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# THESIS

AN EXAMINATION OF THE RELATIONSHIP  
BETWEEN ATOMIC ASORPTION READINGS  
AND ATOMIC EMISSION READINGS  
IN THE JOINT OIL ANALYSIS PROGRAM

by

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March, 1990

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**An Examination of The Relationship Between Atomic Absorption Readings and  
Atomic Emission Readings in The Joint Oil Analysis Program**

by

**Choi, Moon Soo**  
Major, Republic Of Korea Army  
B.S., Republic of Korea Military Academy, 1980

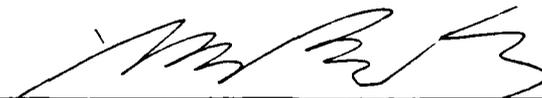
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## ABSTRACT

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## I. INTRODUCTION

### A. BACKGROUND

Over the years, considerable research has been devoted to the development of Spectrometric Oil Analysis (SOA). This is a diagnostic maintenance tool used to maximize personnel safety and minimize cost of material readiness by monitoring the type and concentration of wear metals in lubricating fluid samples. The presence of unusual concentrations of an element in the fluid sample may indicate abnormal wear of the equipment. Once abnormal wear is verified, the equipment may be repaired or removed from service before a major failure occurs. SOA is widely used in the military services to monitor airplane engines, battleships, and military vehicles, etc.

The basic principle of SOA is that when atomic structures of the metallic elements are sufficiently energized, they will emit light of characteristic wavelengths. These wavelengths are always identical for an individual element and no two elements will emit and absorb the same wavelength of light at the same intensities. Two kinds of measuring instruments are used in SOA; Atomic Absorption (AA) spectrometers and Atomic Emission (AE) spectrometers.

Oil analysis by AE spectrometers is accomplished by subjecting the sample to a high voltage spark which energizes the atomic structure of the metallic elements, causing the emission of light. This light is focused into the optical path of the spectrometer, separated according to wavelengths, converted to electrical energy and integrated, giving a number referred to as the parts per million (ppm) concentration for the element.

Atomic absorption spectrometers operate on a different principle. With these instruments, the sample is aspirated into a flame and vaporized. The atomic structure of the elements present becomes sufficiently energized by the high temperature of the flame to absorb light energy. Light energy having the same wavelength of the element being analyzed is radiated through the flame during this excitation and is partially absorbed in the flame. The resultant light is converted to electrical energy and integrated, again producing a number called the ppm concentration for the given element.

The Joint Oil Analysis Program (JOAP) was established in 1976 for the purpose of developing a standardized, mutually beneficial program for all branches of the Department Of Defense (DOD). This program was expected to standardize the usage of SOA for all services, eliminating possible redundancies and inefficiencies. JOAP, in turn, was to institute methods to monitor both internal consistency of repeated readings by the same instrument, as well as consistency from one instrument to another, for all DOD laboratories. In support of this program the Joint Oil Analysis Program Technical Support Center (JOAP-TSC) uses a monthly interlaboratory correlation program to ensure that spectrometers serving DOD remain calibrated and standardized. To accomplish this, JOAP-TSC sends out two "synthesized" oil samples and two "used" oil samples each month to approximately 200 atomic emission laboratories and 40 atomic absorption laboratories for analysis. The "synthesized" oil samples are made from regular service-grade oil, having soluble metallic contaminants added to set desired concentrations; the concentrations used are varied from month to month. The "used" oil samples are meant to be actual used-engine oil samples; however, due to the general

difficulty in obtaining real used-engine oils, especially ones with controllable contaminant levels from month to month, these "used" oil samples are generally made up just like the synthesized samples, and typically contain very little, if any, real used-oil. Each instrument in the program analyzes these same four samples and the results are sent back to JOAP-TSC. This group then compiles the submitted data and issues a monthly report which gives summary descriptions across all laboratories (for both AA and AE instruments), as well as individual laboratory readings for the elements used, and accuracy and repeatability scores for laboratories.

Most of the laboratories use atomic emission spectrometers, but a few Air Force laboratories perform oil analysis on atomic absorption spectrometers. Because of the differences in the operating principles of the two types of spectrometers, the concentration readings obtained from the two instruments tend to be different, when used to analyze the same oil sample. Some Air Force laboratories have both types of instrument; in these cases the AE instrument, which is large and bulky, is generally employed for all analyses at the home base. When a squadron of aircraft is deployed to a remote location, an AA instrument, which is much more portable than AE, is taken along for required oil analyses during deployment. This leads to the situation in which an oil sample from a given engine is analyzed on an AE instrument and the next one (or several) are analyzed on an AA instrument. In addition to this deployment situation, the AA instrument may occasionally be used at home base as a back up when the AE instrument is out of service. Thus it is of practical interest to establish a method of converting concentration readings from one instrument to the other, especially for real used-engine oil samples.

In 1975, the Naval Weapons Engineering Support Activity (NAVWESA), Washington, D.C., demonstrated the feasibility of using statistical regression techniques to establish a conversion formula for AE to AA readings. Lynch and Short [Ref. 1] used limited data from the interlaboratory correlation program to demonstrate this technique. They also recommended a more extensive study to establish this conversion formula. Culler [Ref. 2] also studied this problem in 1975 at the Southwest Research Institute, and recommended an additional conversion formula. In 1982, Jayachandran and Larson [Ref. 3] showed the possibility of estimating a conversion formula by using a simple regression analysis of the mean values of interlaboratory correlation data for the two instruments. Their results show that the fitted straight lines from regression analysis provide accurate representations for the relationship between AA and AE readings for these correlation samples. In addition, they provided a conversion table with estimated parameters. In 1986, they [Ref. 4] reported a more detailed study to establish the relationship between the readings from an AA spectrometer and those for an AE spectrometer with newer data sets from the same source. In this study, it was shown that the correlation coefficients between mean values of AA readings and AE readings are above 0.99 except for one element (molybdenum for used oil samples). However, there were no significant changes noted from the previous study.

## **B. SCOPE OF THE THESIS**

The reports mentioned earlier use only correlation data in deriving the relationship between AA and AE concentration readings. These oil samples are

man-made and typically do not contain actual used-oil in spite of the labels employed. The concentrations of the correlation program samples are controlled by the addition of soluble metallic concentrates to oil samples. Any procedure linking the readings of the two instruments might thus be expected to work well for oil samples whose metallic contaminations are totally in solution.

Actual used-engine oil may, of course, contain tiny metallic particles in suspension. If the oil sample is removed from the engine while the oil is still hot, as is recommended, it is likely that tiny suspended particles will be included in the sample submitted for analysis. It then becomes important to see if any estimated relationships between AE and AA readings, determined from correlation samples with contaminants generally in solution, prove usable for real used-oil samples. This thesis will investigate this point indirectly, by employing actual operating engine oil analysis records reported in Fiscal Year (FY) 1988.

The approach to be employed is the following:

- (1) Correlation program data will be used to get the "best fitting" relation between the AA and AE instruments. This is accomplished by using the trimmed means, for both AA and AE instruments, for 36 months in the period from March 1980 through March 1988 (this data is essentially the same as that employed in Jayachandran and Larson's studies, augmented by a few months in 1987 and 1988) for both synthesized and used-oil samples. All of these samples were analyzed on both types of instrument; the trimmed means (middle 60%) for both instrument types are used. Since both types of

trimmed means are random variables, orthogonal regression [Ref. 5] is employed to estimate the relationship, as opposed to standard least squares employed in Jayachandran and Larson's studies. This approach has the advantage of treating AA and AE symmetrically. The same line is found regardless of which instrument's values are called dependent and which is independent.

- (2) The Air Force has supplied FY88 data on Air Force engines as reported to the B003 data base at Kelly Air Force base, Texas. This data contains all reported oil analyses for all Air Force engines for this year, for all spectrometers employed. A search of this data base has isolated six different type equipment codes for engines, of 3 different types (turbo-jet, turbo-fan, turbo-prop), each of which have a fairly large number of readings from AE and from AA instruments. For any given element, say iron (Fe), one can then examine the distribution of AE readings and the distribution of AA readings (for each type equipment code). Using the linear relation determined in (1) from the correlation data, it then is possible to transform this used-oil AA distribution (from the B003 data base) accordingly; this transformed AA distribution can then be compared with the actual used-oil AE distribution observed. If the relation determined from the correlation samples in (1) is valid for used-oil readings, one would expect this transformed AA distribution to be very similar to the observed AE distribution.

Chapter II discusses the orthogonal regression procedure and applies it to the correlation data; to get a better fit, some spline regressions are also

employed there. Chapter III presents the used-oil distributions derived from the B003 data base. Chapter IV discusses the transformation of the used-oil distributions for AA, employing the equations from Chapter II, and presents several comparisons. Chapter V summarizes the conclusions.

## II. CORRELATION PROGRAM DATA AND ORTHOGONAL REGRESSION

### A. GENERAL

Both AA and AE spectrometers are employed in the DOD Joint Oil Analysis Program. The JOAP-TSC correlation program requires all participating laboratories to analyze the same four samples (two used-oil, two synthesized-oil) each month. This type of data has been employed to explore the relationship between the readings produced by the two types of instruments [Ref. 3, 4, 6]; the readings are highly correlated in general, as noted in these reports.

It is well known that the simple linear regression model generally produces two different lines, depending on which of the two variables is called "dependent" and which is called "independent". That is, if AA readings are regressed on AE readings, the estimated line is not the same as that produced by regressing AE on AA. This in turn implies that conflicting results could occur in trying to investigate the accuracy of such estimated relationships when employed with actual used-engine oils. To circumvent this problem, this thesis employs orthogonal regression procedures.

The data available consists of 64 of the 68 pairs of trimmed means used by Jayachandran and Larson, augmented by 8 later pairs of trimmed means, giving 72 data points in total. These correlation samples were analyzed for contamination levels of 10 different elements (Fe, Ag, Al, Cr, Cu, Mg, Si, Ti, Mo, and Ni). Since the used-engine oil data to be discussed in the following chapters includes only elements Fe, Ag, Al, Cr, Cu, and Mg, these are the

only ones which are discussed in this thesis. Of these, iron (Fe) is by far the most important in terms of applications in JOAP, so the discussion here will explicitly discuss this single element. The same general conclusions are appropriate for all the others.

## B. SUMMARY OF CORRELATION DATA

Appendix A presents the correlation data used in this thesis; given there are the trimmed means of AA and AE readings from all laboratories participating in the interlaboratory correlation program for the specified months. It can be seen from the used-oil sample tables that the mean values of the AA readings are generally lower than the mean values measured by AE instruments, while the synthesized-oil mean values are roughly the same for both instrument types.

Figure 1 displays the scatter plots of synthesized-oil and used-oil correlation data for Fe in two different ways. The top plots present the data in the (AA, AE) plane while the bottom plots use the (AA, AE/AA) plane. The scatter plots of the other metallic elements are presented in Appendix B. Note that the synthesized-oil samples are generally well behaved, with almost perfect linearity exhibited between the AA and AE trimmed means. The 72 samples are fairly uniformly spread over the ranges covered for both instruments, and the readings for both instruments are roughly the same magnitude, with the AE/AA ratios hovering close to 1.

The used-oil trimmed means do not adhere to strict linearity as well, nor are they as uniformly distributed over the ranges covered; the great majority of the 72 samples are concentrated on the low end of the scales for both

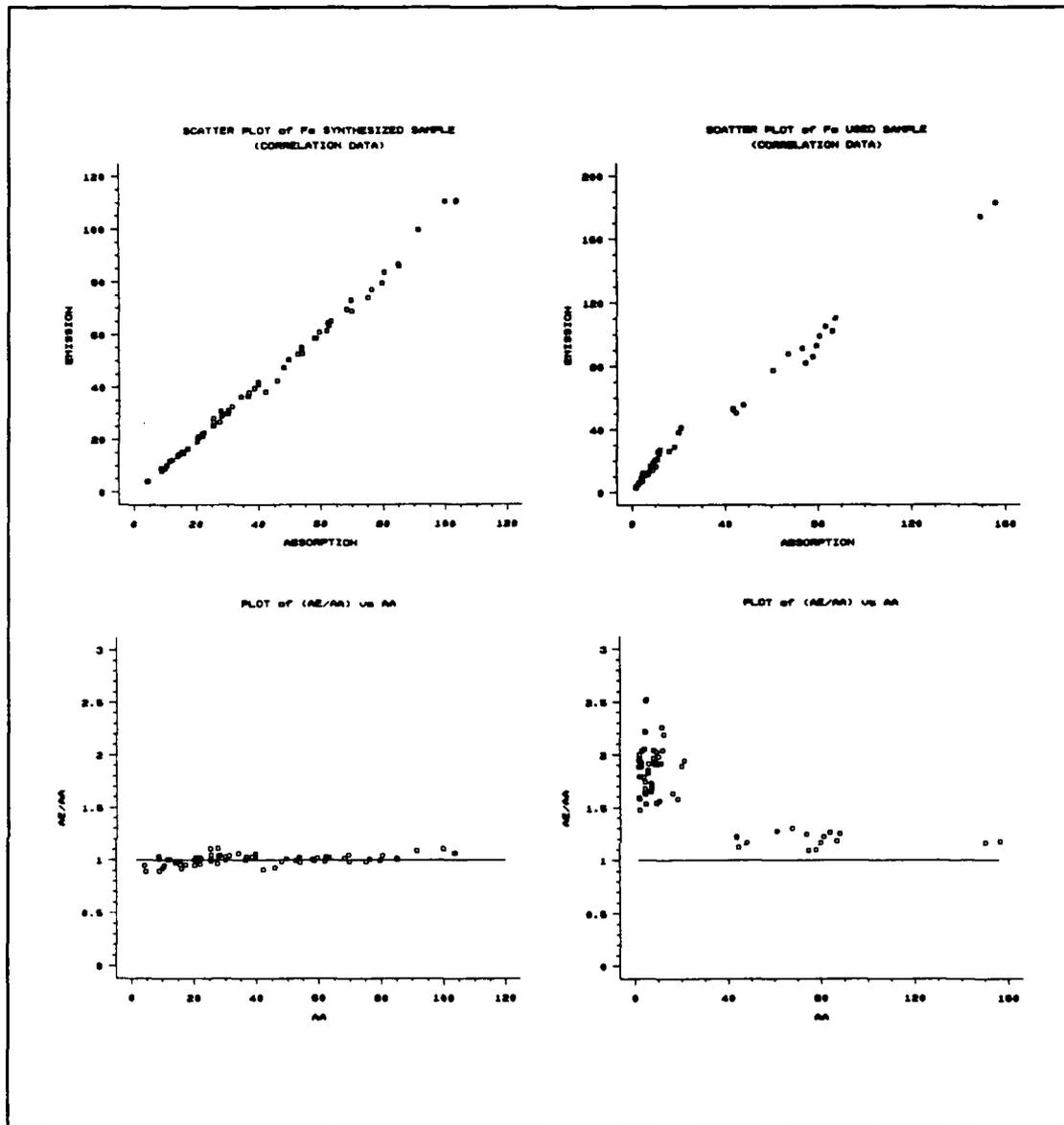


Figure 1. Scatter plots of used and synthesized correlation data and plots of ratio (AE/AA) vs AA

instruments, with the AA means typically being smaller than the AE means. Notice that the AE/AA ratios for low AA values are quite different from those with high AA values, and that they all exceed 1. This different behavior may

be attributed to the presence of actual used-oil in some of these samples (most likely those with small AA readings), but unfortunately no records are available describing the frequency of use of real used-oil in the correlation program.

Because of this nonlinearity of used-oil samples, two different approaches will be tried in deriving an AE-AA relationship which might be useful for real used-oil samples. The first of these is a single orthogonal-regression straight line; the second approach employs a spline with one knot.

### C. ORTHOGONAL REGRESSION ANALYSIS

The usual least squares approach could be employed in estimating a linear relationship between AE and AA means (indeed, this is the approach used in [Ref. 3, 4]); as has been mentioned, this procedure will typically (certainly for the current data) lead to two different regression lines, depending on which instrument is chosen to be the "dependent" variable. Orthogonal regression lines minimize the "perpendicular" distances between the observed data values and resulting lines; the same line is found regardless of which of the two instruments' readings is chosen to be the dependent variable. This methodology can be paraphrased as follows.

#### 1. Single Straight Line

Granted one has  $n$  data pairs  $(x_i, y_i)$ , first let us "center" the coordinate system at the means of the two variables by defining

$$X_i = x_i - \bar{x} \tag{2.1}$$

$$Y_i = y_i - \bar{y}$$

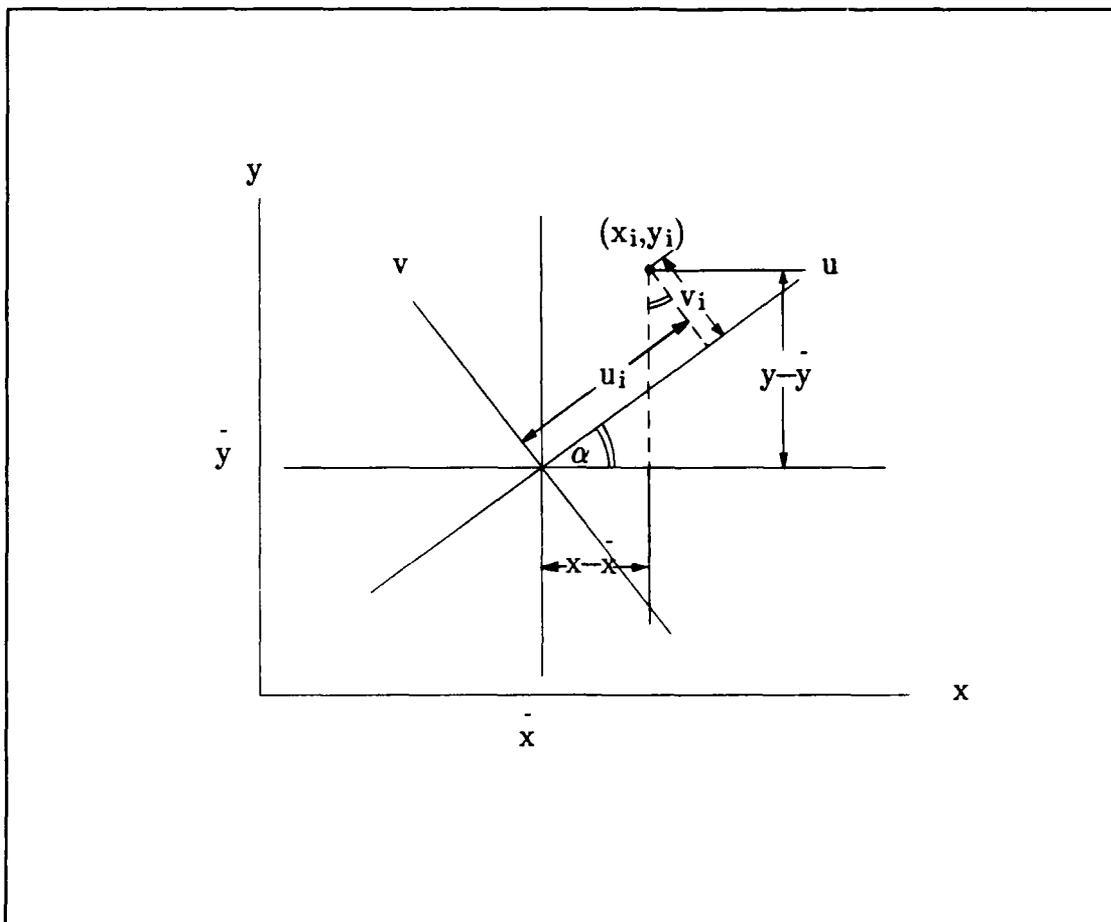


Figure 2. Orthogonal regression analysis (single line)

where  $\bar{x} = \frac{1}{n}\sum x_i$ ,  $\bar{y} = \frac{1}{n}\sum y_i$ . Next, perform a rotation of the coordinate axes through an angle  $\alpha$  (to be determined) by letting

$$u_i = X_i \cos \alpha + Y_i \sin \alpha \tag{2.2}$$

$$v_i = -X_i \sin \alpha + Y_i \cos \alpha$$

as pictured in Figure 2. With the original  $(x_i, y_i)$  values lying roughly on a straight line with positive slope, this transformation then results in the u-axis

passing through the body of the data (with  $0 < \alpha < \frac{\pi}{2}$ ) and the  $v_i$  values represent the perpendicular distances from the original points to the  $u$ -axis. The angle  $\alpha$  thus should be chosen to minimize

$$L = \sum_{i=1}^n v_i^2$$

$$= \sum_{i=1}^n (-X_i \sin \alpha + Y_i \cos \alpha)^2 \quad (2.3)$$

and the resulting minimizing angle  $\hat{\alpha}$  then is defined by equation (2.4).

$$\tan \hat{\alpha} = \frac{(S_{yy} - S_{xx}) + \sqrt{(S_{yy} - S_{xx})^2 + 4S_{xy}^2}}{2S_{xy}} \quad (2.4)$$

where  $S_{xx} = \sum(x_i - \bar{x})^2$ ,  $S_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y})$ , and  $S_{yy} = \sum(y_i - \bar{y})^2$ .

The tangent of this angle  $\hat{\alpha}$  is the slope of the orthogonal regression line in the original coordinate system for  $(x_i, y_i)$ ; thus the orthogonal regression line in the original coordinate system is given by

$$\hat{y} = \bar{y} - \bar{x} \tan \hat{\alpha} + x \tan \hat{\alpha}$$

$$= a + bx \quad (2.5)$$

where  $a = \bar{y} - \bar{x} \tan \hat{\alpha}$ ,  $b = \tan \hat{\alpha}$ .

Applying this orthogonal regression methodology to the correlation data gives the lines plotted in Figure 3; the slopes and y-intercept are given in Table 1, along with the resulting residual sums of squares ( $\sum v_i^2$ ).

**Table 1. ESTIMATED PARAMETERS FOR Fe**

DATA	slope	y-intercept	sum of Sq.
synthesized-oil	-1.314	1.051	112.515
used-oil	4.627	1.151	456.820

The synthesized-oil data is quite well fit by the orthogonal regression line, as certainly could have been anticipated from Figure 1. The used-oil data is not as well represented by the orthogonal regression line, because of the non-linearity evident in Figure 1. This same type of problem also occurs with aluminum (Al) and magnesium (Mg).

The great majority of actual used-oil readings are on the low end of the scales, mostly 20 ppm or less for both instruments. To test the effectiveness of a conversion formula between AA and AE it is thus important to employ an estimated equation which fits the low ppm values well. One way to accomplish this is to use linear splines, connected straight line segments; such broken line segments remain continuous, giving a useful way of comparing actual AA and AE used-oil readings.

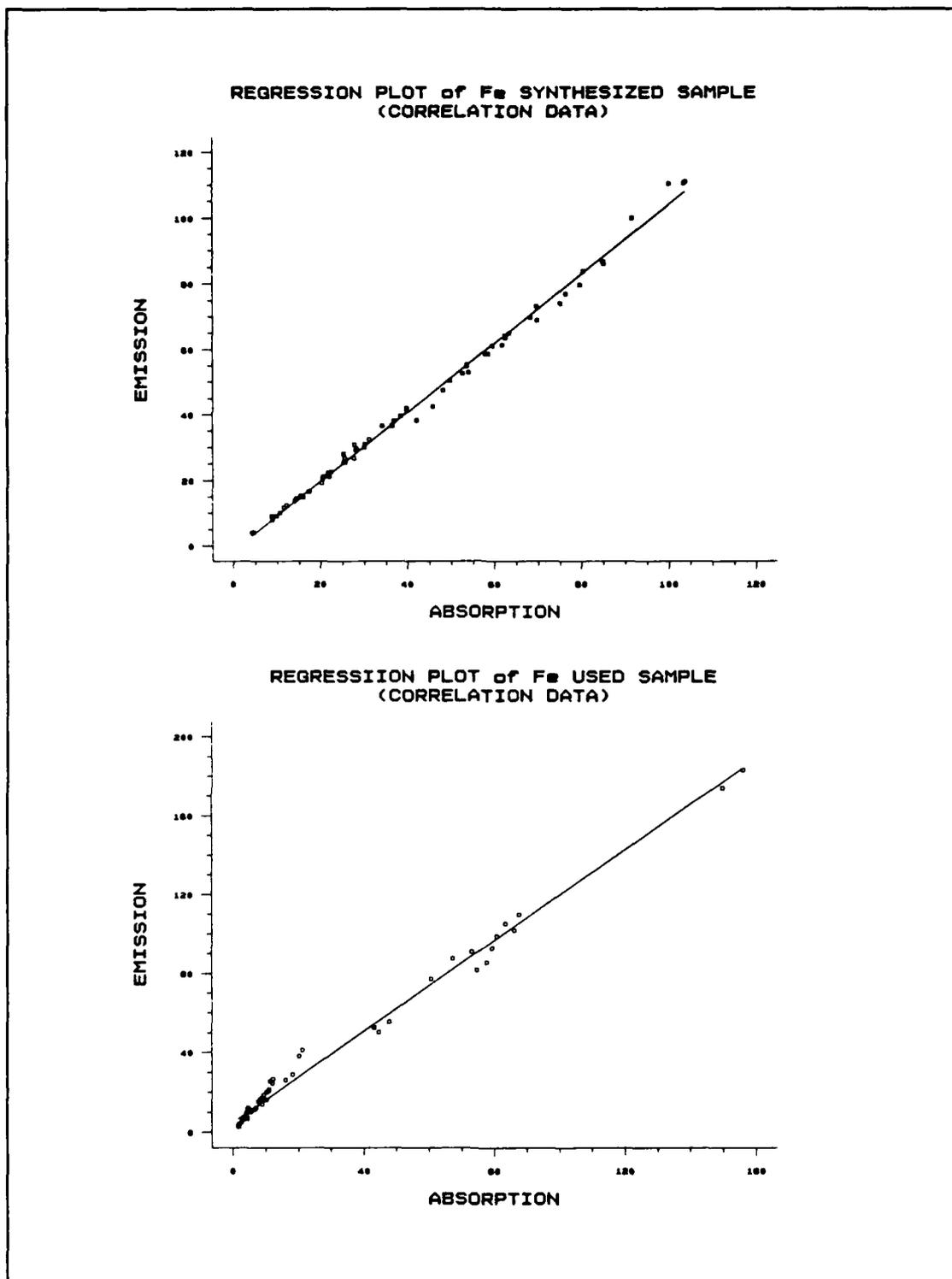


Figure 3. Regression plots of Fe synthesized data and Fe used data (single line)

## 2. Two Line Regression

To derive a useful spline equation, still employing orthogonal regression, the following procedure is followed. Recall the earlier comment that the great majority of actual used-oil readings are on the low end of the ppm scale, so it is most important to get a good fit for low readings. Let  $p$  represent the (fixed) knot location on the AA scale. Then the data to the left of  $p$ , those pairs with AA reading no larger than  $p$ , are used to determine an orthogonal regression line

$$\hat{y} = a + bx$$

as already described. This line passes through the point  $\hat{y} = a + bp$  at the knot location  $p$ .

To fit a second line to the data to the right of the knot, those pairs with AA reading larger than  $p$ , a new coordinate system is employed with origin at  $(p, a + bp)$  (in the original system), using only the data to the right of the knot. The slope parameter for this line is given by

$$\tan \hat{\beta} = \frac{(U_n - U_m) + \sqrt{(U_n - U_m)^2 + 4U_{mn}}}{2U_{mn}} \quad (2.6)$$

where  $U_m = \sum m_i^2$ ,  $U_n = \sum n_i^2$ ,  $U_{mn} = \sum m_i n_i$ ,  $m_i = x_i - p$ , and  $n_i = y_i - (a + bp)$ . A measure of goodness of fit of the resulting spline is given by adding the  $\sum v_i^2$  values for the left and the right lines.

The above procedure was implemented by letting  $p$  (the knot location) equal each of the 66 middle AA observed values in turn, observing

the goodness of fit value for each location. The knot is then located at the AA value with the smallest goodness of fit value. This is summarized below.

- (1) Sort data. AA readings are employed as the independent variable; therefore, after sorting AA readings, AE readings are sorted for tied AA readings.
- (2) Choose knot location, and divide the ordered data into two parts without overlap, then transform the x-axis from 0 to the given knot  $p$ . The given knot is fixed at one of the data points.
- (3) Estimate the orthogonal regression parameters for the first segment with equations (2.4) and (2.5).
- (4) Move the origin to  $x=p, y=a+bp$ . Then estimate the slope parameter for the second segment with equation (2.6).
- (5) Compute the sum of squares for both segments and sum them to get *goodness of fit*.
- (6) Do the above procedures for all possible knots, then compare the goodness of fit for all knots and find the best knot.
- (7) Re-transform the slope and intercept parameters for the best knot to the original coordinate system.

Using the Fe used-oil data set displayed earlier, the spline orthogonal regression routine locates the best knot at  $p=11.3$  (AA reading), giving a sum of squares of 261.615 compared with 456.820 for the single line. The parameters defining these lines are given in Table 2 while Table 3 gives the corresponding results for the Fe synthesized-oil values. It is interesting that

**Table 2. ESTIMATED PARAMETERS FOR Fe USED-OIL**

single line	slope	y-intercept	sum of Sq.
	4.627	1.151	456.820
two lines	left slope	right slope	sum of Sq.
	1.992	1.090	261.615

**Table 3. ESTIMATED PARAMETERS FOR Fe SYNTHESIZED OIL**

single line	slope	y-intercept	sum of Sq.
	-1.314	1.051	112.515
two lines	left slope	right slope	sum of Sq.
	1.014	1.341	61.520

the percent change in the sums of squares is about the same for both types of samples. These spline regressions for the synthesized and used-oil samples are plotted in Figure 4. Notice that the knot location for the synthesized-oil samples is at  $p=79.7$ , much higher than for the used-oil samples.

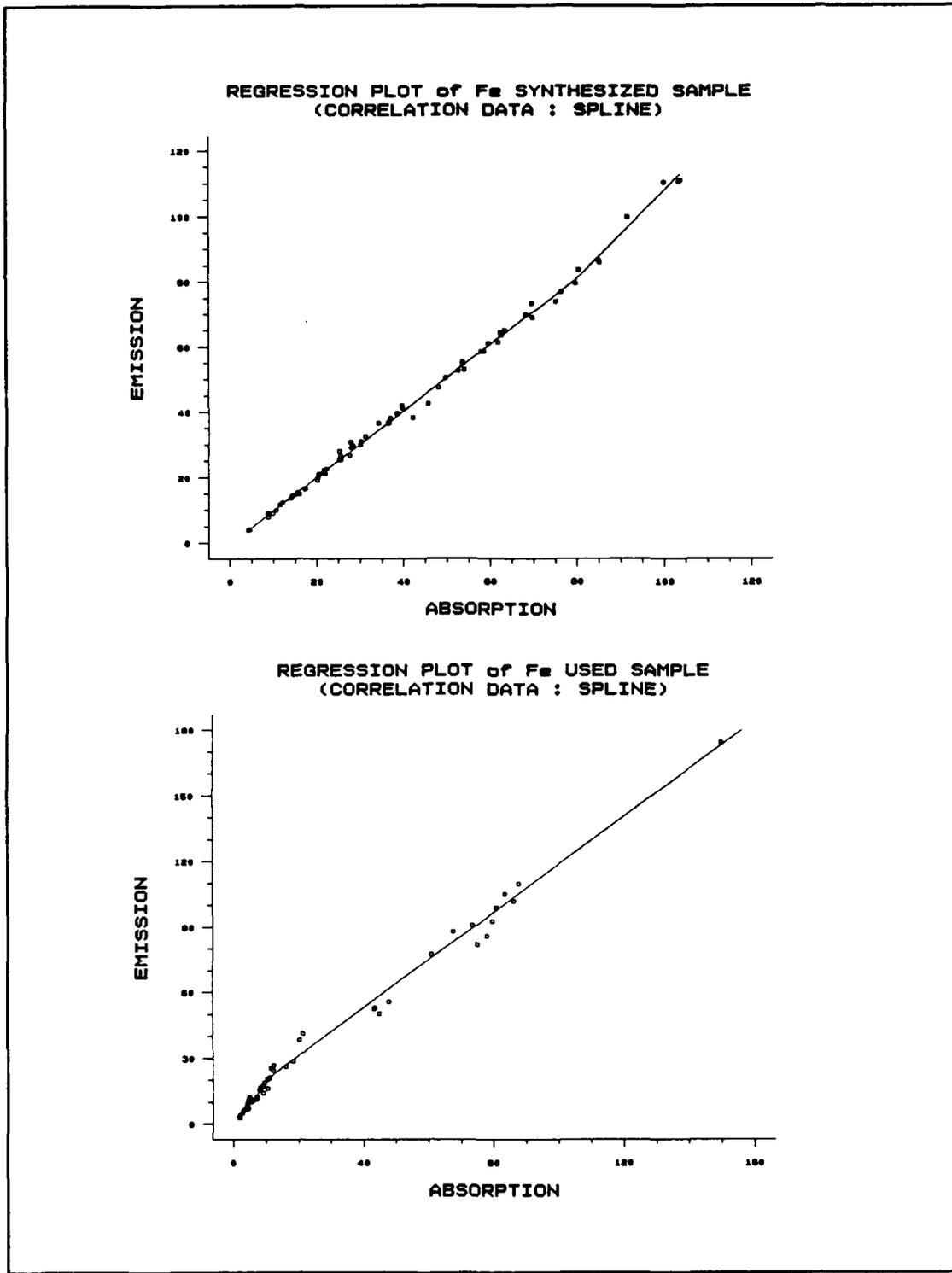


Figure 4. Regression plots of Fe synthesized data and Fe used data (spline)

### III. REAL USED-OIL DATA

#### A. GENERAL

The correlation program data already discussed contains average readings made by both AA and AE spectrometers, on exactly the same prepared samples, half of which are labeled synthesized-oil and half of which are labelled used-oil. These samples allow direct investigation of possible relationships between the readings produced by the two instrument types. Unfortunately there is no comparable source providing readings from both instruments for samples of oil drawn from operating aircraft (real used-oil); thus it is not possible to directly check the reliability of the AA-AE relationships derived from the correlation program samples.

JOAP maintains a data repository for oil analysis records (called the B003 data bank) at Kelly Air Force Base, Texas. All DOD oil analysis laboratories are required to periodically submit all oil analyses records to this data bank. These oil analysis records identify the laboratory and instrument type employed, the particular equipment from which the sample was taken, contaminant readings observed (rounded to the nearest ppm), the laboratory recommendation made, and a number of other items. The Air Force oil analysis program management has provided a data tape containing all Air Force oil analysis records submitted to B003 in FY88. A search of these tapes has located several different Type Equipment Codes (TEC's) which have fairly sizable numbers of analyses performed on both AE and AA instruments during this period. Although none of these oil samples were analyzed on both

instrument types, it is possible, for a specific TEC, to derive the distribution of readings for a given contaminant (say Fe) produced by the AE instruments and to derive this same distribution produced by the AA instruments (for this same TEC). Making the assumption that different engines of the same type should exhibit very similar distributions of contaminant readings, the B003 data can be employed to get AE and AA distributions, for real operating engines; indeed, it then is possible to use the relations derived from the correlation program samples to transform the AA distribution into an "expected" AE distribution (or vice versa). The "expected" AE distribution can then be compared with the actual AE distribution, to examine the validity of the correlation sample relationship for "real" used-oil samples. This approach is followed in this thesis.

## **B. DESCRIPTION OF REAL USED-OIL DATA**

In the thesis, several different types of equipment codes are used; DFKA and EPJA for turbo-fan jet engines, BSQA and BSPA for turbo-jet engines, LLDA and LLLA for turbo-prop engines. When these data tapes were read by the statistical software package SAS [Ref. 7] on an IBM 3033AP mainframe, some filters were applied to the data before summarization. These include deleting those records where the reason for the sample was T, K, M, P, or X : Test Cell, Prior to Maintenance-Removal, Post Maintenance Check, Physical Test and Initial Sample.

To ensure the use of records from actual working aircraft, only those serial numbers with at least 40 records for the year were included, for DFKA, EPJA, BSQA and BSPA. For LLDA and LLLA, because of the smaller number of

records available, all serial numbers with at least 20 records for the year were employed. The SAS outputs for TEC DFKA are presented in Table 5 and Table 6. The data submitted to B003 is not checked for validity and many data transcription errors can be expected; all of the readings of 100 or more are undoubtedly erroneous, as are some of the smaller values. To clean up these data errors a simple rule was employed: keep only the data up to the contaminant level where the SAS computed Cumulative Percentage Rate (CPR) first reaches 100.0. For example, the CPR value first reaches 100.0 at 15 ppm in Table 5. Therefore, the data analyzed are those less than or equal to 15 ppm for this TEC, for Fe. Table 4 presents the original sample sizes for the six TEC's as produced by SAS, and the resulting numbers of records kept after using this rule.

**Table 4. NUMBERS OF RECORDS KEPT FOR Fe**

CODE	TOTAL on TAPE		NUMBER KEPT	
	AE	AA	AE	AA
DFKA	126038	2106	125976	2105
BSPA	26025	10396	26012	10391
BSQA	42289	5828	42268	5826
EPJA	80428	358	80395	358
LLDA	6181	813	6172	813
LLLA	2014	666	2014	666

Table 5. SAS OUTPUT FOR DFKA AE READINGS (Fe)

FE	FREQUENCY	PERCENT	CUMULATIVE FREQUENCY	CUMULATIVE PERCENT
0	30511	24.2	30511	24.2
1	55554	44.1	86065	68.3
2	23610	18.7	109675	87.0
3	8621	6.8	118296	93.9
4	3713	2.9	122009	96.8
5	1739	1.4	123748	98.2
6	1042	0.8	124790	99.0
7	536	0.4	125326	99.4
8	305	0.2	125631	99.7
9	151	0.1	125782	99.8
10	111	0.1	125893	99.9
11	51	0.0	125944	99.9
12	17	0.0	125961	99.9
13	7	0.0	125968	99.9
14	5	0.0	125973	99.9
15	3	0.0	125976	100.0
18	1	0.0	125977	100.0
20	4	0.0	125981	100.0
21	1	0.0	125982	100.0
22	1	0.0	125983	100.0
30	2	0.0	125985	100.0
31	1	0.0	125986	100.0
34	1	0.0	125987	100.0
100	18	0.0	126005	100.0
101	2	0.0	126007	100.0
102	3	0.0	126010	100.0
200	12	0.0	126022	100.0
201	2	0.0	126024	100.0
205	1	0.0	126025	100.0
240	1	0.0	126026	100.0
300	5	0.0	126031	100.0
400	1	0.0	126032	100.0
500	1	0.0	126033	100.0
600	1	0.0	126034	100.0
700	1	0.0	126035	100.0
800	2	0.0	126037	100.0
801	1	0.0	126038	100.0

Table 6. SAS OUTPUT FOR DFKA AA READINGS (Fe)

FE	FREQUENCY	PERCENT	CUMULATIVE FREQUENCY	CUMULATIVE PERCENT
0	228	10.8	228	10.8
1	1435	68.1	1663	79.0
2	405	19.2	2068	98.2
3	37	1.8	2105	100.0
7	1	0.0	2106	100.0

These cumulative percent values can be interpreted as values for a Cumulative Density Function (CDF) for a random variable. Thus, the values given in Table 5 that were kept after eliminating spurious records can be graphed as in Figure 5. The corresponding records for Fe, TEC DFKA, produced by the AA instruments are given in Table 6 and also graphed in Figure 5. (Note the two T scales are different in this figure.)

The phenomenon measured by either type of spectrometer is ideally continuous; both instruments produce readings to 0.1 ppm but are reported to B003 as rounded values to the nearest ppm. Thus the values reported as 0 ppm correspond to readings between 0 and 0.5 ppm, those reported as 1 ppm correspond to readings between 0.5 and 1.5 ppm, etc. In recognizing this fact, it is useful to realize that the step functions given in Figure 5 actually are estimating continuous CDF's at particular points (and no estimate is available in between these points). Thus the CDF reading of 0.242 for 0 ppm for Fe, DFKA, for the AE instrument, can be pictured as the height of this CDF at 0.5 ppm (the end of the interval (0, 0.5)); the readings at the other integers (1, 2, 3, ...) estimate the CDF value at the same integers plus 0.5. These

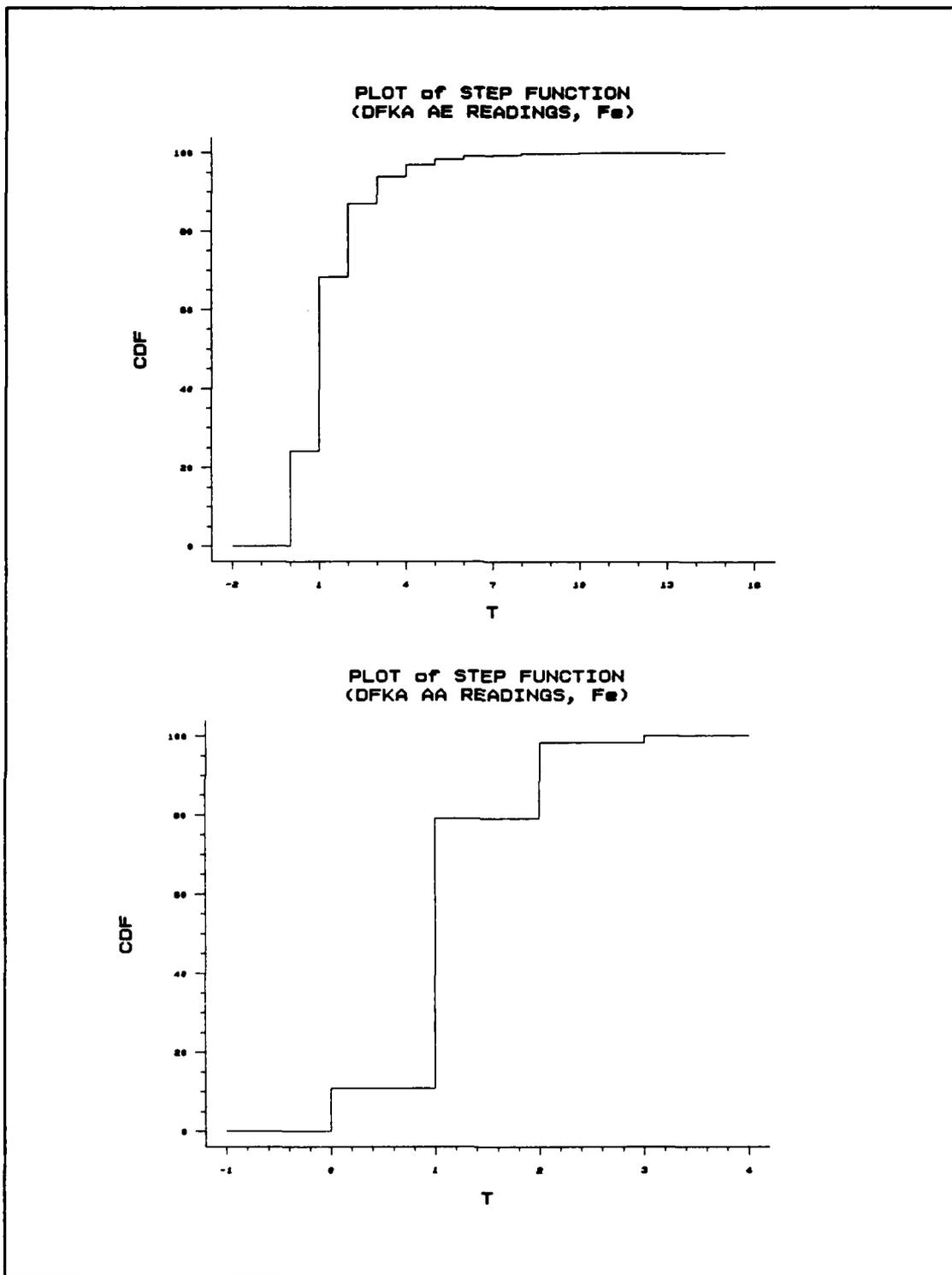


Figure 5. Plots of step function for DFKA Fe

points can then be connected by straight lines to better estimate the underlying continuous CDF's. The resulting "continuous" estimated CDF's are plotted in Figure 6 for DFKA, Fe.

It is obvious from this figure that the CDF's resulting from actual used-engine oil values (for TEC DFKA) are not identical. There are many ways that the AA distribution (or certain aspects of this distribution) could be transformed through the relationship established from the interlaboratory correlation program data, to compare with the counterpart of the actual AE values. Some comparisons of this type are discussed in the following section.

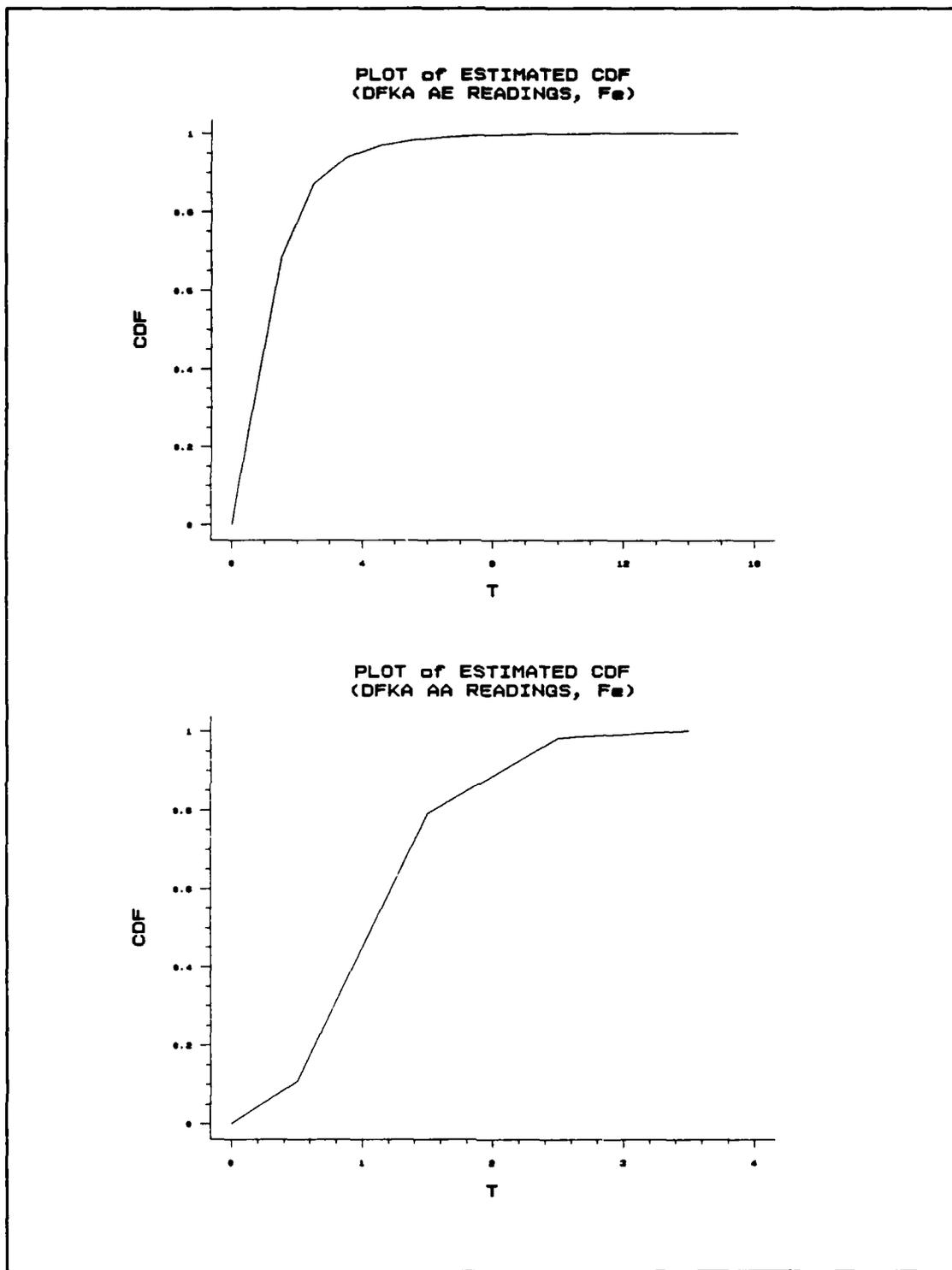


Figure 6. Plots of estimated CDF's for DFKA Fe

## IV. COMPARISONS FOR USED-OIL DISTRIBUTIONS

### A. GENERAL

Recall that we are comparing readings made on AE instruments versus those made on AA instruments as reported to the B003 data base; none of these oil samples were analyzed on both instruments. So long as operating engines (of the same TEC) are involved, it seems reasonable that one should be able to relate the AE distribution of readings produced to the AA distribution of readings produced. This basic idea is employed in what follows. In addition, the distributions of the other metallic elements except Fe are typically not widely enough dispersed to see relationships between the readings of both instruments. Thus just Fe is employed in these comparisons.

### B. COMPARISONS

As seen in Chapter II, the synthesized-oil samples from the correlation program adhere to linearity fairly well, while the used-oil samples are not so well-behaved. Orthogonal regressions over the whole range of the correlation data were derived, for both types of sample, and single-knot splines were also fit to the data. Table 7 gives the number of data values from B003 data base above the best knot location for the 6 TEC's, for all elements. The symbol '-' in this table means that these specified elements are not monitored for the given TEC. As can be seen, very little B003 data lies to the right of the knot location, meaning the equation to the left of the knot is applicable almost

**Table 7. NUMBER OF DATA ABOVE KNOT (AE/AA)**

CODE	Fe	Ag	Al	Cr	Cu	Mg
DFKA	0/0	0/0	0/0	0/0	-	-
BSPA	20/0	0/0	0/0	0/0	0/3	0/0
BSQA	47/0	0/0	0/0	0/0	0/0	0/0
EPJA	0/0	0/0	0/0	0/0	0/0	-
LLDA	0/0	11/0	0/0	0/0	0/0	0/0
LLLA	0/0	2/0	0/0	0/0	0/0	0/0

everywhere for all cases. In the following, only the spline relationships are employed to compare the AA used-oil values with the AE used-oil values from B003.

**1. Mean and Variance**

Most statistical techniques employ simple descriptors of the mean and variance of a set of data. Accordingly, one way of using the actual used-oil readings to see the value of correlation-sample regressions is given by evaluating the mean and variance of the actual AE readings, getting the mean and variance of the actual AA readings and then transforming the AA values to compare with their AE counterparts (or vice versa). As is well known, in the case of an increasing spline function, where

$$\begin{aligned}
 Y &= a+bX && \text{if } X \leq p \\
 Y &= a+bp+cX && \text{if } X > p
 \end{aligned}
 \tag{3.1}$$

then

$$\mu_Y = (a+b\mu_{X \leq p})P(X \leq p) + (a+bp+c\mu_{X > p})P(X > p) \quad (3.2)$$

Explicit formulas relating the variances are quite complicated and are not given. For DFKA, element Fe, the AE mean and variance are 1.3947 and 1.6306, and the AA mean and variance are 1.1463 and 0.3024. The orthogonal regression coefficients for this TEC for both types of sample, are given in Table 8; the resulting transformed AA means and variances are given in Table 9.

**Table 8. SPLINE REGRESSION COEFFICIENTS FOR Fe, DFKA**

DATA	line	slope	y-intercept	Knot (AA)
synthesized-oil	left	1.014	-.251	79.7
	right	1.341	-26.342	
used-oil	left	1.992	-.670	11.3
	right	1.090	9.521	

**Table 9. EXPECTED MEAN AND VARIANCE OF AE READINGS FROM AA TRANSFORMATIONS, DFKA**

EXPECTED VALUES	parameters from	mean	variance
	synthesized-oil	.9111	.3106
	used-oil	1.6137	1.1999

ORIGINAL VALUES	AE	1.3947	1.6306
	AA	1.1463	.3024

As can be seen from Table 9, the synthesized-oil transformed AA mean is smaller than the actual AE mean, while the used-oil transformed mean is larger (and closer). Both of the transformations yield expected variances which underestimate the actual AE variance, but again the used-oil value is closer. This underestimation of the variance is driven by the small variance exhibited by the actual AA distribution. Mean and variance tables for the other TEC's are presented in Appendix D to H. If one formally tests the hypothesis of equality of either means or variance (expected AE versus actual AE), these differences are highly significant, because of the large sample sizes involved.

## 2. Cumulative Density Function (CDF)

Rather than compare simply means and variances, it is quite straightforward to use the spline regression results discussed in Chapter II to transform the observed B003 AA CDF to the expected AE CDF. This expected AE CDF can then be compared to the actual observed B003 AE CDF, for the same TEC and element. In fact either of the two spline regressions derived in Chapter II (from synthesized-oil and from used-oil correlation samples) can be used to get an expected AE CDF for the comparison. Both expected AE CDF's have been compared to the observed AE CDF. For the spline function discussed earlier, the CDF relationship is given by

$$\begin{aligned}
 F_Y(t) &= F_X\left(\frac{t-a}{b}\right) && \text{if } t \leq p \\
 F_Y(t) &= F_X\left(\frac{t-a-bp}{c}\right) && \text{if } t > p
 \end{aligned}
 \tag{3.3}$$

For TEC DFKA, element Fe, the observed B003 used-oil CDF for AA readings is given in Table 10 and the observed used-oil CDF for AE readings is given in Table 11; the synthesized-oil and used-oil spline functions are in Table 8. Transforming Table 10 (AA) using these spline functions gives the two CDF's in Table 12, to compare with the observed used-oil CDF. Graphs of all three are given in Figure 7.

**Table 10. TABLE OF AA READINGS FOR TEC DFKA, Fe**

AA	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	228	.1083	228	.1083
1.5	1435	.6817	1663	.7900
2.5	405	.1924	2068	.9824
3.5	37	.0176	2105	1.0000

**Table 11. TABLE OF AE READINGS FOR TEC DFKA, Fe**

AE	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	30511	.2422	30511	.2422
1.5	55554	.4410	86065	.6832
2.5	23610	.1874	109675	.8706
3.5	8621	.0684	118296	.9390
4.5	3713	.0295	122009	.9685
5.5	1739	.0138	123748	.9823
6.5	1042	.0083	124790	.9906
7.5	536	.0043	125326	.9948
8.5	305	.0024	125631	.9973
9.5	151	.0012	125782	.9985
10.5	111	.0009	125893	.9993
11.5	51	.0004	125944	.9997
12.5	17	.0001	125961	.9999
13.5	7	.0001	125968	.9999
14.5	5	.0000	125973	1.0000
15.5	3	.0000	125976	1.0000

Table 12. SYNTHESIZED-OIL AND USED-OIL TRANSFORMATION,  
DFKA, Fe

AA	CDF value	transformed scale to AE scale	
		synthesized-oil transformation	used-oil transformation
.0	.0000	-.251	-.670
.5	.1083	.256	.326
1.5	.7900	1.270	2.318
2.5	.9824	2.283	4.310
3.5	1.0000	3.297	6.302
max diff. (location)		.2085 ( 1.27)	.1732 ( 1.50)

Because both the synthesized-oil and used-oil regressions have negative y-intercepts, the transformed AA 0 point is negative; this automatically leads to possibly large discrepancies for low ppm values. As can be seen from Figure 7, the used-oil transformation tracks the observed AE distribution much better than the synthesized-oil transformation. The plots and the results of transformation for the other TEC's are presented in Appendix D to H. Kolmogorov-Smirnov type statistics [Ref. 8] are available for comparing two sample CDF's; again because of the very large sample sizes (and in part because of rounding to integer values) both transformed CDF's differ very significantly from the observed AE CDF. This phenomenon remains true for all elements and all TEC's examined.

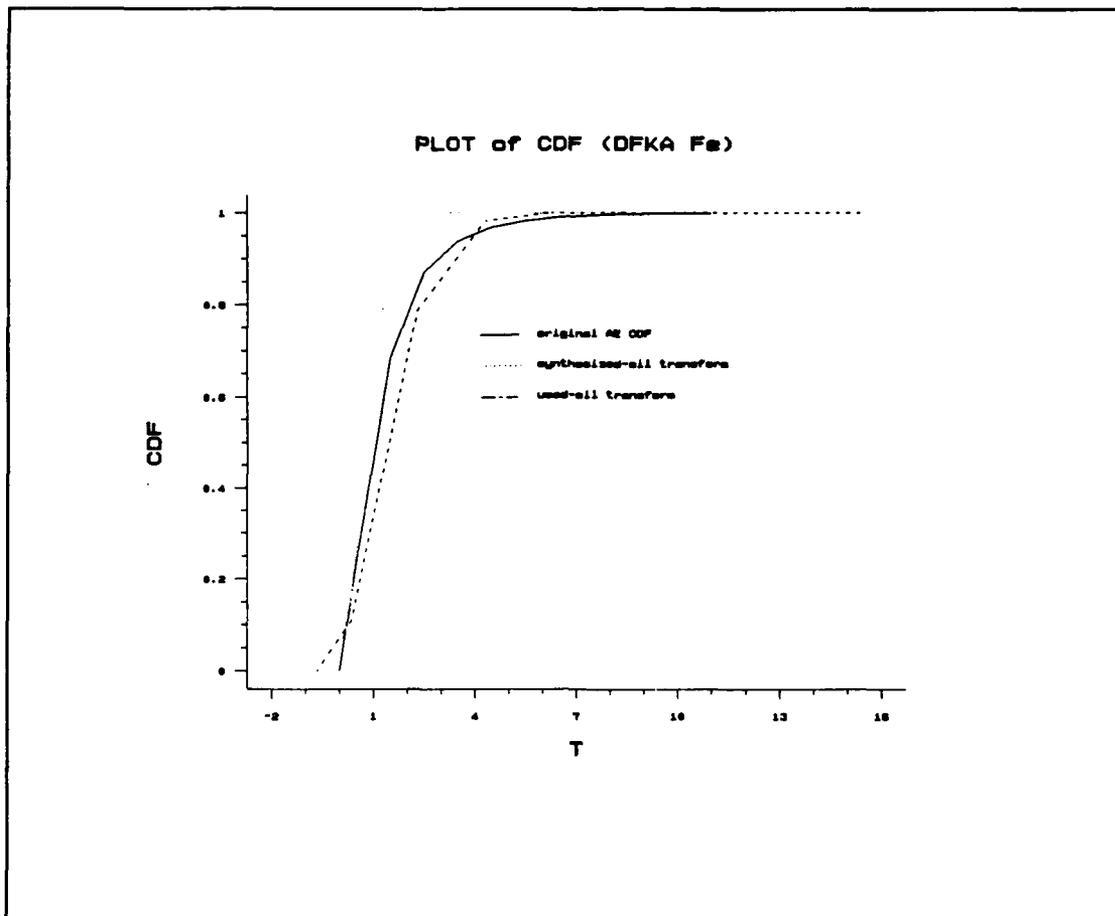


Figure 7. Plots of original AE CDF and expected CDF's from AA CDF

## V. CONCLUSIONS

The purpose of this investigation was to check the validity of correlation program sample relationships between AA and AE readings when employed with real used-oil samples. Since no data bank of used-oil readings, from both instruments, is available for this purpose, an indirect method was employed utilizing records from the B003 data bank.

The used-oil sample relationships uniformly perform better than do the synthesized-oil relationships; this was not expected because these used-oil correlation samples frequently contain no real used-oil. Perhaps the particular months of correlation samples employed here are unusual in that regard; there appears to be no definitive way of determining this.

While the used-oil sample relationships perform better, the results produced leave much to be desired. Any common statistical test comparing the actual AE values to the expected values gotten by transforming the AA values shows them to be extremely different. This is in part due to the large sample sizes available from the B003 data bank. It is also wise to keep in mind the fact that none of these B003 samples were analyzed on both instruments. Perhaps the underlying assumption that the actual contamination levels, for Fe, say, should be the same, for those engines whose samples are sent to AE laboratories as it is for those sent to AA laboratories, is not satisfied.

There are other reasons which may contribute to lack of performance for simple AA-AE relationships. As has been mentioned, the AA instrument uses

a flame to energize the oil sample, while the AE instrument employs a spark. The level of excitation accomplished by these two methods is undoubtedly different, which in turn could affect the readings produced, even though the same oil sample is employed. The fact that one instrument bases its measurement on light emitted, while the other uses light absorbed, may also cause difficulties in relating the two.

Probably the largest source of difficulty in relating AA and AE readings, for real used-oil, is the question of particle size. As an operating engine wears it may very well place tiny particles of metal into the oil; for small enough particles, many of these may in fact remain in suspension in the oil. These particles in suspension are most likely of many different sizes; indeed it is easy to conceive of a particle size distribution for those in suspension. It is quite likely that the two instruments do not "see" identical windows of these particle size distributions. This is caused in part by the different methods of exciting the sample for analysis, and also by the different methods used to introduce the sample to the energy source. The AA instruments aspirate the oil into the source, while the AE instruments fire a spark from a stationary pencil electrode to the top of a rotating disk electrode, which relies essentially on oil viscosity to carry particles to the energy source.

The most reliable way to convert between AA and AE readings, for real used-oil, is necessarily based on analyzing the same samples on both instruments. The used-oil samples employed should contain particle-size distributions typical of those to occur in the engines to be monitored, and may well differ from one TEC from another. This type of study would be fairly expensive to undertake, but may be necessary to get well-performing relationships between the readings of the two types of instruments.

**APPENDIX A. SUMMARY OF CORRELATION DATA**

**Table 13. Fe SYNTHESIZED-OIL SAMPLES**

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	12.2	11.6	12.3	11.7
APR	80.4	69.6	83.7	73.1
MAY	85.2	75.2	86.0	73.9
JUN	15.5	14.6	15.4	14.5
JUL	30.1	28.3	31.0	29.7
AUG	52.6	48.1	52.5	47.5
SEP	21.7	20.6	22.2	21.1
OCT	100.0	91.6	110.3	99.8
NOV	27.7	25.3	30.8	28.0
DEC	8.9	8.8	9.0	9.1
1981 JAN	34.2	31.2	36.5	32.5
FEB	79.7	69.7	79.6	68.8
1983 JUL	22.0	20.2	21.1	19.2
AUG	68.3	61.9	69.6	61.3
SEP	25.5	28.0	26.7	29.1
OCT	25.6	25.3	25.3	25.3
NOV	103.5	103.8	110.5	110.9
DEC	10.6	10.6	10.1	10.0
1984 FEB	62.6	59.6	63.4	60.9
MAR	36.4	36.6	36.7	36.6
APR	17.2	16.1	16.4	14.8
MAY	58.6	54.0	58.5	52.8
JUN	17.4	15.6	16.6	14.9
SEP	53.7	62.3	55.1	64.3
NOV	4.6	4.2	4.1	4.0
DEC	9.9	8.9	9.1	8.0
1985 FEB	30.0	27.6	30.0	26.7
MAR	58.0	63.3	58.5	64.9
APR	22.3	20.3	22.6	20.3
MAY	45.8	42.1	42.5	38.2
JUL	85.0	76.4	86.6	76.9
AUG	36.9	39.7	38.1	42.0
1987 DEC	39.7	38.5	41.1	39.6
1988 JAN	14.1	14.3	13.7	14.2
FEB	53.7	49.7	54.6	50.4
MAR	21.5	20.5	21.3	20.8

Table 14. Fe USED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	83.4	73.2	105.0	90.9
APR	12.2	11.3	26.7	25.5
MAY	156.0	149.7	183.0	174.0
JUN	9.3	8.1	18.8	16.5
JUL	4.8	4.5	12.1	11.3
AUG	86.2	79.5	101.7	92.7
SEP	9.1	8.8	17.3	17.0
OCT	87.6	80.8	109.7	98.7
NOV	4.0	4.0	8.2	8.2
DEC	47.8	44.7	55.5	50.2
1981 JAN	21.2	20.2	41.2	38.2
FEB	5.8	5.5	11.1	10.2
1983 JUL	4.5	4.1	7.4	6.7
AUG	67.3	60.7	87.6	77.2
SEP	18.2	16.1	28.7	26.2
OCT	10.8	10.2	20.7	20.2
NOV	12.0	12.0	24.5	24.5
DEC	4.3	4.2	7.2	7.3
1984 FEB	6.8	7.2	11.2	12.3
MAR	3.7	3.7	6.6	6.6
APR	1.9	1.9	3.0	3.4
MAY	43.3	43.2	52.8	52.4
JUN	1.9	1.7	2.8	2.7
SEP	7.9	8.1	15.5	15.5
NOV	2.6	2.6	5.0	4.9
DEC	2.0	2.0	4.0	3.9
1985 FEB	10.3	9.1	16.1	14.0
MAR	77.7	74.7	85.6	81.9
APR	5.7	5.7	10.5	10.4
MAY	6.9	6.8	11.5	11.2
JUL	4.2	4.1	9.3	9.1
AUG	3.0	3.0	6.1	6.1
1987 DEC	7.2	7.0	12.1	12.1
1988 JAN	1.7	1.7	3.3	3.2
FEB	4.5	4.5	7.3	6.9
MAR	11.1	11.1	21.3	21.2

Table 15. Ag SYNTHESIZED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	53.4	51.1	50.0	48.0
APR	41.8	36.3	40.4	35.3
MAY	45.1	39.1	41.1	35.5
JUN	10.4	9.7	10.4	9.8
JUL	47.8	45.5	46.2	44.4
AUG	31.3	28.5	30.3	29.3
SEP	15.4	14.5	15.9	15.1
OCT	11.7	10.5	11.6	10.5
NOV	27.0	24.2	28.1	25.5
DEC	3.8	3.6	3.6	3.6
1981 JAN	33.9	31.2	36.4	32.3
FEB	44.3	38.7	43.3	37.4
1983 JUL	17.8	16.8	15.9	14.4
AUG	17.5	15.6	15.9	14.1
SEP	10.7	11.8	10.3	11.3
OCT	7.1	6.7	6.3	6.2
NOV	6.2	6.3	6.0	6.0
DEC	6.1	6.0	5.7	5.7
1984 FEB	.2	.2	.0	.0
MAR	2.7	2.7	2.8	2.8
APR	10.4	9.6	9.3	8.4
MAY	8.1	7.1	7.3	6.7
JUN	11.6	10.9	11.0	10.0
SEP	19.2	22.4	18.6	22.0
NOV	27.6	26.0	25.1	23.4
DEC	14.8	13.0	13.4	11.7
1985 FEB	11.6	10.4	10.6	9.3
MAR	16.4	18.0	15.1	16.1
APR	6.8	6.1	6.6	5.9
MAY	19.8	18.2	16.1	14.6
JUL	11.2	10.2	11.4	10.3
AUG	20.7	22.1	19.7	22.0
1987 DEC	11.0	10.5	11.2	10.8
1988 JAN	14.1	14.2	13.9	14.5
FEB	10.3	9.6	10.7	9.9
MAR	16.9	16.3	17.2	16.8

Table 16. Ag USED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	.1	.4	.0	.0
APR	.0	.0	.0	.0
MAY	.6	.4	.1	.5
JUN	.3	.0	.0	.0
JUL	.3	.3	.8	.6
AUG	.4	.5	.1	.2
SEP	3.0	2.9	5.8	5.4
OCT	.9	1.0	.4	.5
NOV	2.0	2.0	3.6	3.5
DEC	.3	.2	.1	.0
1981 JAN	1.2	1.1	2.7	2.5
FEB	6.6	6.2	10.1	9.3
1983 JUL	35.2	32.5	66.8	61.9
AUG	.6	.7	.0	.0
SEP	12.3	11.1	23.1	21.0
OCT	1.0	1.0	1.9	1.9
NOV	.8	.7	.9	.9
DEC	.0	.0	.0	.0
1984 FEB	.7	.8	1.0	1.0
MAR	.0	.0	.0	.0
APR	12.9	11.5	21.6	18.2
MAY	.1	.0	.0	.0
JUN	.0	.0	.0	.0
SEP	.4	.3	.7	.8
NOV	5.0	4.5	9.2	8.5
DEC	1.0	1.0	2.0	2.0
1985 FEB	1.0	1.0	.8	.8
MAR	.0	.0	.0	.0
APR	.0	.0	.3	.3
MAY	.2	.2	.3	.1
JUL	7.2	7.2	10.6	10.4
AUG	10.1	10.0	19.5	19.6
1987 DEC	14.2	14.2	24.7	24.7
1988 JAN	2.3	2.2	4.2	4.1
FEB	1.0	1.2	.8	.8
MAR	.5	.5	.5	.5

Table 17. AI SYNTHESIZED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	50.6	47.9	47.5	45.1
APR	9.9	8.5	12.0	10.4
MAY	11.1	9.3	11.8	9.9
JUN	29.8	27.2	29.6	27.3
JUL	19.9	18.5	21.4	20.3
AUG	18.5	17.0	20.3	18.3
SEP	30.4	28.8	30.2	29.5
OCT	12.6	11.1	15.2	13.6
NOV	40.7	37.8	42.6	38.6
DEC	15.9	15.5	16.0	15.9
1981 JAN	59.8	53.7	58.6	52.2
FEB	49.7	42.0	49.4	42.3
1983 JUL	16.5	15.0	17.5	16.0
AUG	34.8	31.0	36.3	32.0
SEP	27.0	30.0	28.7	30.9
OCT	25.2	25.0	25.3	25.2
NOV	75.4	75.5	76.0	75.7
DEC	6.2	6.1	6.2	6.0
1984 FEB	79.1	76.2	75.3	71.7
MAR	29.4	29.5	29.1	29.1
APR	14.8	13.6	15.5	14.0
MAY	44.3	40.6	42.0	37.8
JUN	15.1	13.8	15.9	14.4
SEP	56.5	65.4	55.7	64.7
NOV	4.7	4.4	4.5	4.1
DEC	9.5	8.5	9.7	8.5
1985 FEB	11.6	10.4	12.4	10.6
MAR	38.5	42.3	39.7	43.5
APR	23.4	21.3	25.9	23.5
MAY	45.4	41.7	41.3	37.9
JUL	64.4	57.5	67.1	58.9
AUG	33.6	36.6	37.8	41.1
1987 DEC	40.6	39.0	43.8	41.9
1988 JAN	30.6	31.6	31.5	32.7
FEB	58.5	53.5	58.3	53.3
MAR	5.7	5.5	5.8	5.5

Table 18. AI USED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	18.2	16.1	24.5	21.5
APR	1.1	.9	.3	.3
MAY	18.9	17.9	22.4	21.7
JUN	.4	.2	.2	.1
JUL	.6	.6	.2	.2
AUG	7.7	6.8	10.4	9.2
SEP	2.8	2.6	2.5	2.2
OCT	9.5	8.7	14.2	13.0
NOV	1.8	1.8	1.6	1.6
DEC	7.9	7.4	10.6	9.6
1981 JAN	31.1	28.6	40.4	36.9
FEB	7.2	7.0	10.7	9.5
1983 JUL	2.4	2.2	.5	.3
AUG	6.0	5.4	8.1	6.9
SEP	4.5	4.0	4.4	3.9
OCT	.7	.7	.0	.0
NOV	.3	.3	.0	.0
DEC	7.3	7.3	8.6	8.7
1984 FEB	1.1	1.1	.0	.0
MAR	1.6	1.6	.1	.1
APR	1.1	1.1	.0	.0
MAY	7.5	7.5	9.3	9.3
JUN	.2	.1	.0	.0
SEP	2.6	2.5	1.1	1.0
NOV	1.1	1.0	.0	.0
DEC	.7	.7	.0	.0
1985 FEB	7.7	7.0	8.8	7.4
MAR	12.8	12.0	17.6	16.8
APR	.8	.7	.0	.0
MAY	2.6	2.5	.9	.6
JUL	.1	.0	.0	.0
AUG	1.3	1.3	.0	.0
1987 DEC	17.0	16.3	24.1	24.0
1988 JAN	2.7	2.7	1.7	1.7
FEB	1.2	1.2	.5	.5
MAR	2.5	2.3	.5	.5

Table 19. Cr SYNTHESIZED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	18.9	17.9	20.3	19.1
APR	10.3	8.8	11.4	9.7
MAY	9.9	8.8	11.0	10.4
JUN	20.3	18.5	22.0	20.1
JUL	29.4	27.8	33.9	32.3
AUG	17.8	16.0	19.4	17.6
SEP	31.1	30.5	32.0	20.9
OCT	11.3	10.2	13.0	11.6
NOV	9.0	8.2	10.7	9.5
DEC	17.1	16.9	16.0	16.0
1981 JAN	55.4	50.3	59.0	52.6
FEB	49.5	43.2	51.7	44.6
1983 JUL	23.7	22.0	23.1	20.9
AUG	23.2	20.6	24.6	21.4
SEP	10.2	11.2	10.8	11.8
OCT	41.5	40.6	40.1	40.0
NOV	25.8	26.0	29.4	29.3
DEC	6.5	6.5	5.9	5.8
1984 FEB	43.0	41.1	43.4	41.3
MAR	12.9	13.0	12.4	12.3
APR	11.6	10.7	11.1	10.0
MAY	24.0	21.8	22.7	20.3
JUN	12.9	11.7	12.6	11.3
SEP	22.9	26.4	24.4	28.1
NOV	2.0	2.0	1.8	1.6
DEC	9.4	8.5	8.8	7.6
1985 FEB	11.8	10.4	11.4	9.8
MAR	21.0	23.0	22.3	24.6
APR	9.7	8.9	10.9	9.7
MAY	38.0	34.8	35.0	32.0
JUL	23.8	21.3	26.7	23.3
AUG	12.3	13.2	14.0	15.3
1987 DEC	22.6	21.6	25.3	24.3
1988 JAN	27.5	27.8	27.8	29.1
FEB	16.2	15.2	18.4	16.8
MAR	1.9	2.1	1.7	1.7

Table 20. Cr USED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	4.6	4.0	8.4	7.1
APR	.0	.0	.4	.3
MAY	5.3	5.0	7.7	7.6
JUN	1.2	1.0	1.9	1.7
JUL	.3	.1	.7	.6
AUG	3.1	2.7	4.2	3.7
SEP	3.7	3.4	4.8	4.3
OCT	3.3	3.1	4.6	4.3
NOV	2.3	2.3	3.7	3.6
DEC	2.3	2.1	2.1	2.0
1981 JAN	36.5	33.6	40.6	36.6
FEB	8.4	8.0	10.8	9.9
1983 JUL	2.0	2.0	3.0	2.7
AUG	4.9	4.5	7.1	6.3
SEP	4.9	4.3	5.9	5.1
OCT	1.0	1.0	1.0	.9
NOV	.6	.6	.7	.6
DEC	.4	.4	.2	.2
1984 FEB	1.4	1.6	1.5	1.7
MAR	.6	.6	.8	.8
APR	.0	.0	.0	.0
MAY	2.8	2.8	3.6	3.6
JUN	2.5	2.5	4.6	4.6
SEP	2.9	2.8	4.0	4.0
NOV	1.0	1.0	1.0	1.0
DEC	.0	.0	.0	.0
1985 FEB	8.6	7.6	11.9	10.3
MAR	3.0	3.0	4.0	3.8
APR	.1	.1	.3	.3
MAY	1.9	1.8	2.5	2.3
JUL	.2	.0	.1	.0
AUG	.0	.0	.0	.0
1987 DEC	18.9	18.7	24.0	23.9
1988 JAN	2.7	2.6	3.5	3.5
FEB	1.3	1.5	1.3	1.3
MAR	1.5	1.5	1.3	1.3

Table 21. Cu SYNTHESIZED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	12.6	12.0	12.4	11.7
APR	80.5	70.0	77.8	67.9
MAY	83.1	75.5	79.1	68.4
JUN	29.0	27.2	28.9	26.8
JUL	21.0	19.3	21.0	19.8
AUG	51.8	47.5	51.1	46.7
SEP	21.8	21.8	23.0	21.8
OCT	102.3	94.2	97.2	88.2
NOV	27.5	25.0	30.2	27.3
DEC	9.0	8.6	9.5	9.4
1981 JAN	33.3	30.2	36.4	32.4
FEB	43.6	38.1	44.0	38.0
1983 JUL	16.8	15.1	17.1	15.3
AUG	46.8	41.7	48.2	42.7
SEP	21.0	22.7	23.5	25.7
OCT	65.5	64.5	60.7	60.5
NOV	12.2	12.3	14.4	14.4
DEC	6.0	5.9	6.2	6.2
1984 FEB	67.7	64.7	67.6	64.7
MAR	30.8	30.7	32.1	32.0
APR	15.3	13.8	15.5	14.1
MAY	13.5	12.4	14.4	12.9
JUN	11.8	10.7	12.2	11.0
SEP	74.9	86.3	73.8	85.6
NOV	30.6	28.4	29.4	27.7
DEC	15.0	13.5	14.9	13.0
1985 FEB	11.6	10.1	11.8	10.1
MAR	74.6	82.1	71.4	78.4
APR	34.5	31.7	36.1	32.7
MAY	45.1	41.7	40.8	37.4
JUL	22.0	19.7	24.5	21.5
AUG	58.2	62.4	57.8	63.3
1987 DEC	50.9	49.3	52.8	51.1
1988 JAN	35.5	36.3	35.8	37.4
FEB	25.0	23.3	26.8	24.4
MAR	76.1	73.1	72.3	71.1

Table 22. Cu USED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	8.6	7.4	10.5	9.0
APR	12.7	12.0	24.5	23.1
MAY	10.3	9.6	12.7	12.4
JUN	1.1	1.0	.1	.0
JUL	.7	.6	1.3	1.2
AUG	5.5	4.8	6.8	5.9
SEP	5.1	4.7	9.2	8.4
OCT	8.3	7.5	5.5	5.0
NOV	2.9	3.0	6.4	6.3
DEC	4.5	4.2	3.4	3.0
1981 JAN	4.0	3.8	7.2	6.4
FEB	3.2	3.1	7.1	6.4
1983 JUL	2.8	2.5	4.0	3.7
AUG	4.7	4.1	6.3	5.4
SEP	5.2	4.5	8.3	7.5
OCT	3.9	3.4	6.6	6.5
NOV	3.1	3.2	6.1	6.1
DEC	22.1	22.3	38.8	38.9
1984 FEB	3.0	3.3	5.3	5.7
MAR	5.7	5.6	11.3	11.2
APR	18.8	16.9	34.2	30.5
MAY	4.3	4.2	5.4	5.3
JUN	1.0	1.0	1.0	1.0
SEP	5.6	5.6	10.3	10.3
NOV	7.6	7.0	14.3	13.6
DEC	5.8	5.8	11.4	11.4
1985 FEB	12.0	10.6	17.8	15.5
MAR	5.3	5.2	6.3	6.0
APR	5.2	5.1	9.0	9.0
MAY	8.2	8.1	12.6	12.3
JUL	1.4	1.3	2.5	2.4
AUG	8.3	8.3	15.5	15.5
1987 DEC	19.5	19.2	34.7	34.8
1988 JAN	3.5	3.5	7.0	6.8
FEB	32.0	29.3	53.6	49.4
MAR	6.7	6.5	12.7	12.8

Table 23. Mg SYNTHESIZED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	56.5	53.6	49.9	48.2
APR	41.5	36.2	41.3	36.7
MAY	44.2	38.8	42.2	36.3
JUN	10.3	9.5	11.4	10.5
JUL	30.0	28.2	30.6	29.2
AUG	28.5	26.0	29.3	26.6
SEP	14.0	13.2	15.1	14.5
OCT	13.0	11.8	14.0	12.5
NOV	45.5	41.5	48.1	44.7
DEC	3.2	3.3	3.8	3.7
1981 JAN	33.9	30.6	39.2	35.3
FEB	67.0	58.9	64.6	56.4
1983 JUL	22.0	20.2	21.6	19.8
AUG	26.9	24.2	28.2	25.1
SEP	72.3	78.6	67.0	73.1
OCT	40.1	39.7	39.0	38.7
NOV	43.0	43.0	44.2	44.3
DEC	10.6	10.4	10.7	10.7
1984 FEB	79.2	75.2	72.0	69.4
MAR	47.0	46.9	45.5	45.5
APR	19.6	18.2	19.9	18.1
MAY	51.0	47.5	48.7	44.8
JUN	16.8	15.3	16.6	15.2
SEP	72.6	83.7	68.3	79.3
NOV	30.2	28.6	29.5	27.9
DEC	15.0	13.5	15.2	13.4
1985 FEB	24.3	21.4	23.6	20.9
MAR	67.4	72.9	61.9	68.1
APR	24.5	22	26.0	23.5
MAY	45.3	42	41.0	38.0
JUL	41.8	37.4	45.4	40.6
AUG	79.6	85.5	74.3	81.5
1987 DEC	30.9	29.9	32.7	31.6
1988 JAN	30.8	31.7	30.8	32.1
FEB	52.9	49.0	50.0	46.7
MAR	21.9	20.7	22.0	21.4

Table 24. Mg USED-OIL SAMPLES

MONTH	ABSORPTION		EMISSION	
	SAMPLE 1	SAMPLE 2	SAMPLE 1	SAMPLE 2
1980 MAR	11.2	9.4	17.3	14.9
APR	.7	.6	1.0	1.0
MAY	18.9	18.0	24.5	23.7
JUN	1.8	1.8	2.1	1.9
JUL	.2	.1	.9	.7
AUG	30.4	27.7	38.1	34.4
SEP	4.9	4.4	10.6	9.7
OCT	152.8	140.3	189.5	170.0
NOV	3.0	2.9	6.9	6.7
DEC	33.1	30.6	40.1	36.6
1981 JAN	4.2	3.9	8.1	7.4
FEB	1.4	1.4	3.1	2.6
1983 JUL	38.6	35.6	70.8	65.1
AUG	72.8	65.9	96.4	85.3
SEP	41.7	39.4	69.6	64.8
OCT	5.8	5.6	10.9	10.5
NOV	1.2	1.2	2.5	2.4
DEC	4.3	4.3	8.5	8.6
1984 FEB	6.1	6.4	11.6	12.6
MAR	3.5	3.6	6.9	6.8
APR	12.6	11.4	27.7	24.8
MAY	5.1	5.0	6.5	6.4
JUN	3.3	3.3	6.4	6.4
SEP	3.4	3.5	6.3	6.3
NOV	9.3	8.9	20.0	19.1
DEC	2.8	2.8	5.4	5.5
1985 FEB	63.7	55.5	109.8	97.6
MAR	8.4	7.9	10.5	10.0
APR	7.3	7.2	14.2	14.2
MAY	14.0	13.9	28.2	27.5
JUL	.4	.3	.7	.7
AUG	19.1	19.0	41.8	41.9
1987 DEC	4.7	4.7	9.1	9.0
1988 JAN	1.5	1.5	3.3	3.2
FEB	34.0	31.2	66.2	60.5
MAR	15.9	16.1	33.7	33.7

APPENDIX B. SCATTER PLOTS OF CORRELATION DATA

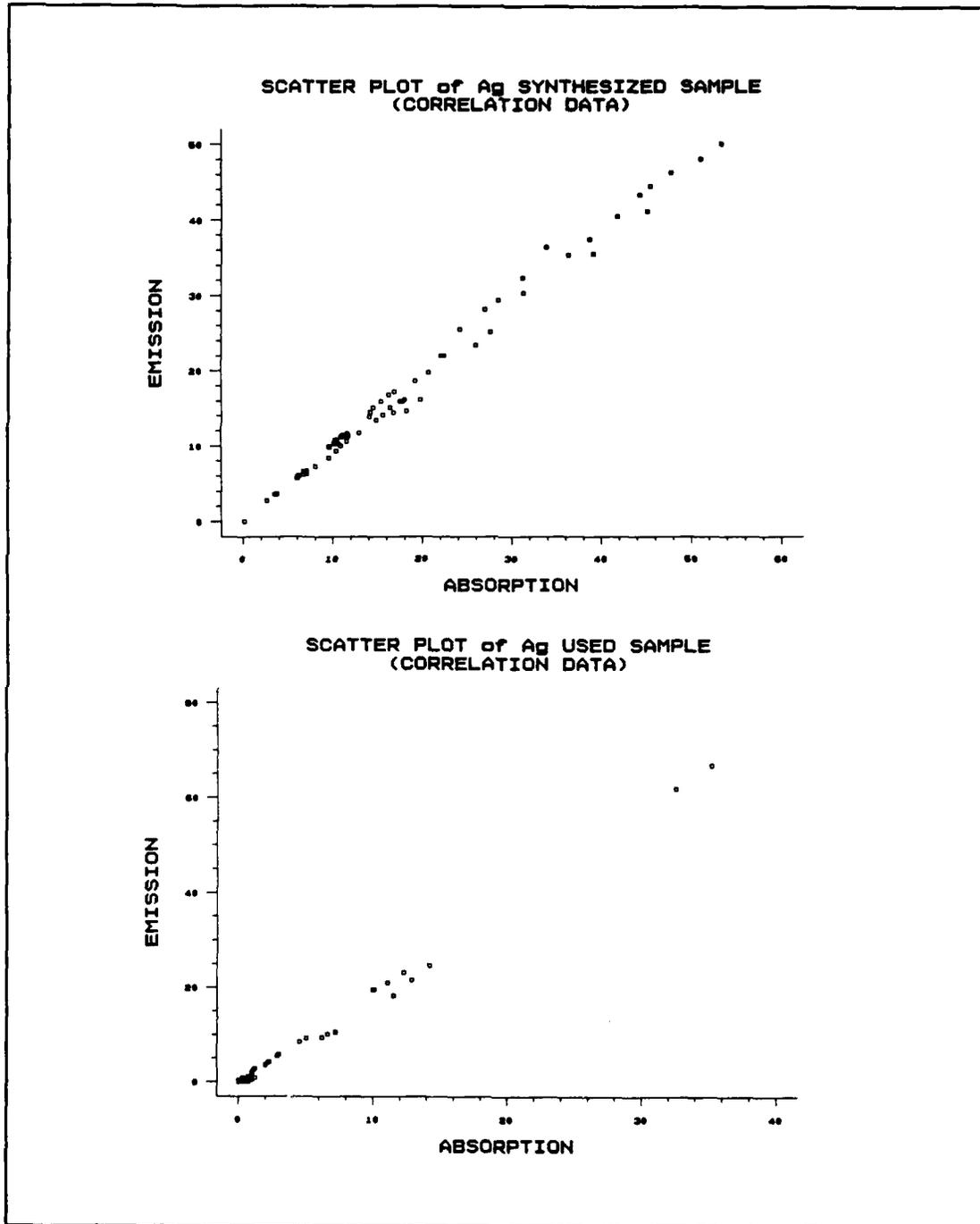


Figure 8. Scatter plots of Ag synthesized and used-oil samples

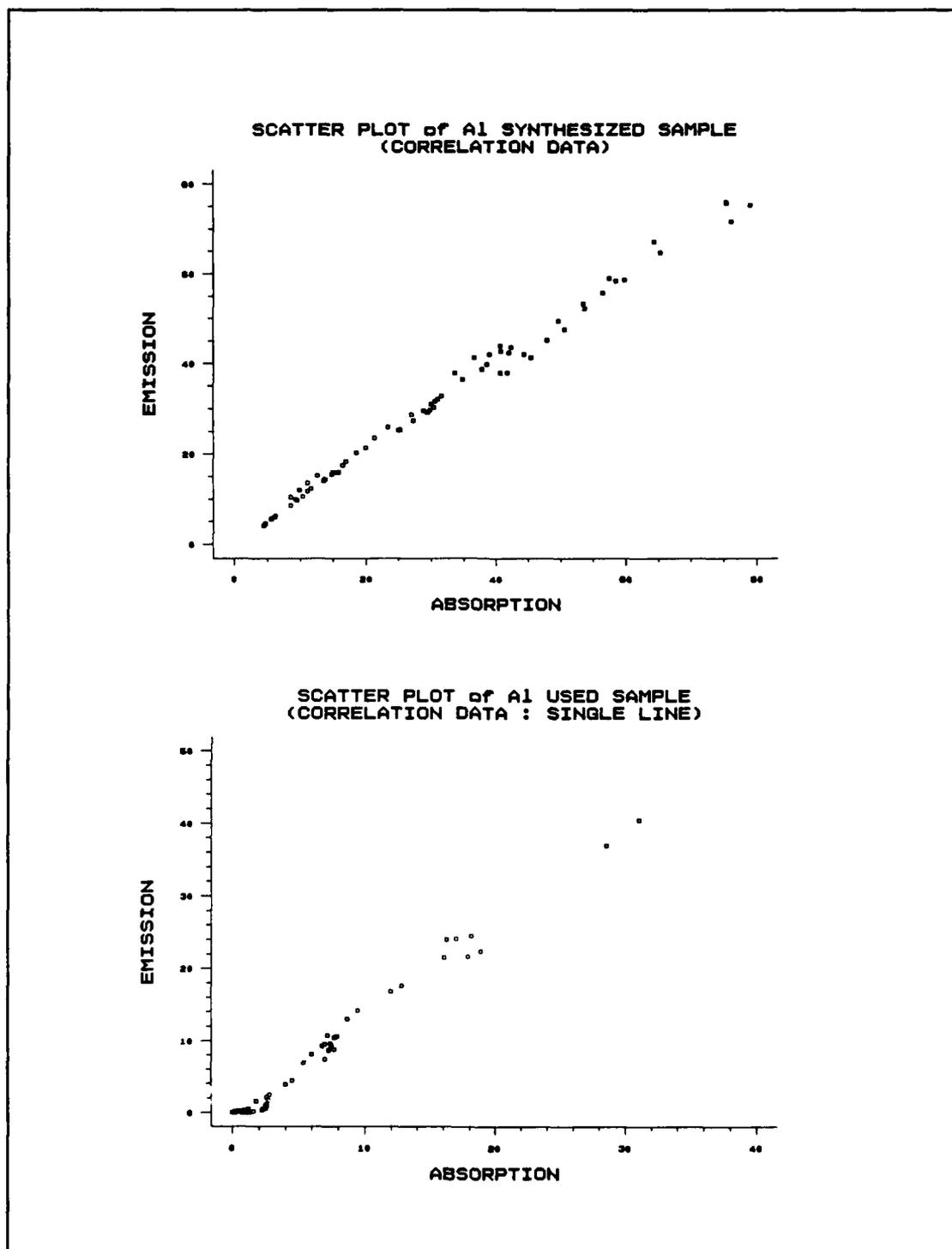


Figure 9. Scatter plots of Al synthesized and used-oil samples

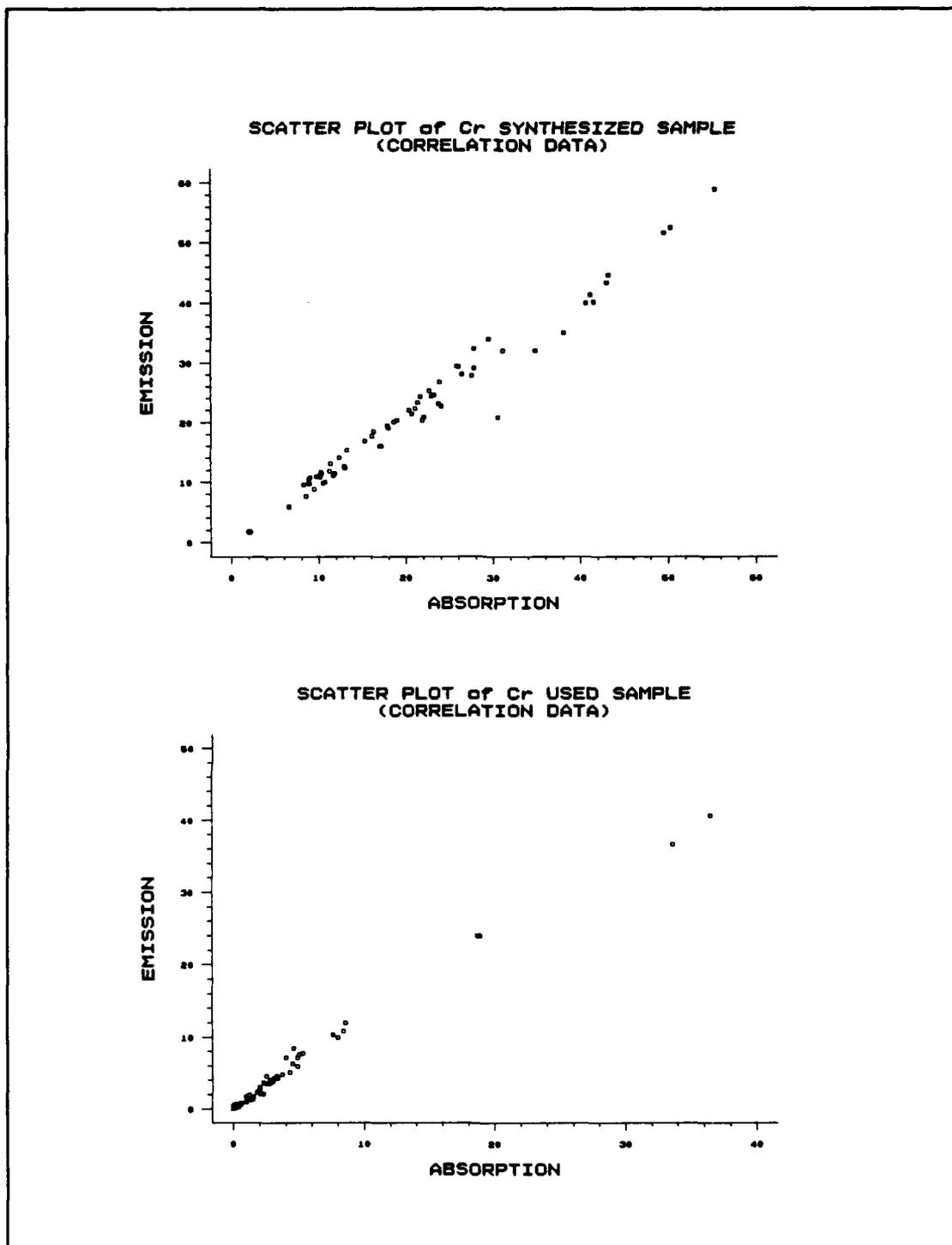


Figure 10. Scatter plots of Cr synthesized and used-oil samples

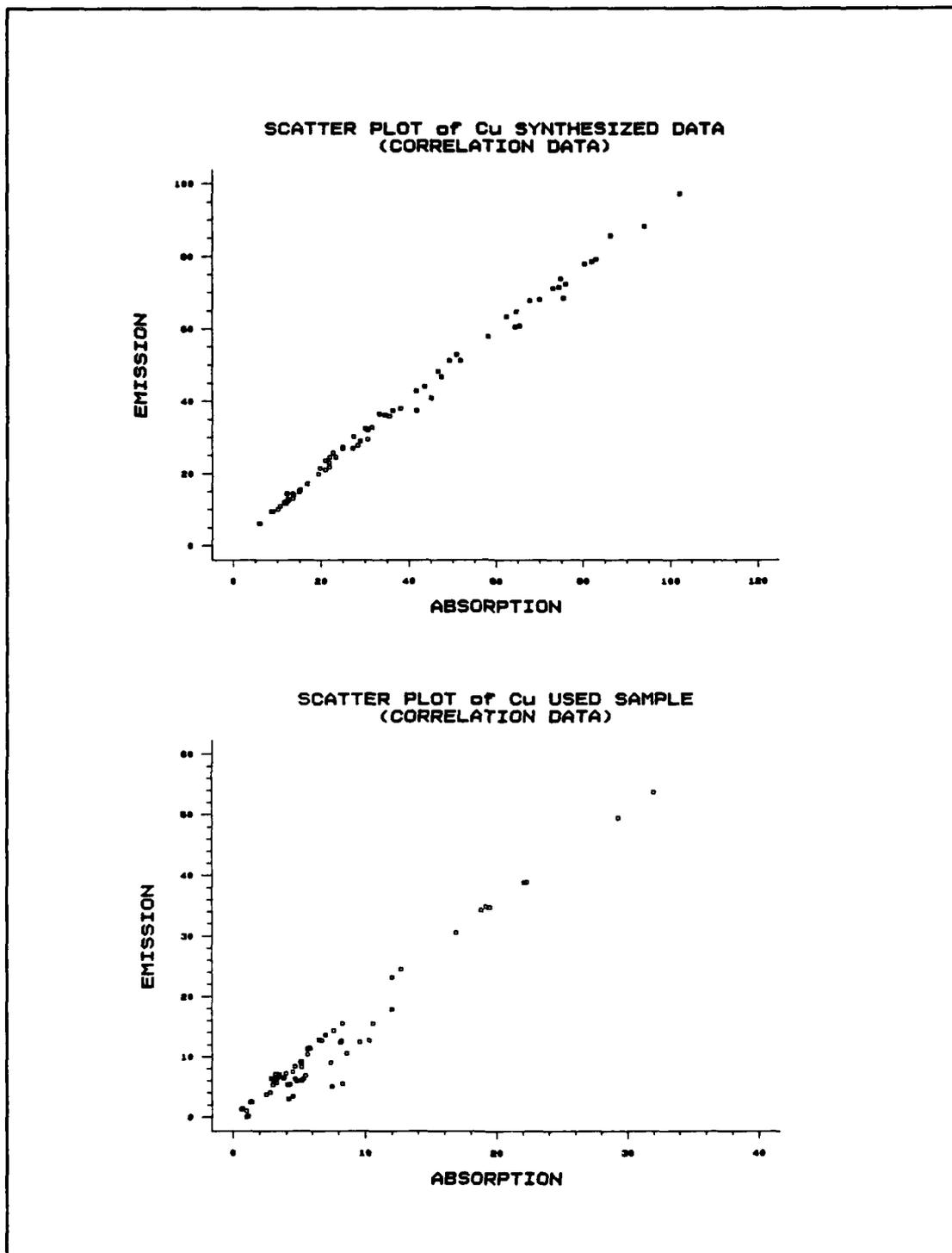


Figure 11. Scatter plots of Cu synthesized and used-oil samples

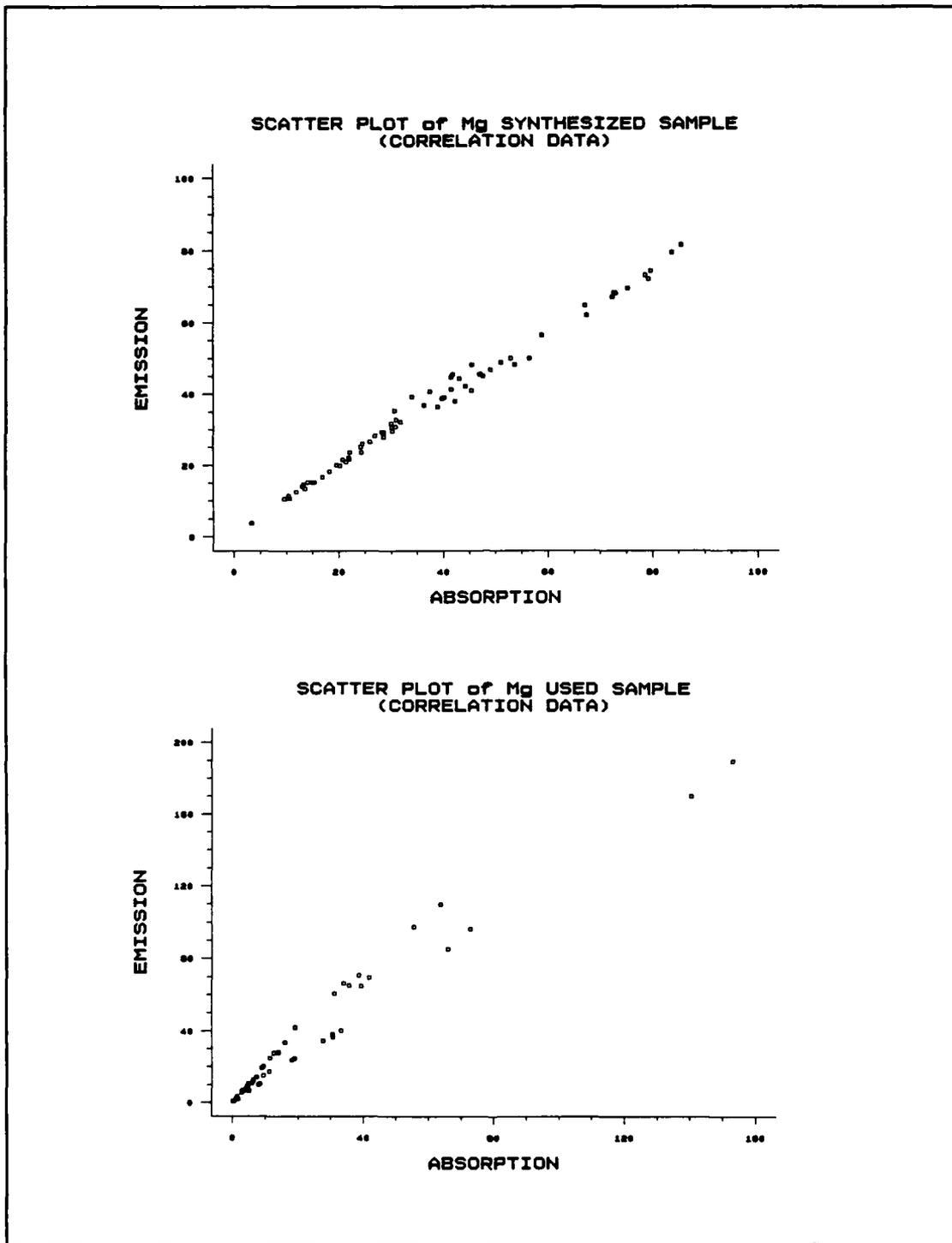


Figure 12. Scatter plots of Mg synthesized and used-oil samples

APPENDIX C. ESTIMATED PARAMETERS (CORRELATION DATA)

Table 25. ESTIMATED PARAMETERS (SINGLE LINE)

IT	synthesized-oil			used-oil		
	y-intercept	slope	sum of squares	y-intercept	slope	sum of squares
Fe	-1.314	1.051	112.515	4.627	1.151	456.820
Ag	-.070	.964	47.018	-.460	1.874	13.065
Al	1.271	.971	99.689	-1.209	1.390	27.692
Cr	.108	1.027	132.439	.437	1.154	22.286
Cu	1.884	.944	102.779	-1.042	1.788	87.242
Mg	2.453	.913	139.482	3.949	1.339	1414.515

Table 26. ESTIMATED PARAMETERS FOR SYNTHESIZED-OIL SAMPLES (SPLINE)

IT	left line		right line		best knot (AA)	sum of squares
	y-intercept	slope	y-intercept	slope		
Fe	-.251	1.014	-26.342	1.341	79.70	61.520
Ag	-.560	1.006	4.745	.849	33.90	43.397
Al	-1.569	1.261	1.972	.956	11.60	92.326
Cr	.404	1.007	-7.469	1.197	41.50	128.525
Cu	-.671	1.094	3.396	.919	23.30	85.146
Mg	.060	1.026	5.160	.865	31.70	110.886

Table 27. ESTIMATED PARAMETERS FOR USED-OIL SAMPLES  
(SPLINE)

IT	left line		right line		best knot (AA)	sum of squares
	y-intercept	slope	y-intercept	slope		
Fe	-.670	1.992	9.521	1.090	11.30	261.615
Ag	-.190	1.689	-2.171	1.964	7.20	9.734
Al	-1.555	1.511	1.826	1.230	12.00	23.033
Cr	.066	1.315	8.813	.852	18.90	9.139
Cu	-1.516	1.878	5.701	1.494	18.80	82.611
Mg	.327	1.707	27.855	1.047	41.70	692.184

APPENDIX D. SUMMARY OF COMPARISON FOR TEC BSPA, Fe

Table 28. EXPECTED MEAN AND VARIANCE OF AE READINGS FROM AA TRANSFORMATIONS, BSPA, Fe

EXPECTED VALUES	parameters from	mean	variance
	synthesized-oil	1.0847	.9738
	used-oil	1.9549	3.7613
ORIGINAL VALUES	AE	2.5812	3.4880
	AA	1.3176	.9478

Table 29. TABLE OF AA READINGS FOR TEC BSPA, Fe

AA	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	1225	.1179	1225	.1179
1.5	6709	.6457	7934	.7635
2.5	1434	.1380	9368	.9015
3.5	579	.0557	9947	.9573
4.5	280	.0269	10227	.9842
5.5	78	.0075	10305	.9917
6.5	59	.0057	10364	.9974
7.5	13	.0013	10377	.9987
8.5	10	.0010	10387	.9996
9.5	4	.0004	10391	1.0000

Table 30. TABLE OF AE READINGS FOR TEC BSPA, Fe

AE	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	479	.0184	479	.0184
1.5	5727	.2202	6206	.2386
2.5	8928	.3432	15134	.5818
3.5	6064	.2331	21198	.8149
4.5	2630	.1011	23828	.9160
5.5	1020	.0392	24848	.9553
6.5	456	.0175	25304	.9728
7.5	249	.0096	25553	.9824
8.5	149	.0057	25702	.9881
9.5	87	.0033	25789	.9914
10.5	73	.0028	25862	.9942
11.5	45	.0017	25907	.9960
12.5	19	.0007	25926	.9967
13.5	20	.0008	25946	.9975
14.5	9	.0003	25955	.9978
15.5	4	.0002	25959	.9980
16.5	4	.0002	25963	.9981
17.5	3	.0001	25966	.9982
18.5	3	.0001	25969	.9983
19.5	4	.0002	25973	.9985
20.5	18	.0007	25991	.9992
21.5	1	.0000	25992	.9992
22.5	1	.0000	25993	.9993
23.5	1	.0000	25994	.9993
24.5	1	.0000	25995	.9993
28.5	1	.0000	25996	.9994
30.5	6	.0002	26002	.9996
31.5	1	.0000	26003	.9997
32.5	1	.0000	26004	.9997
34.5	1	.0000	26005	.9997
35.5	1	.0000	26006	.9998
40.5	6	.0002	26012	1.0000

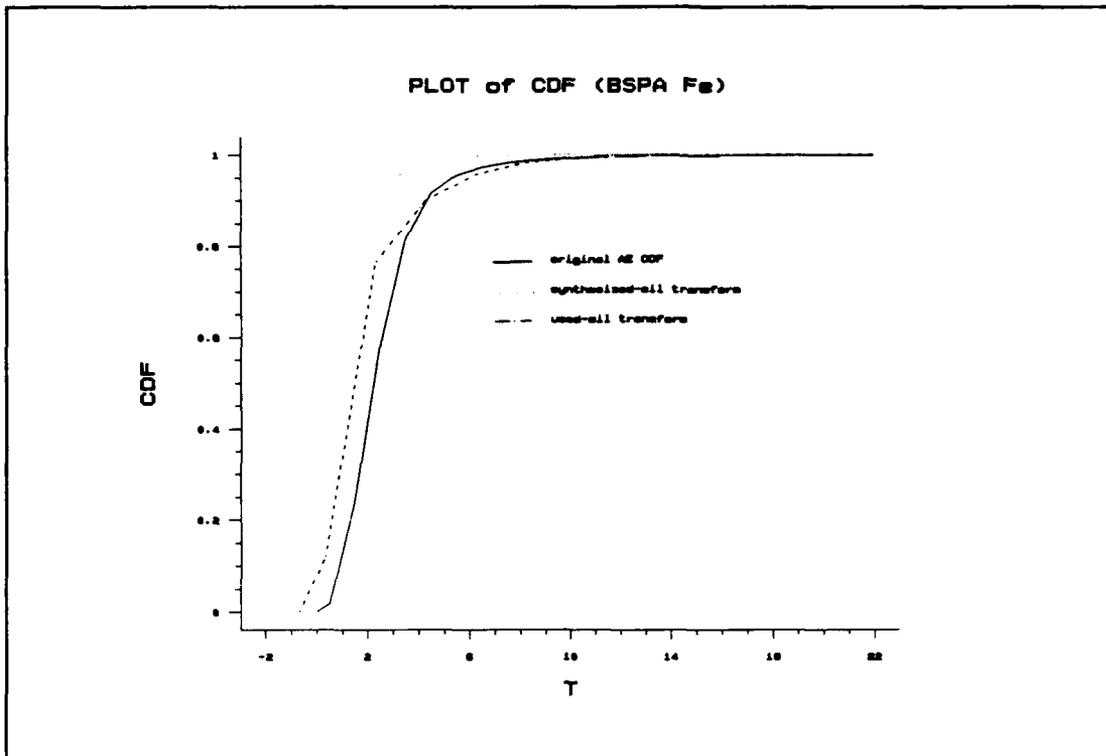


Figure 13. Plots of original and expected AE CDF from AA CDF, BSPA

Table 31. SYNTHESIZED-OIL AND USED-OIL TRANSFORMATION, BSPA, Fe

AA	CDF value	transformed scale to AE scale	
		synthesized-oil transformation	used-oil transformation
.0	.0000	-.251	-.670
.5	.1179	.256	.326
1.5	.7635	1.270	2.318
2.5	.9015	2.283	4.310
3.5	.9573	3.297	6.302
4.5	.9842	4.310	8.295
5.5	.9917	5.324	10.287
6.5	.9974	6.337	12.279
7.5	.9987	7.351	14.271
8.5	.9996	8.365	16.263
9.5	1.0000	9.378	18.255
max diff. (location)		.5757 ( 1.27)	.2598 ( 1.50)

APPENDIX E. SUMMARY OF COMPARISON FOR TEC BSQA, Fe

Table 32. EXPECTED MEAN AND VARIANCE OF AE READINGS FROM AA TRANSFORMATIONS, BSQA, Fe

EXPECTED VALUES	parameters from	mean	variance
	synthesized-oil	1.0941	.5697
	used-oil	1.9735	2.2005
ORIGINAL VALUES	AE	3.0861	4.2790
	AA	1.3269	.5545

Table 33. TABLE OF AA READINGS FOR TEC BSQA, Fe

AA	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	291	.0499	291	.0499
1.5	3964	.6804	4255	.7303
2.5	1185	.2034	5440	.9337
3.5	271	.0465	5711	.9803
4.5	76	.0130	5787	.9933
5.5	29	.0050	5816	.9983
6.5	8	.0014	5824	.9997
7.5	2	.0003	5826	1.0000

Table 34. TABLE OF AE READINGS FOR TEC BSQA, Fe

AE	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	502	.0119	502	.0119
1.5	6856	.1622	7358	.1741
2.5	11827	.2798	19185	.4539
3.5	9903	.2343	29088	.6882
4.5	6065	.1435	35153	.8317
5.5	3266	.0773	38419	.9089
6.5	1691	.0400	40110	.9489
7.5	911	.0216	41021	.9705
8.5	438	.0104	41459	.9809
9.5	271	.0064	41730	.9873
10.5	176	.0042	41906	.9914
11.5	122	.0029	42028	.9943
12.5	80	.0019	42108	.9962
13.5	41	.0010	42149	.9972
14.5	24	.0006	42173	.9978
15.5	17	.0004	42190	.9982
16.5	9	.0002	42199	.9984
17.5	2	.0000	42201	.9984
18.5	2	.0000	42203	.9985
19.5	3	.0001	42206	.9985
20.5	10	.0002	42216	.9988
21.5	5	.0001	42221	.9989
22.5	7	.0002	42228	.9991
23.5	4	.0001	42232	.9991
24.5	6	.0001	42238	.9993
25.5	4	.0001	42242	.9994
26.5	3	.0001	42245	.9995
27.5	2	.0000	42247	.9995
28.5	3	.0001	42250	.9996
29.5	1	.0000	42251	.9996
30.5	6	.0001	42257	.9997
31.5	2	.0000	42259	.9998
32.5	2	.0000	42261	.9998
33.5	4	.0001	42265	.9999
36.5	3	.0001	42268	1.0000

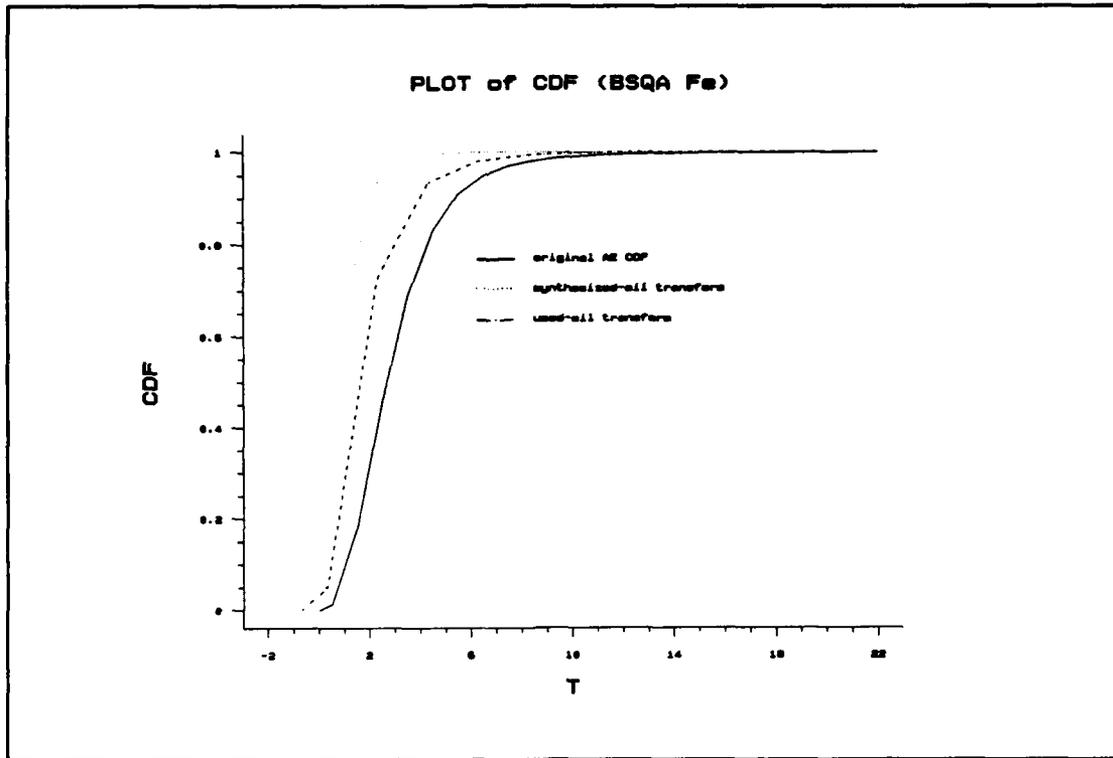


Figure 14. Plots of original and expected AE CDF from AA CDF, BSQA

Table 35. SYNTHESIZED-OIL AND USED-OIL TRANSFORMATION, BSQA, Fe

AA	CDF value	transformed scale to AE scale	
		synthesized-oil transformation	used-oil transformation
.0	.0000	-.251	-.670
.5	.0499	.256	.326
1.5	.7303	1.270	2.318
2.5	.9337	2.283	4.310
3.5	.9803	3.297	6.302
4.5	.9933	4.310	8.295
5.5	.9983	5.324	10.287
6.5	.9997	6.337	12.279
7.5	1.0000	7.351	14.271
max diff. (location)		.6025 ( 1.50)	.3273 ( 2.32)

APPENDIX F. SUMMARY OF COMPARISON FOR TEC EPJA, Fe

Table 36. EXPECTED MEAN AND VARIANCE OF AE READINGS FROM AA TRANSFORMATIONS, EPJA, Fe

EXPECTED VALUES	parameters from	mean	variance
	synthesized-oil	.2156	.1219
	used-oil	.2469	.4710
ORIGINAL VALUES	AE	1.3867	2.0843
	AA	.4602	.1187

Table 37. TABLE OF AA READINGS FOR TEC EPJA, Fe

AA	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	259	.7235	259	.7235
1.5	98	.2737	357	.9972
2.5	1	.0028	358	1.0000

Table 38. TABLE OF AE READINGS FOR TEC EPJA, Fe

AE	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	26052	.3241	26052	.3241
1.5	29684	.3692	55736	.6933
2.5	12477	.1552	68213	.8485
3.5	5987	.0745	74200	.9229
4.5	2945	.0366	77145	.9596
5.5	1403	.0175	78548	.9770
6.5	749	.0093	79297	.9863
7.5	486	.0060	79783	.9924
8.5	252	.0031	80035	.9955
9.5	158	.0020	80193	.9975
10.5	85	.0011	80278	.9985
11.5	58	.0007	80336	.9993
12.5	31	.0004	80367	.9997
13.5	12	.0001	80379	.9998
14.5	16	.0002	80395	1.0000

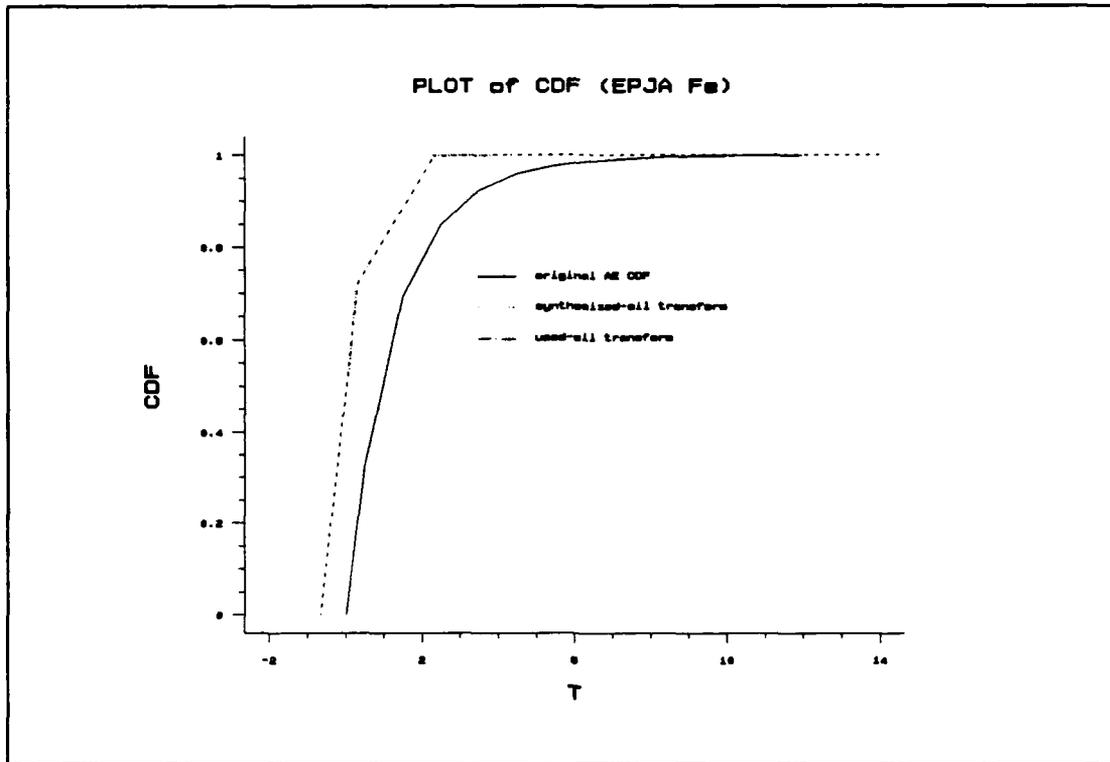


Figure 15. Plots of original and expected AE CDF from AA CDF, EPJA

Table 39. SYNTHESIZED-OIL AND USED-OIL TRANSFORMATION, EPJA, Fe

AA	CDF value	transformed scale to AE scale	
		synthesized-oil transformation	used-oil transformation
.0	.0000	-.251	-.670
.5	.7235	.256	.326
1.5	.9972	1.270	2.318
2.5	1.0000	2.283	4.310
max diff. (location)		.5576 (.26)	.5121 (.33)

APPENDIX G. SUMMARY OF COMPARISON FOR TEC LLDA, Fe

Table 40. EXPECTED MEAN AND VARIANCE OF AE READINGS FROM AA TRANSFORMATIONS, LLDA, Fe

EXPECTED VALUES	parameters from	mean	variance
	synthesized-oil	2.0350	1.7155
	used-oil	3.8227	6.6266
ORIGINAL VALUES	AE	3.8290	8.1638
	AA	2.2552	1.6698

Table 41. TABLE OF AA READINGS FOR TEC LLDA, Fe

AA	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	46	.0566	46	.0566
1.5	164	.2017	210	.2583
2.5	329	.4047	539	.6630
3.5	158	.1943	697	.8573
4.5	94	.1156	791	.9729
5.5	6	.0074	797	.9803
6.5	5	.0062	802	.9865
7.5	4	.0049	806	.9914
8.5	3	.0037	809	.9951
9.5	3	.0037	812	.9988
11.5	1	.0012	813	1.0000

Table 42. TABLE OF AE READINGS FOR TEC LLDA, Fe

AE	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	359	.0582	359	.0582
1.5	877	.1421	1236	.2003
2.5	1179	.1910	2415	.3913
3.5	1027	.1664	3442	.5577
4.5	772	.1251	4214	.6828
5.5	524	.0849	4738	.7677
6.5	411	.0666	5149	.8343
7.5	360	.0583	5509	.8926
8.5	252	.0408	5761	.9334
9.5	149	.0241	5910	.9576
10.5	89	.0144	5999	.9720
11.5	43	.0070	6042	.9789
12.5	42	.0068	6084	.9857
13.5	25	.0041	6109	.9898
14.5	21	.0034	6130	.9932
15.5	17	.0028	6147	.9959
16.5	6	.0010	6153	.9969
17.5	10	.0016	6163	.9985
18.5	5	.0008	6168	.9994
19.5	2	.0003	6170	.9997
20.5	1	.0002	6171	.9998
21.5	1	.0002	6172	1.0000

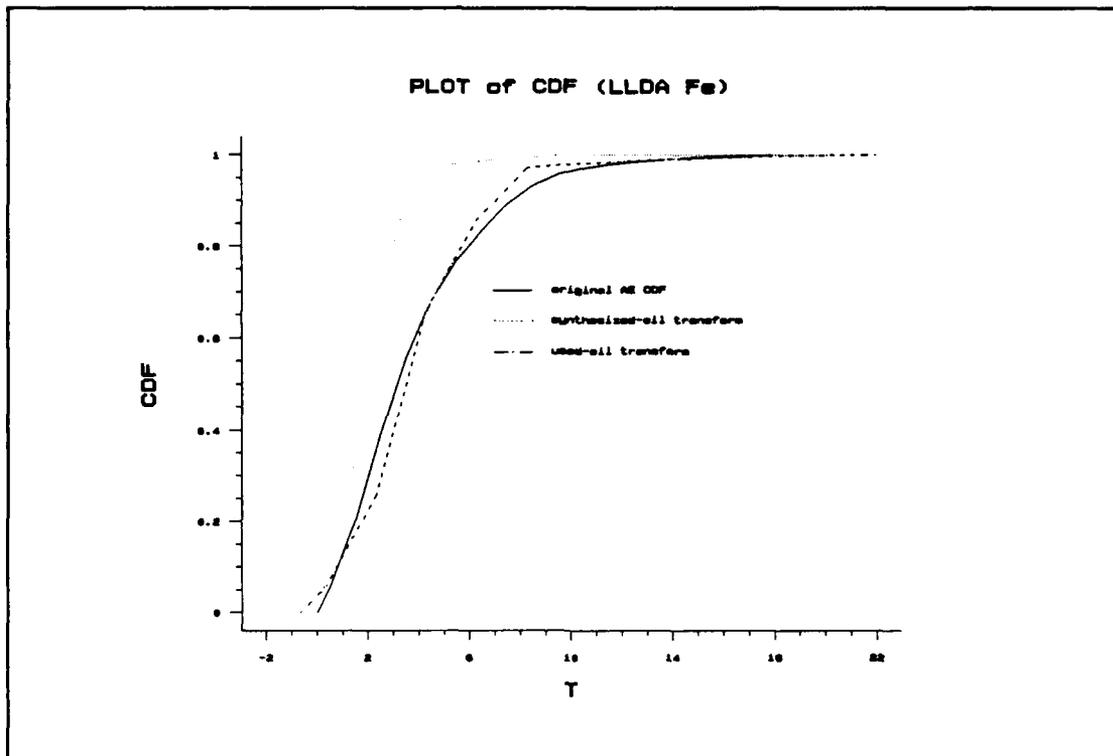


Figure 16. Plots of original and expected AE CDF from AA CDF, LLDA

Table 43. SYNTHESIZED-OIL AND USED-OIL TRANSFORMATION, LLDA, Fe

AA	CDF value	transformed scale to AE scale	
		synthesized-oil transformation	used-oil transformation
.0	.0000	-2.251	-6.670
.5	.0566	.256	.326
1.5	.2583	1.270	2.318
2.5	.6630	2.283	4.310
3.5	.8573	3.297	6.302
4.5	.9729	4.310	8.295
5.5	.9803	5.324	10.287
6.5	.9865	6.337	12.279
7.5	.9914	7.351	14.271
8.5	.9951	8.365	16.263
9.5	.9988	9.378	18.255
11.5	1.0000	11.405	22.059
max diff. (location)		.3335 ( 3.30)	.0983 ( 2.32)

APPENDIX H. SUMMARY OF COMPARISON FOR TEC LLLA, Fe

Table 44. EXPECTED MEAN AND VARIANCE OF AE READINGS FROM AA TRANSFORMATIONS, LLLA, Fe

EXPECTED VALUES	parameters from	mean	variance
	synthesized-oil	1.8958	1.4929
	used-oil	3.5491	5.7668
ORIGINAL VALUES	AE	4.4814	8.8539
	AA	2.1179	1.4532

Table 45. TABLE OF AA READINGS FOR TEC LLLA, Fe

AA	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	26	.0390	26	.0390
1.5	199	.2988	225	.3378
2.5	224	.3363	449	.6742
3.5	158	.2372	607	.9114
4.5	33	.0495	640	.9610
5.5	15	.0225	655	.9835
6.5	4	.0060	659	.9895
7.5	4	.0060	663	.9955
8.5	3	.0045	666	1.0000

Table 46. TABLE OF AE READINGS FOR TEC LLLA, Fe

AE	FREQUENCY	DENSITY	CUMULATIVE FREQUENCY	CDF value
.0	0	.0000	0	.0000
.5	42	.0209	42	.0209
1.5	226	.1122	268	.1331
2.5	372	.1847	640	.3178
3.5	288	.1430	928	.4608
4.5	217	.1077	1145	.5685
5.5	218	.1082	1363	.6768
6.5	211	.1048	1574	.7815
7.5	129	.0641	1703	.8456
8.5	106	.0526	1809	.8982
9.5	67	.0333	1876	.9315
10.5	45	.0223	1921	.9538
11.5	21	.0104	1942	.9643
12.5	33	.0164	1975	.9806
13.5	25	.0124	2000	.9930
14.5	11	.0055	2011	.9985
15.5	1	.0005	2012	.9990
16.5	2	.0010	2014	1.0000

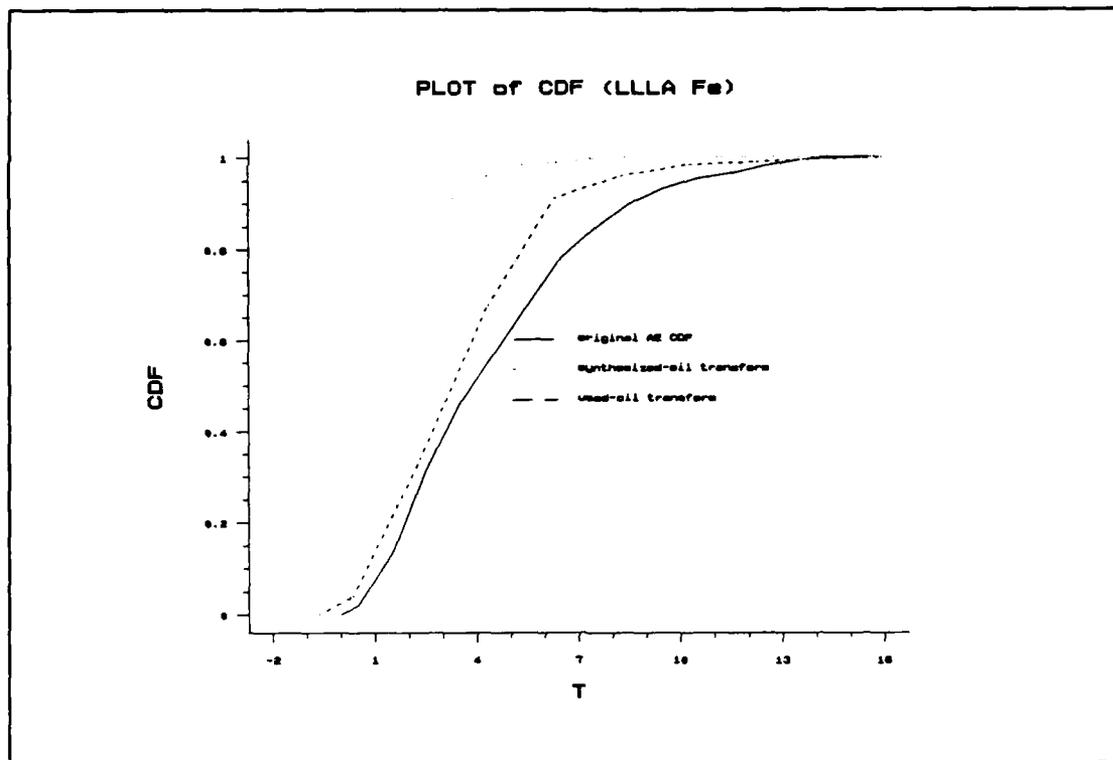


Figure 17. Plots of original and expected AE CDF from AA CDF, LLLA

Table 47. SYNTHESIZED-OIL AND USED-OIL TRANSFORMATION, LLLA, Fe

AA	CDF value	transformed scale to AE scale	
		synthesized-oil transformation	used-oil transformation
.0	.0000	-.251	-.670
.5	.0390	.256	.326
1.5	.3378	1.270	2.318
2.5	.6742	2.283	4.310
3.5	.9114	3.297	6.302
4.5	.9610	4.310	8.295
5.5	.9835	5.324	10.287
6.5	.9895	6.337	12.279
7.5	.9955	7.351	14.271
8.5	1.0000	8.365	16.263
max diff. (location)		.4797 ( 3.30)	.1506 ( 6.30)

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