.$$

$$\text{Sin}^2A_i = H(d\alpha_i)/57.296\ V\ \text{Sin }D_i\text{Cos }D_i{}^2 - \text{Tan}^2D_i$$
$$\text{Sin }A_i = \left\{\left[H(d\alpha_i)/57.296\ V\ \text{Sin }D_i\text{Cos }D_i\right\}^2 - \text{Tan}^2 D_i\right]^{1/2}$$

Taking the inverse of this equation,

$$A_i = \text{Arc Sin}\sqrt{\left[H(d\alpha_i/dt))/57.296\ V\ \text{Sin}\ D_i \text{Cos}\ D_i\right]^2 - \text{Tan}^2\ D_i}$$

$$A_i = \text{Arc Sin}\sqrt{\left[(200)(36)/(57.296)(440)(\text{Sin}\ 30\ \text{Cos}\ 30)\right]^2 - \text{Tan}^2 30}$$

A_i = Arc Sin .318885 = 18.596 degrees.

To check this answer by inserting it into the equation for $d\alpha_i/dt$,

$$d\alpha_i/dt = 57.296\ (V/H)\ \text{Sin}\ D_i \text{Cos}\ D_i \sqrt{\text{Tan}^2\ D_i + \text{Sin}^2\ A_i}$$

$$36 = 57.296\ (440/200) \text{Sin}\ 30\ \cos\ 30 \sqrt{\text{Tan}^2 30 + \text{Sin}^2 18.596}$$

36 =35.999 ≑ 36. The answer, thus, checks out.

Problem 4

Plot a graph of angular velocity in alpha as a function of declination for an aircraft speed of 350 knots at a height above ground of 200 feet. Use the data of Table A1 corrected for 350 knots. Graph velocity for declination from 0 to 45 degrees and azimuths up to 75 degrees. Use declination for the horizontal axis.

Solution

First, convert 350 knots to feet/second. Now, 350 knots is 350 nautical miles/hour. For a nautical mile of 6070 feet, 350 knots is (350)(6070) = 2.1277×10^6 feet/hour = (2.1266×10^6 feet/hour)(3600 seconds/hour) = 590.722 feet/second. The correction factor for the angular velocity table is thus (5/11)(590.722/200) = 1.343. Using azimuths of 1, 15, 30, 45, 60, and 75 degrees, calculate angular velocity in alpha for 0 through 45 degrees declination by multiplying table entries by 1.343. For example, for 45 degrees declination and 45 degrees azimuth, the table entry of 77.190 is multiplied by 1.343 to obtain 103.67 degrees/second. The graph obtained by this procedure is shown in Fig. 13.

Since angular velocity for azimuths over 90 degrees mirrors angular velocity for angles less than 90 degrees, each azimuth curve in Fig. 11 has two labels. For example, angular velocity at 180-30 = 150 degrees is the same as for 30 degrees, except for direction.

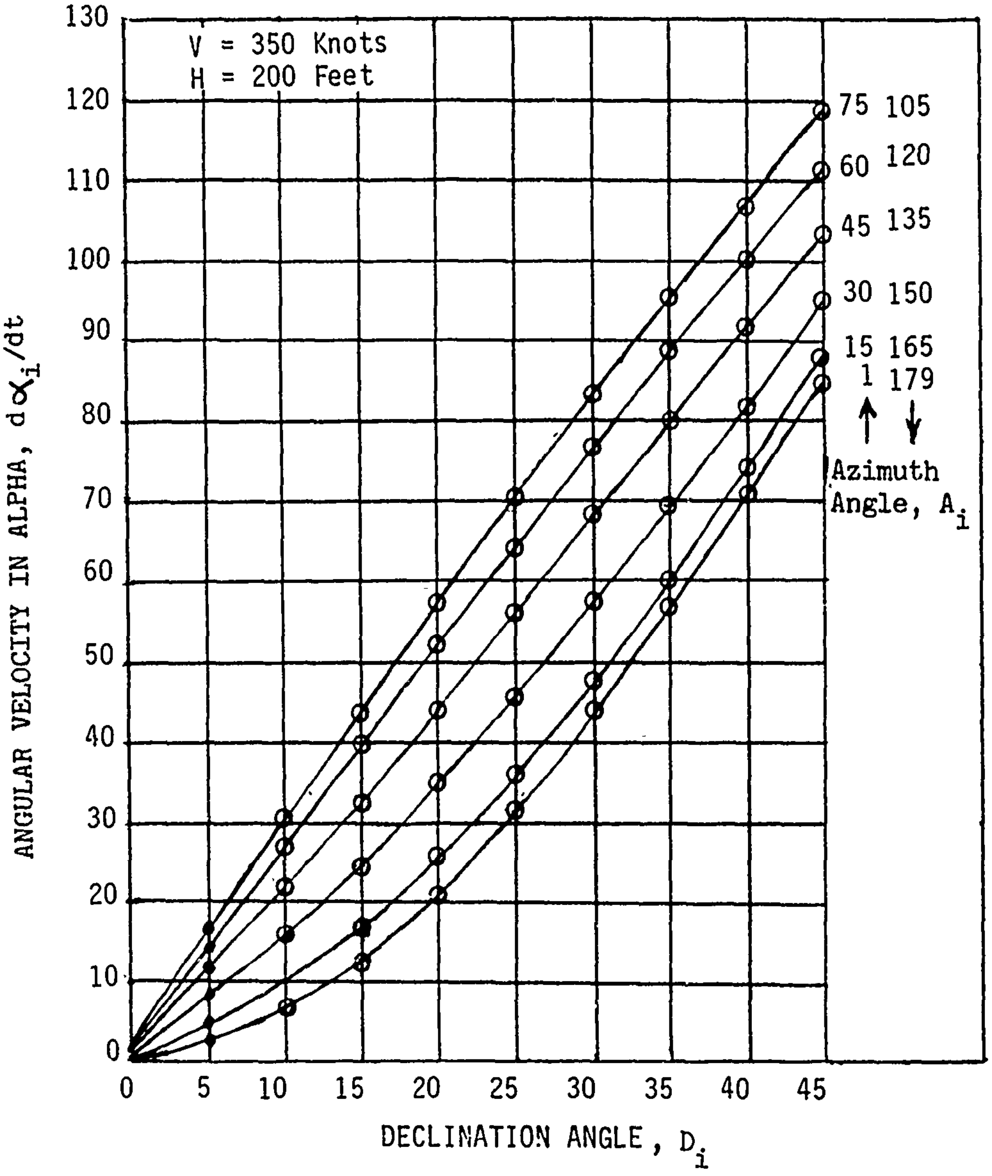

Fig. 13. Angular velocity in alpha for a height of 200 feet and a speed of 350 knots in straight level flight.

Problem 5

In the field of view of an aircraft pilot there is a line that is the locus of points for which the azimuth and declination velocities are equal. Derive the equation of this line. Using the equation, find the azimuth A_i for a declination of 30 degrees and for a declination of 65 degrees for which azimuth and declination velocities are equal. Using equations 21 and 23 of Appendix 1, verify, for both locations, that the angular velocities are equal to each other.

Solution

Equal angular velocities in azimuth and declination mean that $dA_i/dt = dD_i/dt$. By equations 21 and 23, respectively, of Appendix 1,

$$57.296(V/H)\text{Sin } A_i \text{ Tan } D_i = 57.296(V/H)\text{Cos } A_i \text{Sin}^2 D_i$$
$$\text{Sin } A_i \text{ Tan } D_i = \text{Cos } A_i \text{Sin}^2 D_i$$
$$\text{Sin } A_i(\text{Sin } D_i/\text{Cos } D_i) = \text{Cos } A_i \text{Sin}^2 D_i$$
$$\text{Sin } A_i/\text{Cos } A_i = \text{Sin } D_i \text{ Cos } D_i$$
$$\text{Tan } A_i = \text{Sin } D_i \text{Cos } D_i$$

This last equation is a solution to the problem. However, a more convenient equation for plotting may be obtained by using the trigonometric identity Sin D_i Cos D_i = 1/2 Sin $2D_i$. The above equation then becomes Tan A_i = Sin D_i Cos D_i, i.e., Tan A_i = 1/2 Sin $2D_i$. For the two points specified in the problem, one with D_i=30 and one with D_i=60 degrees, respectively, the corresponding azimuth A_i values are A_i = Arc Tan(.5Sin 60) = 23.413, and A_i= (.5 Sin 130) = 20.958 For the first point (A_i=23.413, D_i=30),

dA_i/dt = 57.296 (440/200)Sin 23.412 Tan 30 = 28.92 degrees/second.

dD_i/dt = 57.286 (440/200)Cos 23.412 $\text{Sin}^2 30$ = 28.92 degrees/second.

For the second point(A_i=20.96, D_i= 65),

dA_i/dt = 57.296 (440/200) Sin 20.958 Tan 65= 96.69 degrees/second.

dD_i/dt = 57.296 (440/200) Cos 20.958$\text{Sin}^2 65$ = 96.69 degrees/second.

These calculations show, for each of the two points, that azimuth and declination velocities are equal to each other.

Problem 6

An aircraft is flying straight and level at a height of 220 feet with a ground speed of 220 feet/second. An observer in the aircraft is interested in an object on the ground that is 1,000 feet ahead and is offset from the flight path by 275 feet. Use the equations in Appendix 1 to calculate the object's azimuth and declination, the angle alpha, slant range and ground range. Calculate azimuth and declination velocities using both the equations with ground distance and offset and the equations containing azimuth and declination. Calculate angular velocity in alpha using all seven of the $d\alpha_i/dt$ equations in the section on summary of symbol definitions and equations.

Solution

From triangle B of Fig.12, $\text{Tan } A_i = S/R_i$, so that

A_i = Arc Tan (S/R_i) = Arc Tan (275/1,000) = $\underline{A_i = 15.3763 \text{ degrees}}$.

From Eqn. 3, $\text{Sin } D_i = H/\sqrt{H^2 + S^2 + R_i^2}$ from which

D_i = Arc Sin $(220/\sqrt{220^2 + 275^2 + 1{,}000^2}$ = $\underline{D_i = 11.9764 \text{ degrees}}$.

From Eqn. (10), $\text{Cos}\,\alpha_i = (R_i/H)\text{Sin } D_i$ = (1,000/220)Sin(11.9764) = .943222, so that α_i = Arc Cos .943222 = $\underline{\alpha_i = 19.4001 \text{ degrees}}$.

From Eqn.(1), $r_i = H/\text{Sin } D_i$ = 220/Sin 11.9764 = $\underline{r_i = 1{,}060.20 \text{ feet}}$.

From Eqn.(5), $S = H \text{ Sin } A_i/\text{Tan } D_i$ = 220 Sin 15.3763/Tan 11.9764 = 275.000, which checks the values of A_i and D_i, since S was given as 275 feet.

From triangle A of Fig.12, $\text{Tan } D_i = H/R_i^*$, from which

$R_i^* = H/\text{Tan } D_i$ = 220/Tan 11.0764 = $\underline{R_i^* = 1{,}037.12 \text{ feet}}$.

From Eqn. (21), $dA_i/dt = 57.296(V/H)\text{Sin } A_i \text{Tan } D_i$ =

dA_i/dt = 57.296(440/220) Sin 15.3763 Tan 11.9764 = $\underline{dA_i/dt = 6.44543 \text{ ft}}$.

From Eqn. (20), $dA_i/dt = 57.296\, VS/(S^2 + R_i^2)$ =

$dA_i/dt = (57.296)(440)/(275^2 + 1{,}000^2)$ = $\underline{dD_i/dt = 6.44538 \text{ degrees/second}}$.

Note that the two equations provide the same azimuth velocities.

From Eqn.(23), $dD_i/dt = 57.296\,(V/H)\text{Cos } A_i \text{Sin}^2 D_i$ =

dD_i/dt = (57.296)(440/220) Cos 15.3763 Sin^2 11.9764 = $\underline{dD_i/dt = 4.75770 \text{ degrees/second}}$.

From Eqn. (22), $dD_i/dt = 57.296\, HVR_i/(H^2 + S^2 + R_i^2)\sqrt{(S^2 + R_i^2)}$

$dD_i/dt = (57.296)(220)(440)(1000)/(220^2 + 275^2 + 1{,}000^2)\sqrt{275^2 + 1{,}000^2}$) =

$\underline{dD_i/dt = 4.75766 \text{ degrees/second}}$.

The seven equations for velocity in alpha from the summary section, with inserted values of the variables and constants, are as follows.

(1). $d\alpha_i/dt = 57.296(V/H)\ \text{Sin}\ D_i \text{Cos}\ D_i \sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i} =$

$d\alpha_i/dt = 57.296\ (440/220)\text{Sin}\ 11.9764\ \text{Cos}\ 11.9764\ (\text{Tan}^2 11.9764 + \text{Sin}^2 15.3763)^{1/2} = d\alpha_i/dt = 7.89876$ degrees/second.

(2). $d\alpha_i/dt = 57.296\ V(H^2+S^2)^{1/2} / (H^2+S^2+R_i^2) =$

$d\alpha_i/dt = 57.296(440) \sqrt{220^2+275^2}/(220^2 + 275^2 + 1{,}000^2) =$

$d\alpha_i/dt = 7.89870$ degrees/second.

(3). $d\alpha_i/dt = 57.296(V/H)\ \text{Sin}\ D_i \text{Sin}\ \alpha_i =$

$d\alpha_i/dt = 57.296\ (440/220)\ \text{Sin}\ 11.9764\ \text{Sin}\ 19.4001 =$

$d\alpha_i/dt = 7.89845$ degrees/second.

(4). $d\alpha_i/dt = (\text{Cos}^2 D_i/\text{Sin}\ A_i)(\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i})(dA_i/dt)) =$

$d\alpha_i/dt = (\text{Cos}^2\ 11.9764/\text{Sin}\ 15.3763)(\text{Tan}^2\ 11.9764 + \text{Sin}^2\ 15.3763)^{1/2}(6.44538) =$

$d\alpha_i/dt = 7.89870$ degrees/second.

(5). $d\alpha_i/dt = (1/\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i})\ [(\text{Cos}\ A_i \text{Tan}\ D_i)(dD_i/dt) + (\text{Sin}\ A_i)(dA_i/dt)] =$

$d\alpha_i/dt = (1/\sqrt{\text{Tan}^2\ 11.9764 + \text{Sin}^2\ 15.3763})\ [(\text{Cos}\ 15.3763\ \text{Tan}\ 11.9764)(4.75770) + \text{Sin}\ 15.3763)(6.44538)] =$

$d\alpha_i/dt = 7.89874$ degrees/second.

(6). $d\alpha_i/dt = (1/\text{Sin}\ \alpha_i)[(\text{Cos}\ A_i\ \text{Sin}\ D_i)(dD_i/dt) + (\text{Sin}\ A_i \text{Cos}\ D_i)(dA_i/dt)] = (1/\text{Sin}\ 19.4001)\ [(\text{cos}\ 15.3763\ \text{Sin}\ 11.9764)(4.75770) + (\text{Sin}\ 15.3763\ \text{Cos}\ 11.9764)(6.44538)] =$

$d\alpha_i/dt = 7.89903$ degrees/second.

(7). $d\alpha_i/dt = (1/\sqrt{\text{Tan}^2 D_i + \text{Sin}^2 A_i})[(\text{Cos}\ A_i \text{Tan}\ D_i)(dD_i/dt) + (\text{Sin}\ A_i \text{Cos}\ D_i)(dA_i/dt)] =$

$= (1/\sqrt{\text{Tan}^2 11.9764 + \text{Sin}^2 15.3763})\ [(\text{Cos}\ 15.3763\ \text{Tan}\ 11.9764)(4.75766) + (\text{Sin}\ 15.3763)(6.44538)] =$

$d\alpha_i/dt = 7.89870$ degrees/second.

Note that, within rounding errors, all seven equations provide the same answer.

Problem 7

An object on the ground is directly ahead of an aircraft that is flying at 800 feet/second at a height of 300 feet. At what distance ahead and at what declination will the object be when velocity in alpha reaches 35 degrees/second ?

Solution

By Eqn. (15), $d\alpha_i/dt = 57.296V\sqrt{H^2+S^2} / (H^2+S^2+R_i^2)$. Since S=0,

$$d\alpha_i/dt = 57.296\ VH/(H^2 + R_i^2)$$

$$R_i^2 + H^2 = 57.296VH/(d\alpha_i/dt).$$ Inserting values,

$$R_i^2 + 300^2 = (57.296)(800)(300)/35 = 392{,}886.9$$

$$R_i = (392{,}886.9 - 90{,}000)^{1/2}$$

$$\underline{R_i = 550.352 \text{ feet.}}$$

By Eqn. (6), $R_i = H \cos A_i/\tan D_i$. Here, $A_i=0$, so that

$R_i = H \cos 0/\tan D_i = H/\tan D_i$, from which

$D_i = \text{Arc Tan } (H/R_i) = \text{Arc Tan } (300/550.352) = \underline{D_i = 28.5951 \text{ degrees}}.$

An alternative to the above solution is instructive. When an object is straight ahead, the angle alpha is declination, i.e., $D_i = \alpha_i$.

From Fig. 12 , $\tan D_i = \tan\alpha_i = H/R_i = HR_i^{-1}$. Differentiating,

$d(\tan^2 D_i/dt) = \sec^2 D_i(dD_i/dt) = (dD_i/dt)/\cos^2 D_i = -(H/R_i^2)(dR_i/dt)$.

Now, from above, $R_i = H/\tan D_i$, and, $dR_i/dt = -V$, relative aircraft velocity,

$(dD_i/dt)/\cos^2 D_i = HV/(H^2/\tan^2 D_i) = (V/H) \tan^2 D_i$

$(\tan^2 D_i/\cos^2 D_i = (\sin^2 D_i/\cos^2 D_i)(\cos^2 D_i) = \sin^2 D_i = (H/V)(dD_i/dt)$.

Since $D_i = \alpha_i$ in this problem, $\sin\alpha_i = \sqrt{(H/V)(d\alpha_i/dt)}$.

$d\alpha_i/dt$ = 35 degrees/second = 35/57.296 radians/second. Thus,

$\alpha_i = \text{Arc Sin}\sqrt{(300/800)(35/57.296)}$ = <u>28.5952 degrees, as before</u>.

Since $\tan D_i = H/R_i$, $R_i = H/\tan D_i = 300/\tan 28.5951$ =

$\underline{R_i = 550.035, \text{ as before.}}$

APPENDIX 4

PROBLEMS IN LOW-ALTITUDE HIGH-SPEED FLIGHT

The present paper looks at the angular velocity of ground points and ground objects since, at low altitudes and high speeds, motion rates pose serious problems for pilots and other crewmembers. It must be kept in mind that at low altitudes at high speed, angular motion is only one of several problems. These include the following:

1. Impact of the aircraft with radio and TV towers, power lines, and even hills, is a serious danger, even in broad daylight in good weather.

2. Almost constant observation of the terrain to avoid obstacles is necessary and this makes detection of airborne missiles and hostile aircraft less likely. Also, keeping track of both hostile and friendly aircraft is more difficult. Still another problem at low altitude is inadequate attention to aircraft instruments.

3. Inability to view a large expanse of terrain except at a grazing angle is serious, particularly in unfamiliar territory, because it causes difficulty in navigation. Pilots easily become lost at low altitudes.

4. Buffeting from air turbulence may be present, causing discomfort, fatigue, and possibly motion sickness, while interfering with vision, aiming and tracking.

5. Objects of interest on the terrain may be masked (concealed) by terrain, trees, and buildings. Masked objects may be unmasked at ranges

too close to take effective action against them, or may be unmasked for so short a time that detection probability is low.

6. High angular rates of the terrain and of objects on it, in conjunction with masking of objects, may result in inadequate search time, may prevent either thorough or systematic search, and cause difficulty in aiming at or tracking those objects that are detected.

7. At low altitude, aircraft are vulnerable to hostile aircraft with look-down shoot-down capabilities. Also, attacking aircraft usually have a height advantage.

8. At low altitude, finding or reaching a suitable clear area for landing in case of pilot injury, vehicle damage, or propulsion failure may be difficult or impossible.

9. Over many types of terrain, low altitude ejection from aircraft is very dangerous.

10. High-speed aircraft, particularl, jet-engined aircraft, are not fuel efficient in the dense air at low altitudes. Operating range is restricted and getting back to an airfield may be a problem.

In summary, with low-altitude h. h-speed flight, the ride is rough and fatiguing, air sickness may occur, it's easy to get lost, ground objects move with high angular rates when close, both hostiles and friendlies are difficult to find and keep track of, objects may be masked until too late, aiming and tracking are difficult, impacting ground obstacles is an ever-present danger, either safe landing or ejection in use of trouble is difficult or impossible, and low fuel reserve may preclude return to base as glide distance at low altitude is near zero.

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