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FINAL REPORT

Office of Naval Research Contracts

Reliability and Survivability of Communication Networks

N00014-86-K-0745

UAH Proposal No. 87-99

Period Covered: June, 1987 to September, 1988

Reliability and Survivability of Communication Networks

N00014-89-J-1410

UAH Proposal No. 89-029

Period Covered: January, 1989 to September, 1989

Principal Investigator: Peter J. Slater

Co-principal Investigators: Ashok T. Amin
Kyle T. Siegrist

A. Publications on network reliability:

1. On the expected number of pairs of connected nodes: pair-connected reliabilty, to appear in Computers and Mathematics with Applications.

2. Exact formulas for reliability measures for various classes of graphs, Congressus Numerantium 58, 1987, 43-52.

3. Pair-connected reliability of a tree and its distance degree sequences, Congressus Numerantium 58, 1987, 29-42.

4. On the nonexistence of uniformly optimal graphs for pair-connected reliability, Networks, to appear.

5. A summary of results on pair-connected reliability, Contemporary Mathematics 89, 1989, 145-152.

6. Pair connected reliability of communication networks with vertex failures, Congressus Numerantium 67, 1988, 233-242.

7. The central limit theorem and the law of large numbers for pair-connectivity in Bernoulli trees, Probability in the Engineering and Informational Sciences 3, 1989, 477-491.

8. The optimal unicyclic graphs for pair-connected reliability, submitted for publication.

9. On P_3-optimal graphs, being modified.

Note: Other publications not related to network reliability received acknowledgement of ONR support.

B. Presentations on network reliability.

1. Slater: presented A.1 at the Computer Networks Symposium, Las Cruces, NM, November, 1986.

2. Siegrist: presented A.2 at the 18th Southeastern Conf. on Combinatorics, Graph Theory and Computing, Boca Raton, FL, February, 1987.

3. Amin: presented A.3, IBID.

4. Siegrist, The expected number of pairs of connected vertices in probabilistic graphs, GA Tech., March, 1987.

5. Slater, The pair-connected measure of network reliability, Graphs and Algorithms Conf., Boulder, CO, June, 1987 (other support provided by the NSF).

6. Slater, Cutting numbers for series-parallel graphs, TIMS/ ORSA National Meeting, 1987.

7. Slater: presented A.6 at the 19th Southeastern Conf. on Combinatorics, Graph Theory and Computing, Baton Rouge, LA, February, 1988.

8. Siegrist, The distribution of pair-connectivity for network reliability, IBID.

9. Slater, Pair-connected network reliability, First Cumberland Conf. on Graph Algorithms and Combinatorics, Tullahoma, TN, April, 1988.

10. Slater, On the existence of uniformly optimal graphs for pair-connected reliability with vertex failures, Fourth SIAM Conf. on Discrete Math, San Francisco, CA, July, 1988.

11. Slater, On the study of pair-connected reliability, ONR Conf., Clemson, SC, August, 1988.

12. Slater, Intervals of optimality for pair-connected reliability, Third Carbondale Combinatorics Conf., Carbondale, IL, October, 1988.

13. Amin, On uniformly optimally reliable networks for pair-connected reliability, National University of Singapore, December, 1988.

14. Amin, On P_3-optimal graphs, 2nd International Conf. on Graph Theory, Combinatorics, Algorithms and Applications, San Francisco, CA, July, 1989.

15. Siegrist, Uniformly optimal graphs for pair-connected reliability, SIAM Annual Meeting, San Diego, CA, July, 1989.

C. Graduate students supported:

Ms. D. Grinstead (October, 1987 to August, 1988)

Ms. T. Johnson (Summers 1988 and 1989)

D. Research highlights.

For an (n,m)-graph G (on n vertices and m edges) representing a communications network with each vertex representing a processor and each edge a communications link the majority of studies of reliability and survivability were concerned with global reliability (the probability that the graph *remains* connected) or with 2-terminal reliability (the probability that two specified vertices s and t remain connected). The common model assumed that *vertices* are fail-safe but each edge is inoperable independently with probability $q = 1 - p$.

In [A.1] we introduced a general formula that encompassed the various known reliability/survivability measures and that made clear the probabilistic rating of component failure and the penalty function aspect of measuring the amount of disruption that is created by the failure of certain elements. We introduced the pair-connected measure of reliability, letting PC(G) denote the expected number of pairs of vertices that remain connected. (This concept was independently introduced by Colbourn who used the term "resilience".) Letting p denote the probability that each edge is operable, we have $PC(G; p) = \sum_{i=1}^{m} A_i p^i$.

We have shown that determining PC(G; p) is NP-hard even for the case where G is planar of maximum degree four, and have produced linear algorithms for its determination in special cases (for example, for series-parallel graphs). Indeed, exact formulas have been produced for certain classes of graphs, and in general we have the following theorem.

Theorem. For a fixed value h, the coefficients $A_1, A_2, \ldots, A_h$ in PC(G; p) can be computed in time polynomial in n.

A particularly nice result is the following.

Theorem [A.4] **There does not exist a uniformly optimal (n, m)-graph for pair-connected reliability if $n \leq m \leq \binom{n}{2}-2$.**

Having proven this theorem we began the study of "intervals of optimality", in particular contrasting the stucture of graphs optimal for p near zero versus those optimal for values of p near one.

We also began the study of pair-connectivity for models in which edges are fail-safe but vertices fail. Unlike the above theorem for edge failures, for some but not all values of m there do exist uniformly optimal graphs for the vertex-failure model of pair-connected reliability, and we are continuing our efforts to completely indentify those values of m.

Going beyond studying just the mean of pair-connectivity random variable PC, in work that is continuing we consider deviations from the mean, the law of large numbers, and the central limit theorem for PC as $n \to \infty$.