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Description of sea wave motion and
its influence on the altitude control
of seaskimming missiles

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ABSTRACT (UNCLASSIFIED)

In this report the description and generation of sea wave motion is presented in order to examine the influence of this motion on the altitude performance of a low flying missile. For this study a sea skimming missile is simulated with the 6 DOF Weapon Analysis and Simulation Program (WASP) of the Physics and Electronics Laboratory (FEL-TNO).

rapport no. : FEL-89-A368
titel : Beschrijving van de zeegolfbeweging en de invloed
hiervan op de hoogte regeling van een seaskimming
missile

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SAMENVATTING (ONGERUBRICEERD)

In dit rapport wordt een beschrijving en generatie van zeegolfbewegingen gegeven om het effect van deze beweging op de hoogte prestaties van laag vliegende missies te onderzoeken. Voor deze studie is een seaskimming missile gesimuleerd met het 6 DOF Weapon Analysis and Simulation Program (WASP) van het Fysisch en Elektronisch Laboratorium.

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1 INTRODUCTION

Seaskimming missiles fly as long as possible below the radar horizon of the target ship to reduce the detectability. Therefore the missile has to fly as low as possible at constant height above the mean sea level without ditching the water.

To fly at a constant altitude the missile has to be equipped with an altimeter to measure the missile height above the sea. The altitude control law imbedded in the guidance computer controls the actual distance between the missile and the water surface. In figure 1 the situation above the waves is depicted.

To simulate the flight above the sea waves, these waves have to be modelled and simulated. The sea wave motion can be described by a stochastic process and is determined by i.a. wind speed and depth of the water.

In chapter 2 a description is given of the power spectral density (PSD) function, which describes the power distribution of the sea waves as function of frequency for a stationary and moving observer. In chapter 3 some methods are described for the generation of sea wave elevations. In chapter 4 the influence of sea wave motion on missile performance is analysed. In chapter 5 some conclusions are given.

This study has been carried out under assignment nr. A84KM080 "Missile simulations". Results have been used in various studies for instance the simulations of the threat for the NATO Anti Air Warfare System (NAAWS).

2 POWER SPECTRAL DENSITY FUNCTION FOR STATIONARY AND MOVING
OBSERVER

The sea wave motion can be described by a stochastic process. This process is specified by a power spectral density (PSD) function and a probability density function (PDF).

We will now consider the sea wave PSD function with normal distribution. The state of the sea is described by the sea state number, see table 1. The sea state is defined by a parameter known as the significant wave height H_{3rd} , which is the average of the highest third of all waves recorded in a specified interval, see ref. 1.

The PSD of the sea wave motion is given by the Pierson-Moskowitz (PM) spectrum and is a standard accepted description of this stochastic process. It is a one dimensional spectrum for a stationary (non-moving) observer for the case of a fully developed sea in deep water and is given by :

$$\Phi_{\omega}(\omega) = \frac{0.84}{\omega^5} \cdot e^{\frac{-3.36}{H_{3rd}^2 \cdot \omega^4}} \quad (1)$$

The standard deviation σ equals :

$$\sigma = \frac{H_{3rd}}{4} = \sqrt{\left(\int_0^{\infty} \Phi_{\omega} \cdot d\omega \right)} \quad (2)$$

The significant wave height for sea state (SS) 2 is 0.6 meter and for SS 5 equals 4.0 meter according to table 1. For simulation these two values are most commonly used. The PSD functions for these two states are given in figure 2 and 3.

The above given spectrum (equation 1) is for a non-moving observer. To account for missile velocity the spectrum has to be transformed to a spectrum for a moving observer, because the altimeter moves with the

missile at a speed V as it senses the height above the sea waves. The frequency for a moving observer is shifted according to :

$$\omega_m = \omega + \omega \cdot \frac{U}{C_w} = \omega \cdot \left(1 + \frac{U}{C_w} \right) \quad (3)$$

with C_w the velocity of the wave propagation and U is the horizontal component of the missile speed, towards the direction of the wave propagation. The velocity of wave propagation is :

$$C_w = \frac{g}{\omega} \quad (4)$$

with g the gravity acceleration. So the frequency for a moving observer is :

$$\omega_m = \omega \cdot \left(1 + \frac{U}{g} \cdot \omega \right) \quad (5)$$

The spectrum for a moving observer then becomes :

$$\Phi_{\omega_m}(\omega_m) = \frac{\Phi_{\omega}(\omega)}{1 + \frac{2U}{g} \cdot \omega} \quad (6)$$

The derivation of this spectrum is given in appendix A.

To generate sea waves the process as indicated in figure 4 is used. White normal noise is filtered to get the sea wave elevations. The power spectral density function of this process is given by :

$$\Phi_{\omega_m}(\omega) = |F(j\omega)|^2 \cdot \Phi_N \quad (7)$$

Φ_N is the power spectral density function of white noise.

The problem is now to find a filter that matches with the PM spectrum. In ref. 2 a frequency response $F(j\omega)$ is found according to :

$$F(j\omega) = \frac{b \cdot (j\omega)^4}{[(j\omega)^2 + \sqrt{2} \cdot a \cdot j\omega + a^2]^3} \quad (8)$$

The parameters a and b are defined as :

$$a = \omega_{\Phi_{\max}} / \sqrt[4]{2} \quad (9)$$

$$b = \frac{a^2}{0.385} \cdot \sqrt{\left(\frac{\Phi_{\omega_{\max}}}{\Phi_N} \right)}$$

where $\Phi_{\omega_{\max}}$ is the maximum value of the PSD function of the sea waves for a non-moving observer and $\omega_{\Phi_{\max}}$ is the frequency at which the PSD function for a non-moving observer reaches its maximum value. In appendix B the derivation of $\omega_{\Phi_{\max}}$ is given. A filter which gives a better match of the theoretical PSD (PM-spectrum) has a frequency response $F(j\omega)$ according to :

$$F(j\omega) = \frac{b \cdot (j\omega)^4}{(j\omega + a)^6} \quad (10)$$

where the parameters a and b are now defined as :

$$a = \omega_{\Phi_{\max}} / \sqrt{2} \quad (11)$$

$$b = 1.5^3 \cdot \omega_{\Phi_{\max}}^2 \cdot \sqrt{\left(\frac{\Phi_{\omega_{\max}}}{\Phi_N} \right)}$$

The filter as given in equation (10) will be named FEL filter. In the figures 5 and 6 the results of spectra calculations are given for SS 5 with an observer speed of 340 and 680 m/s, respectively. In these

figures the spectra according to Pierson-Moskowitz (PM), to ref. 2 and to the FEL filter are given. The PM-spectrum is calculated with equation (1). The other spectra are calculated with equation (7). The spectrum made with the FEL filter is closer to the PM spectrum. For sea state 5 the variance of the signal is one. The variance is equal to the integral of the PSD function. The filter from ref. 2 gives a variance for SS 5 equals to 0.66 m^2 and the FEL filter gives a variance value of 0.93 m^2 . A criteria to determine how good different solutions match is the weighted integral squared error (ISE) criterion. The equation for this criterion reads :

$$\text{ISE} = \int_0^{\infty} \text{WF}(\omega) \cdot [\Phi_{\omega}(\omega) - \Phi_{\text{PM}}(\omega)]^2 \cdot d\omega \quad (12)$$

$\text{WF}(\omega)$ is the weight function.

The ISE is dependent of observer speed. The ISE at 340 m/s for ref. 2 is approximately 5 times as large as the ISE for the FEL filter if the weighting function is one. If the weighting function equals six from zero to $\omega_{\Phi_{\text{max}}}$ and one for the other part of the frequency spectrum than the ISE for both filters is the same.

The FEL filter gives a better match according to the integral square error criterion and will therefore be used for the generation of the sea wave elevations.

3 METHODS FOR GENERATION OF SEA WAVE ELEVATIONS

Sea wave elevations or sea wave heights are calculated with the process described in figure 4, but there are different methods for doing this. Two methods are described here, we call the first method generating sea wave elevations on-line and the second method generating sea wave elevations off-line. In the off-line method an on-line correction for missile speed is applied.

The first method, which was formerly used, is based on the calculation of a batch of sea wave elevations as seen by a moving observer for one second with a sample time of 0.01 sec, so this batch contains 100 samples. This is done every second again with newly calculated filter coefficients because of possible fluctuating missile speed. This method has several drawbacks. Firstly it is very time consuming because every second again the filter has to be settled before the filter can generate sea wave elevations. Due to the large time constants involved 1100 samples have to be generated of which only 100 are used. Secondly it is less accurate. The PSD function for this method, calculated from an autocorrelation function, is given in figure 7 (observer speed is 340 m/s). The data file contained 20000 samples. As can be seen low frequencies are introduced. In appendix C the derivation of the PSD function from N data points is described.

The second method is based on the generation of sea wave elevations for one speed only, with a correction afterwards for the influence of the actual speed, if the actual speed and the speed used in the generation differ.

In figure 8 the spectrum is given, also computed from an autocorrelation function (observer speed is 340 m/s). If figure 7 and 8 are compared, it is clear that method 2 is more accurate, because less low frequencies are introduced.

The term "travelled distance" is introduced to correct for missile speed. This correction is necessary if this speed differs from the one

used for the generation off-line. The travelled distance is a function of missile speed and actual time of flight.

To illustrate this method an example is given. With the off-line method sea wave elevations are generated for sea state 2 and a missile speed of 170 m/s. The sampling time is 0.01 seconds. To prevent aliasing the sampling frequency must be larger than 2 times the highest frequency in the spectrum. Every sample corresponds with a distance of 1.7 meter. This means that the sea wave elevations are now only a function of travelled distance.

Now suppose that the missile is flying at 340 m/s. From the PM spectrum a highest frequency of 450 rad/s or 72 Hz follows. The sample time must therefore be smaller than 0.007 sec, to prevent aliasing. The sampled distance is 1.7 meter. This means that a sample must be taken from the data file if the missile has travelled 1.7 meter. So if the simulation sample time is 0.005 sec then every simulation step a sample must be taken from the data file.

This method isn't exactly correct, because the assumption is made that the PSD function is linearly dependent of missile speed only. This simplification means it is assumed that the sea waves which are overflown have a constant (i.e. not time dependent) shape like terrain has whereas sea waves move with a frequency dependent speed. According to these assumption the transformed frequency is, if the speed is x times the speed V , that was used to make the data file :

$$\omega_m = x \cdot \omega + \frac{x \cdot V \cdot \omega^2}{g} \quad (12)$$

instead of

$$\omega_m = \omega + \frac{x \cdot V \cdot \omega^2}{g} \quad (13)$$

For an observer speed of 340 m/s and SS 5 an error is made of 0.5% in shifted frequency if the missile flies 2 times as fast.

The power contained in the spectrum is the same in both cases, only the spectrum is not shifted correctly, however with small error. To prove that the power does not change the next derivation is given.

If the PSD for an observer speed of V in the frequency band $[\omega_1, \omega_2]$ equals $\Phi(\omega)$ then the PSD for an observer speed of xV in the frequency band $[x\omega_1, x\omega_2]$ equals $\Phi_x(\omega_x) = 1/x \cdot \Phi(\omega/x)$.

The power contained in this frequency band is :

$$\int_{\omega=\omega_1}^{\omega=\omega_2} \Phi(\omega) \cdot d\omega = \int_{\omega=\omega_1}^{\omega=\omega_2} 1/x \cdot \Phi(\omega) \cdot (dx\omega) \quad (14)$$

with $\omega_x = x\omega$ follows :

$$\int_{\omega=\omega_1}^{\omega=\omega_2} \Phi(\omega) \cdot d\omega = \int_{\omega_x=\omega_x^1}^{\omega_x=\omega_x^2} 1/x \cdot \Phi(\omega_x/x) \cdot (d\omega_x) = \int_{\omega_x=\omega_x^1}^{\omega_x=\omega_x^2} \Phi_x(\omega_x) \cdot d\omega_x \quad (15)$$

This method was tested for sea state 2 and 5.

The test consists of 4 steps. These steps are :

1. Generate a data file containing sea wave elevations for a missile speed of 340 m/s with a sample time of 0.01 sec.
2. Generate a data file containing sea wave elevations for a missile speed of 680 m/s with a sample time of 0.01 sec.
3. Compute the PSD function for the data file generated at step 2.
4. Compute the PSD function for the data file generated at step 1, but each step a sample skipped.

In step 1 a datafile is generated with a sampling distance of 3.4 meter. In step 2 a datafile is generated with a sampling distance of 6.8 meter. For sea state 5 a sampling distance of 13 meter is still enough, so it is oversampled. For the generation of the PSD in step 4 samples can therefore be skipped instead of taking every sample as function of travelled distance.

Figure 9 presents the computed PSD functions for a speed of 680 m/s. From this figure it can be concluded that the described method is sufficiently accurate for use in simulation studies.

4 THE INFLUENCE OF SEA WAVE MOTION ON MISSILE PERFORMANCE

With the six degree of freedom (DOF) Weapon Analysis and Simulation Program (WASP) a seaskimming missile is simulated. For a description of WASP see ref. 3.

The missile is flying at nearly constant speed at 5 meter above mean sea level. A sea state of 5 is assumed, which corresponds to a significant wave height of 4 meter or a standard deviation of 1 meter.

Some airframe parameters are given in table 2. The time constant of the innerloop (acceleration autopilot) is 0.25 seconds and nearly constant during flight. The structure of the altitude autopilot is given in figure 10. The time constant of the altitude loop is in the order of 2 to 3 seconds for the parameters given below.

Due to altimeter and seeker restrictions, the roll angle has to have a constant value during flight. Therefore the roll control system is much faster than the pitch and yaw channels.

Four tests are performed. For each test the autopilot has the same innerloop branches, but differs in feedback filter time constants for the measured height and filtered height rate.

The altimeter is assumed to have no biases or measurement errors.

The cut-off frequencies of the filters are important, because the high frequencies in the spectrum can be filtered out. For a missile speed of 340 m/s and sea state 5 the peak in the PSD function is at 13.5 rad/s. The time constant of the height rate filter was 0.2 sec in batch 1 and 2 and 0.143 sec in batch 3 and 4. The difference between batch 1 and 2 is the filter used in the altitude feedback loop. In batch 1 the measured altitude is not filtered and in batch 2 the measured altitude is filtered by a first order network with a time constant of 0.2 seconds. The same applies to batch 3 and 4 but now with a filter time constant of 0.143 seconds.

For each test an undisturbed run and three disturbed runs were made. The mean, variance and standard deviation of missile altitude and angle of pitch are presented in table 3. These values are calculated from the disturbed runs using the nondisturbed runs as reference.

To illustrate the effects of the choice of altitude loop parameters on the missile performance some comparative figures are given.

In figures 11 and 12 horizontal flight profiles are given for a run from batch 2 and batch 3, respectively together with the undisturbed runs.

Due to the reduction in missile mass during the flight the seaskimming altitude increases a little bit with time.

In figure 13 the input (calculated sea wave elevations) of the altimeter is given. In figure 14 the output of the altimeter for a run of batch 2 is depicted. In figure 15 the angle of pitch for a run of batch 2 is given and in figure 16 for a run of batch 3. The dashed line in both figures are the undisturbed reference runs.

The choice of the altitude loop parameters is sometimes determined by contradicting requirements. If the altitude loop has to be fast a lot of noise will be introduced in the system and the influence of the sea waves on altitude performance will be large. This can have influence on the commanded seaskimming altitude.

5 CONCLUSIONS

The off-line generation of a data file containing the sea wave elevations as a function of travelled distance can be used to simulate the sea surface as sensed by an altimeter aboard a missile.

The sampling time must be correctly chosen. This is dependent of missile speed and sampling distance in the off-line generated data file.

The choice of altimeter loop parameters can have much influence on the missile vertical flight profile. This choice is sometimes determined by contradicting requirements.

WAVE AND SEA SCALE FOR FULLY ARISEN SEA														
SEA STATE	SEA-GENERAL		WIND				SEA							
	DESCRIPTION	BEAUFORT WIND FORCE	DESCRIPTION	RANGE (KNOTS)	WIND VELOCITY (KNOTS)	WAVE HEIGHT FEET			SIGNIFICANT RANGER PERIODS (SECONDS)	PERIOD OF MAXIMUM ENERGY OF SPECTRUM	T (AVERAGE PERIOD)	L (AVERAGE WAVE LENGTH)	MINIMUM FETCH (NAUTICAL MILES)	MINIMUM DURATION (HOURS)
						AVERAGE	SIGNIFICANT	AVERAGE TO HIGHEST						
0	Sea like a mirror.	0	Calm	< 1	0	0	0	0	-	-	-	-	-	-
	Ripples with the appearance of scales are formed, but without foam crests.	1	Light Air	1-3	2	0.05	0.08	0.10	5-12 sec	0.7	0.5	10 ft.	5	18 min.
1	Small wavelets, short but pronounced, crests have a glassy appearance, but do not break.	2	Light Breeze	4-6	5	0.18	0.29	0.37	0.4-2.8	2.0	1.4	6.7 ft.	8	39 min.
	Large wavelets, crests begin to break. Foam of glassy appearance. Perhaps scattered white horses.	3	Gentle Breeze	7-10	8.5	0.6	1.0	1.2	0.8-5.0	3.4	2.4	20	9.8	1.7 hrs.
2	Small waves, becoming larger; fairly frequent white horses.	4	Moderate Breeze	11-16	10	0.88	1.4	1.8	1.0-6.0	4	2.9	27	10	2.4
					12	1.4	2.2	2.8	1.0-7.0	4.8	3.4	40	18	3.8
3	Small waves, becoming larger; fairly frequent white horses.	4	Moderate Breeze	11-16	13.5	1.8	2.9	3.7	1.4-7.6	5.4	3.9	52	24	4.8
					14	2.0	3.3	4.2	1.5-7.8	5.6	4.0	59	28	5.2
4	Moderate waves, taking a more pronounced long form, many white horses are formed. (Chance of some spray).	5	Fresh Breeze	17-21	16	2.9	4.6	5.8	2.0-8.8	6.5	4.6	71	40	6.6
					18	3.8	6.1	7.8	2.5-10.0	7.2	5.1	90	55	8.3
5	Large waves begin to form, the white foam crests are more extensive everywhere. (Probably some spray).	6	Strong Breeze	22-27	19	4.3	6.9	8.7	2.8-10.6	7.7	5.4	99	65	9.2
					20	5.0	8.0	10	3.0-11.1	8.1	5.7	111	75	10
6	Large waves begin to form, the white foam crests are more extensive everywhere. (Probably some spray).	6	Strong Breeze	22-27	22	6.4	10	13	3.4-12.2	8.9	6.3	124	100	12
					24	7.9	12	16	3.7-13.5	9.7	6.8	160	130	14
7	Sea heaves up and white foam from breaking waves begins to be blown in streaks along the direction of the wind. (Spray begins to be seen).	7	Moderate Gale	28-33	24.5	8.2	13	17	3.8-13.6	9.9	7.0	164	140	15
					26	9.6	15	20	4.0-14.5	10.5	7.4	188	180	17
8	Moderately high waves of greater length, verge of crests break into spindrift. The foam is blown in well marked streaks along the direction of the wind. Spray affects visibility.	8	Fresh Gale	34-40	28	11	18	23	4.5-15.5	11.3	7.9	212	230	20
					30	14	22	28	4.7-16.7	12.1	8.6	250	280	23
9	Very high waves with long overhanging crests. The resulting foam is in great patches and is blown in dense white streaks along the direction of the wind. On the whole the surface of the sea takes a white appearance. The rolling of the sea becomes heavy and checkered. Visibility is affected.	10	Whole Gale	48-55	30.5	14	23	29	4.8-17.0	12.4	8.7	258	290	24
					32	16	26	33	5.0-17.9	12.9	9.1	285	340	27
10	Exceptionally high waves. Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	11	Storm	56-57	34	19	30	38	5.5-18.5	13.6	9.7	322	420	30
					36	21	35	44	5.8-19.7	14.5	10.3	363	500	34
11	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	11	Storm	56-57	37	23	37	46.7	6.20-5	14.9	10.5	376	530	37
					38	25	40	50	6.2-20.8	15.4	10.7	392	600	38
12	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	12	Hurricane	64-71	40	28	45	58	6.5-21.7	16.1	11.4	444	710	42
					42	31	50	64	7.23	17.0	12.0	492	800	47
13	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	13	Hurricane	64-71	44	36	58	73	7.24-2	17.7	12.5	524	860	52
					46	40	64	83	7.25	18.6	13.0	590	1000	57
14	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	14	Hurricane	64-71	48	44	71	90	7.5-26	19.4	13.8	610	1100	63
					50	49	78	99	7.5-27	20.2	14.3	700	1300	69
15	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	15	Hurricane	64-71	51.5	52	83	106	8.28-2	20.3	14.7	726	1360	73
					52	54	87	110	8.28-5	21.0	14.9	790	1500	79
16	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	16	Hurricane	64-71	54	59	95	121	8.29-5	21.8	15.4	810	1600	81
					56	64	103	130	8.5-31	22.6	16.0	910	1800	85
17	Sea completely covered with long white patches of foam blowing in direction of wind. Everywhere edges of wave crests are blown into froth. Visibility reduced.	17	Hurricane	64-71	59.5	73	116	148	10.22	24	17.0	955	2100	91
					64	80	128	164	10.25	25	17.5	1000	2200	95

Mach number	0.9	--
Mass at start	700.	kg
IX at start	10.7	kgm ²
IY at start	1582.7	kgm ²
IZ at start	1582.7	kgm ²
Reference Area	0.0962	m ²
Mass flowrate	1.25	kg/sec

Table 2: Some missile parameters.

batch	mean (meter)	variance (meter ²)	std deviation (meter)
1	0.003	0.00081	0.028
2	0.001	0.00026	0.016
3	0.005	0.00121	0.035
4	0.002	0.00050	0.022

Table 3a : Some results of the influence of sea wave motion on the missile altitude.

batch	mean (deg)	variance (deg ²)	std deviation (deg)
1	-0.018	0.408	0.63
2	-0.003	0.079	0.28
3	-0.028	0.647	0.80
4	-0.008	0.195	0.44

Table 3b: Some results of the influence of sea wave motion on the missile angle of pitch.

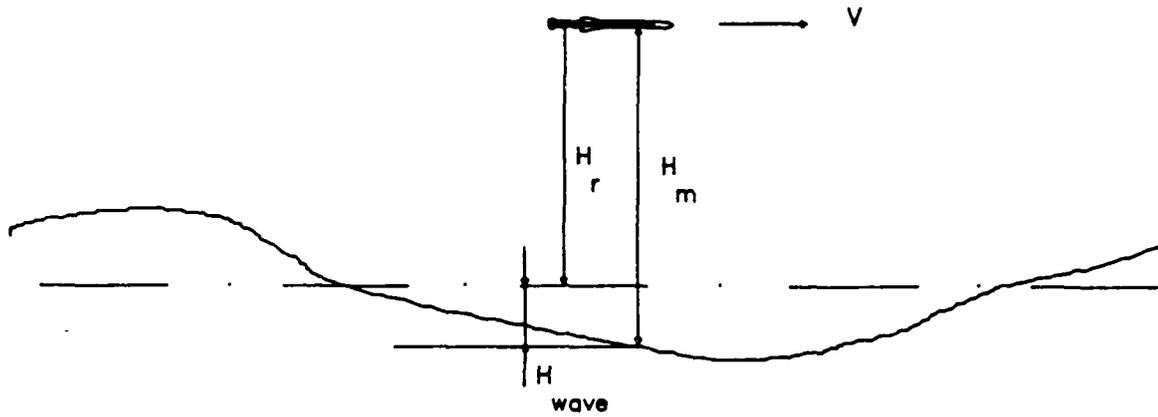


Fig. 1 : Missile flying above the sea waves at a sea skimming altitude H_r .

Fig. 2 : PSD of sea waves for SS 2
(stationary observer)

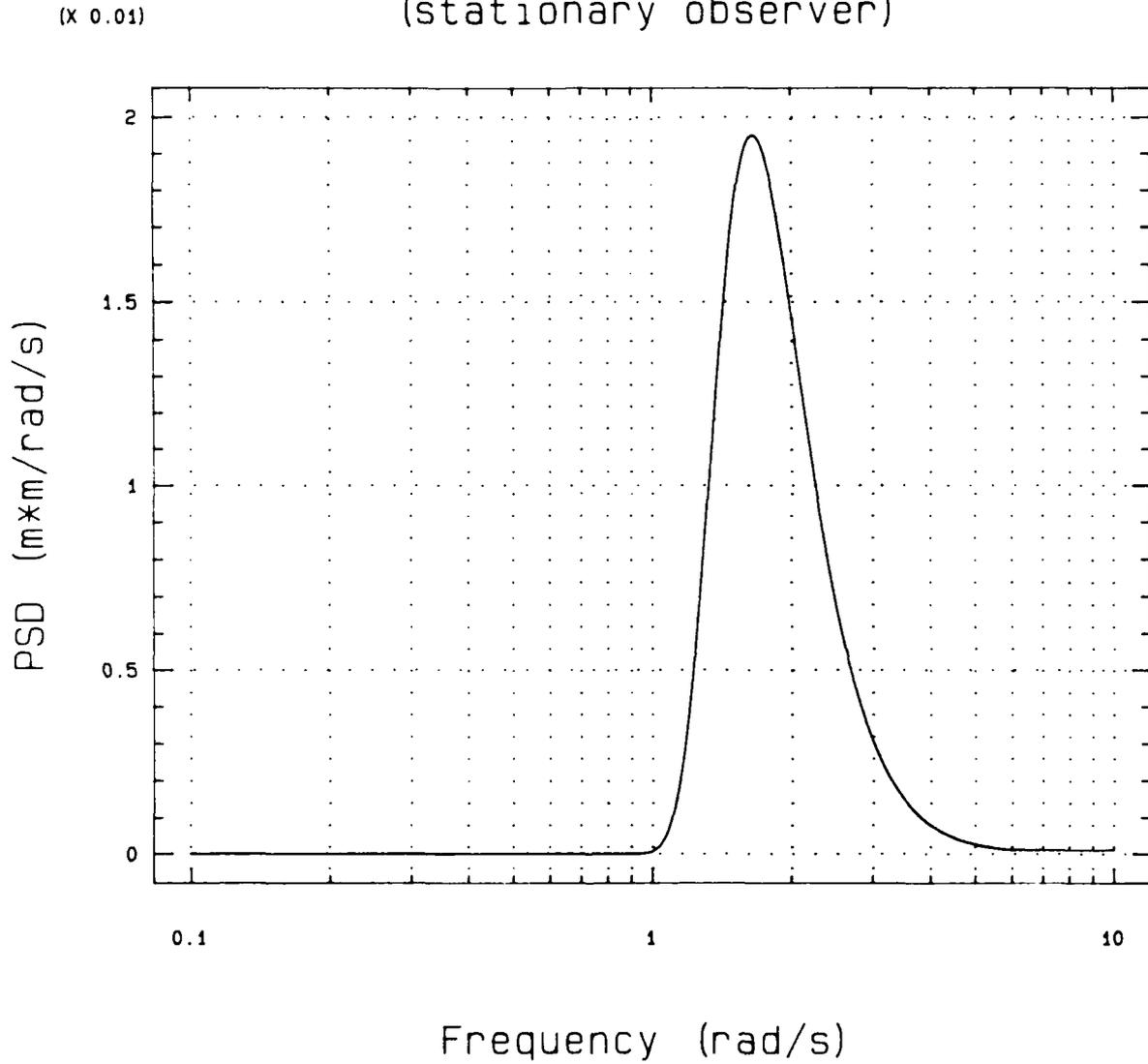
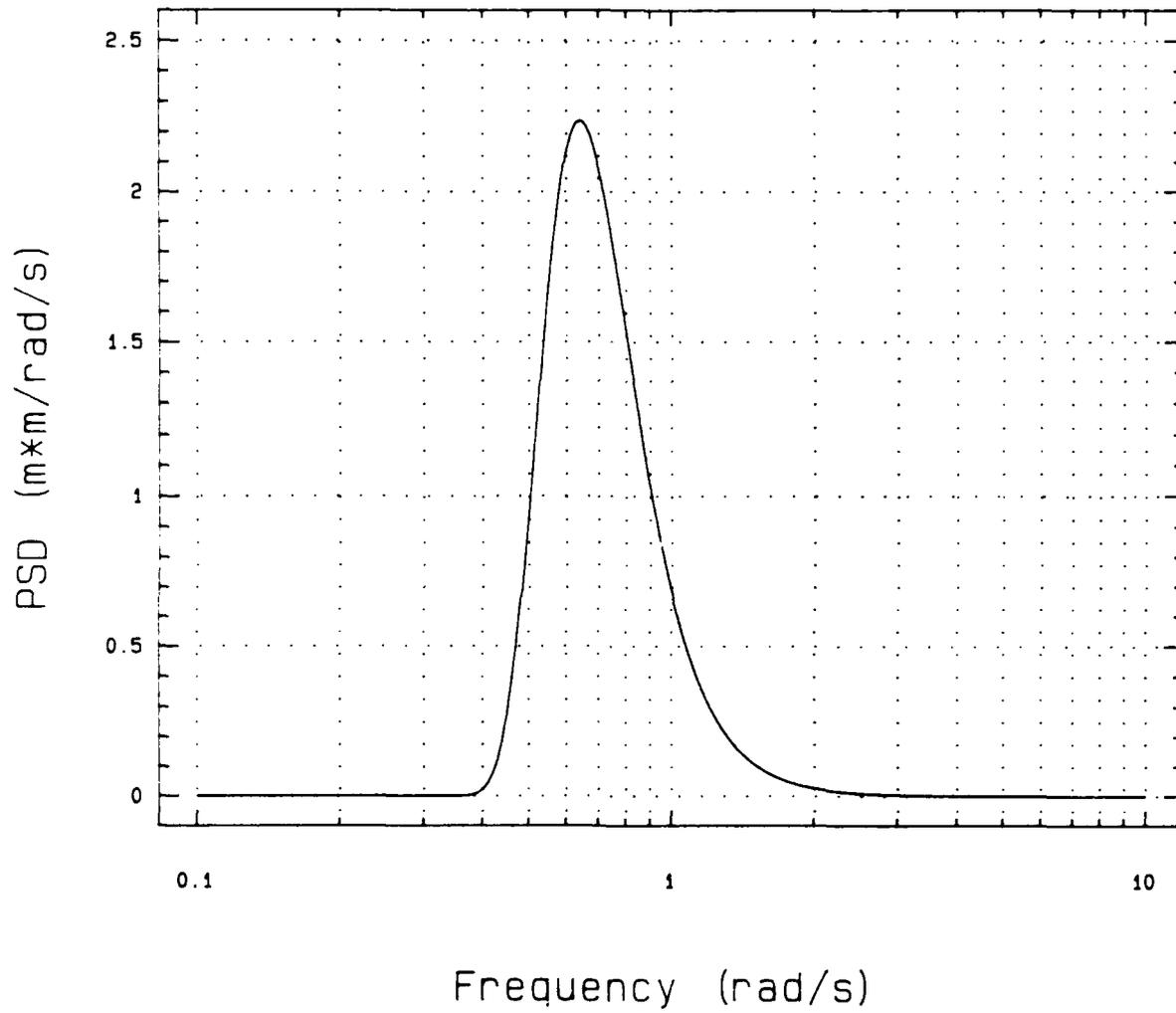


Fig. 3 : PSD of sea waves for SS 5
(stationary observer)



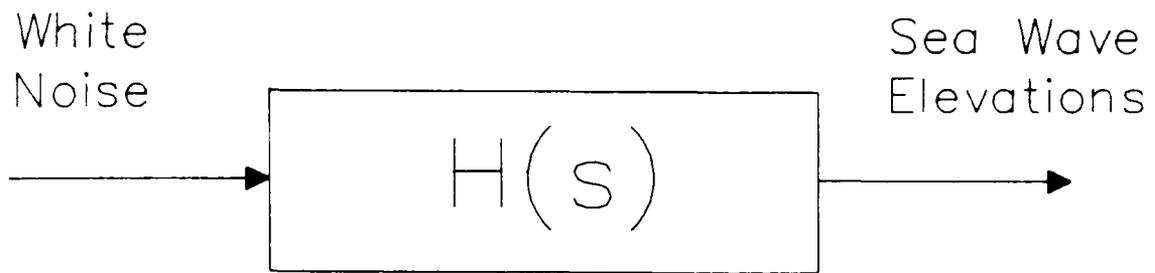


Figure 4 : Proces for the generation of sea wave elevations

Fig. 5 : Some PSD functions of sea waves
for SS 5 (moving observer $V = 340$ m/s)

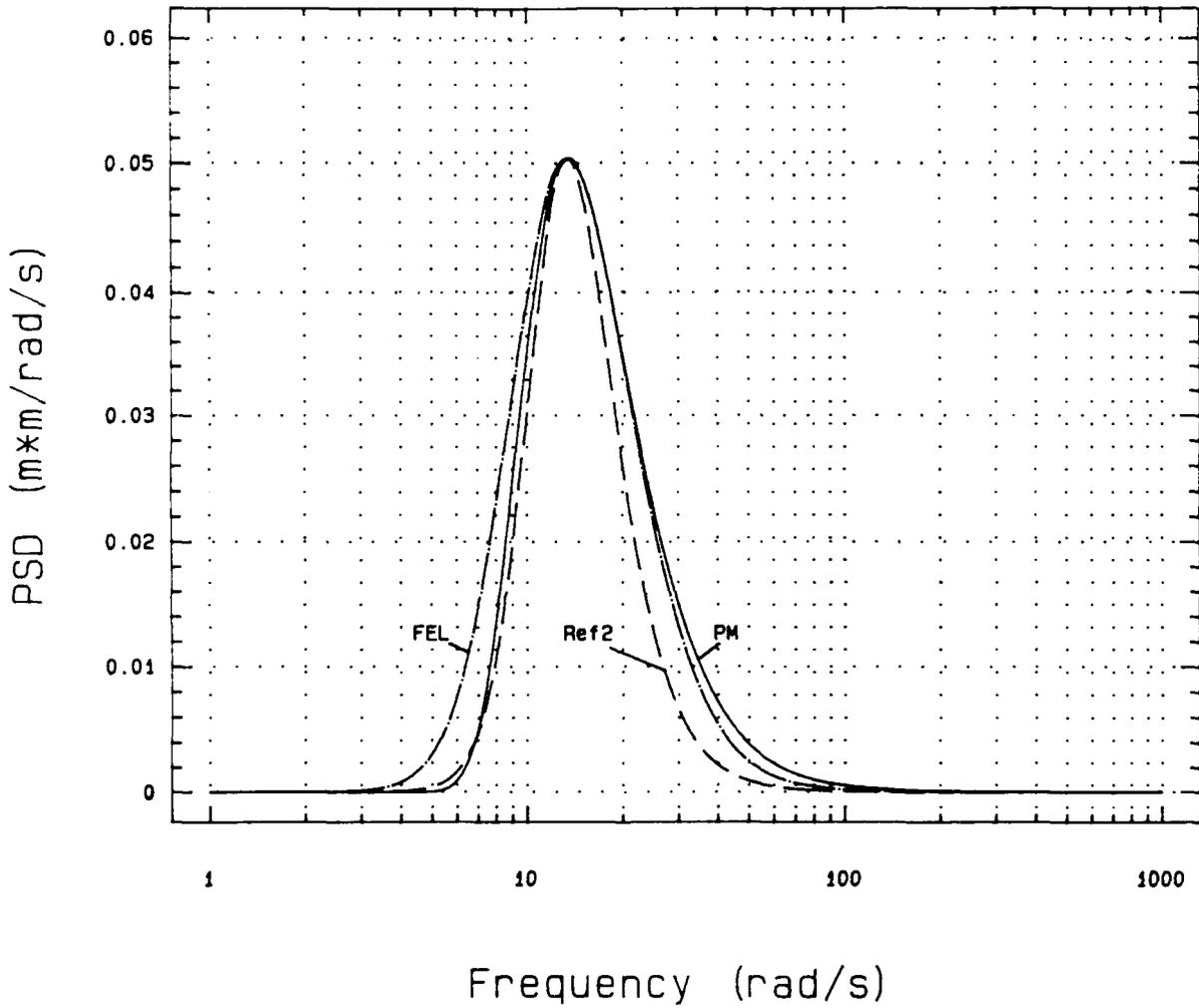


Fig. 6 : Some PSD functions of sea waves
for SS 5 (moving observer $V = 680$ m/s)

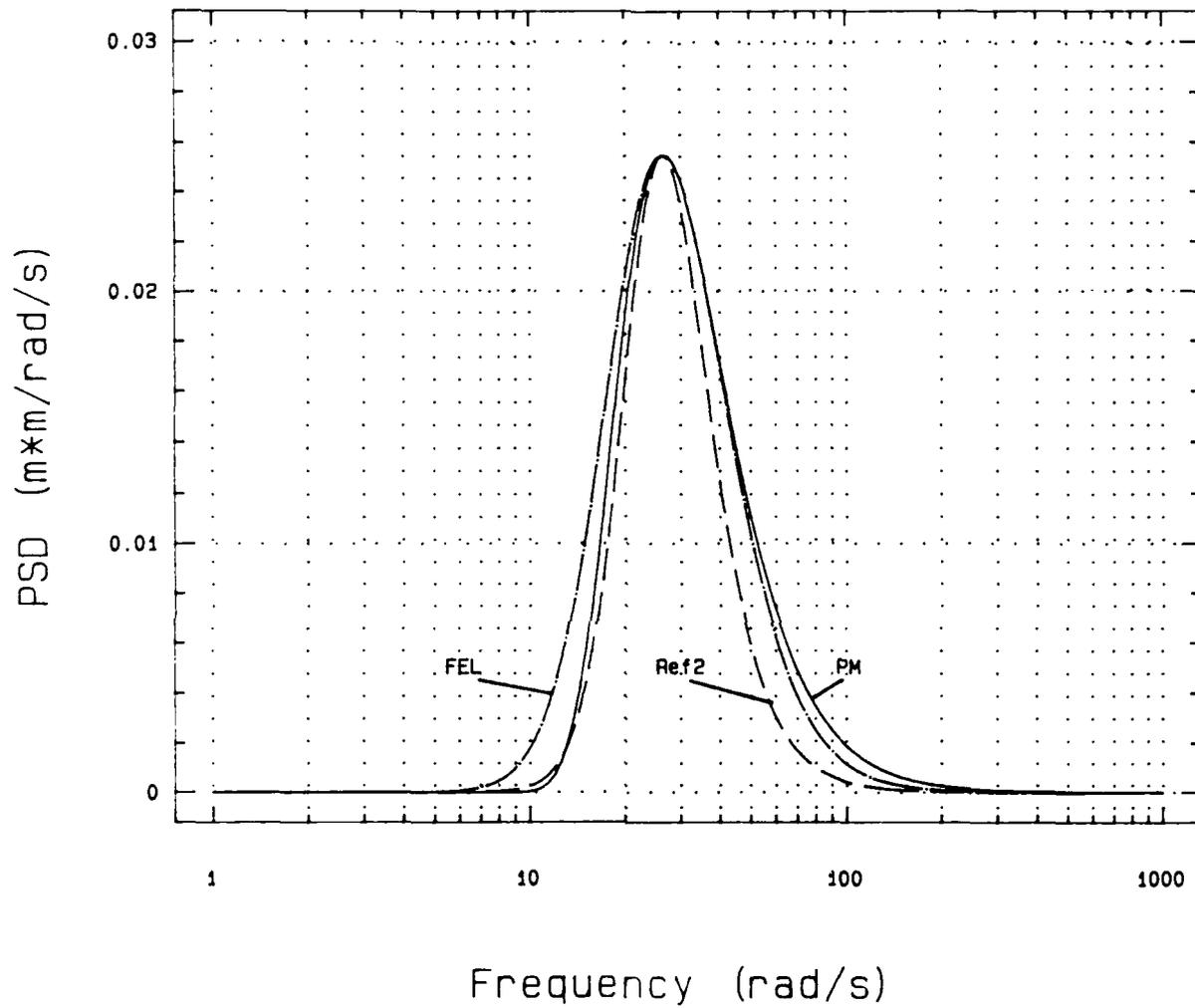


Fig. 7 : PSD function generated with the
on-line method

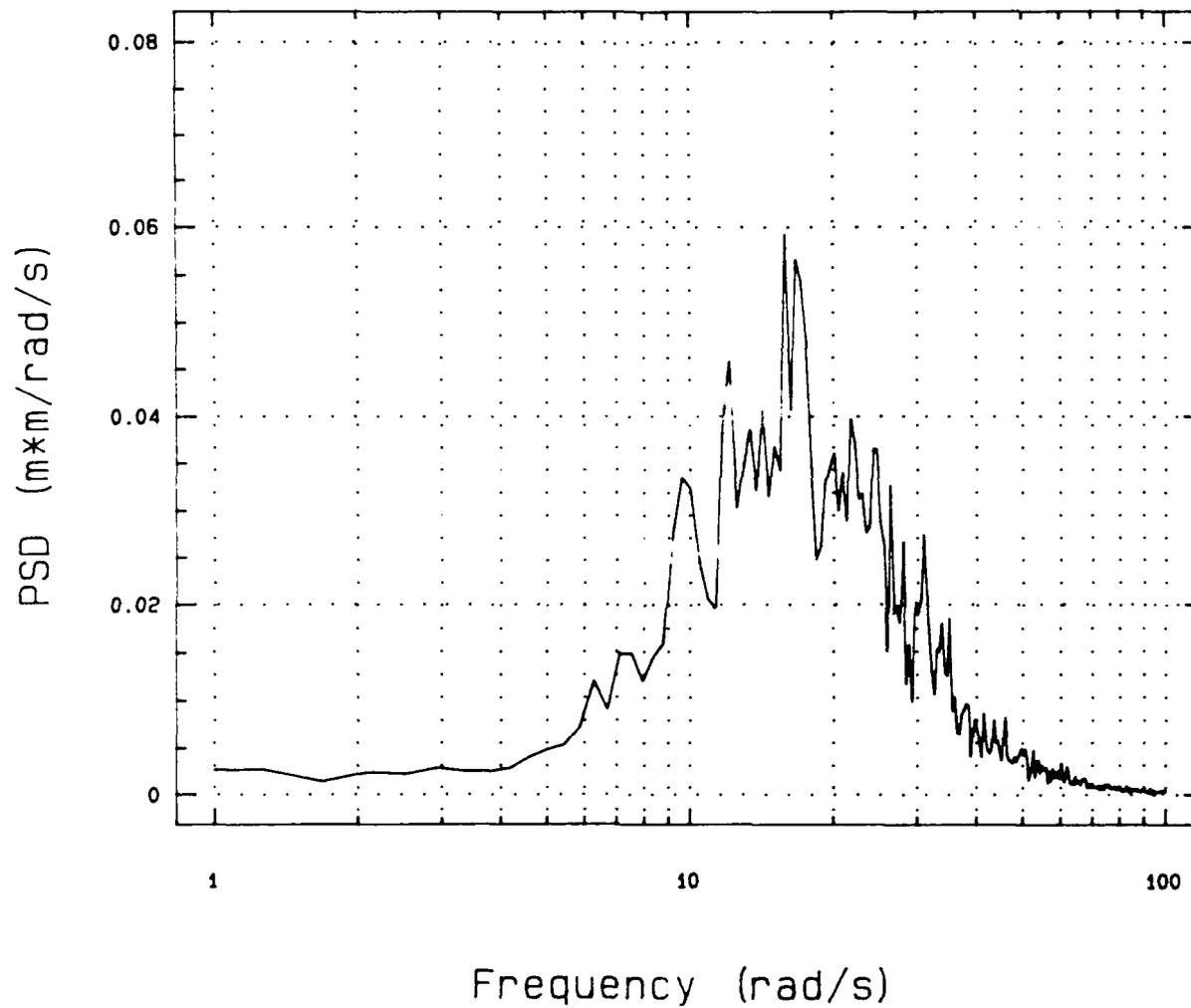


Fig. 8 : PSD function generated with
off-line method.

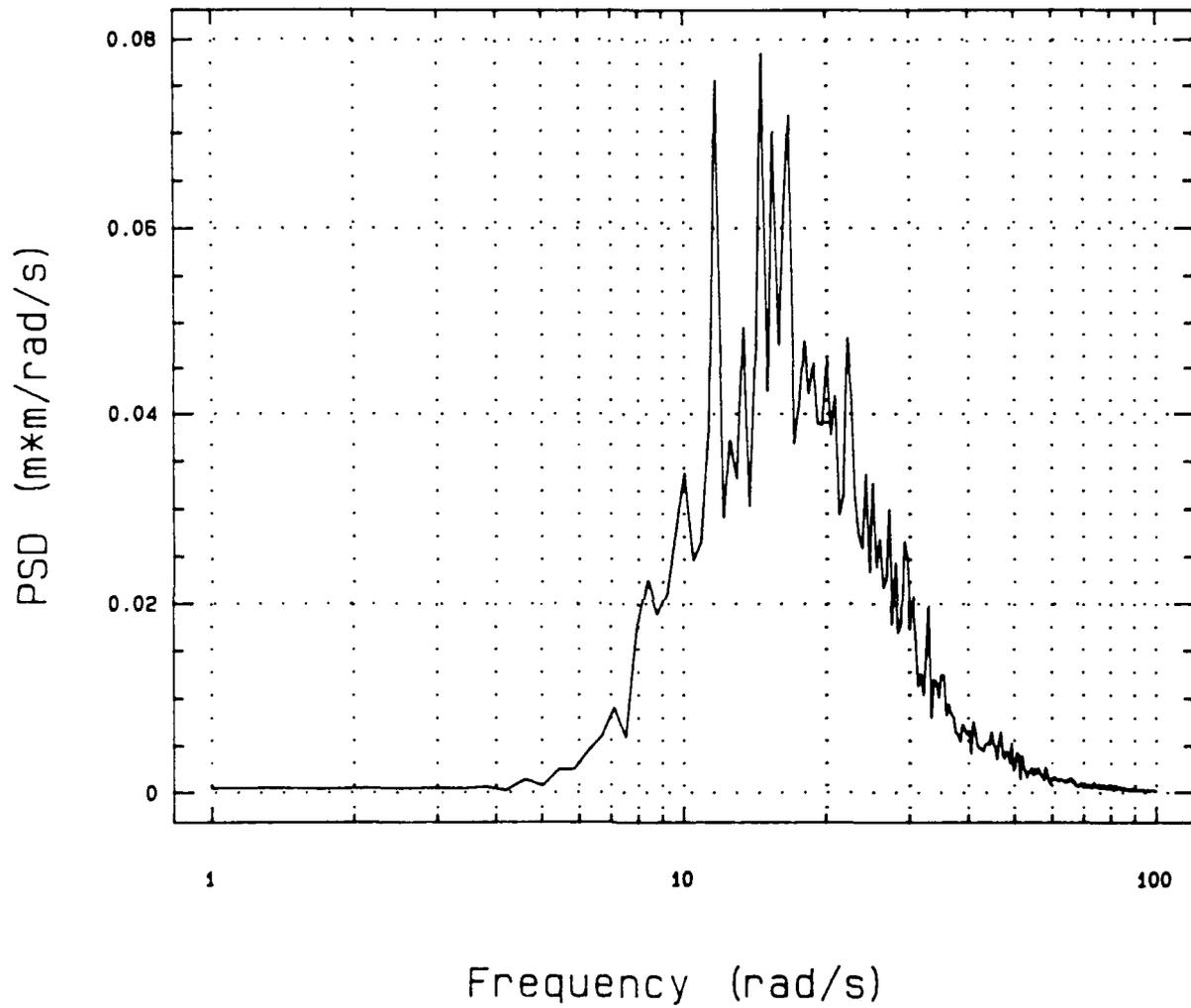
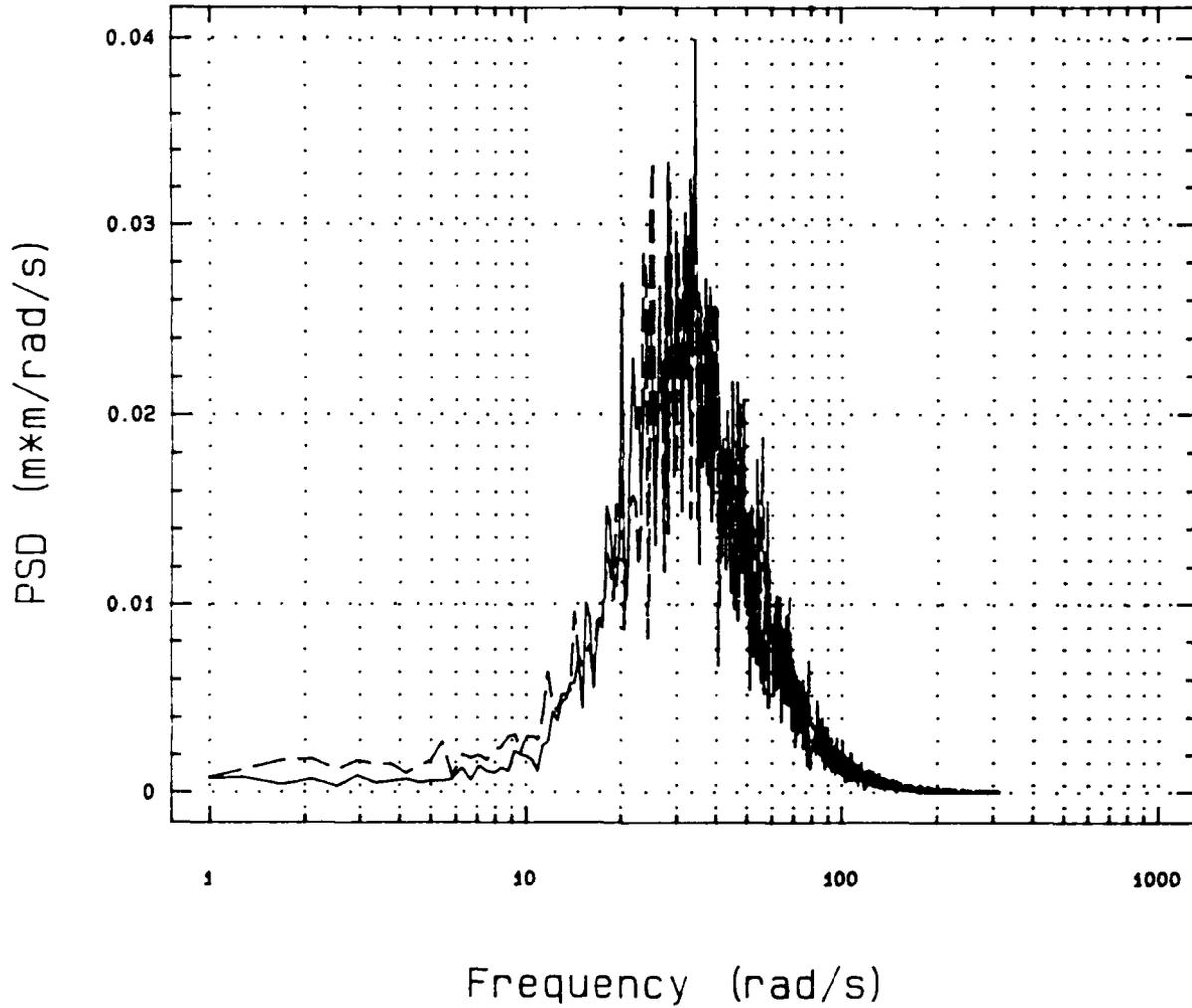


Fig. 9 : PSD functions for SS 5 and an
observer speed of 680 m/s.— 680

-- 340-2V



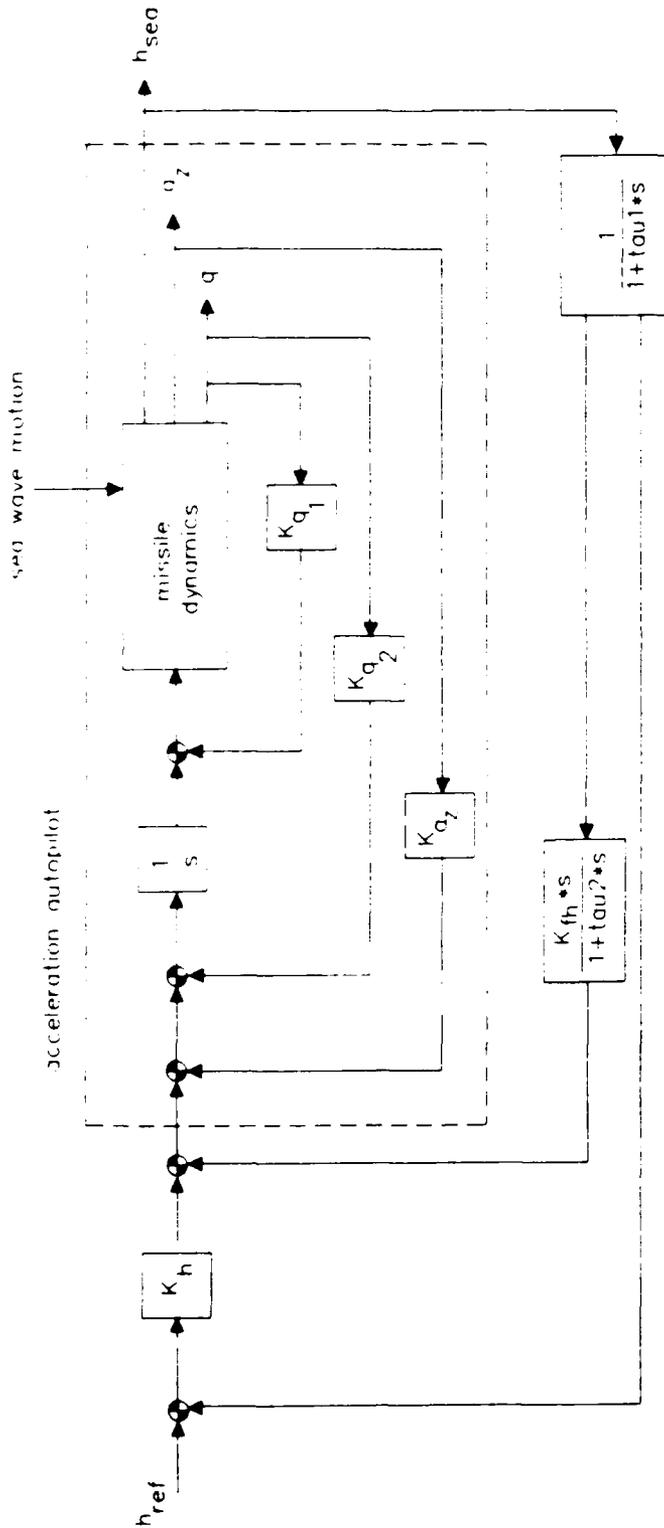


Fig. 10 : Altitude autopilot

Fig. 11 : Horizontal flight profile of a
seaskimmer (batch 2)

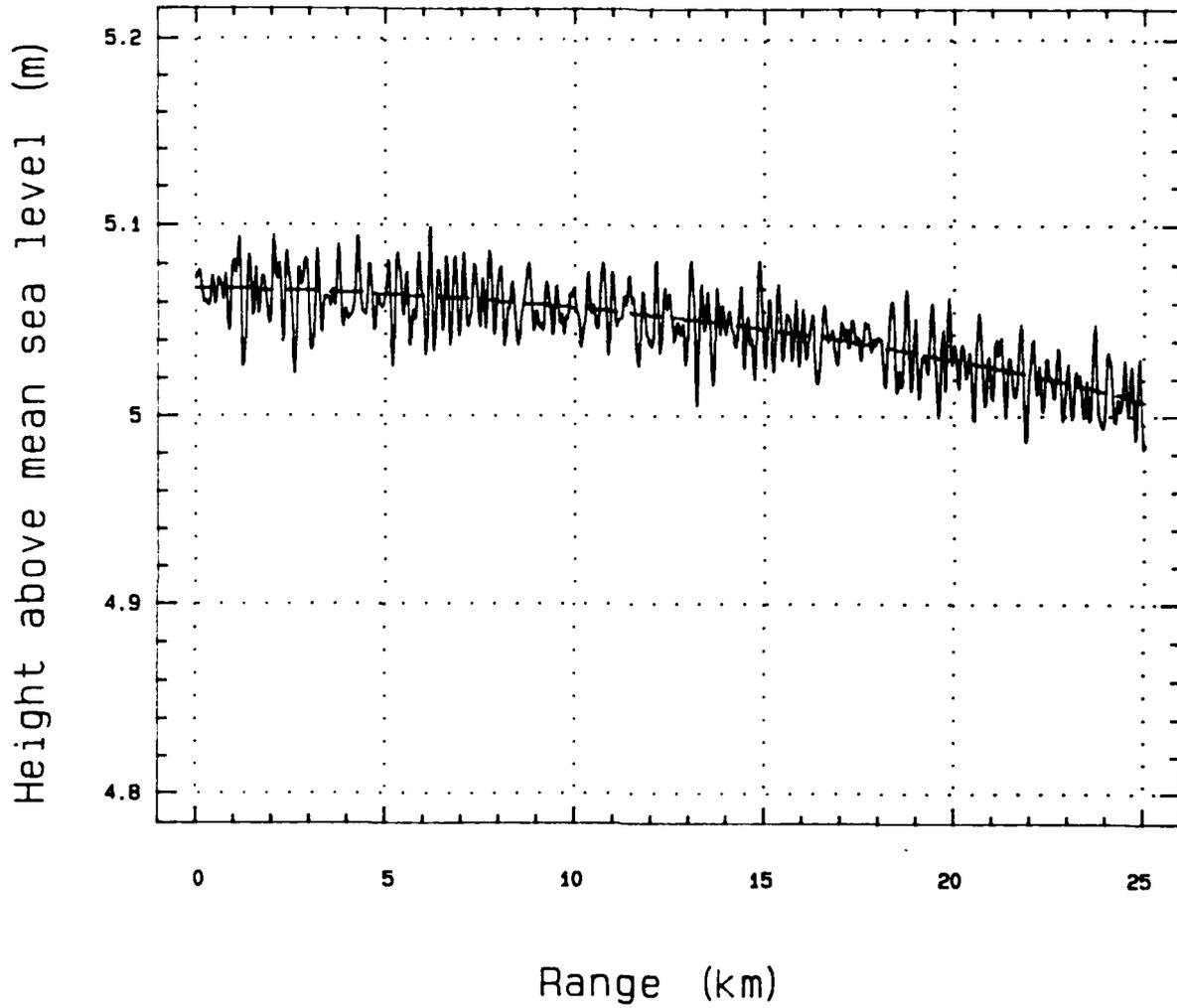


Fig. 12 : Horizontal flight profile of a seaskimmer (batch 3)

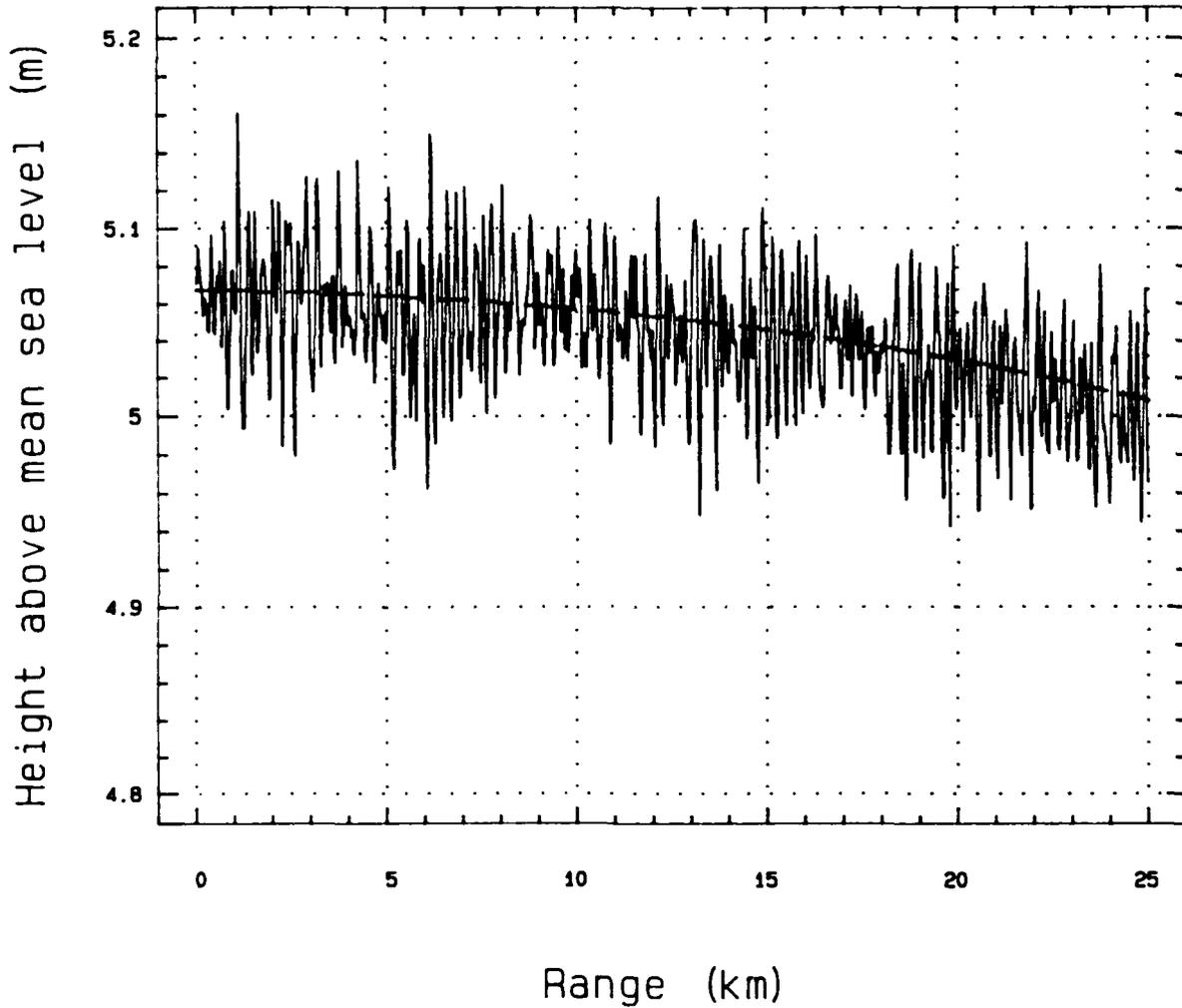


Fig. 13 : Input signal of altimeter

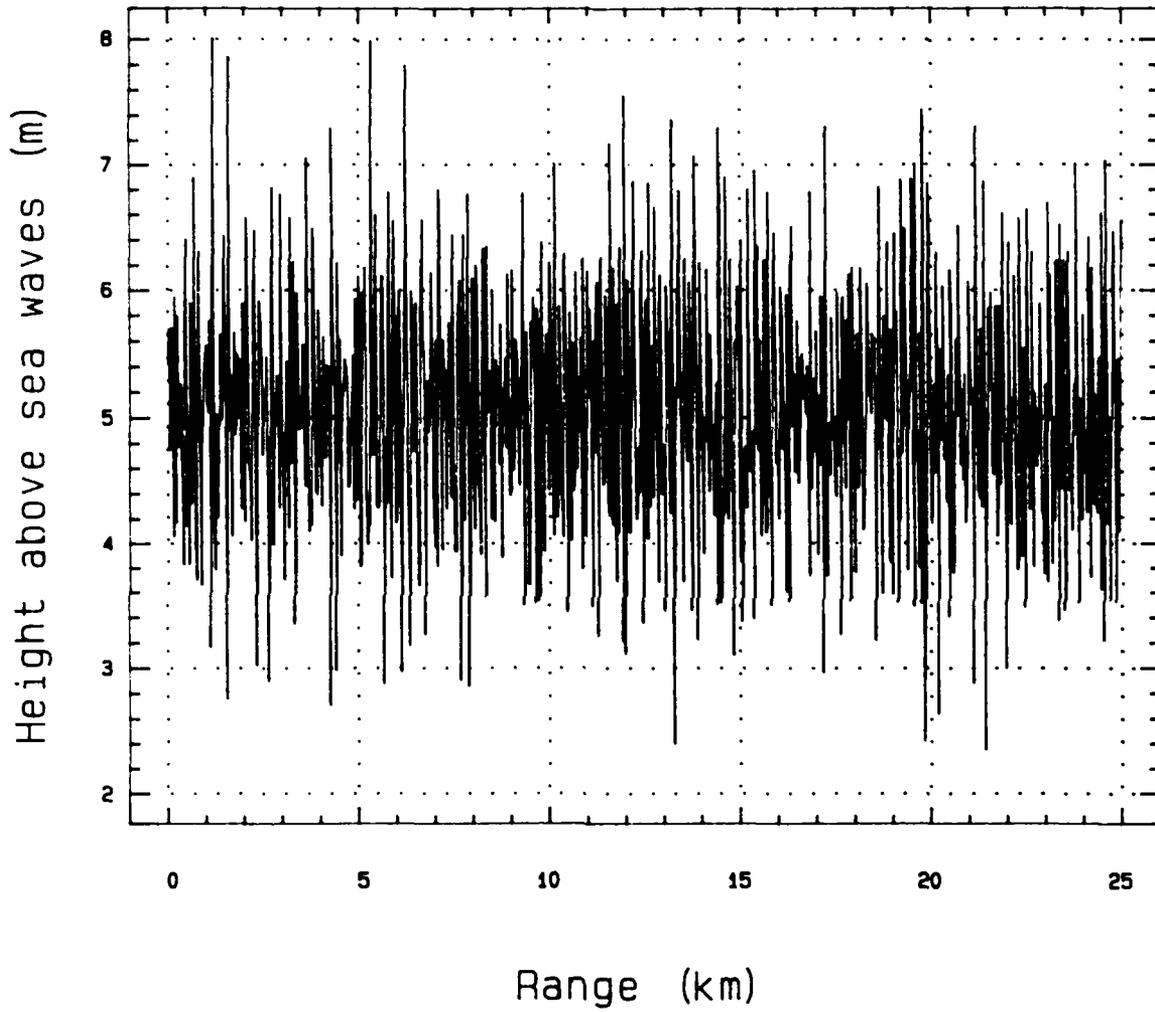


Fig. 14 : Output of altimeter
(batch 2)

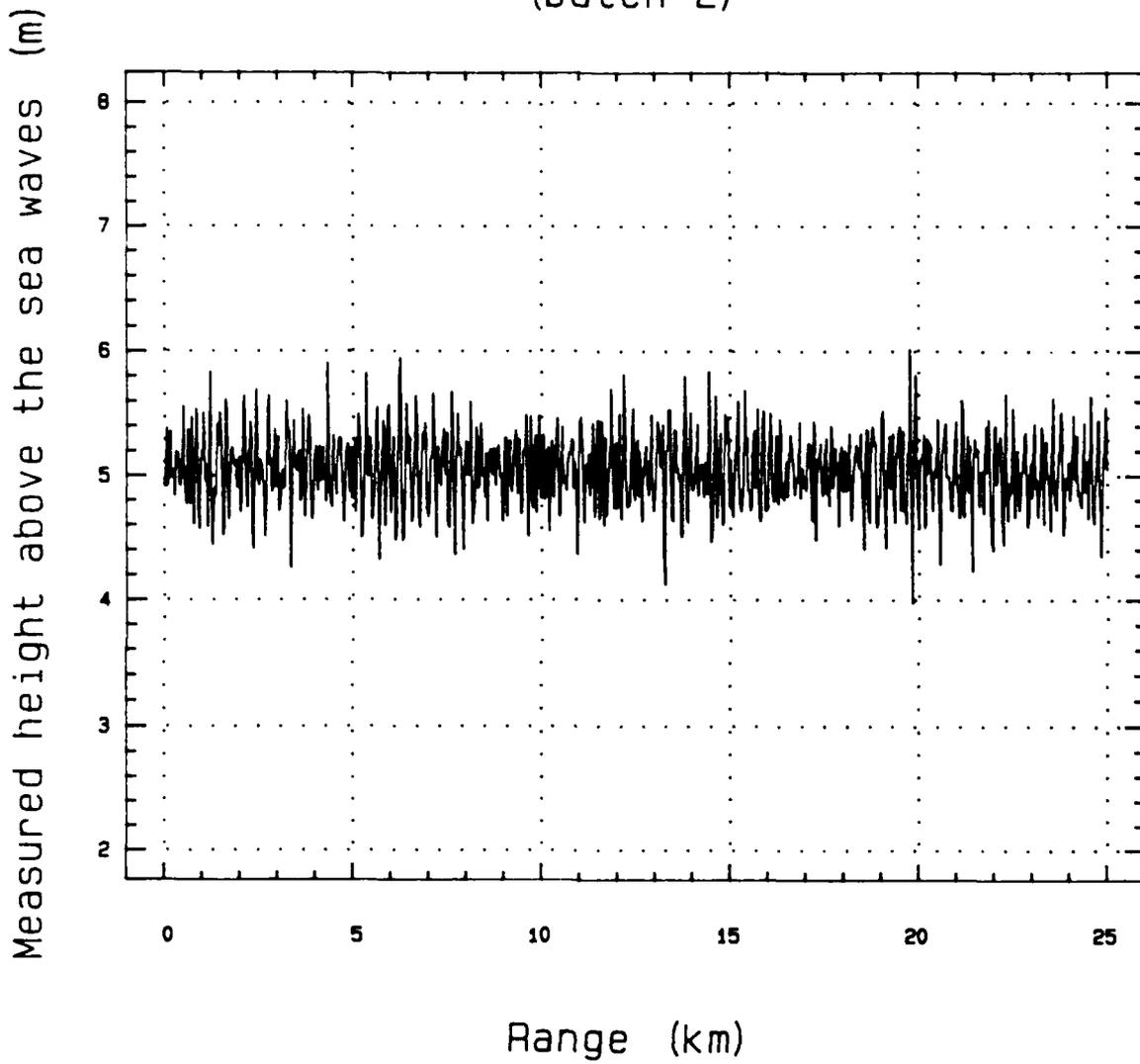


Fig. 15 : Influence of sea wave motion
on missile angle of pitch (batch 2)

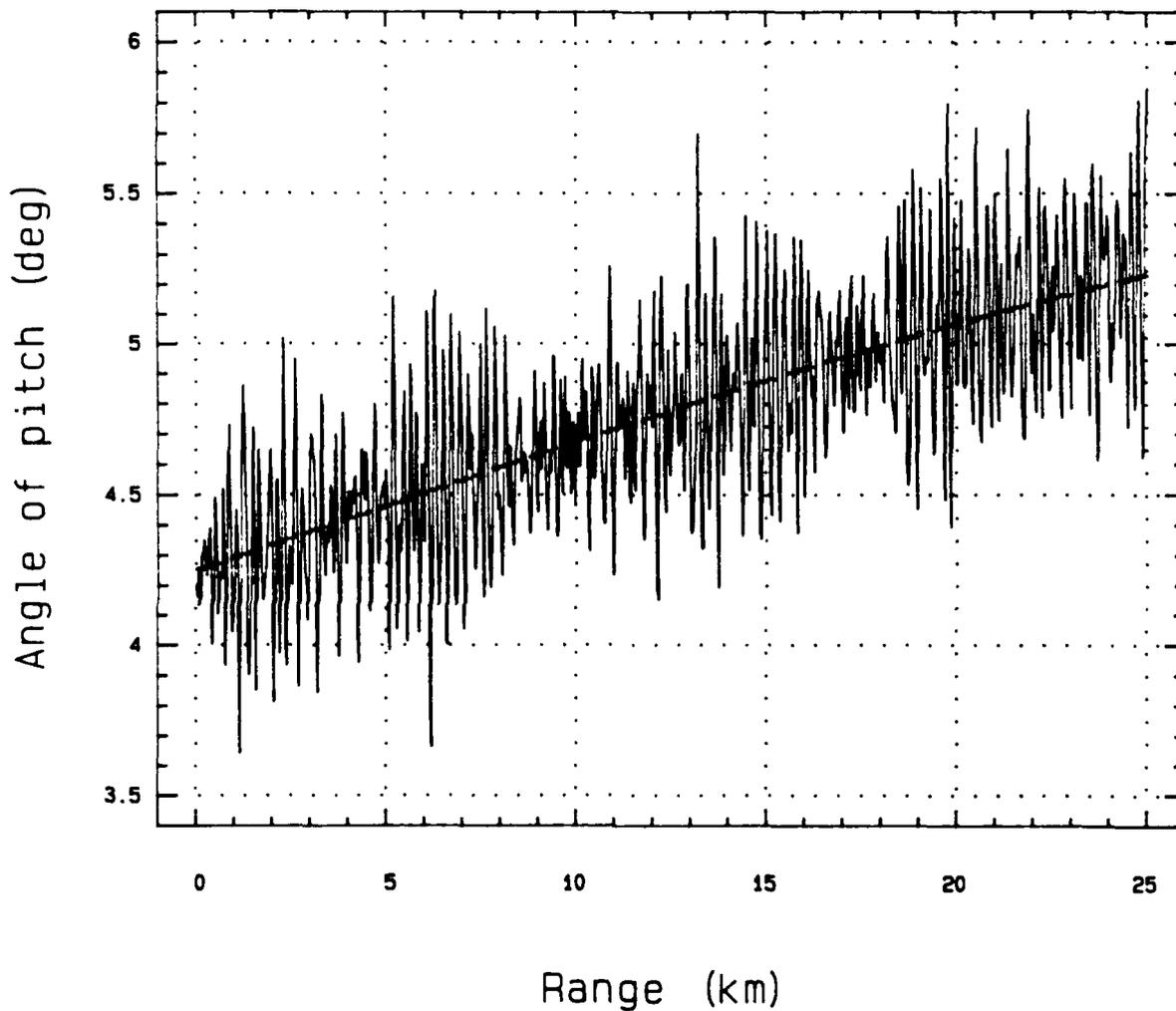
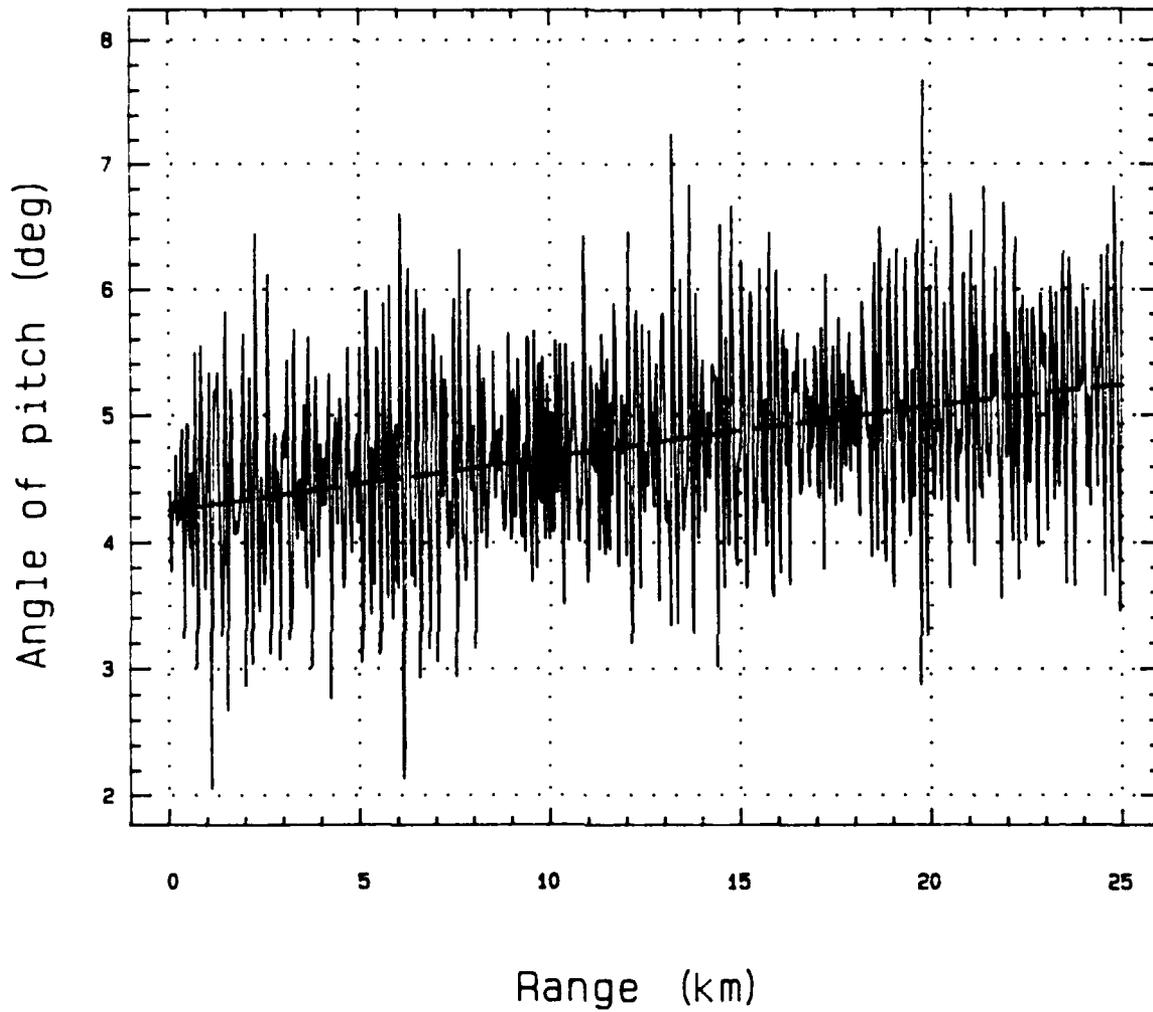


Fig. 16 : Influence of sea wave motion
on missile angle of pitch (batch 3)





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PSD FOR A MOVING OBSERVER

The power contained in the frequency band $[0, \bar{\omega}]$ of the stationary observer must be the same as the power contained in the frequency band $[0, \bar{\omega}_m]$ of the moving observer if $\bar{\omega}$ and $\bar{\omega}_m$ are related by equation (5). This leads to the following relation :

$$\int_{\omega_m=0}^{\bar{\omega}_m} \Phi_m(\omega_m) \cdot d\omega_m = \int_{\omega=0}^{\bar{\omega}} \Phi_{st}(\omega) \cdot d\omega \quad (\forall \bar{\omega} \in [0, \infty)) \quad (A-1)$$

Differentiating equation (5) gives :

$$d\omega = \frac{d\omega_m}{1 + \frac{2U}{g} \cdot \omega} \quad (A-2)$$

Substituting equation (A-2) in (A-1) gives :

$$\int_{\omega_m=0}^{\bar{\omega}_m} \Phi_m(\omega_m) \cdot d\omega_m = \int_{\omega_m=0}^{\bar{\omega}_m} \frac{\Phi_{st}(\omega)}{1 + \frac{2U}{g} \cdot \omega} \cdot d\omega_m \quad (A-3)$$

Differentiating both sides of equation (A-3) with respect to $\bar{\omega}_m$ gives :

$$\Phi_m(\omega_m) = \frac{\Phi_{st}(\omega)}{1 + \frac{2U}{g} \cdot \omega} \quad (A-4)$$

MAXIMUM VALUE OF THE PSD

Determine the frequency at which the power spectral density function has its maximum value.

The PSD function is :

$$\Phi_{\omega}(\omega) = \frac{0.84}{\omega^5} \cdot e^{\frac{-3.36}{H_{3rd}^2 \cdot \omega^4}} = \frac{k_1}{\omega^5} \cdot e^{\frac{k_2}{\omega^4}} \quad (B-1)$$

This function has its maximum if the derivative of equation (B-1) equals zero. The derivative is :

$$\dot{\Phi}_{\omega}(\omega) = \frac{-5 \cdot k_1}{\omega^6} \cdot e^{\frac{k_2}{\omega^4}} + \frac{k_1}{\omega^5} \cdot e^{\frac{k_2}{\omega^4}} \cdot \frac{-4 \cdot k_2}{\omega^5} \quad (B-2)$$

or

$$\dot{\Phi}_{\omega}(\omega) = e^{\frac{k_2}{\omega^4}} \cdot \left[\frac{-5 \cdot k_1 \cdot \omega^4 - 4 \cdot k_1 \cdot k_2}{\omega^{10}} \right] \quad (B-3)$$

From equation (B-3) follows if $\dot{\Phi}$ equals zero :

$$-5 \cdot k_1 \cdot \omega^4 - 4 \cdot k_1 \cdot k_2 = 0 \quad (B-4)$$

So the frequency at which the PSD function has its maximum value is :

$$\omega_{\Phi_{max}} = 1.28 / \sqrt{H_{3rd}} \quad (B-5)$$

PSD FROM N TIME SAMPLES

The calculation of the power spectral density (PSD) from a time dependent variable u of which N equidistant samples are given is presented.

The fourier transform of $u(t)$ is $F(\omega)$:

$$F(\omega) = \int_{-\infty}^{\infty} u(t) \cdot e^{-j\omega t} dt \quad (C-1)$$

the inverse transform is

$$u(t) = 1/2\pi \cdot \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega t} d\omega \quad (C-2)$$

Because $u(t)$ is only sampled in the interval $[-T/2, T/2)$ equation (C-1) is approximated by:

$$F(\omega) = \int_{-T/2}^{T/2} u(t) \cdot e^{-j\omega t} dt \quad (C-3)$$

T is the observation time.

If $u(t)$ is sampled every Δt seconds, the sampled variable $\hat{u}(t)$ equals :

$$\hat{u}(t) = \sum_{n=-N/2}^{N/2-1} u(t) \cdot \delta(t - n \cdot \Delta t) \cdot \Delta t \quad (C-4)$$

with $\Delta t = T/N$.

The fourier transform of $\hat{u}(t)$ is :

$$\begin{aligned}
 F_{\hat{u}}(\omega) &= \int_{-T/2}^{T/2} \hat{u}(t) \cdot e^{-j\omega t} \cdot dt = \int_{-T/2}^{T/2} \sum_{n=-N/2}^{N/2-1} u(t) \cdot \delta(t - n \cdot \Delta t) \cdot \Delta t \cdot e^{-j\omega t} \cdot dt = \\
 &= \Delta t \cdot \sum_{n=-N/2}^{N/2-1} u(n \cdot \Delta t) e^{-j\omega n \Delta t} \quad (C-5)
 \end{aligned}$$

By renumbering the samples with $u_n = u((n-1) \cdot \Delta t)$, $n=1, \dots, N$, such that the first sample is u_1 , equation (C-5) becomes :

$$F_{\hat{u}}(\omega) = \Delta t \cdot e^{j\omega(N\Delta t/2)} \cdot \sum_{n=1}^N u_n \cdot e^{-j\omega(n-1)\Delta t} \quad (C-6)$$

with $\omega_m = 2\pi(m-1)/T = 2\pi(m-1)/(N\Delta t)$ and $m = 1, \dots, N$ follows :

$$\begin{aligned}
 f_m = F_{\hat{u}}(\omega_m) &= \Delta t \cdot e^{j\pi(m-1)} \cdot \sum_{n=1}^N u_n \cdot e^{-j2\pi(m-1)(n-1)/N} = \\
 &= -(-1)^m \cdot \Delta t \cdot \sum_{n=1}^N u_n \cdot e^{-j2\pi(m-1)(n-1)/N} \quad (C-7)
 \end{aligned}$$

f_m is the discrete Fourier transform component of frequency ω_m of the series :

$$(u_n ; n=1, \dots, N)$$

The component f_m can be written as :

$$f_m = -(-1)^m \cdot (a_m + jb_m) \cdot \Delta t \quad (C-8)$$

Using Euler's formula ($e^{jx} = \cos(x) + j \cdot \sin(x)$), the components a_m and b_m become :

$$a_m = \sum_{n=1}^N u_n \cdot \cos[(m-1)(n-1)2\pi/N]$$

$$b_m = - \sum_{n=1}^N u_n \cdot \sin[(m-1)(n-1)2\pi/N]$$
(C-9)

As can readily be seen, the information about the Fourier transform is contained in the first $N/2$ values of $a_m, b_m : (a_m, b_m ; m=1, \dots, N/2)$, because the spectrum of a band limited signal, sampled every Δt seconds vanishes for ω values higher than $\pi/\Delta t$. From a transform point of view it is evident that if the time signal has N real samples, so has the transformed time signal. This is because of the invertibility property of the Fourier transform.

The amplitude spectrum is found from :

$$|f_m| = \sqrt{(a_m^2 + b_m^2)}$$
(C-10)

With a fast Fourier transform routine the first $N/2$ coefficients can be calculated. To determine the PSD, the autocorrelation function must first be calculated. The sample autocorrelation function $R_N(n)$ of u , based on N samples, is determined with :

$$R_N(n) = \frac{1}{N-n} \cdot \sum_{j=1}^{N-n} u_{j+n} \cdot u_j$$
(C-11)

with $n = 0, \dots, n_{\max}$ and $R_N(-n) = R_N(n)$.

The PSD function is the Fourier transform of the autocorrelation function. When the autocorrelation based on a finite number of samples is used in the Fourier transform, an approximation $S_N(\omega)$ of the PSD is obtained (see ref. 4) :

$$S_N(\omega) = \sum_{n=1}^{2 \cdot n_{\max} + 1} R_N(n - n_{\max} - 1) \cdot e^{-j\omega n \Delta t} \quad (\text{C-12})$$

With a fast Fourier transform routine the approximated PSD function can be determined using equations (C-10), (C-11) and (C-12).

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IN THIS REPORT THE DESCRIPTION AND GENERATION OF SEA WAVE MOTION IS PRESENTED IN ORDER TO EXAMINE THE INFLUENCE OF THIS MOTION ON THE ALTITUDE PERFORMANCE OF A LOW FLYING MISSILE. FOR THIS STUDY A SEASKIMMING MISSILE IS SIMULATED WITH THE 6 DOF WEAPON ANALYSIS AND SIMULATION PROGRAM (WASP) OF THE PHYSICS AND ELECTRONICS LABORATORY (FEL-TNO).

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