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NCEL

Technical Note

February 1990

By L. J. Malvar and G. E. Warren

Sponsored By Naval Facilities Engineering Command

AD-A223 547

MIXED MODE CRACK PROPAGATION IN CONCRETE

ABSTRACT Two smeared crack approaches to fracture of concrete in mixed mode are implemented in two-dimensional nonlinear concrete elements: (1) tensile stress transfer across cracks and (2) tensile plus shear stress transfer across cracks. To corroborate the analytical model a notched beam under mixed mode loading is then analyzed. In both cases, the stiffnesses normal and parallel the crack were modified to insure a positive definite stiffness matrix. Stresses were corrected and set as functions of the crack slip and crack opening. Equilibrium iterations were implemented to redistribute stress. In both cases, acceptable agreement was found between analytical predictions and experimental results. The consideration of shear stress transfer yielded better predictions, but requires consideration of a non-symmetrical stiffness matrix.

NAVAL CIVIL ENGINEERING LABORATORY PORT HUENEME CALIFORNIA 93043-5003

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METRIC CONVERSION FACTORS

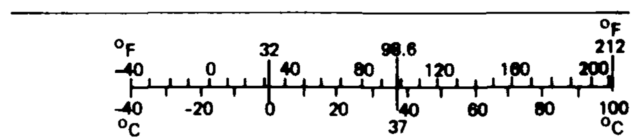
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-018

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	February 1990	Final Oct 88 - Sep 89

4. TITLE AND SUBTITLE

MIXED MODE CRACK PROPAGATION IN CONCRETE

5. FUNDING NUMBERS

PE - 61153N
WU - DN665019

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESSE(S)

Naval Civil Engineering Laboratory
Port Hueneme, CA 93043-5003

8. PERFORMING ORGANIZATION REPORT NUMBER

TN-1807

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESSE(S)

Naval Facilities Engineering Command
Alexandria, Virginia 22332

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Two smeared crack approaches to fracture of concrete in mixed mode are implemented in two-dimensional nonlinear concrete elements: (1) tensile stress transfer across cracks and (2) tensile plus shear stress transfer across cracks. To corroborate the analytical model a notched beam under mixed mode loading is then analyzed. In both cases, the stiffnesses normal and parallel to the crack were modified to insure a positive definite stiffness matrix. Stresses were corrected and set as functions of the crack slip and crack opening. Equilibrium iterations were implemented to redistribute stress. In both cases, acceptable agreement was found between analytical predictions and experimental results. The consideration of shear stress transfer yielded better predictions, but requires consideration of a non-symmetrical stiffness matrix.

14. SUBJECT TERMS

Fracture mechanics, nonlinear, concrete, mixed mode, smeared cracking

15. NUMBER OF PAGES	16. PRICE CODE
47	

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

CONTENTS

PURPOSE

The Naval Facilities Engineering Command (NAVFAC) through the Naval Civil Engineering Laboratory (NCEL) has initiated a project to develop fracture mechanics methodology for design application of reinforced concrete elements in tensile and shear stress states. In a preceding study, analytical modeling methodology of Mode I (opening) was detailed in two and three dimensions, and Mode II crack propagation (shearing) addressed. In this report Mode II modeling methodology is developed and a benchmark mixed mode problem is analyzed. This report supports the project "Fatigue and Fracture of Concrete" in the NAVFAC 6.1 Basic Research Program YR023-03-01, Structural Modeling.

The modifications implemented in the computer program ADINA have been compiled in the appendixes.

INTRODUCTION

Although it is generally recognized that crack initiation in concrete occurs in Mode I (opening), crack propagation is more likely to take place in mixed mode, i.e., involving Mode I and II (shearing), or III (tearing).

Mixed mode crack propagation involves considering the transfer of tensile and shear forces across cracks. Constitutive relations representing the transferred stresses were evaluated (Ref 1). In the present report these constitutive relations are implemented in a general purpose finite element program developed by ADINA R&D Inc. (Ref 2). A benchmark experiment by Arrea and Ingraffea (Ref 3 and 4) is then modeled, with and without considering transfer of shear stresses.

PROBLEM

The mixed mode problem considered is depicted in Figure 1, and concrete properties used are reported in Table 1. In many cases the problem was approached without considering shear transfer across the crack. Initial attempts at modeling the shear transfer using a constant shear retention factor β (typically $\beta \leq 0.1$) yielded results with almost no softening after peak load (Ref 5 and 6). Better representations were obtained either assuming the existence of a Mode II fracture energy (Ref 7), or using a predetermined crack path (Ref 8).

In this study the consideration of a shear transfer model is attempted and its effects observed.

TENSILE STRESS TRANSFER

The transfer of tensile stresses across a crack had already been implemented with a smeared crack approach (Ref 9) using the Crack Band Model (CBM) (see Appendixes A, B, and C). This tension softening behavior involved a negative stiffness, C_s, for the cracked element. The CBM was implemented assuming zero stiffness (actually a very small value was used to avoid a singular stiffness matrix) and then resetting the stresses as a function of the crack opening. These stresses are then redistributed during equilibrium iterations. The stress transferred versus crack width relationship is tabulated in Table 2 (Ref 1) in nondimensional form. The fracture energy, G_f, is related to the stress versus displacement relationship by:

$$G_f = w_o \; f_t \int_0^1 \sigma/f_t \; d(w/w_o)$$

where w = crack width or crack opening

σ = stress transferred at crack width w

w_o = crack width beyond which no stress is transferred

f_t = tensile strength

In a first analysis on the mixed mode problem the CBM alone was used in order to evaluate the importance of considering shear transfer.

The latest version of ADINA (Ref 2) acknowledges the importance of strain softening by including a linear stress release as a function of strain after cracking (Ref 10). However, this stress release is not explicitly linked to fracture energy, and the authors have shown that linearizing the highly nonlinear post peak stress versus strain relationship negatively affects results (Ref 9). Hence, this feature of the latest ADINA version was not used in the present project.

ARC-LENGTH PROCEDURE

The solution of the finite element incremental equations of motion was first attempted using the spherical arc-length and the constant increment of external work procedures described in Reference 11. The post-peak numerical analysis of this experiment has shown to be highly unstable (Ref 5). The adopted approaches did not yield converged equilibrium states past peak load and, thus, were modified.

A type of indirect displacement control (Ref 12) was then adopted: in the arc-length procedure the norm of displacement (involving all nodal points) was replaced by the distance between the two points at the edges of the notch (Appendix C). The vertical component of this distance is referred to as CMSD (Crack Mouth Sliding Displacement). During the test, the CMSD is a monotonically increasing parameter that stabilized the algorithm. In the experiment, the CMSD had been used as feed-back control parameter.

FAILURE ENVELOPES

The failure envelopes used in ADINA (Ref 13) are largely based on biaxial concrete strength experimental results by Kupfer et al. (Ref 14). In the plane stress analytical model, the crack path showed sensitivity to the tensile envelope representation close to the tension/tension zone ($\sigma_1>0$, $\sigma_2>0$, $\sigma_3=0$) (Figure 2). The existing linear envelope in the tension/compression zone was then modified to better match experimental results. The following power relationship was used:

$$\sigma'_t = \sigma_t \left[1 - \left(\frac{{}^t\sigma_i}{\sigma'_c}\right)^n \right]$$

where σ_t = uniaxial cut-off tensile stress

σ'_t = uniaxial cut-off tensile stress under multiaxial conditions

σ'_c = uniaxial compressive failure stress under multiaxial conditions

${}^t\sigma_i$ = principal stress in direction i at time t

$n = 1$ if $\sigma'_c \geq 8000$ psi (563 kp/cm^2)

$= 1 + 0.0002(8000-\sigma'_c)$ if $\sigma'_c < 8000$ psi

Both linear and power envelopes are shown in Figure 2, together with Kupfer et al. results for $\beta p = 315$ kp/cm^2 (4450 psi). βp is the uniaxial compressive strength of 50 by 50 by 200 mm (2 x 2 x 7.9 in.) prisms. The current ADINA addresses this deficiency, but corrects it in a different way (Ref 10):

$$\sigma'_t = \sigma_t \left[1 - 0.75 \frac{{}^t\sigma_i}{\sigma'_c} \right]$$

From Figure 2 it is apparent that the present modification yields a better match. Program modifications are reported in Appendix D.

FINITE ELEMENT MODEL

The finite element mesh used is depicted in Figure 1. Loads of 0.13P and P were applied at points A and B, respectively. In the computer program this is accomplished using an automatic step incrementation method, where the level of externally applied loads is adjusted automatically. In the experiment, a single total load of 1.13P was applied on a steel beam bearing on rollers at points A and B. The point of application of that total load will be referred as point C.

SHEAR TRANSFER

Cracks in reinforced concrete are able to transmit shear forces across crack faces. This transfer is traditionally neglected on the assumption that this would be a conservative simplification. However, Bazant et al. showed that this assumption can be an over simplification (Ref 15 and 16). Crack dilation occurs with shear slip. However, crack dilation is prevented by forces normal to the crack faces, which will have to be compensated by tensiles forces in the reinforcement across the crack.

Shear stresses can be transferred across a crack in three ways: (1) aggregate interlock as a result of the roughness of the crack faces, (2) dowel action or shear resistance of the reinforcement across the crack, and (3) the axial tensile force component in the reinforcement oblique to the plane of cracking.

For members with low reinforcement and for small crack widths, aggregate interlock is the main mechanism of shear transfer. Tests carried out on beams without web reinforcement showed that aggregate interlock accounted for up to 75 percent of the shear transfer (Ref 17). Hence, most attention will be given to this first mechanism of transfer.

SHEAR TRANSFER MODEL

Three accepted empirical models which represent the nonlinear relationships between shear stress and slip are: the Rough Crack Model (RCM) in its original form (Ref 11), or in a modified form (MRCM) (Ref 18), and the Two-Phase Model (TPM) (Ref 19 and 20). The constitutive laws of the MRCM are as follows:

$$\sigma_{nn} = -a_{12} \left(\frac{r}{1+r^2}\right)^{0.25} \sqrt{\delta_n \sigma_{nt}} \qquad \text{(always compressive)} \qquad (1)$$

$$\sigma_{nt} = \tau_o \left(1 - \sqrt{\frac{2\delta_n}{d_a}}\right) r \, \frac{a_3 + a_4 |r|^3}{1 + a_4 r^4} \qquad (2)$$

in which δ_n = crack opening ($\delta_n \geq 0$)

δ_t = relative slip

σ_{nn} = interface normal stress

σ_{nt} = interface shear stress

$r = \delta_t / \delta_n$

$a_{12} = 0.62$

$$a_3 = 2.45/\tau_o$$

$$a_4 = 2.4(1-4/\tau_o)$$

$$\tau_o = 0.25\ f'_c$$

and

$$\begin{bmatrix} d\sigma_{nn} \\ d\sigma_{nt} \end{bmatrix} = \begin{bmatrix} B_{nn} & B_{nt} \\ B_{tn} & B_{tt} \end{bmatrix} \begin{bmatrix} d\delta_n \\ d\delta_t \end{bmatrix} \quad (3)$$

where $B = \begin{bmatrix} B_{nn} & B_{nt} \\ B_{tn} & B_{tt} \end{bmatrix}$ is the crack stiffness matrix.

The derivation of B is shown in Appendix E.

IMPLEMENTATION IN FINITE ELEMENT PROGRAM

Transfer of shear stresses was implemented by combining the MRCM and the CBM. The incremental flexibility matrix due to the solid concrete and including strain softening in tension is given by (Ref 21):

$$\{d\varepsilon\} = D^{sc}\ \{d\sigma\}$$

or

$$\begin{bmatrix} d\varepsilon_{nn} \\ d\varepsilon_{tt} \\ d\varepsilon_{nt} \end{bmatrix} = \begin{bmatrix} 1/E & -\mu/E & 0 \\ -\mu/E & 1/E & 0 \\ 0 & 0 & 1/G \end{bmatrix} \begin{bmatrix} d\sigma_{nn} \\ d\sigma_{tt} \\ d\sigma_{nt} \end{bmatrix} \quad (4)$$

where μ = Poisson's ratio.

In addition, since we assume strain softening in tension to be present, the slope C_s of the strain softening branch has to be taken into account. The crack stiffness is then:

$$C^{cr} = \begin{bmatrix} wB_{nn}+C_s & wB_{nt} \\ wB_{tn} & wB_{tt} \end{bmatrix} \quad (5)$$

For very small values of the crack opening, C_s is large, but B_{nn} is almost zero; whereas, when the crack opening reaches about 0.1mm, the opposite holds.

The incremental stiffness matrix can be obtained as follows:

$$D = D^{sc} + C^{cr-1}$$

$$C = D^{-1}$$

yielding

$$C = \frac{1}{1+wB_{tt}/G + (1-\mu)(wB_{nn}+C_s+\Phi w^2/G)/2G} \cdot \tag{6}$$

$$\begin{bmatrix} \Phi w^2/G+wB_{nn}+C_s & \mu(\Phi w^2/G+wB_{nn}+C_s) & wB_{nt} \\ \mu(\Phi w^2/G+wB_{nn}+C_s) & \Phi w^2/G+wB_{nn}+C_s+2(1+\mu)wB_{tt}+E & \mu wB_{nt} \\ wB_{tn} & \mu wB_{tn} & (1-\mu)\Phi w^2/2G+wB_{tt} \end{bmatrix}$$

where $\Phi = B_{nn}B_{tt} - B_{nt}B_{tn}$.

This yields an incremental stiffness matrix which is not symmetrical and is not guaranteed to be definite positive.

Since ADINA considers only symmetric matrices, the solution was attempted using a modified stiffness, and then correcting the stresses at every iteration for each load step increment. It was then assumed that:

$$\mu = 0$$

and to insure definite positiveness

$$C_{11} = 0^+ \qquad \text{if } C_{11} < 0$$

$$C_{31} = C_{13} = \sqrt{C_{11}C_{33}} - 0^+$$

where 0^+ is a small positive number.

MODEL REPRESENTATION

To evaluate the effects of shear transfer, a 100 by 100 by 100 mm (4 x 4 x 4 in.) concrete finite element was first cracked in tension, then sheared in the perpendicular direction (Figure 3), in displacement control. Given the nodal displacements, strains at the Gauss points are evaluated, then an iterative process determines crack slip, crack dilatation, and concrete deformation, using formulas (1), (2), C_s, and:

$$\delta_n = (\varepsilon_n - \sigma_{nn}/E)w$$

$$\delta_t = (\varepsilon_t - \sigma_{nt}/G)w$$

The model behavior is predicted using all three formulations (RCM, MRCM, TPM). For each case, Figure 4 shows the shear and normal loads transferred. The TPM values were capped to the maximum predicted by the RCM. It is observed that the dilatancy induces vertical compression (along the z axis). If reinforcing bars perpendicular to the cracks were present, the dilatancy would increase the tension in the bars at the crack locations.

From Figure 4 it is apparent that all three models yield very similar shear transfer capacity, but the normal stress due to dilatation is significantly higher for the RCM. Since more normal stress experimental data appears to back the MRCM and TPM, the RCM was discarded. In the mixed mode analysis, the more recent MRCM formulation was chosen, since it presents no discontinuity in the stress gradient.

RESULTS

In order to evaluate the importance of modeling stress transfer across cracks, the analytical model was first run with no transfer, i.e., assuming total stress release right after cracking. Since the standard algorithms did not converge, the indirect displacement method was used, with a very low fracture energy (0.0002 N/mm) equivalent of a sudden stress release. Results for this first run are shown in Figure 5.

The analysis was then carried out considering only tensile stress transfer across the cracks (CBM). Finally, the MRCM was added and a new analysis completed (CBM+MRCM). Results for both cases are shown in the form of load versus CMSD (Figure 5), and load versus vertical displacement at point C (Figure 6). The vertical displacement at point C was derived by linear interpolation of the vertical displacements of points A and B. Data points indicating the reported range of experimental results (Ref 3) are shown in Figure 5.

Convergence of the arc-length algorithm was only obtained for carefully chosen control parameters. These parameters control the size of the step in the load-CMSD space (ALFA), the maximum number of iterations allowed for each time step (ITEMAX), the maximum displacement at control point E (DISPP), and energy convergence criteria (ETOL) (Ref 11). In each case they were respectively:

Parameter	No Transfer	CBM	CMB+MRCM
ITEMAX	45	45	30
ETOL	10^{-6}	10^{-6}	5.10^{-4}
DISPP	-0.015	-0.015	-0.015
ALFA	0.4	0.4	0.5

The crack pattern for the last loading step is indicated in Figure 7 (CBM case). Figure 8 shows the deformed shape obtained for the last step (CBM case).

DISCUSSION

Figure 6 indicates that the displacement at point C presents a sharp snap-back past peak load. This explains why displacement control at that point cannot yield the post peak response. The displacement at both points A and B shows a similar behavior, which explains why the norm of displacement in the arc-length procedure was unsuccessful.

Figure 5 shows that considering tensile stress transfer alone yields a conservative behavior prediction. The maximum load is underestimated by about 20 percent, and the post peak load carrying capacity is lower. However, the shape of the strain softening portion is similar. A higher value of G_f would yield a better match to the experimental peak load and post peak response (Ref 8).

The crack pattern (Figure 7) still differs from the reported experimental crack path. It was, however, observed that a small variation in the mesh size, or initially larger load step sizes, would affect the path or result in bifurcation points. Similarly, stiffer bearing plates would bring the crack path closer to the notch plane. The crack path would easily follow any of the different directions indicated in Figure 9. This would explain the discrepancies in crack paths found by different authors (Ref 22, 23, 24, and 25) (using a similar but symmetrical specimen). For example, the experimental crack path obtained in Reference 23 coincides with the analytical crack pattern shown in Figure 7.

Should tensile stress transfer not have been considered, the maximum load carrying capacity of the analytical model would have been reached as soon as the first tensile cracks formed (around 50 kips) (Figure 5). This is obviously an inadequate representation of the experimental behavior.

Transfer of both tensile and shear stress is considered best in matching experimental behavior. The peak load is higher and the post peak behavior is closer to experimental results. However, in order to obtain the complete post peak behavior, a nonsymmetrical stiffness matrix would have to be considered. This would present additional difficulties, such as (1) implementation in a new program with a nonsymmetrical solver, and (2) increase in computation time. The increased accuracy has to be weighed against the increased cost in implementing shear transfer. In this case, the crack pattern remained similar to the previous one.

CONCLUSIONS

The consideration of shear stress transfer across the propagating cracks yielded a better prediction of the experimental results. However, the resultant stiffness matrix is nonsymmetrical and would require implementation in a program with a nonsymmetric solver. This would enhance the convergence of the indirect displacement control algorithm.

The exclusive consideration of tensile stress transfer yielded good results up to peak load. Beyond this point, the loads are underestimated, although the shape of the unloading branch matches the experimental trend.

This could be an acceptable representation of mixed mode behavior as long as it is kept in mind that a conservative post peak behavior will be obtained. Finally, it was shown that inadmissible results are obtained if both tensile and shear stresses are assumed to completely vanish upon cracking.

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Table 1. Concrete Properties

Fracture Energy	G_f = 0.055 N/mm
Compressive Strength	$f'c$ = 45.5 N/mm^2
Tensile Strength	f_t = 2.80 N/mm^2
Modulus of elasticity	E = 24.8 GPa

Table 2. Stress - Crack Width Relationship

w/w_o	σ/f_t
0.00	1.0000
0.05	0.7082
0.10	0.5108
0.15	0.3817
0.20	0.2986
0.25	0.2446
0.30	0.2080
0.40	0.1596
0.60	0.0904
0.80	0.0361
1.00	0.0000

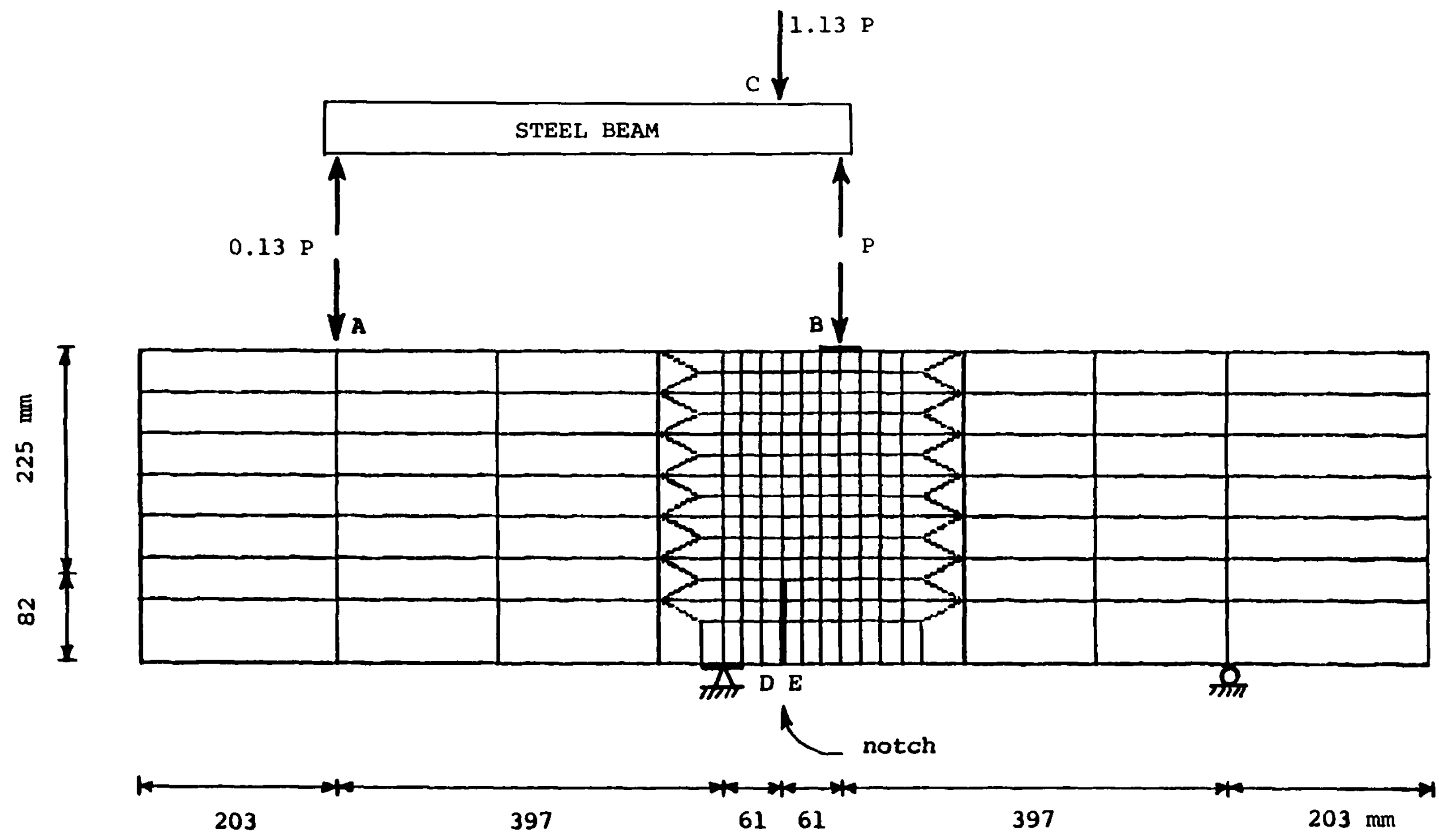


Figure 1. Experimental setup and finite element mesh.

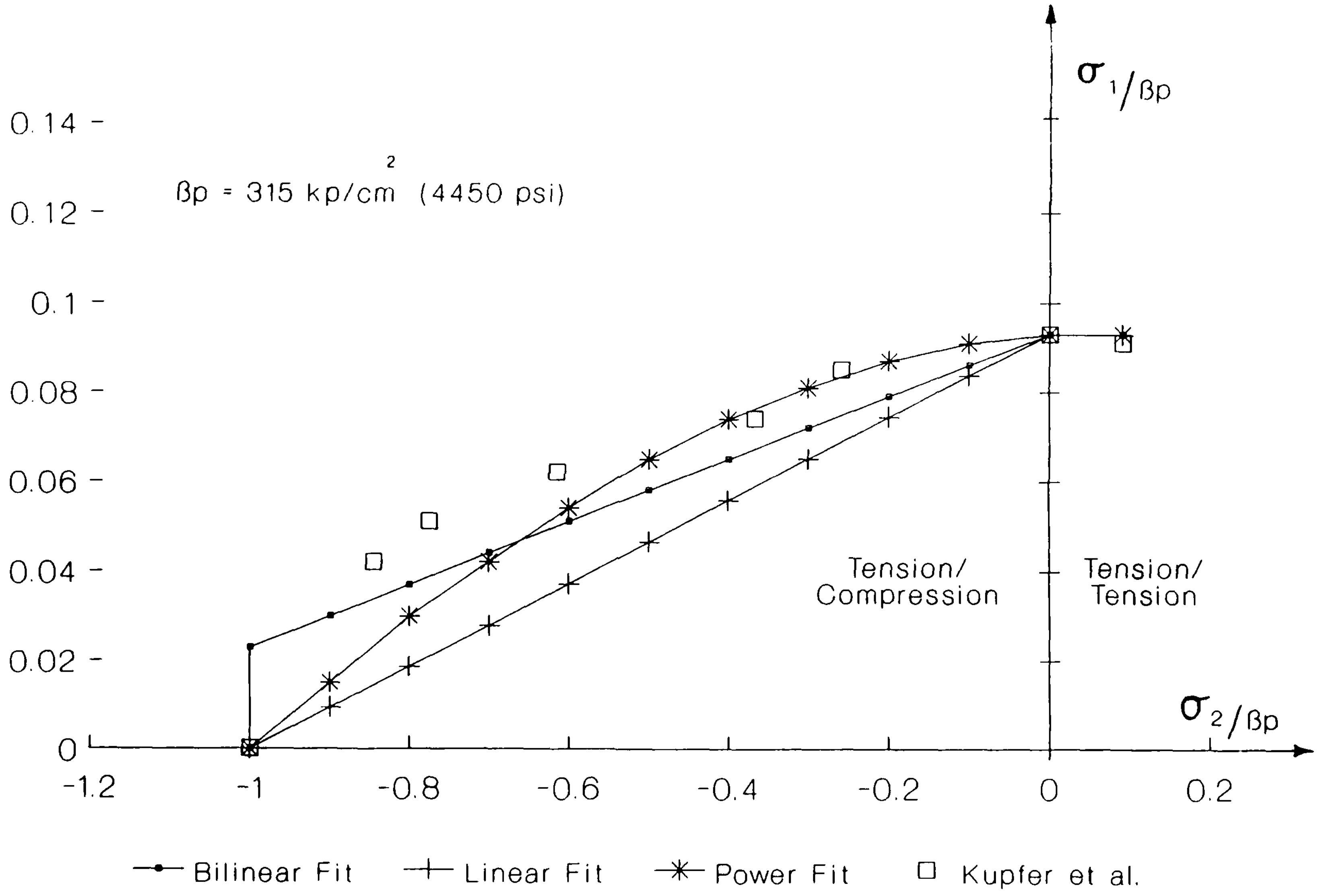


Figure 2. Tension-compression failure envelope.

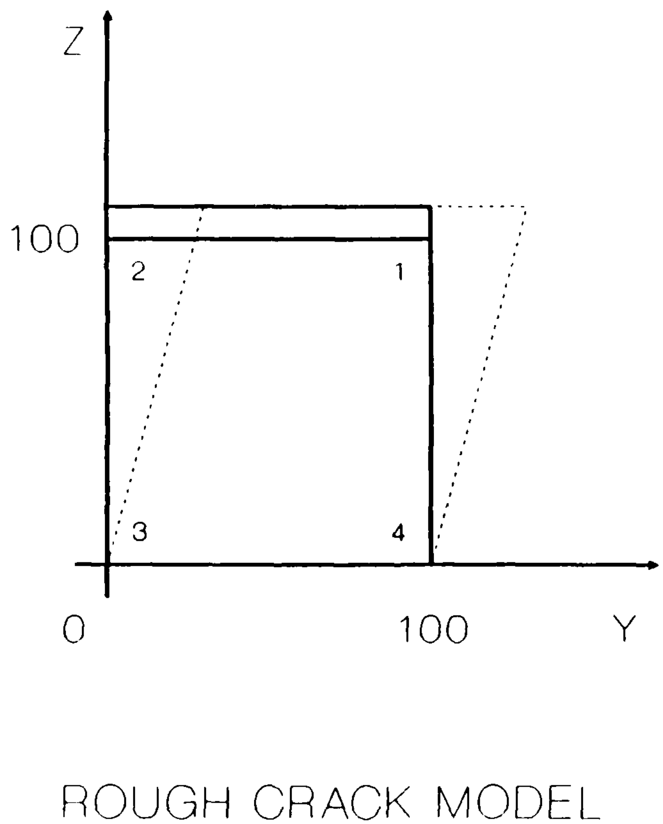
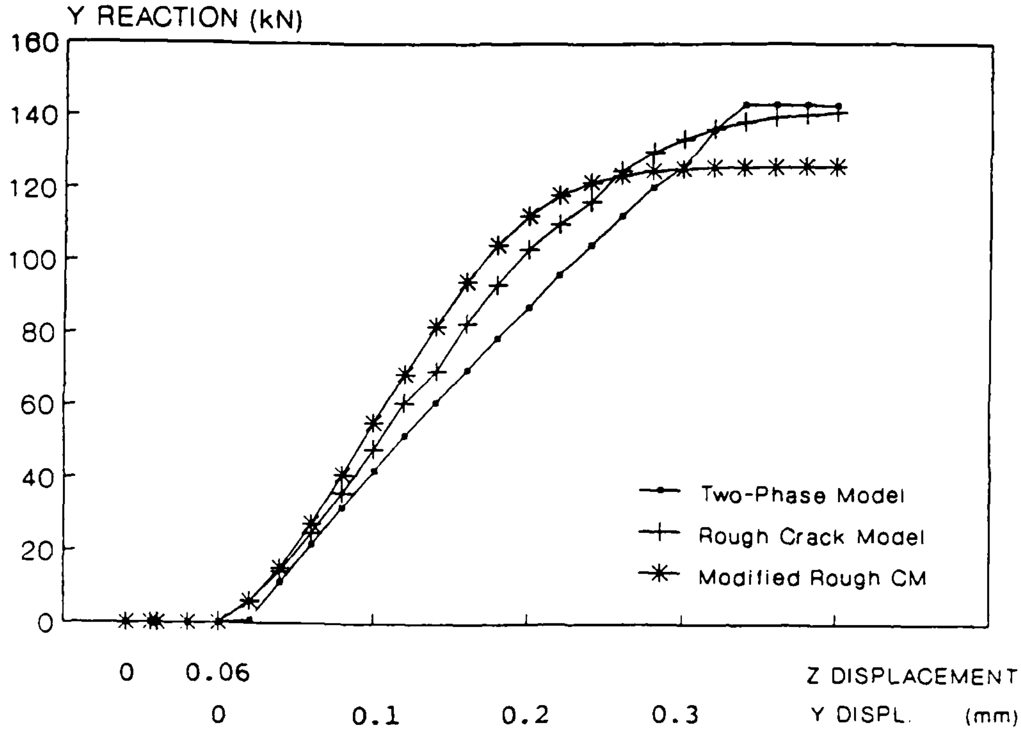
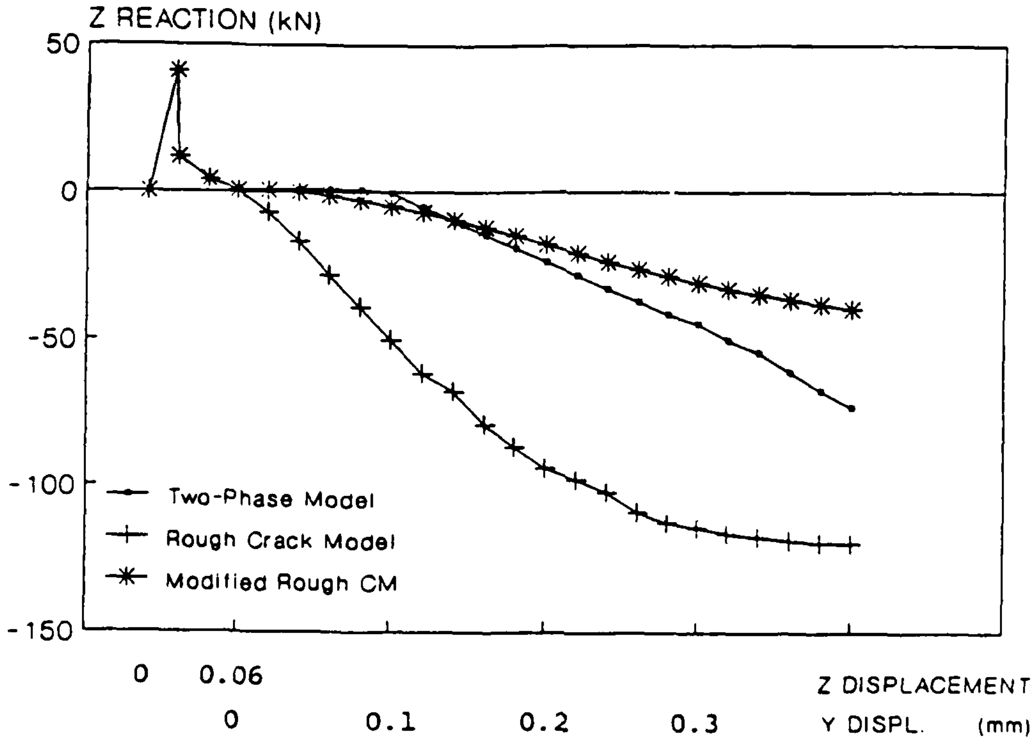


Figure 3. Shear transfer experiment.



(a) Y Reaction (kN)



(b) Z Reaction (kN)

Figure 4. Shear transfer models.

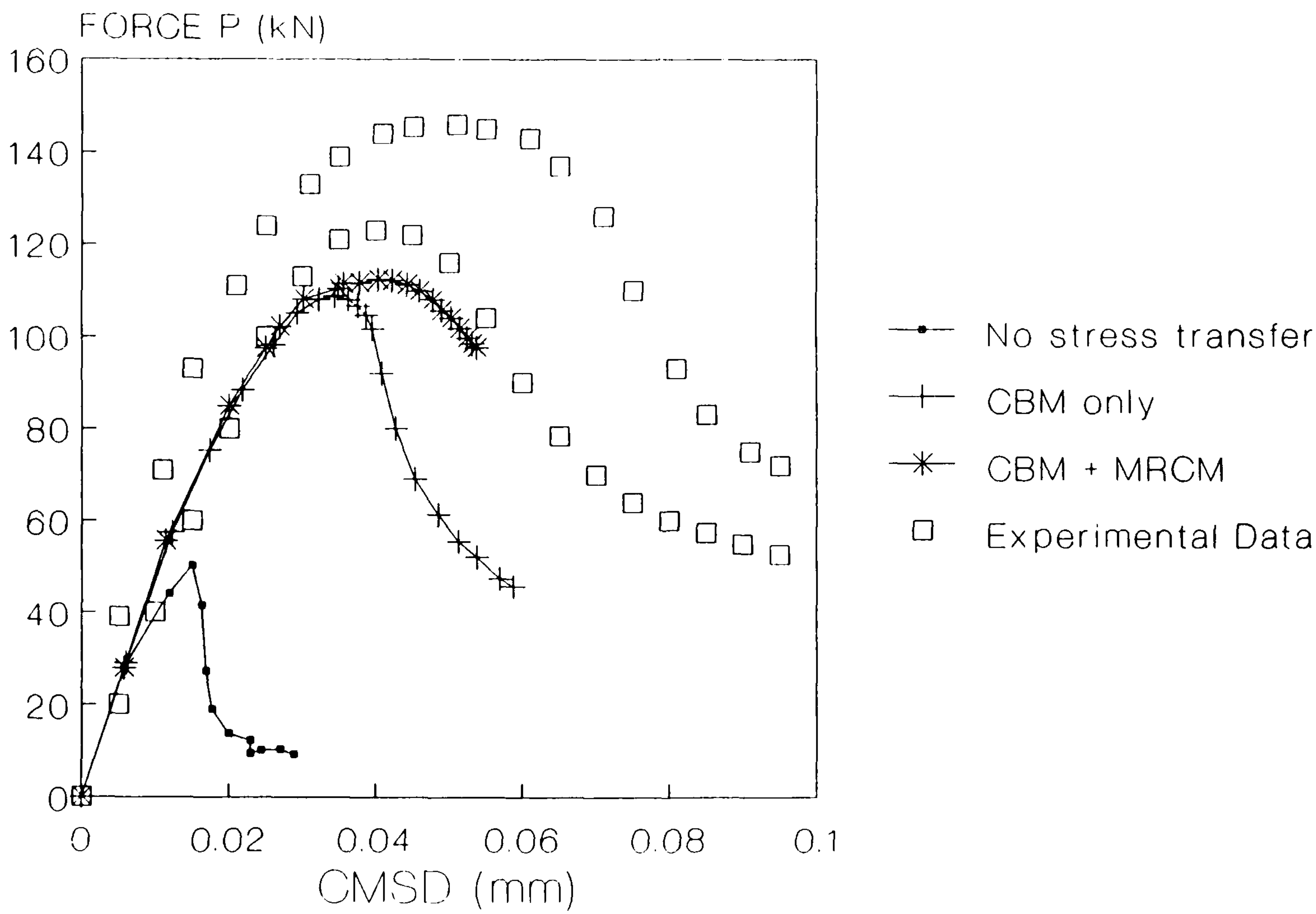


Figure 5. Load versus CMSD plots.

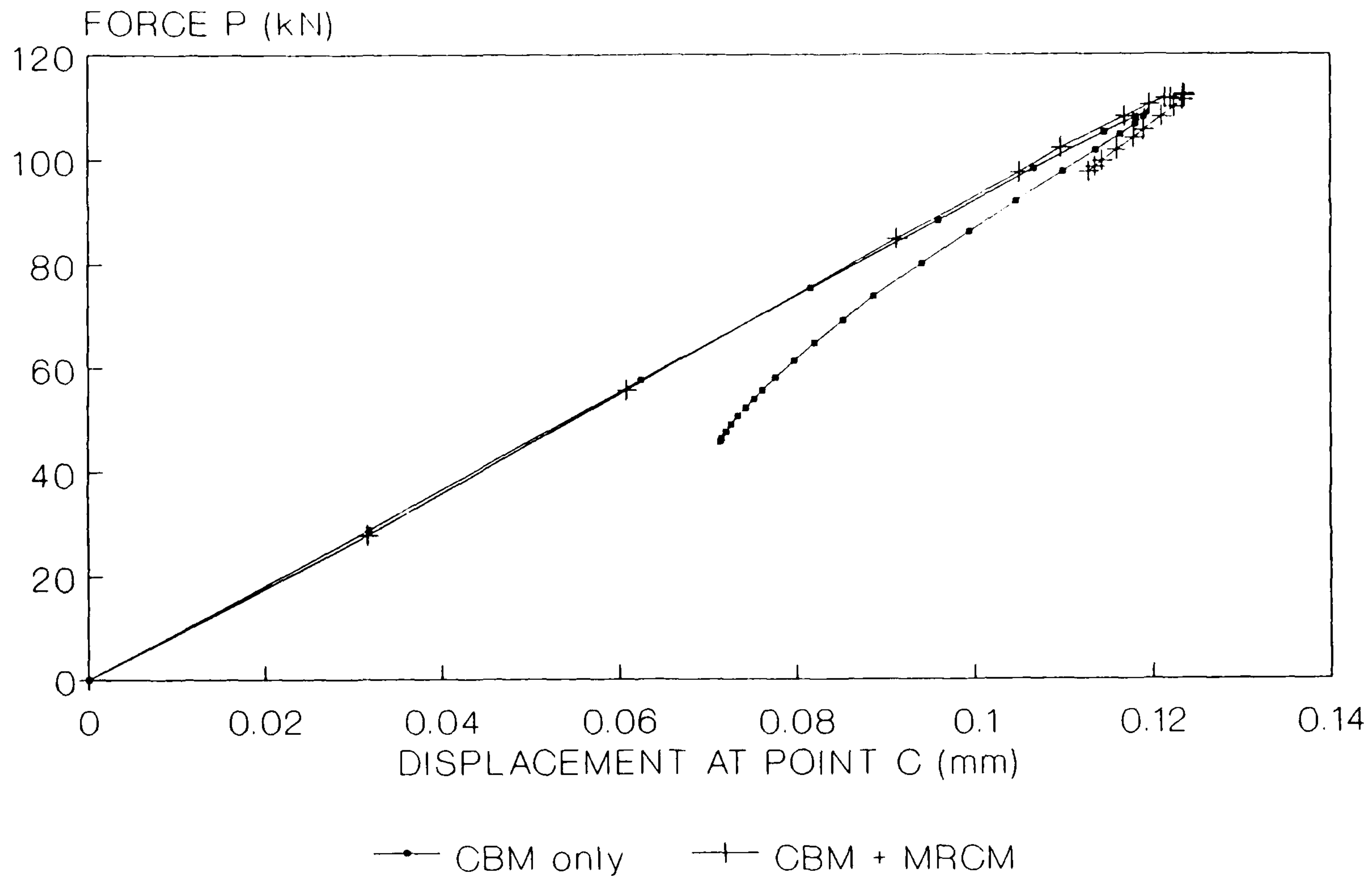


Figure 6 Load versus load point vertical displacement.

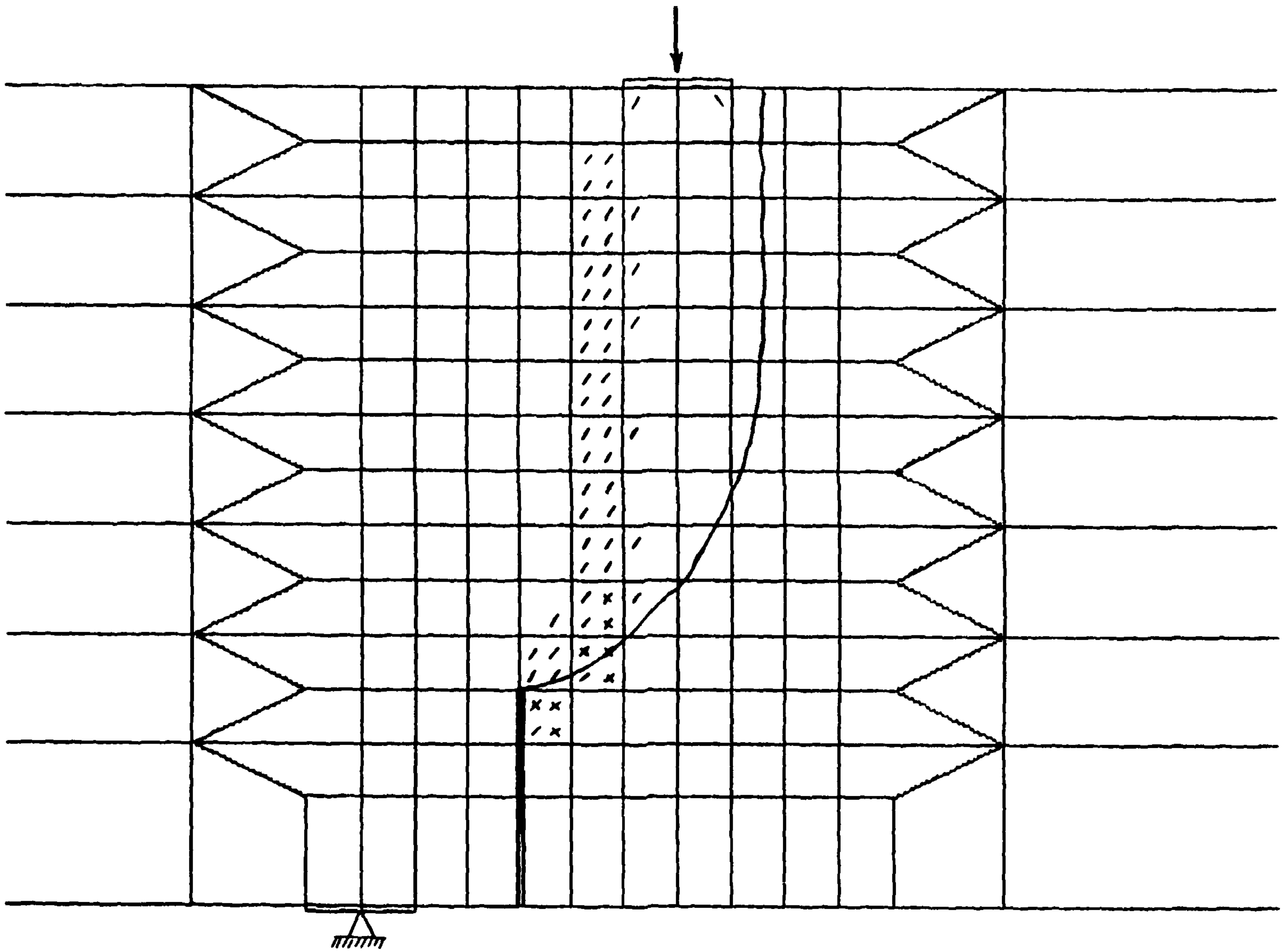


Figure 7. Crack pattern.

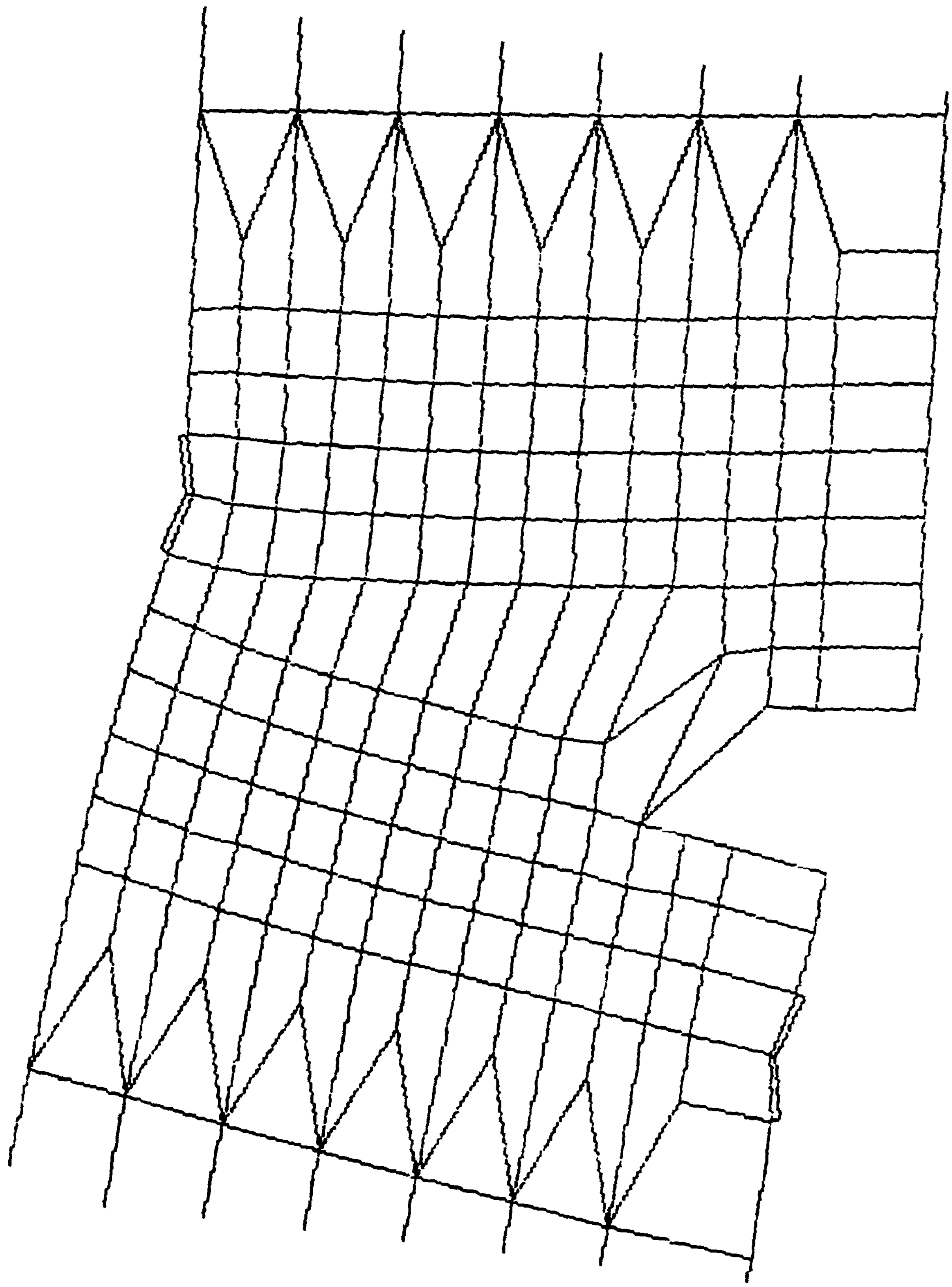


Figure 8. Deformed shape at failure.

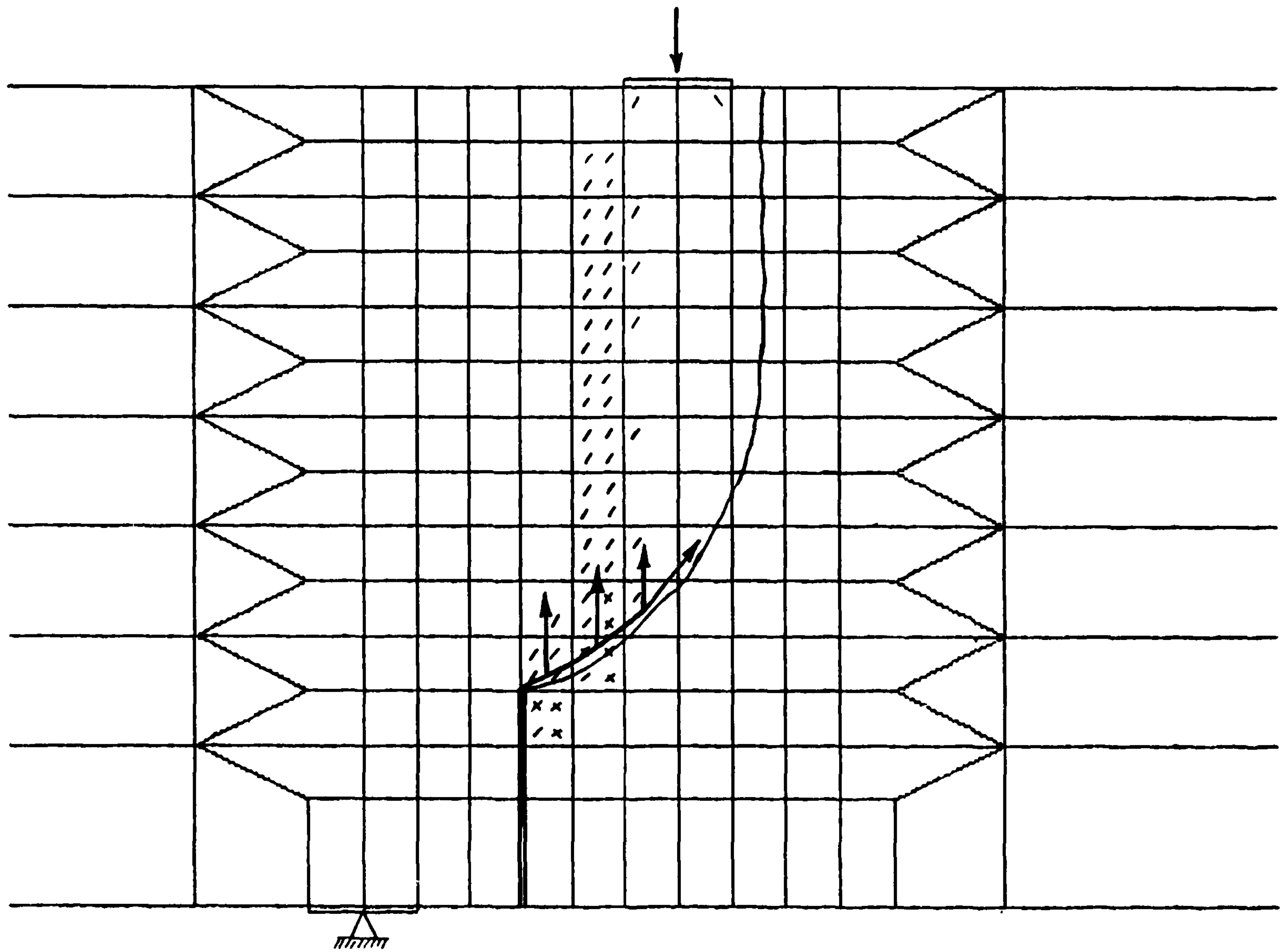


Figure 9. Alternate crack paths

Appendix A

CBM, 2-D

The following are changes implemented in the two-dimensional element formulation; namely, in the subprograms TODMFE.F77 and ELT2D4.F77 for tensile stress transfer. Although not mentioned in this report, formulation for the Rotating Crack Model is also included (see Ref 1).

Version 84.NL3 of the computer program ADINA was used. Text record identifiers were inserted in Columns 72 to 80, using HISTORIAN PLUS Software Control System.

CHANGES IN TODMFE.F77

```
                                                                    Change at
                                                                    or after:

     1      IDWAS/ 0, 0, 0,18,18, 0,10,15,15,33,33, 0, 0,26,6*0/,    TODMFE93

      COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM                  TDFE  42

      IF (MODEL.EQ.5) READ(IIN,1005) ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM TDFE 101

 1005 FORMAT (I5,4F10.0)                                             TDFE1219

      COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM                  MATRT214

      WRITE (6,2236) ISCODE,WWCC,ELWW,GGFF,DDAA                      MATRT244
      IF (IRCM.EQ.0) WRITE (6,2237)
      IF (IRCM.GE.1) WRITE (6,2238)

 2236 FORMAT(/38H (BB) CODE FOR TENSILE STRESS TRANSFER ,I5,          MATRT726
     1       /38H        1=LINEAR SOFTENING              ,
     2       /38H        2=CORNELISSEN'S SOFTENING       ,
     3       /38H       SOFT BAND WIDTH (WWCC)           ,F10.5,
     4       /38H       SOFT ELEMENT WIDTH (ELWW)        ,F10.5,
     5       /38H       FRACTURE ENERGY (GGFF)           ,F10.8,
     6       /38H       MAXIMUM AGGREGATE SIZE (DDAA)    ,F10.5)
 2237 FORMAT(/41H       ONLY PERPENDICULAR CRACKS ALLOWED   )
 2238 FORMAT(/41H       ROTATING CRACK MODEL IS USED        )
```

CHANGES IN ELT2D4.F77

```
      IDW=18*ITWO                                                        ELT2D438

      DIMENSION PROP(1),WA(18,1),YZ(1),NOD5(1),NODS(1),TEMPV1(1)        ICDMOD16

      DO 10 I=1,18                                                      ICDMOD26

      COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM                      CDMOD 50

     1         CRKSTR(6),STRESS(4),STRAIN(4),C(4,4),NODS(1),TEMPV1(1), CDMOD 53

     2         TEMPV2(1),YZ(1),NOD5(1),WA(1),DUMWA(18)                  CDMOD 54

      DO 1 I=1,18                                                       CDMOD 66

      IF (IRCM.GE.1 .AND. ANGLE.LT.3.61D2) GO TO 13                     CDMOD135

      GO TO 14                                                          CDMOD150
   13 CONTINUE
      CALL CRAKID (STRESS,STRAIN,PGRAV,CRKSTR,RKLD,RKUN,GLD,SP33,
     1             ANGLE,EP,NUMCRK,MODEL,1)
   14 CONTINUE

   47 CALL DCRACK (C,SIG,ANGLE,MODEL,ITYP2D,NUMCRK,1,1,CRKSTR)          CDMOD270

      CALL DCRACK (C,STRESS,ANG,MODEL,ITYP2D,NUMCRK,1,2,CRKSTR)         CDMOD302

      CALL DCRACK (C,STRESS,ANGLE,MODEL,ITYP2D,NUMCRK,2,2,CRKSTR)       CDMOD350

      CALL DCRACK (C,STRESS,ANGLE,MODEL,ITYP2D,NUMCRK,1,2,CRKSTR)       CDMOD374

      CRKSTR(4)=EP(1)                                                   CDMOD415
      CRKSTR(5)=EP(2)
      CRKSTR(6)=EP(3)

      CALL DCRACK (C,STRESS,ANGLE,MODEL,ITYP2D,NUMCRK,1,2,CRKSTR)       CDMOD422

      CALL DCRACK (C,STRESS,ANGPRI,MODEL,ITYP2D,NUMCRK,1,2,CRKSTR)      CDMOD427

      CALL DCRACK (C,STRESS,ANG,MODEL,ITYP2D,NUMCRK,2,1,CRKSTR)         CDMOD590

      DO 210 I=1,18                                                     CDMOD596
      COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM                      CRAKID13

      DIMENSION STR(4),EPS(4),CRKSTR(6),SP1(1),SP31(1),SP32(1),SP33(1), CRAKID15

      IF (IRCM.GE.1) GO TO 11                                           CRAKID16

      GO TO 107                                                         CRAKID45
```

```
C
   11 IF (KKK.GE.2) GO TO 12
C
C     FIND DIRECTION OF  PRINCIPAL STRAINS
C
      AA=(EPS(1) + EPS(2))*0.5
      BB=(EPS(1) - EPS(2))*0.5
      CC=SQRT(BB*BB + EPS(3)*EPS(3))
      EPSL(1)=AA + CC
      EPSL(2)=AA - CC
      EPSL(3)=0.D0
      EPSL(4)=EPS(4)
      ANGLE=4.5D1
      IF (EPS(3).EQ.0.D0) ANGLE=0.1D-3
      IF (ABS(BB).LT.0.1D-6) GO TO 12
      DUM=ABS(EPS(3)/BB)
      ANGLE=57.296*ATAN(DUM)
C
      IF (BB.LT.0.D0 .AND. EPS(3).GT.0.D0) ANGLE=180. - ANGLE
      IF (BB.LT.0.D0 .AND. EPS(3).LE.0.D0) ANGLE=180. + ANGLE
      IF (BB.GT.0.D0 .AND. EPS(3).LE.0.D0) ANGLE=360. - ANGLE
      ANGLE=ANGLE/2.
C
C     FIND STRESSES PERPENDICULAR AND PARALLEL TO CRACK
C
   12 CONTINUE
      PI=4.D0*ATAN(1.D0)
      TANG=ANGLE
      IF (TANG.LT.-5.41D2) TANG=TANG + 722.
      IF (TANG .LT. (-1.8D2)) TANG=TANG + 361.
      IF (TANG.GT.1.8D2) TANG=TANG - 180.
      GAM=2.*ABS(TANG)*PI/180.
     SG=SIN(GAM) CG=COS(GAM)
     IF (KKK.EQ.3) GO TO 107
C
     R11=(STR(1) + STR(2))*0.5
     R12=(STR(1) - STR(2))*0.5
     SIGP(1)=R11 + R12*CG + STR(3)*SG
     SIGP(2)=R11 - R12*CG - STR(3)*SG
     SIGP(3)=0.D0
     SIGP(4)=STR(4)
C
     IF (KKK.EQ.2) RETURN

     COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM                  DCRACK 8

     DIMENSION C(4,4),SIG(4),D(4,4),T(4,4),DSIG(4),CRKSTR(6)        DCRACK 9
```

```
      IF (IRCM.EQ.0) GO TO 12                                            DCRACK59
      IF (EP(1).NE.EP(2))
     1    C(3,3) = (SIGP(1)-SIGP(2))/(2*(EP(1)-EP(2)))
      IF (EP(1).EQ.EP(2)) C(3,3) = 1.D-8
   12     CONTINUE
C     RELEASE APPROPRIATE STRESSES                                       DCRAC204
C                                                                        DCRAC205
   98 NF=NUMCRK + 1                                                      DCRAC206
      GO TO (140,120,110,155,100,100,100), NF                            DCRAC207
  100 CALL DSOF (4,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)            DCRAC208
      IF (NUMCRK - 5) 140,120,110                                        DCRAC209
  110 CALL DSOF (2,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)            DCRAC210
  120 SIGP(3)=SIGP(3)                                                    DCRAC211
      CALL DSOF (1,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)            DCRAC212
C                                                                        DCRAC213
C     ROTATE STRESSES TO GLOBAL AXES                                     DCRAC214

      SUBROUTINE DSOF (IJ,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)     CDMOD620
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
      COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM
      DIMENSION SIGP(4),EP(4),CRKSTR(6),CORN(11,3)
      IF (CRKSTR(IJ).GT.0.D0) GOTO 5
      SIGP(IJ)=FALSTR
      RETURN
    5 CONTINUE
C
      DATA (CORN(I,1),I=1,11)/0.,.05,.1,.15,.2,.25,.3,.4,.6,.8,1.0/
      DATA (CORN(I,2),I=1,11)/1.,.7082,.5108,.3817,.2986,.2446,
     1                         .2080,.1596,.0904,.0361,0.0/
      JJ=IJ
      IF (JJ.EQ.4) JJ=3
      KK=JJ+3
      EEPP=EP(IJ)
      IF (EP(IJ).GT.CRKSTR(KK)) CRKSTR(KK)=EP(IJ)
      IF (EP(IJ).LT.CRKSTR(KK)) EEPP=CRKSTR(KK)
      ISS=ISCODE-2
      IF (ISS) 10,20,30
C
   10 CONTINUE
      EETT=1/(1/E-(2*GGFF)/(SIGMAT**2*WWCC))
      SIGP(IJ)=FALSTR+EETT*(EEPP-CRKSTR(JJ))
      IF (EP(IJ).LT.CRKSTR(KK)) SIGP(IJ)=EP(IJ)/EEPP*SIGP(IJ)
      IF (SIGP(IJ).GT.FALSTR) SIGP(IJ)=FALSTR
      IF (SIGP(IJ).LT.0.D0) SIGP(IJ)=0.D0
      SIGP(3)=0.D0
      RETURN
C
```

```
   20 CONTINUE
      EO=GGFF/(WWCC*0.19704*SIGMAT)
      DO 21 I=1,11
      CORN(I,3)=CORN(I,1)+CORN(I,2)*CRKSTR(JJ)/EO
      IF (EEPP/EO.LT.CORN(I,3)) GO TO 22
   21     CONTINUE
   22     AA=(CORN(I-1,2)-CORN(I,2))/(CORN(I-1,3)-CORN(I,3))
      BB=CORN(I-1,2)-AA*CORN(I-1,3)
      SIGP(IJ)=FALSTR*(AA*EEPP/EO+BB)
      IF (EP(IJ).LT.CRKSTR(KK)) SIGP(IJ)=EP(IJ)/EEPP*SIGP(IJ)
      IF (SIGP(IJ).GT.FALSTR) SIGP(IJ)=FALSTR
      IF (SIGP(IJ).LT.0.D0) SIGP(IJ)=0.D0
      SIGP(3)=0.D0
      RETURN
C
   30     CONTINUE
      RETURN
C
      END
```

Appendix B

CBM, 3-D

Three-dimensional element formulation.

CHANGES IN THREDM.F77

```
                                                                    Change at
                                                                    or after:

    1     IDWAS / 0, 0, 0, 25,25, 0,14,21,21,47,47,38,8*0/,          THRED100

    COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,1RCM                   THDFE 46

    IF (MODEL.EQ.5) READ(IIN,1009) ISCODE,WWCC,ELWW,GGFF,DDAA,1RCM  THDFE102

 1009 FORMAT (I5,4F10.0)                                            THDF1190

    COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,1RCM                   MATWRT14

    WRITE (6,2239)                                                  MATWR243

 2239 FORMAT(/38H (BB) CODE FOR TENSILE STRESS TRANSFER,I5,         MATWR596
    1      /38H      1=LINEAR SOFTENING            ,
    2      /38H      2=CORNELISSEN'S SOFTENING     ,
    3      /38H     SOFT BAND WIDTH (WWCC)         ,F10.5,
    4      /38H     SOFT ELEMENT WIDTH (ELWW)      ,F10.5,
    5      /38H     FRACTURE ENERGY (GGFF)         ,F10.8,
    6      /38H     MAXIMUM AGGREGATE SIZE (DDAA)  ,F10.5)
```

CHANGES IN ELT3D4.F77

```
    IDW=25*ITWO                                                     ELT3D444

    DIMENSION PROP(1),WA(25,1),XYZ(1),NOD9(1),NODS(1),TEMPV1(1)     ICMOD316

    DO 10 I=1,25                                                    ICMOD326

    1        CRKSTR(6),STRESS(6),STRAIN(6),C(6,6),RLMN(3,3),NODS(1), CMOD3D54

    1        TEMPV1(1),TEMPV2(1),XYZ(1),NOD9(1),WA(1),DUMWA(25)     CMOD3D55
```

```
      DO 1 I=1,25                                                    CMOD3D67

   47 CALL DCRAK3 (C,SIG,RLMN,MODEL,NUMCRK,1,1,CRKSTR)               CMOD3261

      CALL DCRAK3 (C,STRESS,RLMN,MODEL,NUMCRK,1,2,CRKSTR)            CMOD3286

      CALL DCRAK3 (C,STRESS,RLMN,MODEL,NUMCRK,2,2,CRKSTR)            CMOD3340

      CRKSTR(4)=EP(1)                                                CMOD3362
      CRKSTR(5)=EP(2)
      CRKSTR(6)=EP(3)

      CALL DCRAK3 (C,STRESS,RLMN,MODEL,NUMCRK,1,2,CRKSTR)            CMOD3363

  159 CALL DCRAK3 (C,STRESS,RLMN,MODEL,NUMCRK,1,2,CRKSTR)            CMOD3414

      CALL DCRAK3 (C,STRESS,RLMN,MODEL,NUMCRK,1,2,CRKSTR)            CMOD3420

  130 CALL DCRAK3 (C,SIG,RLMN,MODEL,NUMCRK,2,1,CRKSTR)               CMOD3561

      DO 210 I=1,25                                                  CMOD3567

      DIMENSION STR(4),EPS(4),CRKSTR(6),SP1(1),SP31(1),SP32(1),SP33(1), CRAKID15

      DIMENSION C(4,4),SIG(4),D(4,4),T(4,4),DSIG(4),CRKSTR(6)        DCRACK 9

C     RELEASE APPROPRIATE STRESSES                                   DCRAK165
C                                                                    DCRAK166
      NF=IK + 1                                                      DCRAK167
      GO TO (140,120,110,100,155), NF                                DCRAK168
  100 CALL DSOF3 (3,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)       DCRAK169
  110 SIGP(6)=SIGP(6)                                                DCRAK170
      CALL DSOF3 (2,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)       DCRAK171
  120 SIGP(5)=SIGP(5)                                                DCRAK172
      SIGP(4)=SIGP(4)                                                DCRAK173
      CALL DSOF3 (1,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)       DCRAK174
C                                                                    DCRAK175
C     ROTATE STRESSES TO GLOBAL AXES                                 DCRAK176

      SUBROUTINE DSOF3 (IJ,SIGP,FALSTR,EP,CRKSTR,E,VNU,SIGMAT,SIGMAC)   CMOD3590
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
      COMMON /SOFT/ ISCODE,WWCC,ELWW,GGFF,DDAA,IRCM
      DIMENSION SIGP(4),EP(4),CRKSTR(6),CORN(11,3)
C
      IF (CRKSTR(IJ).GT.0.D0) GOTO 5
      SIGP(IJ)=FALSTR
      RETURN
```

```
    5 CONTINUE
      DATA (CORN(I,1),I=1,11)/0.,.05,.1,.15,.2,.25,.3,.4,.6,.8,1.0/
      DATA (CORN(I,2),I=1,11)/1.,.7082,.5108,.3817,.2986,.2446,.2080,
     1                        .1596,.0904,.0361,0.0/
      JJ=IJ
      KK=JJ+3
      EEPP=EP(IJ)
      IF (EP(IJ).GT.CRKSTR(KK)) CRKSTR(KK)=EP(IJ)
      IF (EP(IJ).LT.CRKSTR(KK)) EEPP=CRKSTR(KK)
C
      ISS=ISCODE-2
      IF (ISS) 10,20,30
C
   10 CONTINUE
      EETT=1/(1/E-(2*GGFF)/(SIGMAT**2*WWCC))
      SIGP(IJ)=FALSTR+EETT*(EEPP-CRKSTR(JJ))
      IF (EP(IJ).LT.CRKSTR(KK)) SIGP(IJ)=EP(IJ)/EEPP*SIGP(IJ)
      IF (SIGP(IJ).GT.FALSTR) SIGP(IJ)=FALSTR
      IF (SIGP(IJ).LT.0.D0) SIGP(IJ)=0.D0
      IF (IJ-2) 12,11,11
   11 SIGP(6)=0.D0
   12 SIGP(5)=0.D0
      SIGP(4)=0.D0
      RETURN
C
   20 CONTINUE
      EO=GGFF/(WWCC*0.19704*SIGMAT)
      DO 23 I=1,11
      CORN(I,3)=CORN(I,1)+CORN(I,2)*CRKSTR(JJ)/EO
      IF (EEPP/EO.LT.CORN(I,3)) GO TO 24
   23 CONTINUE
   24 AA=(CORN(I-1,2)-CORN(I,2))/(CORN(I-1,3)-CORN(I,3))
      BB=CORN(I-1,2)-AA*CORN(I-1,3)
      SIGP(IJ)=FALSTR*(AA*EEPP/EO+BB)
      IF (SIGP(IJ).GT.FALSTR) SIGP(IJ)=FALSTR
      IF (SIGP(IJ).LT.0.D0) SIGP(IJ)=0.D0
      IF (IJ-2) 22,21,21
   21 SIGP(6)=0.D0
   22 SIGP(5)=0.D0
       SIGP(4)=0.D0
       RETURN
C
   30     CONTINUE
      RETURN
C
      END
```

Appendix C

GENERAL MODIFICATIONS

The following are changes implemented in the rest of the program, namely in the subprograms ADINA.F77, ADINI.F77 and ADINA2.F77. Only the modified spherical constant arc-length scheme is allowed and only Full Newton iterations without line search are carried out. If NODQL is chosen between 3 and 100, a subset of NODQL nodes is used in the norm of displacement. If NODQL is 2, the distance between two points is used instead of the norm of displacement.

CHANGES IN ADINA.F77

```
      COMMON /DICS/ DISPM4,ADNOM,ADMAX,ADCOM,NODQ,NDID,NEDPM4,N1CRLO,IARADINA189
     1              ,NODQL,NEDPML(100,7)

      IF (IRSM4.EQ.2) KSTOP=1                                        ADINA994
```

CHANGES IN ADINI.F77

```
     1              ,NODQL,NEDPML(100,7)                             ADINI 33

     1READ (IIN,1004) NODQ,NDID,DISPM4,ADNOM,ADMAX,ICOMA,IAR,NODQL   ADINI166
      IF(NODQL.EQ.0 .OR. METHOD.NE.4) GO TO 70
      NNODQL=INT(FLOAT(NODQL-1)/10.)+1
      DO 69 I=1,NNODQL
   69 READ (IIN,1007) (NEDPML(10*(I-1)+K,1),K=1,10)
   70 CONTINUE
      IF (METHOD.EQ.4) NEWREF=1                                      ADINI718
      IF (NODQL.EQ.0) GO TO 46                                       ADINI801
      WRITE (6,2067) NODQL
      DO 446 I=1,NNODQL
  446 WRITE (6,2068) (NEDPML(10*(I-1)+K,1),K=1,10)

 1004 FORMAT (2I5,3F10.0,3I5)                                        ADIN1218

 1007 FORMAT (10I5)                                                  ADIN1220

 2067 FORMAT (/5X,                                                   ADIN1577
     155HNODE SUBSET FOR DISPLACEMENT NORM, TOTAL NODES (NODQL)=,I5,/)
 2068 FORMAT (15X,10I4)
```

CHANGES IN ADINA2.F77

```
     1               ,NODQL,NEDPML(100,7)                             LOADMS55

      IF (NODQL.EQ.0) GO TO 13                                        LOADM103
      DO 13 II=1,NODQL
      NIDL=N5 - 1 + ((NEDPML(II,1)-1)*NDOF
      DO 22 IN=1,6
      IF (IDOF(IN).EQ.0) GO TO 22
      NIDL=NIDL+1
      NEDPML(II,IN+1) = IA(NIDL)
   22 CONTINUE
   13 CONTINUE
      IF(NODQL.EQ.0) GO TO 14                                         LOADM109
      WRITE (6,5000)
      DO 15 I=1,NODQL
   15 WRITE (6,5001) NEDPML(I,1),(NEDPML(I,K),K=2,7)
   14 CONTINUE

 5000 FORMAT(/34H NODE SUBSET FOR DISPLACEMENT NORM,                  LOADM268
     1      /34H      NODE   EQUATION NUMBERS      )
 5001 FORMAT(6X,I4,4X,6I4)

C     IF (PEOLD.GT.BIG*PEINIT) GO TO 210                              EQUIT254

      GO TO 230                                                       EQUIT259

     1               ,NODQL,NEDPML(100,7)                             ASTIM423


     1               ,NODQL,NEDPML(100,7)                             ASTCHE71

      DUALL=3.*DUALL                                                  ASTCH151

      IF (NODQL.NE.0) GO TO 500                                       ASTCH186

      COMMON /DICS/ DISPM4,ADNOM,ADMAX,ADCOM,NODQ,NDID,NEDPM4,N1CRLO,IARDOPRFM14
     1               ,NODQL,NEDPML(100,7)

      IF (NODQL.EQ.2) GO TO 160                                       DOPRFM21
      IF (NODQL.NE.0) GO TO 150

  150 PD=0.D0                                                         DOPRFM46
      DO 151 I=1,NODQL
      DO 151 J=2,7
      IF (NEDPML(I,J).EQ.0) GO TO 151
      PD=PD+AA(NEDPML(I,J))*BB(NEDPML(I,J))
  151 CONTINUE
      PD=PD*(NALLEQ/(NODQL+1))
      PR=0.D0
      RETURN
```

```
160 PD=0.D0
    DO 161 J=2,7
    IF (NEDPML(1,J).EQ.0) GO TO 161
    PD=PD+(AA(NEDPML(1,J))-AA(NEDPML(2,J)))
   1     *(BB(NEDPML(1,J))-BB(NEDPML(2,J)))
161 CONTINUE
    PD=PD*(NALLEQ/3)
    PR=0.D0
    RETURN

   1              ,NODQL,NEDPML(100,7)                               ALSTEP 9

   1              ,NODQL,NEDPML(100,7)                               ALSET 10

    NODQL=0                                                          ALSET 37

    DO 2 I=1,100                                                     ALSET 39
    DO 2 J=1,7
  2 NEDPML(I,J)=0

   1              ,NODQL,NEDPML(100,7)                               NEWDAV36
```

Appendix D

FAILURE ENVELOPE

Concrete Strength	Power
8000 psi (563 kp/cm^2) (55.2 MPa)	1.00
6000 psi (422 kp/cm^2) (41.4 MPa)	1.40
4000 psi (281 kp/cm^2) (27.6 MPa)	1.80
2000 psi (141 kp/cm^2) (27.6 MPa)	2.20

CHANGES IN ELT2D4.F77

```
      FALSTR=SIGMAT*(1. - (P3/SIGMAC)**POWR)                    PRNCPL86

      IF (P1.GE.0.D0) FALSTR=SIGMAT*(1. - (P2/SIGCP)**POWR)     PRNCP121
     1                       *(1. - (P3/SIGCP)**POWR)

      IF (MODEL.EQ.5 .AND. SIG(2).LT.0.D0)                      CRAKI183
     1 FALSTR=SIGMAT*(1. - (SIG(2)/SIGMAC)**POWR)               CRAKI184
```

CHANGES IN ELT3D4.F77

```
  120 FALSTR=SIGMAT*(1. - (P3/SIGMAC)**POWR)                    PRNCP232

      IF (P1.GE.0.D0) FALSTR=SIGMAT*(1. - (P2/SIGCP)**POWR)     PRNCP266
     1                       *(1. - (P3/SIGCP)**POWR)

      IF (MODEL.EQ.5 .AND. SIG(3).LT.0.D0)                      CRAK3120
     1 FALSTR=SIGMAT*(1. - (SIG(3)/SIGMAC)**POWR)               CRAK3121
```

Appendix E

DERIVATION OF CRACK STIFFNESS MATRIX B

The MRCM formulation can be rewritten as

$$\sigma_{nn} = -a_{12} r \sqrt{\delta_n} \sigma_{nt}/h$$

$$\sigma_{nt} = \tau_o (1-\sqrt{2\delta_n/d_a}) r (f/g)$$

where:

$$f = a_3 + a_4 \, |r^3|$$

$$g = 1 + a_4 \, r^4$$

$$h = (1+r^2)^{0.25}$$

and by derivation:

$$f_n = \partial f/\partial n = -3a_4 |\delta_t{}^3/\delta_n{}^4|$$

$$f_t = \partial f/\partial t = 3a_4 \delta_t |\delta_t/\delta_n{}^3|$$

$$g_n = \partial g/\partial n = -4a_4 (\delta_t{}^4/\delta_n{}^5)$$

$$g_t = \partial g/\partial t = 4a_4 (\delta_t{}^3/\delta_n{}^4)$$

$$h_n = \partial h/\partial n = (1+r^2)^{-0.75} (-2\delta_t{}^2/\delta_n{}^3)/4$$

$$h_t = \partial h/\partial t = (1+r^2)^{-0.75} (2\delta_t/\delta_n{}^2)$$

The crack stiffness terms are then:

$$B_{nn} = -a_{12}\left((-h\delta_t/\delta_n{}^2 - h_n r)\sqrt{\delta_n}\sigma_{nt}/h^2 + r\delta_n{}^{-0.5}\sigma_{nt}/2h \; r\sqrt{\delta_n}B_{tn}/h \right)$$

$$B_{nt} = -a_{12}\left((-h_t r + h/\delta_n)\sqrt{\delta_n}\sigma_{nt}/h^2 + r\sqrt{\delta_n}B_{tt}/h \right)$$

$$B_{tt} = \tau_o(1-\sqrt{2\delta_n/d_a}) \left| f/\delta_n g + r(f_t g - fg_t)/g^2 \right|$$

$$B_{tn} = \tau_o\left(-fr/(g\sqrt{2d_a\delta_n}) + (1-\sqrt{2\delta_n/d_a}) \left[(f_n g - fg_n)r/g^2 - f\delta_t/(g\delta_n{}^2) \right] \right)$$

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