



Wave Propagation in the Presence of Interface Layers in Composites

S. K. DATTA

Department of Mechanical Engineering and CIRES, University of Colorado, Boulder, CO 80309-0427 (U.S.A.)

A. H. SHAH and W. KARUNASENA

Department of Civil Engineering, University of Manitoba, Winnipeg R3T 2N2 (Canada)

P. OLSSON and A. BOSTRÖM

Division of Mechanics, Chalmers University of Technology, Gothenburg (Sweden)

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Abstract

Effect of interface layers on wave propagation in a composite medium has been investigated in this paper. Two systems are considered: scattering of waves by an inclusion with a thin interface layer and guided waves in a laminated plate with thin interface layers. In the first case the presence of interface layers is found to modify the scattering cross-section significantly. In the second case it is found that guided waves in a laminated plate are slowed down considerably in the presence of soft interface layers. The results of this study can be used to characterize interface layers by ultrasonic techniques.

1. Introduction

In composites (particularly metal matrix composites reinforced by fibers or particles), it is often the case that there is an interface layer surrounding the fibers or particles induced by processing conditions. This layer has properties different from those of the matrix and the reinforcing phase. The strength and fracture behavior of the composite is significantly influenced by this interface layer. It is, however, difficult to characterize this layer non-destructively. Ultrasonic techniques provide a means to characterize mechanical properties of the interface regions. In this paper we present results showing the effect of interface layers on scattering cross sections of spheroidal inclusions. The null-field or T matrix method [1, 2] has been used to study the scattering problem. The interface layer is modelled as a thin shell, the equations of motion of which enter

the boundary conditions on the surface of the inclusion. In an earlier study, Mal and Bose [3] considered the effect of thin viscous layers on long-wavelength scattering by spherical inclusions. More recently Datta *et al.* [4, 5] have studied scattering by spherical inclusions surrounded by thin interface elastic layers. In both these studies it is assumed that the tractions are continuous across the layer, whereas displacements satisfy jump conditions that are linear in the thickness of the layer. The resulting approximate boundary conditions are based on the assumption that inertial and curvature effects are negligible. The present study shows that both these have a strong effect on the scattering behavior.

As another example of the influence of interface layers on wave propagation in composites we have examined guided wave propagation in a composite laminate with interface bond layers. A stiffness method [6, 7] is used to study the dispersion of guided waves with or without bond layers. It is found that the dispersion depends strongly on bond layer properties.

2. Scattering by an inclusion

Consider an inclusion with a thin interface layer of thickness h , embedded in an elastic matrix. Let S be the surface of the inclusion with its outward pointing unit normal n . The layer is assumed to be sufficiently thin that all boundary conditions can be applied on S . The density and Lamé constants of the inclusion are taken to be ρ_1 , λ_1 and μ_1 respectively. The corresponding quantities for the layer and the matrix are ρ_0 , λ_0 ,

μ_0 and ρ, λ, μ respectively. A time dependence of the form $\exp(-i\omega t)$ is assumed throughout.

In the null-field approach, all pertinent fields are expanded in spherical vector waves. For brevity we present here only an outline of the method. The details can be found elsewhere [1, 8, 9]. In the matrix the incident regular waves are denoted by $\text{Re } \psi_n$ and the outgoing scattered waves by ψ_n , where n is the quadruple index. The corresponding transmitted waves in the inclusion are $\text{Re } \bar{\psi}_n'$. The incident and scattered waves are then expressed as

$$\mathbf{u}^{\text{in}} = \sum_n a_n \text{Re } \psi_n \quad (1)$$

$$\mathbf{u}^{\text{sc}} = \sum_n f_n \psi_n \quad (2)$$

Here a_n is assumed known and f_n is to be determined, *i.e.* the problem is to determine the transition matrix elements $T_{nn'}$ that give the relationship between a_n and f_n :

$$f_n = \sum_{n'} T_{nn'} a_{n'} \quad (3)$$

Following the usual procedure in the null-field approach the *outer integral representation* gives relations between a_n, f_n and the surface displacement \mathbf{u}_+ and traction \mathbf{t}_+ :

$$a_n = -\frac{ik_s}{\mu} \int_S (\mathbf{u}_+ \cdot \mathbf{t}(\psi_n) - \mathbf{t}_+ \cdot \psi_n) dS \quad (4)$$

$$f_n = \frac{ik_s}{\mu} \int_S (\mathbf{u}_+ \cdot \mathbf{t}(\text{Re } \psi_n) - \mathbf{t}_+ \cdot \text{Re } \psi_n) dS \quad (5)$$

Here k_s is the shear wavenumber in the matrix.

The index $+$ denotes the limit from the outside and \mathbf{t} is the traction operator. The surface field on the inclusion is expanded in the regular spherical waves of the inclusion

$$\mathbf{u}_- = \sum_n \alpha_n \text{Re } \psi_n' \quad (6)$$

and the inner integral representation then gives an expansion of the surface traction on the inclusion as

$$\mathbf{t}_- = \sum_n \alpha_n \mathbf{t}'(\text{Re } \psi_n') \quad (7)$$

where \mathbf{t}' is the traction operator of the inclusion.

The problem is now to express \mathbf{u}_+ and \mathbf{t}_+ in eqns. (4) and (5) in terms of \mathbf{u}_- and \mathbf{t}_- and to this end the properties of the layer must be taken into account. As h is assumed small (compared with both the wavelengths and the radii of curvature of S) a consistent approach is to start from the three-dimensional equations for the layer and approximate to order h wherever possible to obtain a theory that is exact to order h . For a rotationally symmetric inclusion this leads to

$$\left(\frac{\partial \mathbf{u}}{\partial n} \right)_{\text{tan}} = \frac{(\mathbf{t}_-)_{\text{tan}}}{\mu_0} - \hat{\mathbf{n}} \times (\nabla_s \times \mathbf{u}_-) \quad (8)$$

$$\hat{\mathbf{n}} \cdot \frac{\partial \mathbf{u}}{\partial n} = \frac{1}{\lambda_0 + 2\mu_0} (\hat{\mathbf{n}} \cdot \mathbf{t}_- - \lambda_0 \nabla_s \cdot \mathbf{u}_-) \quad (9)$$

and

$$\frac{\partial \mathbf{t}}{\partial n} = \mathbf{B}(\mathbf{u}_-, \mathbf{t}_-) \quad (10)$$

The expression for \mathbf{B} is lengthy and can be found in ref. 10. It contains terms representing the inertial and curvature effects. It should be noted that a most common assumption that is made dealing with thin layers [5] is that the normal traction is continuous across the layer. This means that \mathbf{B} is set equal to zero. Also, curvature effects represented by the last two terms in (8) and (9) are dropped.

Using equations (4)–(6) it is now possible to derive the transition matrix elements $T_{nn'}$. This is rather complicated and will be omitted here. Because of the complexity of using the exact $O(h)$ approximation to $T_{nn'}$, it seems worthwhile to look for some simplifications. As mentioned before, a commonly made simplifying assumption would be to neglect all the inertial and curvature effects in (8)–(10). Another simplification would be to use a membrane-shell-type approximation for the thin layer. The latter simplifies the expression for \mathbf{B} to

$$\mathbf{B}'(\mathbf{u}, \mathbf{t}) = -\rho_0 \omega^2 \mathbf{u} + \nabla_s \sigma'(\mathbf{u}) \quad (11)$$

This expression can be simplified further by keeping only the first term on the right-hand side which represents the inertial effect. This gives

$$\mathbf{B}'' = -\rho_0 \omega^2 \mathbf{u} \quad (12)$$

In ref. 9, results were presented for the scattering by a spherical inclusion with a thin surrounding layer using the exact solution to the problem and

the approximations (11) or (12). It was found that use of (12) gave very good results at low frequencies and the predictions using (11) and (12) started to diverge at high frequencies. In Section 4 we present some results for the scattering of P waves by an SiC inclusion with interface layer in an aluminum matrix, showing the comparison between the predictions of the full $O(h)$ approximation and the three simplifying approximations mentioned above.

3. Rayleigh-Lamb waves in a laminated plate

Consider a laminated plate where each lamina is made up of aligned continuous fiber reinforced material and there is a thin bonding material layer between adjacent laminae. In this paper we consider a cross-ply plate ($0^\circ/90^\circ/\dots/90^\circ/0^\circ$) and assume that the waves are propagating either in the 0° or in the 90° direction.

On the assumption that the wavelength is much larger than the fiber diameter and the spacing between the fibers, each lamina can be modelled as a transversely isotropic medium with the symmetry axis aligned with the fiber direction. In that case it is possible to derive an exact dispersion equation for propagation of harmonic waves in the plane of the plate. However, for a larger number of laminations this equation is very complicated and requires extreme care in locating the roots. For this reason, we have presented before [6, 7, 11] a stiffness method that applies to an arbitrary number of layers.

In this method each lamina is divided into several sublayers. In each sublayer the displacement is assumed to have the form

$$u(x, y, z, t) = U(z) \exp\{i(kx + ly - \omega t)\} \quad (13)$$

where we may choose the x axis parallel to the 0° fibers and the z axis in the thickness direction of the lamina. Choosing the origin in the midplane of the sublayer, $U(z)$ is approximated by polynomial interpolation functions with coefficients

as the displacements and tractions at the top and bottom of the sublayer. This assures continuity of these quantities at the interfaces between the sublayers and the laminae. It has been shown [6, 7, 11] that this method (method I) gives very accurate results even at high frequencies. However, this method leads to a more complicated eigenvalue (algebraic) problem than if only displacement continuity at the interfaces is enforced. In the latter case, one loses the high accuracy that is obtained using method I. A compromise between the two is that used in refs. 12 and 13, called method II, where the interpolation polynomials involve displacements at the top, middle and bottom surfaces. It has been shown [6] that this last method gives good results except at high frequencies. Because method II is somewhat simpler to use when a large number of laminae has to be considered we have used that in this paper. Since both method I and method II have been well documented before, we shall omit the details here. In Section 4 we present numerical results showing the effect of interfaces on the dispersion behavior.

4. Numerical results and discussion

4.1. Effect of interface layer on scattering

In this section we present some results for the scattering cross-section of spheroids. We consider incident longitudinal waves. As an example we take a SiC particle embedded in an aluminum matrix. The material properties of the matrix are

$$\lambda + 2\mu = 110.5 \text{ GPa}$$

$$\mu = 26.7 \text{ GPa}$$

$$\rho = 2.706 \text{ kg m}^{-3}$$

Those of the particle are

$$\lambda_1 + 2\mu_1 = 474.2 \text{ GPa}$$

$$\mu_1 = 188.1 \text{ GPa}$$

$$\rho_1 = 3.181 \text{ kg m}^{-3}$$

TABLE 1 Comparison of analytical and numerical results for the total scattering cross-section for a plane P wave incident on an SiC sphere in aluminum

$k_0 a$	Analytical $h = 0.1a$	Full order h	Membrane	Inertial	Only jump in u	Order h without geometrical effects	Analytical $h = 0$
0.1	4.232×10^{-5}	4.211×10^{-5}	3.800×10^{-5}	3.741×10^{-5}	2.346×10^{-5}	2.414×10^{-5}	2.671×10^{-5}
0.5	2.212×10^{-2}	2.201×10^{-2}	2.142×10^{-2}	2.067×10^{-2}	1.1264×10^{-2}	1.313×10^{-2}	1.432×10^{-2}
1.0	0.2339	0.2328	0.2443	0.2355	0.1408	0.1482	0.1585
1.5	0.7504	0.7469	0.7559	0.7234	0.4675	0.4874	0.5256
2.0	1.5574	1.5474	1.4113	1.3379	0.9831	1.0029	1.110

Table 1 shows the comparison between the results of various approximations and the exact calculations for a spherical inclusion. The properties of the layer have been taken to be the mean of those of the inclusion and matrix. The thickness h is taken to be $0.1a$, a being the radius of the sphere. It is seen from this table that both geometrical and inertial effects are important in the determination of the scattering cross-section. Other conclusions that can be drawn from this table are as follows.

(1) Full order h predictions are pretty close to the exact results for low and intermediate frequencies.

(2) A simplified approximation obtained by keeping only the inertial effects is fairly good.

(3) The approximations resulting from keeping the jump in u (as is done commonly) is inferior to both the membrane and the inertial approximations. In fact, this results in a prediction that is lower than that for an inclusion without an interface layer. It is found that the prediction of an $O(h)$ approximation without the geometrical effects is also lower.

(4) The presence of the interface layer around the particle changes the scattering cross-section measurably. The implication of this for a particle-reinforced composite material would be to change the phase velocity and attenuation of ultrasonic waves. Thus, measurements of these quantities would allow characterization of the interface. However, our study suggests that, in order to carry out the inversion correctly, it is necessary to include the effects of curvature and inertia of the layer. A simple spring model neglecting these effects may lead to erroneous conclusions.

In order to demonstrate the inertial effects we present in Fig. 1 a comparison between the predicted scattering cross-sections of a prolate spheroid with an aspect ratio b/a of 2 to 1 for a longitudinal wave incident at 45° to the symmetry axis. These results are obtained using the membrane-type approximation and for the following three types of interface layers: type 1 is

$$\lambda_0 = \frac{\lambda + \lambda_1}{2}$$

$$\mu_0 = \frac{\mu + \mu_1}{2}$$

$$\rho_0 = \frac{\rho + \rho_1}{2}$$

type 2 is

$$\lambda_0 + 2\mu_0 = 36.8 \text{ GPa}$$

$$\mu_0 = 8.9 \text{ GPa}$$

$$\rho_0 = \rho$$

and type 3 is

$$\lambda_0 + 2\mu_0 = 36.8 \text{ GPa}$$

$$\mu_0 = 8.9 \text{ GPa}$$

$$\rho_0 = 1.353 \text{ kg m}^{-3}$$

It is seen from these values that as the density decreases, so does the scattering cross-section. The substantial increase in the scattering cross-section in the presence of the interface layer is also noticeable.

4.2. Effect of an interface layer on dispersion of guided waves in a laminated plate

We next present some results showing the measurable changes in the phase velocity disper-

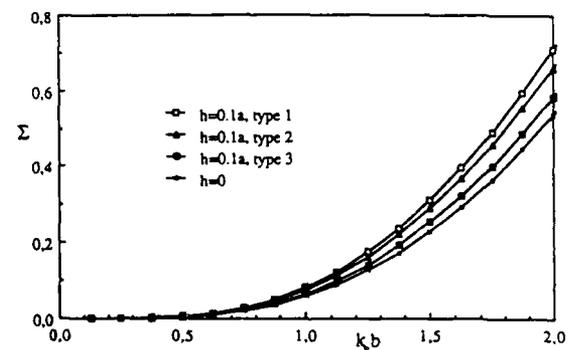


Fig. 1. Total scattering cross-section of a prolate ($b/a = 2/1$) spheroid for an incident P wave propagating at 45° with the symmetry axis.

TABLE 2 Properties of 0° and 90° laminae and of the interface layer

	C_{11} ($\times 10^{11} \text{ N m}^{-2}$)	C_{22} ($\times 10^{11} \text{ N m}^{-2}$)	C_{12} ($\times 10^{11} \text{ N m}^{-2}$)	C_{44} ($\times 10^{11} \text{ N m}^{-2}$)	C_{55} ($\times 10^{11} \text{ N m}^{-2}$)
0° lamina	1.0673	0.1392	0.0644	0.0350	0.0707
Interface	0.0865	0.0865	0.0475	0.0195	0.0195
90° lamina	0.1392	1.0673	0.0644	0.0707	0.0350

sion in a cross-ply laminated plate caused by interface bond layers. We consider graphite-fiber-reinforced laminae. The effective elastic properties of a lamina and the interface bond layers are listed in Table 2.

Figures 2-7 show the variations in phase velocities with frequency in a laminated plate with 19 laminae with and without interface layers. Figures 2 and 3 are for propagations along x (0°) and y (90°) directions respectively when there are no interface layers. Figures 4 and 5 show the effect of interface layers. The interface layer thickness is taken to be one tenth of a lamina thickness and the ratio of the densities of a layer and a lamina is 0.67. To show the effect of density (inertia) we show in Figs. 6 and 7 the dispersion

curves when the density ratio is 1.5. In all these figures the non-dimensional phase velocity and frequency are defined as

$$c = \frac{\omega/k}{\{(C_{55}/\rho)_0\}^{1/2}}$$

$$\Omega = \frac{\omega H}{2\{(C_{55}/\rho)_0\}^{1/2}}$$

where H is the total thickness of the plate.

The following conclusions can be drawn from these figures:

(1) It is seen from Figs. 2 and 3 that at low frequencies the dispersion for propagation along 0°

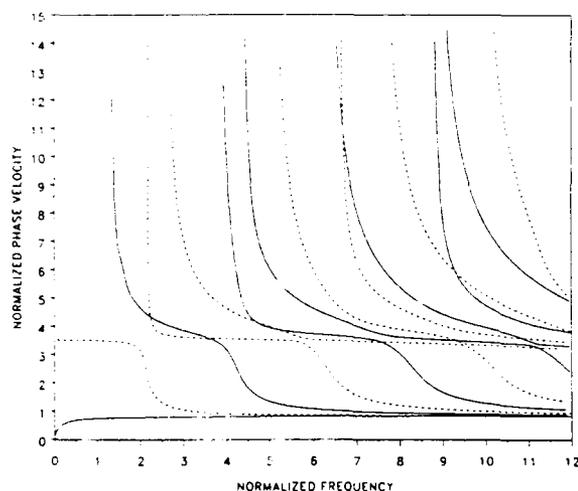


Fig. 2. Dispersion of guided waves in a $0^\circ/90^\circ/0^\circ$... laminated (19 layers) plate for propagation in the 0° direction. ---, symmetric modes; —, antisymmetric modes; no interface.

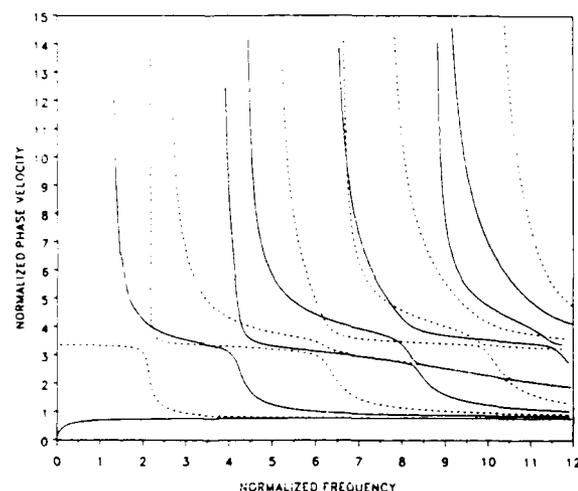


Fig. 3. Same as in Fig. 2 for propagation in the 90° direction.

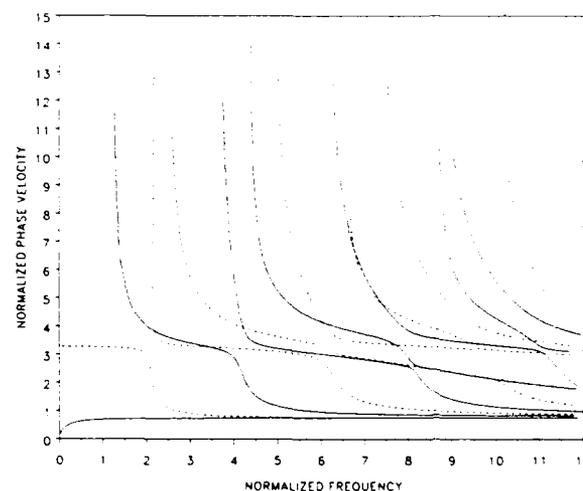


Fig. 4. Dispersion of guided waves in a $0^\circ/90^\circ/0^\circ$... laminated plate (19 laminae) with interface layers between 0° and 90° laminae. The ratios of the thicknesses and densities of a lamina and a layer are 10 and 1.5 respectively. Propagation is in the 0° direction. ---, symmetric modes; —, antisymmetric modes.

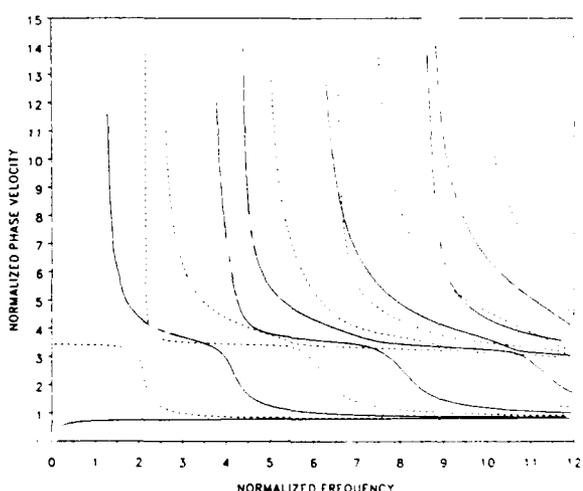


Fig. 5. Same as in Fig. 4 for propagation in the 90° direction.

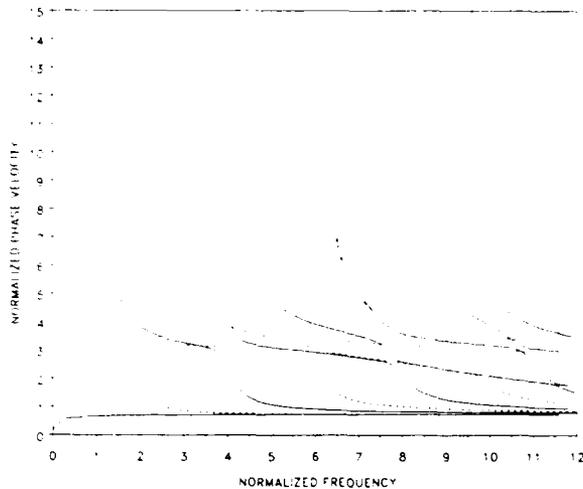


Fig. 6. The configuration is the same as in Fig. 4, but the density ratio is 0.67.

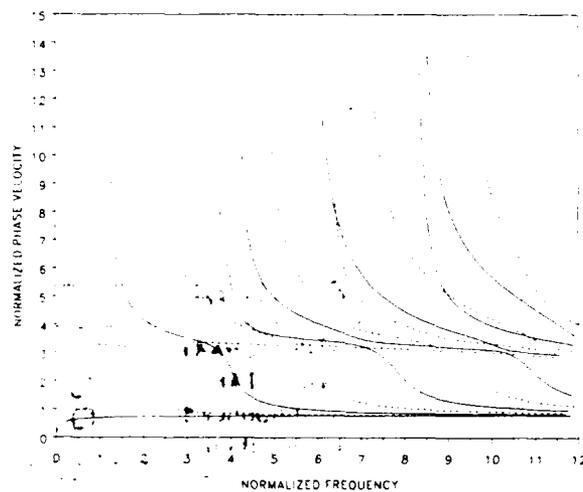


Fig. 7. Same as in Fig. 6 for propagation in the 90° direction.

and 90° directions is the same. Thus the plate behaves quasi-isotropically at low frequencies. However, at high frequencies the dispersions for propagation in these two directions are quite different.

(2) Comparison of Figs. 4 and 5 with Figs. 2 and 3 shows that the presence of the interface layers lowers the phase velocities and the cut-off frequencies. The effect is amplified at high frequencies.

(3) Finally, Figs. 6 and 7 show that raising the density of the interface layers does not change the dispersion of the first few modes appreciably. However, with increasing frequency the differences between Figs. 4 and 5 and Figs. 6 and 7 become appreciable. At certain frequency bands

these differences are quite large. This suggests that, in order to extract the bond layer properties, judicious choice of frequency and modes can be made to obtain optimum results.

5. Conclusion

The effect of interface and interphase layers on wave propagation in a composite medium has been studied in this paper. Two problems have been considered: scattering by a reinforcement particle and guided waves in a laminated plate. The results of this investigation show that in a particulate-reinforced composite the presence of an interface layer between the particles and the matrix would cause significant changes in the damping of ultrasonic waves. Also, phase velocity dispersion in a laminated plate changes measurably in the presence of interface bond layers. It is found in both cases that the density of the layer plays an important role in modifying the wave propagation characteristics. In addition, for curved interfaces the curvature effect should be taken into account. Thus, in order to characterize interfaces properly, it would be necessary to take into account properly not only the stiffnesses but also the inertial and curvature effects.

Acknowledgments

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