INTRA-PULSE FREQUENCY DIVERSITY (IPFD) LAB DEMONSTRATION

Syracuse University

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13. ABSTRACT (Maximum 200 words)

This correspondence suggests a new approach to wideband adaptive beamforming for correlated signal and interference. Unlike other approaches, such as spatial smoothing, the new approach solves the signal cancellation problem by employing the idea of frequency domain smoothing. Advantages of frequency domain smoothing method show that proper spatial filtering can be achieved by frequency domain smoothing, whether the desired signal and the interference are correlated or not.

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I. THE DEMONSTRATION

Objective

Obtain higher resolution in the doppler frequency domain for improved target detectability in distributed clutter.

Approach

Intra-pulse frequency diversity waveform is used to increase the number of independent data samples for pulse-doppler processing without increasing the observation length. These independent data samples are coherently processed via the newly developed C"M-HFDLP method [1] to achieve a stable high-resolution doppler spectrum.

Data Acquisition and Processing

The RADC L-band facility and AP-120B array processor were used. See [2] for details of the waveform design, configuration and programs. Matrix operations were performed on the complex baseband radar data to provide coherent processing of the doppler spectrum by eigenvalue decomposition.

Contents of This Demonstration

(1) The intra-pulse frequency diversity waveform which drives the L-band transmitter: three subpulses of three microsecond each with the carrier frequencies of -6, -1 and +4 MHz (with respect to the reference frequency).

(2) The spectrum of the above waveform.

(3) The range-doppler plot with the conventional FFT based pulse-doppler processing (8 pulses with rectangular window): the weak target in the range-extended strong clutter is not detectable since the doppler frequency separation is smaller than the Fourier resolution limit.

(4) Same as (3) above with Hamming window for lower sidelobes: target is still not detectable.

(5) The range-doppler plot with the new method: the target is clearly detectable.

(6) The "zoomed-in" range-doppler plot with the new method.

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II. OTHER ACCOMPLISHMENTS

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Ph.D. Dissertation Completed:

C.C. Li, Active High-Resolution Multiple-Target Direction Finding, Syracuse University, 1988.

Copies of the above publications are attached.
I. INTRODUCTION

High-resolution direction finding of angularly closely spaced targets has received increasing attention in the past few years. Many methods have been suggested and are still under further investigation, e.g., MEM, MLM, MUSIC, modified FBLP (Min-Norm), ESPRIT, etc. Unfortunately, the detection, resolution, and estimation performances of these methods often suffer severe degradation in the cases of completely correlated target-signal returns caused by severe multipath or "smart" jamming.

The degradation can stem from the fact that the covariance matrix of the target signals is nearly singular or even completely singular. One class of methods to remove such singularity is the so-called spatial smoothing [2-7]. For example, the full sensor array can be divided into partially overlapping subarrays to average the covariance matrices of the correlated targets, or one can use two approaches, i.e., the frequency-domain smoothing [1] and the non-frequency-domain smoothing such as various spatial smoothing [2-7], or multidimensional search of signal-subspace [8, 9]. A performance comparison of the two approaches for the fluctuating target cases under equal transmitted-energy constraint is presented. Both theoretical analysis and simulations are used to study the performance of the detection (determination of number of targets) and angle estimation. It is found that the frequency-domain smoothing can significantly outperform the non-frequency-domain smoothing under the equal transmitted-energy constraint.

For high-resolution active direction finding of completely correlated targets, one can use two approaches, i.e., the frequency-domain smoothing [1] and the non-frequency-domain smoothing such as various spatial smoothing [2-7], or multidimensional search of signal-subspace [8, 9]. A performance comparison of the two approaches for the fluctuating target cases under equal transmitted-energy constraint is presented. Both theoretical analysis and simulations are used to study the performance of the detection (determination of number of targets) and angle estimation. It is found that the frequency-domain smoothing can significantly outperform the non-frequency-domain smoothing under the equal transmitted-energy constraint.

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targets and introduce our notations. In Section III we compare the potential performances of angle estimation of the frequency-domain smoothing and non-frequency-domain smoothing, via evaluating the Cramér-Rao Lower Bounds (CRLBs) associated with the two approaches under the equal transmitted-energy constraint. Such a study is to show the effectiveness identically distributed. Thus, the data set for processing consists of the two smoothing approaches with regard to the same Doppler frequency shift have little range For convenience of discussion we assume that arrival (AOAs) \( \theta_1 \neq \theta_2 \) and the second arriving signal being a delayed version of the first, i.e.,

\[
s_j(t) = \rho s_1(t - t_0)
\]

where \( \rho \) is a real constant and \( t_0 \) the path delay much smaller than that of the range cell. Let \( \sigma_j(1) \) and \( \sigma_j(2) \) be the phases of the target complex amplitudes \( s_j = [s_j(1) s_j(2)]^T \) at \( f_j \). From (4) it is seen that

\[
\sigma_j(2) = \sigma_j(1) - 2\pi f_j t_0, \quad j = 1,2,\ldots, J
\]

i.e., \( \sigma_j(1) \) and \( \sigma_j(2) \) are linked by a constant, in contrast to the two independent target reflection cases where \( \sigma_j(1) \) and \( \sigma_j(2) \) can be modeled as independent random variables.

For convenience of discussion we assume that the sensors are identical and uniformly spaced with separation \( D = \lambda_0/2 \) where \( \lambda_0 \) is the wavelength corresponding to \( f_0 \), the central frequency of the whole frequency band. The array reference point is chosen to be the center of the aperture with all AOAs referenced to the broadside of the linear array. The Rayleigh angular resolution limit of such an array is about \( \Omega = 2/M \) rad.

\[\text{SNR}_j(k) = E\{s_j(k)^2\}/\text{var}\{w_j(m)\}, \quad j = 1,2,\ldots, J, \quad k = 1,2,\ldots, d. \]
For a real, jointly normal distributed data set, the Fisher information matrix for bound calculation can be expressed as [16]

\[
[J(\theta)]_{u,v} = \frac{1}{2} \text{tr} \left[ R^{-1}(\theta) \frac{\partial R(\theta)}{\partial \theta_u} R^{-1}(\theta) \frac{\partial R(\theta)}{\partial \theta_v} \right] + \text{Re} \left( \left[ \frac{\partial m(\theta)}{\partial \theta_u} \right]^H R^{-1}(\theta) \left[ \frac{\partial m(\theta)}{\partial \theta_v} \right] \right)
\]

where \( R(\theta) \) is the covariance matrix as a function of the parameter set vector \( \theta, m(\theta) \) the mean vector as a function of \( \theta \), and \( \theta_u, \theta_v \) denote the \( u \)th and \( v \)th parameter, respectively. For a jointly complex-normal distributed data set, the above equation can be modified as

\[
[J(\theta)]_{u,v} = \text{tr} \left[ R^{-1}(\theta) \frac{\partial R(\theta)}{\partial \theta_u} R^{-1}(\theta) \frac{\partial R(\theta)}{\partial \theta_v} \right] + 2 \text{Re} \left( \left[ \frac{\partial m(\theta)}{\partial \theta_u} \right]^H R^{-1}(\theta) \left[ \frac{\partial m(\theta)}{\partial \theta_v} \right] \right)
\]

with \( \text{Re}(\cdot) \) being the real part of a complex number.

In the case of two completely correlated targets, the unknown parameters of the data set with a conventional signaling are the two spatial frequencies

\[
\omega_k = 2\pi(D/\lambda_0)\sin\theta_k, \quad k = 1, 2 \quad (8)
\]

the signal power \( \sigma_1^2 \) and \( \sigma_2^2 \), the noise power \( \sigma_n^2 \), and the phase difference between the two signals which depends on the delay \( \tau_0 \). The unknown parameters with frequency diversity signaling include the signal powers \( \sigma_{1,1}^2(j), \sigma_{2,1}^2(j) \), noise power \( \sigma_n^2(j) \), the delay \( \tau_0 \), and two target AOA related parameters.

We still use \( \omega_1 \) and \( \omega_2 \) for these two parameters since the spatial frequencies at \( f_j, j = 1, 2, \ldots, J \) are deterministic functions of these two even if \( f_0 \) is not necessarily equal to one of \( f_j, j = 1, 2, \ldots, J \).

Let \( \sigma_{2,1}^2 \) be the CRLB to the variance of an \( \omega_1 \) or \( \omega_2 \) estimate. In the remainder of this paper and Figs. 2-5, the abbreviation CRLB is used to denote \( 10\log_{10}(1/\sigma_{2,1}^2) \) as a rather common practice.

We consider the situations where \( \rho = 1 \) and \( \tau_0 \) is corresponding to a small fraction of the range cell, i.e., \( \tau_0 \ll 1/B'_{\text{max}} \), where \( B'_{\text{max}} \) denotes the largest bandwidth of the subpulses. In practice, these situations are found to be the most important and difficult to handle [10, ch. 3 and 4]. Fig. 2 shows the CRLBs as a function of \( t_0f_0 \) for \( J = 7 \) subbands with the relative bandwidth \( B/f_0 = 10 \) percent and 30 percent, respectively. The CRLBs oscillate with a damping factor proportional to \( B/f_0 \) but settle at a level independent of \( B/f_0 \). In contrast, the dotted lines in Fig. 2 indicate the maximum and minimum of the CRLB for the non-frequency-domain smoothing, which would oscillate in between with the period equal to 0.5. From this figure, we can see that the CRLB for
1) The potential AOA estimation performance of the frequency-domain smoothing approach is expected to always be better than that of the non-frequency-domain smoothing approach.

2) The larger the bandwidth, the better the potential AOA estimation performance can be of the frequency-domain smoothing approach.

3) Only a small number of subbands $J$ are necessary for the potential AOA estimation performance of the frequency-domain smoothing approach to be independent of the unknown delay $t_0$.

In the next section we study the estimation performances of a typical frequency-domain smoothing method and a typical non-frequency-domain smoothing method.

IV. ESTIMATION PERFORMANCE OF TWO SMOOTHING METHODS

For simplicity we choose the MUSIC-based coherent signal-subspace processing method described in [11, 17] and the MUSIC-based spatial smoothing method of [4]. Since there is little analytical result available about the estimation performance of the spatial smoothing method, we conduct the comparison via statistical simulation. The following parameters are set up for the simulation.

1) the number of targets $d = 2$ with $\rho = 1$, i.e., $\text{SNR}(1) = \text{SNR}(2)$;

2) the relative bandwidth $B/f_0 = 30$ percent with the number of subbands $J = 7$;

3) the number of sensors $M = 8$;

4) the spatial frequencies $\omega_1 = 0.8227$ and $\omega_2 = 0.430$ which correspond to an angle separation equal to $0.5\pi$;

5) the subarray size $L = 6$ for the spatial smoothing method in [4];

6) the required preliminary estimate of approximate center of the spatial frequencies with the frequency-domain smoothing method of [11] $J = (\omega_1 + \omega_2)/2$;

7) the equal transmitted-energy constraint as of (6);

8) the number of pulses $N = 32$.

Fig. 5 compares the mean square errors (MSEs) of the two smoothing methods for $d_1$ at various $t_0$ with $t_0 = 6, 6.25$, and $6.5$, respectively. Again, $10\log_{10}(1/\text{MSE})$ is actually plotted. Fifty independent runs are used at each SNR. The MSEs of frequency-domain smoothing with the three different values of $t_0$ are found to be approximately the same and thus only the one with $t_0 = 6$ is plotted in Fig. 5. Also included in Fig. 5 are the corresponding CRLBs. The frequency-domain smoothing method is seen to significantly outperform the spatial smoothing method.
Fig. 5. MSEs of $\hat{\psi}_1$ with frequency-domain smoothing ($B/f_0 = 30\%$, $f = 7$) and spatial smoothing versus SNR: completely correlated targets, $M = 8$, $d = 2$, $\omega_1 = 0.8227$, $\omega_2 = 0.430$, $L = 6$, $N = 32$.

and reaches the CRLBs quite closely. We note that with $t_0f_0 = 6.25$ the spatial smoothing method fails to reach the corresponding CRLB even at very high SNRs. A possible cause of this may stem from the fact that with this spatial smoothing method, not all of the off-diagonal elements of the estimated correlation matrix can be made use of.

Since the frequency-domain smoothing method needs a preliminary estimate $\hat{\omega}_i$, it is interesting to know its effect on the MSE. It has been shown in [17, 18] that the performance of the frequency-domain smoothing approach is not sensitive to this preliminary estimate.

V. DETECTION PERFORMANCE COMPARISON

The above two MUSIC based smoothing methods both need to first determine the number of targets $d$ before performing AOA or spatial frequency estimation. One may use AIC or MDL types of methods [19, 20] to conveniently accomplish this. For completely correlated targets, [21] gives a spatial smoothing based modification of the AIC/MDL method, while a frequency-domain smoothing based modification is presented and analyzed in [17]. In this section we compare the probability of correct determination of the number of targets with the two modified methods for two completely correlated, closely spaced fluctuating targets under the constraint of equal transmitted-energy. An analytical detection performance study can be found in [17] for the frequency-domain smoothing based method, but a corresponding study for the spatial smoothing based method is not available to our knowledge. Therefore, we conduct the detection performance comparison via simulation. The parameters used in this simulation are the same as those in the last section. Since the probability of underestimating the number of targets is the dominant detection error for the cases of completely correlated and/or closely spaced signals, and since our previous experience with detection performance analysis has resulted in favor of choosing the AIC rather than MDL penalty function [17, 22, 23], we use the AIC penalty function here.

Fig. 6 shows the probability of correctly determining the number of targets with the spatial smoothing based method [21] and frequency-domain smoothing based method [17] under equal transmitted-energy constraint, using 50 independent runs. In this figure, the probabilities for the spatial smoothing based method with $t_0f_0 = 6, 6.25, 6.5$, respectively, are plotted as a function of SNR. For the frequency-domain smoothing based method, the corresponding three curves are almost the same. So only the one with $t_0f_0 = 6$ is included. The detection performance of the spatial smoothing based method is very sensitive to the values of $t_0f_0$ (or the phase difference $\phi = 2\pi t_0f_0$). Though the detection performance of the frequency domain based method is measured about 2 dB poorer in terms of SNR than the spatial smoothing based method with $t_0f_0 = 6.5$, it performs significantly better than the other two cases of the spatial smoothing based method. Therefore, one may conclude that the frequency-domain smoothing based detection method is more able to provide a well-performed solution for determination of the number of completely correlated, closely spaced fluctuating targets.

VI. CONCLUSION AND DISCUSSION

For high-resolution active direction finding of completely correlated fluctuating targets the results are in favor of using the frequency-domain smoothing approach, which consists of the frequency diversity signaling and a coherent wideband processing...
algorithm such as the one described in [11, 17], rather than the non-frequency-domain smoothing approach which uses the conventional (no diversity) signaling. A narrowband processing algorithm such as the spatial smoothing method of [4, 21]. The study of the estimation performance bounds also indicate that the frequency-domain smoothing can be expected to achieve a much better performance than any non-frequency-domain smoothing, especially for an array with a small number of sensors. We note that for nonfluctuating targets, [24] contains a study which leads to the same conclusion in favor of using the frequency-domain smoothing. We also note that the frequency-domain smoothing can be used with arrays of non-uniformly-spaced, non-identical sensors. It should be pointed out that passive direction finding of completely correlated wideband sources has been successfully solved by using the idea of the frequency-domain smoothing [11, 17].

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Some remarks on the above analysis are in order now.

Remark 1: Since the estimate \( P(w_1, w_2) \), given by (4a), (8), and (9), satisfies condition 1) for \( |I| \leq 2 \) and condition 2) for \( |I| > 2 \), respectively, we can conclude that the estimate \( P(w_1, w_2) \) given by Kimura and Honoki's hybrid approach coincides with the true ME estimate when \( \hat{P}(w_1) \) is positive definite, and the elements of the first row of its inverse \( \hat{P}^{-1}(w_1) \) satisfy (26).

Remark 2: If the original estimate \( \hat{P}(w_1) \) is of Toeplitz, \( \hat{P}(w_1) \) is directly taken as \( \hat{P}(w_1) \); in the other case, \( \hat{P}(w_1) \) is obtained by averaging \( \hat{P}(w_1) \); see [1], eq. (20). In a way similar to our above analysis, it can be straightforwardly concluded that for a multichannel process \( X(\tau) \), the power spectrum estimate \( \hat{P}(w_1, w_2) \) is the true ME estimate when every element \( \hat{P}(w_1, w_2) \) of the inverse \( \hat{P}^{-1}(w_1, w_2) \) has the form \( \sum_{-\infty}^{\infty} e^{i w_1 k} \exp \{ jw_2 \}, \) for \( |I| > L \). This can be regarded as another form of (26) for the multichannel 1-D process. It is worthwhile to point out even if the estimate \( \hat{P}(w_1, w_2) \) is the true ME estimate, the final estimate \( P(w_1, w_2) \) obtained by using \( \hat{P}(w_1, w_2) \) is not necessarily the ME estimate, as will be stated in the next remark.

Remark 3: Since the positivity of \( \hat{P}(w_1) \) is available for the case of cyclic and skew-cyclic Toeplitz \( R(\tau) \), Kimura and Honoki have conjectured that their final estimate \( P(w_1, w_2) \) coincides with the true ME estimate for such a case. The above analysis conclusion shows that Kimura and Honoki's conjecture is not true in general since the cyclic and skew-cyclic Toeplitz \( R(\tau) \) cannot, in general, guarantee that all the elements of the first row of the inverse \( \hat{P}^{-1}(w_1) \) satisfy (26). To show this point, we present a counterexample at Kimura and Honoki's conjecture.

It is always possible to find a multichannel process \( X(\tau) \) such that it has the following power spectrum:

\[
\hat{P}(w_1) = \begin{bmatrix} 5 + 2 \cos w_1 & 25 + 20 \cos w_1 + 4 \cos (2w_1) \\ 25 + 20 \cos w_1 + 4 \cos (2w_1) & -2 \sin w_1 \\ -2 \sin w_1 & 5 + 2 \cos w_1 \end{bmatrix}
\]

By using definition (11), it is easy to show \( \hat{P}(w_1) \) is of cyclic Toeplitz. Since there are \( f_{11}(w_1) = f_{33}(w_1) \) and \( f_{22}(w_1) = f_{33}(w_1) \) between the elements of \( \hat{P}(w_1) \), we see that

\[
R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{P}(w_1) \exp \{ jw_1 \} \, dw_1
\]

is also of cyclic Toeplitz. Hence, \( \hat{P}(w_1) \) is directly taken as \( \hat{P}(w_1) \), and

\[
\hat{P}(w_1) = \hat{P}(w_1) = \begin{bmatrix} 5 + 2 \cos w_1 & 25 + 20 \cos w_1 + 4 \cos (2w_1) \\ 25 + 20 \cos w_1 + 4 \cos (2w_1) & -2 \sin w_1 \end{bmatrix}
\]

Note that the given \( \hat{P}(w_1) \) is positive definite and is simply shown to be the ME estimate of Remark 2, and the corresponding \( R(\tau) \) is of cyclic Toeplitz, but the final spectrum estimate \( P(w_1, w_2) \) obtained by Kimura and Honoki's hybrid approach from such \( \hat{P}(w_1) \) does not coincide with the true ME estimate since for the case of \( k = 0 \) and \( |I| > L \),

\[
\sum_{n=-\infty}^{\infty} b_{\nu} b_{\nu}^{*} = \sum_{n=-\infty}^{\infty} h_{n}^{*} h_{n}
\]

is the sum of infinite terms, this implies that

\[
\sum_{n=-\infty}^{\infty} b_{\nu} b_{\nu}^{*} = \sum_{n=-\infty}^{\infty} h_{n}^{*} h_{n}
\]

for \( k \leq K, |I| > L \).

IV. CONCLUSIONS

Some important theoretical problems remain to be solved in Kimura and Honoki's hybrid approach to high-resolution 2-D spectrum analysis. The most important problem is probably to know when the final estimate is close to the true ME estimate. In this correspondence, we have analyzed such a problem, and our work has provided a "theoretical" and "practical" solution to this problem. Using our result, one can easily know if the final estimate given by Kimura and Honoki's hybrid approach does coincide with the true ME estimate.

We have also presented a counterexample, and have shown that Kimura and Honoki's conjecture on the above problem is not true.

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Adaptive Beamforming for Correlated Signal and Interference: A Frequency Domain Smoothing Approach

J. X. ZHU and H. WANG

Abstract—This correspondence suggests a new approach to wideband adaptive beamforming for correlated signal and interference. Unlike other approaches, such as spatial smoothing, the new approach solves the signal cancellation problem by employing the idea of frequency domain smoothing. Advantages of frequency domain smoothing over spatial smoothing are identified in this correspondence. Preliminary performance studies of a simple frequency domain smoothing method show that proper spatial filtering can be achieved by its frequency domain smoothing, whether the desired signal and the interference are correlated or not.

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1. Introduction

Signal cancellation is a problem of adaptive beamforming (ABF) when the desired signal from the look direction and the interference are highly correlated. One cure is the so-called spatial smoothing method [11], by which the full sensor array is divided into partially overlapping subarrays to enable an average of the covariance matrices of the subarray output vectors. Such an average was shown to be able to destroy the correlation between the signal and interference. Unfortunately, spatial smoothing can only be applied to arrays of uniformly spaced, identical sensors. Besides, a tradeoff has to be made for a given array between the size of the subarrays and the number of subarrays, since the interference rejection performance with spatial smoothing depends on both parameters. Moreover, if the number of sensors is not large enough, the number of subarrays available for spatial smoothing might be too small to lead to any significant improvement.

A rather different approach for wide-band ABF is given in [2], which does not suffer from the signal cancellation since it employs the "complete nulling" criterion at every subband. Such a criterion, however, may sacrifice too much signal power, resulting in an array output signal-to-interference-plus-noise ratio (SINR) lower than that potentially achievable, especially when the interference sources are not very strong and their angles of arrival are close to that of the desired signal [31].

In this correspondence, we present a new approach to wide-band ABF for correlated signal and interference. The key idea is to perform a smoothing operation in the frequency domain, which was originally presented in the context of high-resolution direction finding of multiple wide-band sources [4].

Fig. 1 shows a general configuration of a frequency domain implementation of wide-band bandpass ABF. For simplicity of presentation, we consider a linear array. The sensors do not need to be uniformly spaced, nor do their patterns need to be identical. Following each sensor is a bank of narrow-band filters covering the whole frequency band of interest. The bandwidth of each sub-band is assumed to be a few percent of its central frequency \( f_j \), \( j = 1, 2, \ldots, J \). In that for each sub-band, the narrow-band array output representation is valid. The ABF weight vectors \( w_j \), \( j = 1, 2, \ldots, J \) are updated according to the chosen optimization criterion and interference environment. Several criteria all result in the form of

\[
 w_j = \alpha_j R_j^{-1} g_j(\theta_j) \quad j = 1, 2, \ldots, J
\]

where \( \alpha_j \) is a criterion dependent constant, \( R_j \) is the correlation matrix of the complex output vector \( x_j(t) \) of the \( j \)-th subband array, and \( g_j(\theta_j) \) is the \( j \)-th subband array direction vector at the look angle \( \theta_j \). When the interference is completely or partially correlated to the desired signal from \( \theta_j \), the signal cancellation occurs, and the claimed optimum performance is completely lost due to the violation of the basic assumption.

It should be pointed out that if the existence of the correlation between the signal and interference is known in advance, then one should take advantage of the signal-correlated interference to achieve an even higher SINR, instead of simply trying to suppress all interference [5]. However, such a priori knowledge is not always available, nor always easy to obtain from the received data. Therefore, the need still remains for finding \( w_j \), \( j = 1, 2, \ldots, J \) such that the interference suppression performance is much better than that using (1) when the signal and interference are correlated, and close to that using (1) when the signal and interference are not correlated.

In the following discussion we assume:

1) the angle of arrival of the desired signal \( \theta \) is available and used as the array look direction \( \theta_0 \);
2) the interference sources, correlated or uncorrelated with the desired signal, occupy the same frequency band as the desired signal, and
3) the receiver noise is uncorrelated sensor to sensor.

II. Frequency Domain Smoothing

It is easy to see that the correlation matrix of the \( j \)-th subband array output has the form

\[
 R_j = A_j A_j^H + \sigma_j^2 I
\]

where \( A_j \) is the direction matrix associated with the desired signal and \( d = 1 \) interference sources, \( R_j = d \times d \) is the correlation matrix of the desired signal and interference, and \( \sigma_j^2 I \) is the correlation matrix of the receiver noise of the \( j \)-th subband. When there is a complete correlation between the desired signal and an interference source, \( R_j = 1 \), \( 2, \ldots, J \) becomes singular and the so-called signal cancellation occurs.

As pointed out in [4], the frequency-domain-smoothed correlation matrix \( \hat{R}_j = \hat{U}_j^H R_j \hat{U}_j \) is nonsingular in general. This fact leads to the opportunity of employing a frequency-domain-smoothed "correlation matrix" to replace \( R_j \) of (1), so that the beamformer can deliver a reasonably good spatial filtering performance insensitive to whether the desired signal and interference are (completely) correlated or not.

Parallel to the direction finding problem using the idea of frequency domain smoothing [4], a class of adaptive wide-band beamformers with frequency domain smoothing can be developed for different applications. In this correspondence, however, we are only interested in presenting one example which is suitable for a number of practical applications in radar, sonar, and spread-spectrum communication.

To implement the smoothing operation on \( R_j \), \( j = 1, 2, \ldots, J \), a frequency domain transformation represented by \( T \) must be performed on the estimate of the correlation matrix \( R_j \), such that

\[
 T_j A_j = A_{\text{II}}, \quad j = 1, 2, \ldots, J
\]

where \( A_{\text{II}} \) is the direction matrix of the array at the central frequency \( f_0 \) (\( f_0 \) may be equal to one of \( f_j \), \( j = 1, 2, \ldots, J \)). Obviously, there are many possible choices of \( T \), which could achieve (3), but all of them would need the angles of arrival of the interference which are not available to the beamforming subsystem. Therefore, approximations of \( T \) of (3) have to be used in practice.

One possible way is to do a preselected angle approximation to the perfect transformation matrix \( T \). Let \( \theta_j \) be the look direction of the array, and \( \theta_{\text{II}}, \theta_j = \theta_j, \quad m = 1, 2, \ldots, M - 1 \) be \( M - 1 \) different angles uniformly covering the whole angular domain of concern. Conceptually summarized below are the steps used to implement the frequency-domain-smoothed adaptive beamformer with a preselected approximation of the transformation:

1) Form \( \hat{R}_j \), the estimate of the correlation matrix \( R_{\text{II}} \), from the \( K \) sampled complex output vectors \( x_j(t), j = 1, 2, \ldots, J \);
2) perform the frequency domain transformation to obtain

\[
 \hat{R}_j = \sum T_j \hat{R}_j T_j^H
\]

where

\[
 T_j = A_j A_j^H
\]
with
\[
A_j = [a(\theta_1), a(\theta_2), \ldots, a(\theta_M)]^T.
\]

3) form the frequency-domain-smoothed "correlation matrix" estimate for each subband
\[
\tilde{R}_j = \left( \tilde{T}_j \right) \tilde{R}(\tilde{T}_j)^T.
\]

4) obtain the beamforming weight vector for the jth subband
\[
w_j = \tilde{R}_j^{-1} a(\theta_j), \quad j = 1, 2, \ldots, J.
\]

We note that significant reduction in computing for \(T_j\) and \(\tilde{T}_j\) can be expected by exploiting the structure of \(A_j\). In the following section, we will present our preliminary performance study of the above method.

III. PRELIMINARY PERFORMANCE STUDY

In this section we will show the beam patterns of both frequency domain smoothing and spatial smoothing. For simplicity, we consider an array of \(M = 10\) uniformly spaced, omnidirectional sensors with the space between sensors equal to one-half of the wavelength corresponding to \(f_0\), and we assume that the desired signal, interference, and receiver noise all have flat spectra in the same frequency band with a relative bandwidth \(B/f_0 = 0.30\). The whole band is covered by \(J = 10\) subbands. The angle of arrival of the desired signal is taken to be \(\theta_d = 35^\circ\). Two interference sources are present. The first one is completely correlated with the desired signal, being a delayed version with delay \(t_i = 15/f_0\). The second interference source is a strong jammer which is uncorrelated with the desired signal. The angles of arrival of the two interference sources are \(20^\circ\) and \(30^\circ\), respectively.

In Fig. 2, the solid line is the beam pattern of the frequency domain smoothing method. The two dashed lines are the beam patterns of the spatial smoothing method with the subarray size equal to 4 and 6, respectively, and the beam pattern without any smoothing is plotted as the dotted line. The protection of frequency domain smoothing against both interference sources is seen to be much better than that of spatial smoothing.

Since the beam pattern of frequency domain smoothing is the sum of the J subband beam patterns, the sharp nulls at the angle of the strong interference indicates that the J subband beam patterns must have nulls almost at the same angle even though the transformation used is just an approximation.

Comparing the two beam patterns of spatial smoothing, one can see that the protection against the correlated interference can be gradually improved by reducing the size of the subarrays. Such a reduction of the subarray size for more spatial smoothing not only is limited by the number of interference sources, but also degrades the nulling performance at the angle of the strong interference due to the loss of angular resolution with the reduced subarray size [6]. In contrast, frequency domain smoothing is free from this problem as shown in Fig. 2.

If the interference exists only over part of the frequency band, (7) and (8) indicate that this frequency domain smoothing method will produce nulls in all J subband beam patterns toward the partial-band interference. To avoid such unnecessary nulls in the interference-free subbands, one may simply apply Steps 1)-(4) to those subbands containing interference, and use the conventional beamforming method for the interference-free subbands. If the interference sources have a colored spectrum, a more sophisticated method of frequency domain smoothing needs to be developed.

IV. CONCLUSION

The frequency domain smoothing approach suggested in this correspondence can outperform the spatial smoothing approach.

Fig. 2. Beam patterns of the frequency domain smoothing method and spatial smoothing method.

especially for small sensor arrays. The features of this new approach include the following:

1) it can be applied to arrays with arbitrary geometry and sensor patterns;
2) it does not sacrifice the angular resolution for smoothing;
3) it does not need a large number of sensors to collect the smoothing operation.

We note that [2], [7], and [8] contain some interesting results on the construction of the approximations to the linear transform of (3).

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Effects of Sensor Position and Pattern Perturbations on CRLB
For Direction Finding of Multiple Narrow-Band Sources

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ABSTRACT

This paper studies the effects of sensor pattern and sensor position perturbations on the angle estimation performance bound for multiple narrow-band sources. The Cramer-Rao Lower Bound is used with a probabilistic modeling of the perturbations. The CRLBs of a linear uniform array under sensor position and pattern perturbations are evaluated in detail for the case of two narrow-band sources.

I. INTRODUCTION

In many applications where a sensor array is used for angle estimation of multiple sources/targets, one has to consider the effects of the sensor pattern and sensor position perturbations. Though some efforts have been made to develop robust high-resolution estimates [1]-[3], there is a need to understand the behavior of the estimation performance bound under the perturbations, so that one can see how much room left for developing more robust estimate.

In this paper, the behavior of estimation performance bound under sensor position and pattern perturbations is investigated. The Cramer-Rao Lower Bound (CRLB) is chosen since it is simple to evaluate, and since all available high-resolution direction finding methods require either large number of snapshots or high SNR, for which the CRLB has been seen to provide a reasonably tight bound on the mean-square-error (MSE) of the angle estimate. The perturbations on sensor positions and patterns are modeled as Gaussian random variables. We note that previous work on evaluation of the CRLB under perturbations, such as [4]-[6], involves the cases of single source and multiple sources disjoint in the frequency domain.

II. MODEL FORMULATION

Consider an array with arbitrary but known nominal geometry as shown in Fig.1. Let the number of sensors be M, each at position \( \tilde{r}_m = (x_m, y_m) \). \( \tilde{r}_m = (x_m, y_m) \), with respect to the chosen reference point. Denote the sensor pattern of the m-th sensor as \( g_m(\theta) \). Let \( \mathcal{J}_1 \), \( \mathcal{J}_2 \), \ldots, \( \mathcal{J}_d \) be the wave-number vectors of d narrow-band plane waves of the same central frequency \( f_0 \). The array output complex vector \( \tilde{X} \), Mx1, can be expressed as

\[ \tilde{X} = \mathbf{A} S + N \]  \hspace{1cm} (1)

where \( \mathbf{A} = [g_1, g_2, \ldots, g_M] \) is the direction matrix with the direction vector

\[ g_i(\theta) = \{g_i(\theta_1) \exp(j2\pi f_0 (-\theta_1)), \ldots, g_i(\theta_d) \exp(j2\pi f_0 (-\theta_1)) \}^T, \hspace{1cm} i=1,2,\ldots,d \]  \hspace{1cm} (2)

\( S = [S_1, S_2, \ldots, S_d]^T \) the arriving signal vector modeled as zero-mean Gaussian random vector with a diagonal covariance matrix, and \( N = [N_1, N_2, \ldots, N_d]^T \) the receiver noise modeled as zero-mean Gaussian random vector with its covariance matrix equal to \( \sigma_N^2 \). \( S \) and \( N \) are assumed to be independent.

From Fig.1, we have

\[ \tilde{g}_m(-\theta_i) = \left( x_m \cos \theta_i + y_m \sin \theta_i \right) / \lambda, \hspace{1cm} m=1,2,\ldots,M, i=1,2,\ldots,d \]  \hspace{1cm} (3)

In the presence of sensor position perturbation, the m-th sensor position vector becomes \( \tilde{r}_m = (x_m + \Delta x_m, y_m + \Delta y_m) \), where \( \Delta x_m \) and \( \Delta y_m \) are the sensor position perturbations in x direction and y direction respectively. The sensor position perturbations \( \Delta x_m, \Delta y_m = n=1,2,\ldots,M \) are modeled as i.i.d. Gaussian random variables of zero mean. We assume that the standard deviation of the position perturbations is much smaller than the spaces among the sensors.

The perturbed complex gains, i.e., the perturbed sensor patterns, are modeled as \( g_m(\theta) \pm \Delta g_m(\theta) \) where \( g_m(\theta) = m=1,2,\ldots,M \) are the nominal patterns and \( \Delta g_m(\theta), m=1,2,\ldots,M \) are i.i.d. zero mean complex Gaussian random perturbations. Again, the standard deviation is assumed to be much smaller than the nominal complex gain \( g_m(\theta) \).

In order to see the amplitude and phase perturbation effects separately, a model for the case of pattern amplitude perturbation only is also set up, in which the nominal pattern \( g_m(\theta) \).

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When multiple snapshots are used for estimation, the independence of snapshots is assumed and the perturbations are assumed to remain unchanged among all snapshots.

III. CRAMER-RAO LOVER BOUND UNDER PERTURBATIONS

If we only consider position perturbations, i.e., assume no pattern perturbations, then the unknown real parameter set \( \Theta \) contains

\[
\Theta = (\theta_1, \theta_2 \ldots \theta_n, \theta_1', \theta_2' \ldots \theta_n') ,
\]

i.e., a total of \( 2d \times 2n \) real parameters. For the case of pattern perturbations only, the unknown real parameter set \( \Theta \) is

\[
\Theta = (\theta_1, \theta_2 \ldots \theta_n, \rho_1', \rho_2' \ldots \rho_n')
\]

i.e., a total of \( 2d \times n \) real parameters. For the case of pattern amplitude perturbations only, \( \Theta_m(\Theta) \), \( m=1,2,3 \ldots M \) are set up as i.i.d. real zero mean Gaussian random variables in section II. Therefore the unknown real parameter set in this case is

\[
\Theta = (\theta_1, \theta_2 \ldots \theta_n, \rho_1', \rho_2' \ldots \rho_n')
\]

a total of \( 2d \times 2n \) real parameters.

With the presence of random parameters, the Fisher information matrix is given by [7]

\[
\mathbf{J}(\Theta) = \mathbf{J}_1(\Theta) + \mathbf{J}_2(\Theta)
\]

where the elements of \( \mathbf{J}_1 \) and \( \mathbf{J}_2 \) are

\[
\mathbf{J}_1(\Theta)_{i,j} = -E_x(e^{2i \log(\Pr(X_i|\Theta_i))})
\]

\[
\mathbf{J}_2(\Theta)_{i,j} = -E_x(e^{2i \log(\Pr(X_i|\Theta)))}
\]

Under the assumption that the position and pattern perturbations are small, \( \mathbf{J}_1(\Theta) \) can be approximated by the corresponding Fisher information matrix for non-random \( \Theta \) at the nominal values [4], i.e.,

\[
\mathbf{J}_1(\Theta)_{i,j} = -E_x(e^{2i \log(\Pr(X_i|\Theta)))}) |_{\Theta_i=\Theta_i} \]

where \( \Theta_i \) is the parameter set at the nominal value. For Gaussian signals and noises Eq. (10) becomes [4]

\[
|\mathbf{J}_1(\Theta)|_{i,j} = tr[\mathbf{R}_{x}^{-1}(\Theta) \mathbf{R}_{x}^{-1}(\Theta) \mathbf{R}_{x}^{-1}(\Theta) \mathbf{R}_{x}^{-1}(\Theta)]\end{equation}

where \( \mathbf{R} \) is the covariance matrix of the data vector \( \mathbf{X} \).

Because the first \( 2d+1 \) parameters are non-random, \( \mathbf{J}_2(\Theta) \) has its first \( 2d+1 \) rows and columns equal to zero. Let \( \mathbf{A}_{\text{pos},\text{rea},\text{ima},\text{amp}} \) be the covariance matrix of the position perturbations, \( \mathbf{A}_{\text{pos},\text{rea},\text{ima},\text{amp}} \) the covariance matrix of the pattern perturbations (real and imaginary part), and \( \mathbf{A}_{\text{pos},\text{rea},\text{ima},\text{amp}} \) the covariance matrix of the pattern amplitude perturbation. For sensor position perturbation only, we have

\[
\mathbf{J}_2(\Theta) = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right],
\]

and for pattern perturbation only,

\[
\mathbf{J}_2(\Theta) = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right],
\]

and for pattern amplitude perturbation only,

\[
\mathbf{J}_2(\Theta) = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right],
\]

In the case of multiple snapshots, the probability density function \( \Pr(X_i|\Theta) \) in Eq. (8) would be joint density function \( \Pr(X_i,X_j,X_k|\Theta) \), where \( K \) is the number of snapshots. Under the assumption that all snapshots are independently identically distributed, \( \Pr(X_i,X_j,X_k|\Theta) = [\Pr(X_i|\Theta)^k] \) and Eq. (8) becomes

\[
|\mathbf{J}_1(\Theta)|_{i,j} = -K \cdot E_x(e^{2i \log(\Pr(X_i|\Theta)))}).
\]

The \( \mathbf{J}_1(\Theta) \) will remain the same as in Eq. (9) because of the assumption that the perturbations remain unchanged among all snapshots.

In the following section, we will numerically evaluate the CRLB for a linear uniform array to see how the sensor position and pattern perturbations affect the estimation bound.

IV. CASE STUDY

We consider an array of 8 sensors with half wavelength spacing, which presents a Rayleigh angular resolution of 16.4° (degree). For simplicity, the nominal sensor patterns are assumed to be omnidirectional, i.e., \( \varphi(\Theta) = 0 \) for all sensors. Let \( \Theta_1 \) and \( \Theta_2 \) be the angles of arrival of two narrow band sources of the same central frequency. Again, for simplicity, we fix \( \Theta_3 \) at
0°, and then \( \theta_0 = \theta_1 - \theta_2 = \theta_3 \). In the following figures, the CRLB for \( \theta_0 \) estimation under pattern amplitude, pattern amplitude and phase, and position perturbations, denoted as CRLBa, CRLBg and CRLBp respectively, are compared with the CRLB for \( \theta_0 \) estimation without perturbations (CRLBp). For pattern amplitude perturbations the standard deviation is denoted as \( \sigma_0 \). For pattern amplitude and phase perturbations, which are modeled as i.i.d. zero mean complex Gaussian random variables, we denote the standard deviation of its real part or imaginary part as \( \sigma_p \), i.e., the standard deviation of the complex pattern perturbation is \( \sigma_p \). For sensor position perturbations, we let the standard deviation of the perturbations in x direction and y direction be the same and denoted as \( \sigma_0 \).

Figure 2 shows the effect of the pattern amplitude perturbations on the bound (CRLBp) as a function of \( \theta_0 \), the angular separation between the two narrow-band sources. For \( \theta_0 \) larger than the resolution cell (=16.4°), the CRLBa is almost the same as CRLBp, i.e., the amplitude perturbation of the sensor gains has almost no effect on the estimation performance potential. In the high-resolution region (69°<16.4°), however, the larger the amplitude perturbation, the poorer the estimation performance potential, which implies some extra difficulty to overcome in order to achieve high resolution.

To overcome such an extra difficulty for high resolution, Fig. 3 shows that one possible way is to increase the signal-to-noise ratio (SNR). In this sense we may loosely consider that the pattern amplitude perturbation has a similar effect on the performance potential as the receiver noise.

Figure 4, Figure 5 and Figure 6 show the CRLB under both the amplitude and phase perturbation on the sensor pattern (CRLBg). When perturbations on the phases of the complex sensor gain are added, the CRLBg behaves quite differently from that without phase perturbation. Figure 4 shows the bound as a function of the angle separation \( \theta_0 \) with different \( \sigma_p \), while Figure 5 as of SNR with different \( \theta_0 \) and Figure 6 as of SNR with different \( \sigma_p \). Both Figure 5 and Figure 6 indicate that with the phase perturbations on the sensor gain, the CRLBg will level off as SNR increases. That is, the CRLBg cannot be reduced to arbitrarily small by increasing SNR. Under sensor pattern perturbation, therefore, it is the phase perturbation that significantly affects the estimation performance.

Figures 7-9 show the CRLB under sensor position perturbations (CRLBp). We can see that the effects of position perturbation are almost the same as that of pattern perturbation. Again, when noise is no longer a dominant factor, it is the position perturbation that will limit the estimation performance.

Fig. 7 plots CRLBp as a function of the angular separation \( \theta_0 \). In the low-resolution region (69°<16.4°), the degradation from the CRLBp is almost independent of \( \theta_0 \) and determined only by the position perturbation standard deviation \( \sigma_0 \). In the high-resolution region (69°<16.4°), however, the degradation from CRLBp is \( \theta_0 \)-dependent and becomes smaller when \( \theta_0 \) is smaller. Noting the sharp increase of CRLBp in the high-resolution region, we should realize that the smaller degradation with smaller \( \theta_0 \) merely means the more dominant factor which the receiver noise shows with smaller \( \theta_0 \).

Fig. 10 shows the effect of the number of snapshots on the CRLB under position perturbations (CRLBp), from which the increase of the number of snapshots increases the CRLBp levels off.

V. CONCLUSIONS

Both sensor pattern and position perturbation can seriously degrade the potential estimation performance. With proper joint estimation scheme, the effect of the pattern amplitude perturbation might be compensated by increasing the signal to noise ratio. When the signal to noise ratio is high the position perturbation or pattern phase perturbation becomes the dominant factor and the CRLBp or CRLBg will level off. In such situations, one might not be able to reduce the CRLBp or CRLBg to an arbitrarily small number by further increase of the signal to noise ratio, even with some joint estimation scheme. Under the pattern or position perturbation, the increase of the number of snapshots is expected to offer only limited help as the SNR.

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