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13. ABSTRACT *(Maximum 200 words)*

Renormalization group methods (RNG) have been applied to large eddy simulations of wall regions of channel flows and spectral element RNG simulations of flows in complex geometries were explored. The results predicted wall region streaks accurately at much less spatial resolution than earlier methods. The methods were extended to compressible flows. They have been used to show that the shock region is characterized by large negative values of the divergence indicating tube-like structures. High enstrophy regions reside outside the shock regions. High vorticity regions in incompressible flow tend to be concentrated in tubes, while in compressible flows they tend to be concentrated in sheets. RNG was also applied to k-e modelling of the flow over a backstep. Full simulations were also completed for large Reynolds number turbulence.

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In work under this grant there has been significant progress on a number of key fluid dynamics problems. The results have been presented in numerous publications, including:

Renormalization Group Analysis of Turbulence

1. Long-Time, Large-Scale Properties of the Random-Force-Driven Burgers Equation (V. Yakhot and Z.-S. She), Phys. Rev. Lett. **60** (18) (1988).

2. Computational Test of the Renormalization Group Theory of Turbulence (V. Yakhot, S. A. Orszag, and R. Panda), J. Sci. Comp., **3**, 139 (1989).

3. Deviations from the Classical Kolmogorov Theory of the Inertial Range of Homogeneous Turbulence (V. Yakhot, Z.-S. She, and S. A. Orszag), Phys. Fluids A **1**, (2) 289-293 (1989).

4. Space-Time Correlations in Turbulence: Kinematical vs Dynamical Effects (V. Yakhot, S. A. Orszag, and Z.-S. She), Phys. Fluids A **1**, 184-186 (1989).

5. Analysis of the ε-Expansion in Turbulence Theory: Approximate Renormalization Group for Diffusion of a Passive Scalar in a Random Velocity Field (V. Yakhot and S. A. Orszag), to be published (1989).

6. Application of Renormalization Group Methods to Turbulence (V. Yakhot, S. A. Orszag, and W. Dannevik), in Current Trends in Turbulence Research, (ed. by H. Branover, M. Mond, and Y. Unger), AIAA Proceedings **112**, 224-251 (1988).

7. Scale Invariant Solutions of the Theory of Thin Turbulent Flame Propagation (V. Yakhot), Combustion Sci. and Technology (1988).

8. Correspondence Principle for Turbulence: Application to the Chicago Experiment on High Rayleigh Number Bénard Convection, to be published (V. Yakhot) (1989).

9. Intermittency of Turbulence (Z.-S. She, E. Jackson, and S. A. Orszag), to appear in Proc. of Symp. in Pure Math (1989).

These papers explored the renormalization group (RNG) theory of turbulence from a number of different directions. Basic theory is covered in Papers 1, 5, and 6. In addition to the explanation of many of the basic constants and transport ideas of classical turbulence theory, the RNG theory suggests several novel new features, including the depletion of nonlinearity in turbulent flows and the alignment of small-scale structures along large eddies leading to the equivalence of Eulerian and Lagrangian correlation measures. Applications to Burgers' equation are studied in Paper 1. These papers lead to novel and effective descriptions of these flows. In the case of heat transfer, basic engineering descriptions are obtained that are proving quite useful in practice. The theory is successfully tested computationally in Paper 2. In Papers 3, 4, and 9, some novel predictions are made concerning intermittency effects in turbulence and the relations between Eulerian and Lagrangian time correlations and explored in greater depth. In particular, the RNG theory predicts intermittency effects that are opposite in direction to most classical models; the RNG predictions are consistent with a number of recent experiments. In Paper 7, the RNG theory is extended to treat turbulent flames in pre-mixed turbulent flows. This theory gives an elegantly simple, practical, and seemingly accurate description of these complex flow phenomena. In Paper 8, the theory is extended to buoyant convection and used to explain the recent results of Libchaber, Kadanoff, et. al. for high Rayleigh number Bénard convection.

Direct and Large-Eddy Simulations of Turbulence Structure

10. Renormalization Group Formulation of Large-Eddy Simulations (A. Yakhot, S. A. Orszag, V. Yakhot, and M. Israeli), to appear (1989).

11. Near-Wall Structures in Large Eddy Simulations of Turbulent Channel Flow (A. Yakhot, S. A. Orszag, and V. Yakhot), Phys. Fluids, submitted (1989).

12. Turbulence in a Randomly Stirred Fluid (R. Panda, V. Sonnad, E. Clementi, S. A. Orszag, and V. Yakhot), Phys. Fluids, **A1**, 1045-1053 (1989).

13. Scale Dependent Intermittency and Coherence in Turbulence (Z.-S. She, E. Jackson, and S. A. Orszag), J. Sci. Comp., **3**, 407 (1988).

14. Energy and Dissipation Range Spectra in the Inertial Range of Homogeneous Turbulence (V. Yakhot, Z.-S. She, and S. A. Orszag), in Recent Advances in Computational Fluid Dynamics, Lecture Notes in Engineering, (ed. by C. C. Chao, S. A. Orszag, W. Shyy), 112-124, Springer-Verlag (1989).

15. Large-Eddy Simulation of a Turbulent Channel Flow (A. Yakhot, V. Yakhot, S. A. Orszag, R. Panda, and M. Israeli), in Current Trends in Turbulence Research, (ed. by H. Branover, M. Mond, and Y. Unger), AIAA Proceedings **112**, 252-261 (1988).

16. Instability Mechanisms in Shear-Flow Transition (B. Bayly, S. A. Orszag, and T. Herbert), Ann. Rev. Fluid Mech. 20, 359-391 (1988).

17. Group Summary: Computation (S. A. Orszag), to appear in Proc. of NASA Langley Workshop on Transition (1990).

18. Spectral Element-RNG Simulations of Turbulent Flows in Complex Geometries (G. Karniadakis, A. Yakhot, S. Rakib, S. A. Orszag, V. Yakhot), in 7th Symposium on

Turbulent Shear Flows, Stanford, pp. 7.2.1-7.2.6 (1989).

These papers describe extensive supercomputer simulations to explore the structure of turbulent flows. In Paper 10, the RNG theory is extended to formulate the subgrid scale transport approximations for use in large-eddy simulations (LES). Results are reported in Papers 10, 11, and 15 concerning RNG/LES calculations of turbulent shear flows. A major result here is that the RNG/LES methods do an excellent job of calculating near wall structures, including wall streaks, at much less spatial resolution than earlier methods and without any *ad hoc* approximations used to interpolate between molecular and eddy viscosity. In Papers 12, 13, and 14, direct numerical simulations are used to test the correspondence principle of the RNG theory and to study small-scale eddy dynamics in turbulence, including intermittent flow phenomena. Paper 16 is a review paper on the secondary instability theory of transition, while Paper 17 reviews the future of computations in transition studies. Paper 18 gives a variety of applications of the RNG theory to complex flows.

The following provides some additional details on four of the problems that were studied with support from the contract. They are: (1) Formulation of RNG methods for compressible turbulence; (2) Turbulence structure at large Reynolds numbers; (3) Compressible turbulence simulations; and (4) Spectral element solutions of complex flows in complex geometries. Some of this work is still in progress so the descriptions given below are status reports.

1. Formulation of RNG Methods for Compressible Turbulence

To accomplish this, we have generalized the correspondence principle that allows the establishment of an RNG recursion relation with a calculable fixed point to the case of compressible flows thus leading to a new set of RNG-based sub-grid and transport models for compressible turbulence. To illustrate how this is done, we summarize the dynamics as follows.

The equation of motion for the universal range (or ranges) in a compressible flow are:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{\Phi}$$

$$\frac{\partial \mathbf{\Phi}}{\partial t} = -\nabla \frac{\mathbf{\Phi} \cdot \mathbf{\Phi}}{\rho} - c_s^2 \nabla \rho + \left[\eta \nabla^2 + \left(\frac{\eta}{3} + \xi\right) \nabla \nabla \bullet\right] \frac{\mathbf{\Phi}}{\rho} + \mathbf{f}$$

where $\mathbf{\Phi} = \rho \mathbf{v}$ is the momentum of the fluid η and ξ are viscosities and $c_s^2 = \frac{\partial P}{\partial \rho}$ in the sound velocity. The random force added into the right side of the Navie. Stokes equation is defined by correlation function:

$$< f_i(k,\omega) f_j(k'\omega') >= (2\pi)^{d+1} 2 D_0 \delta_{ij} k^{-y} \delta(\mathbf{k} + \mathbf{k}') \delta(\omega + \omega') \tag{1}$$

In contrast to the correspondence principle for incompressible turbulence the force defined by (1) does not obey the condition $\nabla \cdot \mathbf{f} = 0$. The key step of the derivation is the plausible assumption that with weak compressibility of the fluid, shocks are not formed. Thus we assume that

$$\rho = \rho_0 + n$$

and that

$$\frac{n}{\rho_0} \ll 1 \tag{2}$$

The equations of motion become:

$$\frac{\partial n}{\partial t} = -\nabla\Phi$$

$$\frac{\partial\Phi_i}{\partial t} = -\nabla_j\Phi_i\Phi_j(1-n+n^2) - \nabla_i n c_s^2 + \nu_0 v_j \nabla_j \Phi_i(1-n+n^2) + \mu_0\nabla_i\nabla_j\Phi_j(1-n+n^2) + f_i \tag{3}$$

where $\nu_0 = \frac{\eta}{\rho_0}$ and $\mu_0 = \frac{\eta}{3\rho_0} + \frac{\xi}{\rho_0}$ are the corresponding kinematic viscosities. The Fourier-transform of the first of equations (3) gives:

$$n = \frac{\Phi_i(k,\omega)k_i}{\omega} \tag{4}$$

Substituting (4) into (3) the following equation of motion is derived:

$$\Phi_j(\hat{k}) = G_{ij}(\hat{k})f_i(\hat{k}) + \lambda G_{i,j}(\hat{k})\int dq Q_{ij\ell}(k,P,k-P)\Phi_j(k-p)\Phi_\ell(q)dq + \lambda^2\int dq\, dp\, S_{ij\ell m}$$

$$(k,P,q,k-p-q)\Phi_j(P)\Phi_\ell(q)\Phi_m(k-p-q)d\hat{p}\,d\hat{q} \tag{5}$$

with $G_{ij}(\hat{k})$ and $Q_{ij\ell}, S_{ij\ell m}$ simply found from the Fourier transformed equation of motion. This equation has an additional non-linearity $O(\Phi^3)$ which is a new feature of the theory.

It is easy to show that the term $O(\Phi^2)$ gives a correction to the viscosity while $O(\Phi^3)$ non-linearity leads to a renormalization of the sound velocity thus introducing k-dependent sound dispersion. Application of the RNG procedure to this equation allows us to evaluate correlation functions in compressible turbulence and, which is most important, to derive a new set of sub-grid and transport models. Some key results are that $c_s \sim k^{-1}$ as $k \to 0$ so large-scale motions are effectively incompressible and the energy spectrum flattens from $k^{-5/3}$ to k^{-1}.

2. Turbulence Structure at Large Reynolds Numbers

We have investigated the statistical dynamics of coherent vortex structures in developed turbulence in order to begin to understand depletion of nonlinearity and coherent flow structures, and to assess the relevance of simple vortex models. For example, alignment of the vorticity and velocity with the principal axes of the rate of strain are studied in detail in pseudo-spectral numerical simulations of moderate Reynolds number turbulence. It is shown that coherent features of small scale flows manifested in the high positive mean of the middle eigenvalue $\alpha > \beta > \gamma$ of the rate-of-strain, $\overline{\beta}$, and in high alignment of $\boldsymbol{\omega}$ with the associated eigenvector, $\mathbf{s}_\beta$, have physically separate origins. The dynamical aspect of alignment is further studied using a statistical sampling analysis in a Lagrangian frame following trajectories of fluid particles. It is shown that high alignment results from the generation of a local quasi-two-dimensional structure associated with strong vorticity which induces a rotation of $\mathbf{s}_\beta$ towords $\boldsymbol{\omega}$. In addition, we have observed a tendency in developed turbulence for velocity vectors to lie in the plane formed by the two principal stretching directions. This provides a mechanism for depletion of nonlinearity in turbulence. To understand more deeply vorticity dynamics, we have studied in depth Crow instability and vortex tube dynamics and topology.

More precisely, the instantaneous effect of the strain at a given point is to disproportionately increase the α-component of $\boldsymbol{\omega}$, leading to increased alignment with $\mathbf{s}_\alpha$. In the absence of other effects, we would expect that $\boldsymbol{\omega}$ would typically align with $\mathbf{s}_\alpha$. However, the induced rotation of the principal axis by the vorticity is an important dynamical event which gives rise to the observed alignment of ω with s_β. When the vorticity is strong, vorticity lines tend to be locally straight and parallel. Due to the incompressibility condition, vorticity amplitudes also tend to be constant along vortex lines. In addition, the direction in which the vorticity has the longest correlation length is parallel to the vorticity vector. The suppression (or substantial weakening) of spatial dependence in that one direction gives precisely a locally (quasi) two-dimensional flow. This also suggests that elongated structures may form along the vorticity vector direction, especially when vorticity amplitude becomes high.

3. Compressible Turbulence Simulations

We have developed an advanced pseudospectral computer code to solve the compressible Navier-Stokes equation for an ideal gas. With 64^3 modes, the code requires less than 30 sec per time step (31 FFT's per time step). After several thousand steps, a statistical quasi-equilibrium state is achieved. Typical values of the Taylor micro-scale Reynolds number are about 50 and the mean Mach number (based on the fluctuating velocity and the sound speed) is about 0.8.

Some results obtained to date include: (1) The shock region is characterized $\nabla \cdot u$ taking on large negative values, which leads to tube-like structures; (2) High-enstrophy regions seem to be outside the shock regions; (3) Exchange of energy between the inertial energy and the compression parts of the kinetic energy occurs periodically in time. Recently, we have made a most remarkable comparison between incompressible and compressible turbulence. We have found that high vorticity regions in incompressible flow tend to be concentrated in tubes, which high vorticity regions in compressible turbulence at $Ma \approx 1$ tend to be located on sheets. This major effect of compressibility requires more investigation.

4. Spectral Element Solutions of Complex Flows in Complex Geometries

We have combined RNG modeling techniques with spectral element discretization procedures to formulate an algorithm appropriate for simulating high Reynolds number turbulent flows in complex geometries. Three different approaches of modeling are followed based on RNG algebraic, differential $k-\varepsilon$, and subgrid-scale models for the turbulent viscosity. Results obtained for the fully developed channel flow, and for the separated flow over a backwards- facing step suggest that all three formulations are suitable for turbulent flow simulations. The implementation of RNG models within the framework of a high-order discretization scheme (i.e., spectral element methods) is essential, as it results in resolution of fine turbulence structures even with the simple differential $k-\varepsilon$ model at relatively small number of degrees of freedom.

$k-\varepsilon$ Modeling of Turbulent Flow Over a Backwards-Facing Step

The geometry for this flow and the corresponding spectral element mesh are chosen to be identical with the geometry used in experiments of Kim, et. al. (1978). The outflow length is taken to be twenty times the step height H. A total of $K = 92$ elements of order $N = 9$ were employed in the discretization. The Reynolds number $R_* = \frac{U_* H}{\nu_0}$ is 1870 (or $R_c = 45,000$). In this simulation the computational domain includes the walls, where we impose the zero flux condition for the disipation rate ε; at the outflow Neumann conditions are specified for all field variables. The inflow conditions match the measured profiles in the experiment of Kim, et. al. In addition to the large separated recirculation zone, eddies of smaller size of opposite rotation appear at the step test section floor juncture consistent with experimental findings (Abbot). To the best of our knowledge no other simulation has resolved such fine structures previously using a single or a zonal modeling approach. Furthermore, the length of the recirculation zone is computed to be $L = 7.3H$ in close agreement with the experimental value (Kim 1978). Most studies to date fall short of predicting the correct value due to errors both in numerics and turbulence models.

Large-Eddy Simulations of Turbulent Channel Flow

In the next example we employ the RNG subgrid-scale model to simulate turbulent channel flow at $R_* = 185$. This simulation corresponds to identical conditions as the direct simulation recently reported by Kim, Moin and Moser (1987). In our simulation we have used however 60 times fewer grid points. In particular, for this case we employ a global spectral discretization based on Fourier expansion in streamwise and spanwise directions, and Chebyshev expansion in the inhomogeneous direction $(16 \times 64 \times 64)$. The initial conditions are based on three-dimensional Tollmien-Schlichting waves. The results of our simulation are essentially identical with the results of the direct simulation and in close agreement with the experiments (Eckelman). The agreement extends also to higher order statistics as well as to flow structure and the streak spacing (we computed $\lambda_* \approx 100$); all previous LES have failed to predict the correct value of streak separation.