NEW PRESSURE GRADIENT EQUATIONS FOR LUMPED-PARAMETER INTERIOR BALLISTIC CODES

FREDERICK W. ROBBINS
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New Pressure Gradient Equations for Lumped-Parameter Interior Ballistic Codes

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Gradients, Gradient Equations, Interior Ballistics, Lumped-Parameter, Chamberage, Two-Phase Flow, RGA, IBRGA
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1. INTRODUCTION

During the analysis of 120-mm gun firings, designed to look at the interior ballistic characteristics of combustible cartridge cases (Robbins, Koszoru, and Minor 1986), XNOVAKTC (XKTC) (Gough 1980) was noted to be in agreement with measured pressure-time curves as well as with pressure-difference curves, while IBHVG2 (Anderson and Fickie, 1987) gave a calculated maximum breech pressure which was 42 MPa higher than measured. Parametric studies were performed using XKTC to attempt to attribute this disparity to the various processes omitted from IBHVG2. The boattail intrusion was calculated to account for 14 MPa, with effects of flamespreading and intergranular stress accounting for 3 MPa each. Subsequent calculations (Robbins 1986) indicated chambrage, propellant packaging, wave dynamics, and multiphase effects (the solid propellant velocity lag and concomitant formation of an ullage region between the projectile base and the propellant bed) as contributors to the differences between the lumped-parameter and two-phase interior ballistic codes. In this report, we show that the influence of chambrage and propellant velocity lag on the pressure gradient may be represented in analytical form. We compare the analytical pressure gradient with that predicted by XKTC and assess the extent to which the effects of chambrage and propellant velocity lag account for the differences in ballistic predictions.

The NOVA codes, of which XKTC is the latest version, have been used with uncompromised data bases to model gun systems with much success (Robbins, Koszoru, and Minor 1986; Robbins 1983; Robbins and Horst 1984). Since XKTC calculates the pressure gradient from first principles and agrees with gun firings, XKTC is assumed correct. Accordingly, all the lumped-parameter computer runs, with different gradient equations, are compared with equivalent XKTC computer runs.

1.1 Models. Gradient equations have been developed by Gough (Gough, no date) to look at (1) the effects of chambrage, (2) the effects of propellant velocity lag, and (3) both effects at the same time. The governing partial differential equations, assumptions, and definitions are given for each effect followed by the resultant gradient equations and an equation for the total kinetic energy of the propellant and gas. The details of the solution technique and mathematical procedure are documented in Appendices 1-3.

1.2 Influence of Chambrage. For the chambrage gradient equation, the propellant is assumed to be uniformly distributed between the breech and the base of the projectile, and the variation in area is assumed to be confined to the chamber. The basic results presented in this section have been previously derived (Vinti 1942) and, more recently, in a somewhat different context, (Morrison and Wren 1988).

The continuity and momentum equations for unsteady flow of a homogeneous inviscid substance are

\[
\frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial \rho A u}{\partial z} = 0 \quad \text{(C.01)}
\]
and
\[ \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial z} + g_e \frac{\partial p}{\partial z} = 0. \]  
(C.02)

Making the Lagrange assumption
\[ \frac{\partial p}{\partial z} = 0 \]  
(C.03)
and using
\[ \rho = \frac{p}{V(z_p)} \]  
(C.04)
results in the pressure distribution
\[ p(z) = p(0) + a(t)J_1(z) + b(t)J_2(z), \]  
(C.05)
where
\[ a(t) = \frac{CA_b}{g_e V^2(z_p)} \left[ \frac{A_b V^2_p}{V(z_p)} - V_p \right], \]  
(C.06)
\[ b(t) = - \frac{CV^2_p A_b^2}{2g_e V^2(z_p)}, \]  
(C.07)
\[ J_1(z) = \int_{0}^{z} \frac{V(z)dz}{A(z)}, \]  
(C.08)
and
\[ J_2(z) = \frac{V^2(z)}{A^2(z)}. \]  
(C.09)

Substituting the projectile acceleration \( V_p \) given by
\[ V_p = \frac{g_e A_b}{M_p} [P_b - P_{ps}], \]  
(C.10)
into (C.05) results in a pressure distribution in terms of the pressure at the base of the projectile \( P_b \)
\[ p(z) = p(0) + a_1(t)J_1(z) + a_2(t)J_1(z)P_b + b(t)J_2(z), \]  
(C.11)
where
\[ a_1(t) = \frac{CA_b}{g_e V^2(z_p)} \left[ \frac{A_b V^2_p}{V(z_p)} + \frac{g_e A_b}{M_p} P_{ps} \right], \]  
(C.12)
and
\[ a_2(t) = - \frac{CA_b^2}{M_p V^2(z_p)}. \]  
(C.13)
With the mean pressure $P_m$ defined as

$$P_m = \frac{\int_0^{Z_p} P(z)A(z)dz}{\int_0^{Z_p} A(z)dz}$$

(C.14)

and using (C.11) and identifying $P(0)$ with $P_B$, then the base and breech pressures are given in terms of the mean pressure by

$$P_m + a_2(t)J_1(z_p) + b(t)J_2(z_p) = \frac{a_2(t)J_3(z_p)}{V(z_p)} - \frac{b(t)J_4(z_p)}{V(z_p)}$$

(C.15)

and

$$P_B = 1 - a_2(t)J_1(z_p) + \frac{a_2(t)J_3(z_p)}{V(z_p)}$$

(C.15)

where

$$J_3(z_p) = \int_0^{Z_p} A(z)J_1(z)dz$$

(C.17)

and

$$J_4(z_p) = \int_0^{Z_p} A(z)J_2(z)dz.$$  

(C.18)

The kinetic energy of the gas/solid mixture is

$$KE = \frac{A_2^2 V_c^2 CJ_4(z_p)}{2g_s V^3(z_p)}.$$  

(C.19)

1.3 Influence of Propellant Velocity Lag. For the two-phase gradient equation developed in this section, the chamber and tube are assumed to have the same, uniform diameter. The coupling of the two-phase effects with chambering is addressed in the next section. The propellant is assumed to be uniformly distributed between the breech face and the leading edge of the bed with the leading edge of the bed being initially at the base of the projectile. However, once the projectile begins to move, the leading edge of the propellant bed is not assumed to remain in contact with the base of the projectile. Instead, the motion of the leading edge of the bed is explicitly modeled by reference to the equation of motion of the solid phase which includes the influence of both pressure gradient (buoyancy force) and interphase drag. The existence of a region of
ullage between the propellant bed and the base of the projectile is also recognized explicitly. The requirement that the gas-phase density be everywhere uniform causes the gas velocity at the leading edge of the propellant bed \( (U_g) \) to have a much more complex time dependence than that in the simple Lagrange analysis. Instead of being a fixed fraction of the projectile velocity at all times, the value of \( U_g \) may be significantly greater than the projectile velocity during the pressurization phase since the ullage requires a net compression to maintain equilibrium with the conditions in the mixture region where combustion is occurring. The complex time dependence of \( U_g \) will be seen to have important repercussions in respect to the behavior of the pressure gradient. It is believed that this is the first time that these effects have been modeled analytically.

The continuity equations for the mixture region are

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = \dot{m}(t) 
\]

\( \rho \frac{\partial e}{\partial t} - \frac{\partial (1 - e) \rho u_p}{\partial z} = \dot{m}(t) \).  

Making assumptions analogous to the Lagrange assumption

\[
\frac{\partial \rho}{\partial z} = 0 \quad \text{(gas density constant throughout the tube)} 
\]

\[
\frac{\partial e}{\partial z} = 0 \quad \text{(porosity constant throughout the mixture region)} 
\]

and noting the assumption \( \dot{m}(t) \) is only a function of time, then

\[
u_p = U_p \left[ \frac{z}{z_b} \right] 
\]

and

\[
u = U_g \left[ \frac{z}{z_{b*}} \right]. 
\]

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

\[
\varepsilon (U_g - U_p) = U_{g*} - U_p. 
\]

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption L.3.
Consider the momentum equation for the mixture region,

\[ \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial t} + \rho g \frac{\partial z}{\partial t} = - f_s + \rho u \frac{\partial p}{\partial z} \]  \hspace{1cm} (L.7)

and

\[ \frac{\partial (1-\epsilon) \rho_p u_p}{\partial t} + \frac{\partial (1-\epsilon) \rho_p u_p^2}{\partial t} + (1-\epsilon) \rho \frac{\partial p}{\partial z} = f_s - \rho u_p \]  \hspace{1cm} (L.8)

where

\[ f_s = \frac{(1-\epsilon)}{D_p} \rho (u-u_p)^2 f_{sc} \]  \hspace{1cm} (L.9)

and for

\[ REN = \frac{\rho D_p |U_e - U_p|}{\mu} \]

\[ \lambda = \left[ 0.5 + \frac{a_0}{a_0} \right]^{1.17} \]

\[ \epsilon_o = 1 - C \frac{\rho_p V_0}{\rho (1-\epsilon_o)} \]

Then

\[ f_{sc} = \begin{cases} \frac{2.5 \lambda}{REN \epsilon_o} f_s & \epsilon < \epsilon_o \\ \frac{2.5 \lambda}{REN \epsilon_o} \left( \frac{(1-\epsilon) \epsilon_o}{\epsilon} \right)^{0.45} f_s & \epsilon \geq \epsilon_o \end{cases} \]  \hspace{1cm} (L.10)

Defining \( \phi = \phi - \rho A_B L/C \),

\[ \frac{d^2 \ln \rho}{dt^2} = c_i(t) - \frac{\rho A_B^2 P_B}{V_p M_p} + \frac{\rho A_B^2 P_{ref}}{V_p M_p} \]

where

\[ c_i(t) = \dot{m} \left( \frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{V^2_p}{V^2_F} - \frac{m^2}{m^2} \]
and

\[ V_f = V_o + A_b(z_p - z_m) - \frac{c}{\rho_p} + \frac{m}{\rho_p}, \]

\[ \dot{m} = \rho_p S \frac{dx}{dt}, \quad (L.12) \]

\[ \dot{m} = \rho_p S \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2} \]

\[ = \rho_p S \frac{dS}{dx} \left( \frac{dx}{dt} \right)^2 + \rho_p S \frac{d^2x}{dt^2}, \quad (L.13) \]

with

\[ \frac{dx}{dt} = aP_m^n \quad (L.14) \]

and

\[ \frac{d^2x}{dt^2} = aP_m^{n-1} \frac{dP_m}{dt}, \quad (L.15) \]

where \( dP_m/dt \) is determined numerically, or from

\[ \frac{dP_m}{dt} = \frac{\frac{d\rho_c}{d\tau} \cdot \rho_c}{V_p} + \frac{T_m}{V_p} \]

and

\[ \dot{V}_p = \frac{2A_b}{M_p} P_m (P_m - P_{\text{ref}}). \quad (L.16) \]

Then

\[ P(z) = P_{br} - \frac{(k_{11}P_m + k_{12})}{2} z^2, \quad (L.17) \]

where

\[ k_{11} = \frac{C^* \left( 1 - \frac{A_b}{V_p} \right) \frac{A_b}{V_p}}{z_s V(z_s)} \quad (L.18) \]

and

\[ k_2 = \frac{1}{1 - \frac{v_c}{c}} \quad (L.19) \]
and
\[ k_{12} = \frac{\phi_1 C k_2}{g \sigma z_b V(z_b)} - k_{11} P_{ra}, \quad (L.20) \]

\[ \phi_1' = \phi U_{s} - \phi U_r - \frac{\phi \varepsilon}{\varepsilon^2} \left( V_p + L \frac{dln \rho}{dt} - U_p \right) \]
\[ + \frac{\phi f L}{\varepsilon} \frac{dln \rho}{dt} + \frac{2 \phi U_2}{z_b} \left( U_s - U_p \right) \]
\[ + \frac{\phi_2}{D_p \varphi_r} \left( U_s - U_p \right)^2 f \varphi_r + \frac{L \phi C(t)}{\varepsilon}, \quad (L.21) \]

and
\[ \phi_2 = 1 - \phi - \phi_1 \left( 1 - \varepsilon \right). \quad (L.22) \]

For the ullage region, the momentum equation for the gas is
\[ \frac{dP}{dz} = - \frac{\rho}{g} \left( \frac{du}{dt} + u \frac{du}{dz} \right), \quad (L.23) \]

and, therefore, the pressure in the ullage region is
\[ P(z) = P(z_b) - \frac{\rho}{g} \left[ (V_p + z_b \Delta)(z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right], \quad (L.24) \]

where
\[ \Delta = \frac{d^2ln \rho}{dt^2} - \left( \frac{dln \rho}{dt} \right)^2. \]

Defining
\[ P_m = \frac{\int_{z_p}^{z_p} P(z) A(z) dz}{\int_{z_p}^{z_p} A(z) dz}, \quad (L.25) \]

then
\[ P_b = \frac{P_m - D}{G}, \quad (L.26) \]
and

\[ P_{Br} = A P_B + B, \quad (L.27) \]

where

\[
D = \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B}{M_p} \frac{p_{L2}}{2z_p} + \frac{k_{12} z_b^2}{2z_p} \left( \frac{z_b}{3} + L \right) + \frac{\rho L^2 A_B}{2z_p M_p} \frac{p_{L2}}{2z_p} \\
+ \frac{\rho L^2}{2 g_o} \left( 1 - \frac{2L}{3z_p} \right) \left( c_i (t) - \left( \frac{d \ln p}{dt} \right)^2 \right) \\
+ \frac{\rho L^2}{2 M_p V_p} \left( 1 - \frac{2L}{3z_p} \right) P_{res}. \quad (L.28)
\]

\[
G = 1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_0^2 \rho L^2}{2 V_p M_p} \left( 1 - \frac{2L}{3z_p} \right) - \frac{k_{11} z_b^2}{2z_p} \\
- \frac{k_{11} L z_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p}. \quad (L.29)
\]

\[
A = 1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_0^2 \rho L^2}{2 V_p M_p}. \quad (L.30)
\]

and

\[
B = \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B}{M_p} P_{res} \\
+ \frac{\rho L^2}{2 g_o} \left( c_i (t) - \left( \frac{d \ln p}{dt} \right)^2 \right) + \frac{A_0^2 \rho L^2}{2 V_p M_p} P_{res}. \quad (L.31)
\]

The kinetic energy of the propellant and gases is

\[
KE = \frac{A_0^2 z_b^2}{6 g_o} \left[ \epsilon \rho U_e^2 + (1 - \epsilon) \rho_p U_p^2 \right] \\
+ \frac{\rho A_B L}{6 g_o} \left[ 3 V_p^2 + 3 V_p L \frac{d \ln p}{dt} + L^2 \left( \frac{d \ln p}{dt} \right)^2 \right].
\]

The mean pressure over the mixture region is

\[
P_{max} = P_{Br} - \left( \frac{k_{11} P_B + k_{12}}{6} \right) z_b^2.
\]
1.4 Combined Influence of Chambrage and Propellant Velocity Lag. The Robbins-Gough-Anderson (RGA) gradient equation, which combines the influences of chambrage and multiphase flow, assumes that the variation in area is confined to the propellant chamber and that the propellant initially fills the chamber. The cross-sectional area within the ullage is accordingly uniform and equal to $A_B$ at all points. Nomenclature is as in the previous gradient equations.

The continuity equations for the mixture region are

\[
\frac{\partial p}{\partial t} + \frac{1}{A} \frac{\partial \mu A}{\partial z} = \dot{m}(t) \tag{R.1}
\]

\[
\rho_r \frac{\partial e}{\partial t} - \frac{1}{A} \frac{\partial (1-e)p_r \mu A}{\partial z} = \dot{m}(t). \tag{R.2}
\]

Making assumptions analogous to the Lagrange assumption, namely

\[
\frac{\partial p}{\partial e} = \frac{\partial e}{\partial z} = 0 \quad \text{in the mixture region} \tag{R.3}
\]

and

\[
\frac{\partial p}{\partial z} = 0 \quad \text{in the ullage region} \tag{R.4}
\]

and noting the assumption $\dot{m}(t)$ is only a function of time, then

\[
u_r = \frac{U_r A_B V(z)}{V(z_b) A(z)} \tag{R.5}
\]

and

\[
u = \frac{U_r A_B V(z)}{V(z_b) A(z)}. \tag{R.6}
\]

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

\[\varepsilon (U_e - U_r) = U_e - U_r\]

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption R.3.
Consider the momentum equations for the mixture region,

\[ \frac{1}{A} \left[ \frac{\partial A \rho_u}{\partial t} + \frac{\partial A \rho u^2}{\partial z} \right] + \varepsilon_0 \frac{\partial P}{\partial z} = -f_s + \dot{m}_u \] \hspace{1cm} (R.7)

and

\[ \frac{1}{A} \left[ \frac{\partial A(1-\varepsilon)\rho_u}{\partial t} + \frac{\partial A(1-\varepsilon)\rho u^2}{\partial z} \right] + (1-\varepsilon)\varepsilon_0 \frac{\partial P}{\partial z} = f_s - \dot{m}_u, \] \hspace{1cm} (R.8)

where

\[ f_s = \frac{(1-\varepsilon)}{D_p} \rho (u - u_p)^2 \] \hspace{1cm} (R.9)

and for

\[ REN = \frac{pD_p(U_u - U_p)}{\mu} \]

and for

\[ \lambda = \left[ 0.5 + \frac{\alpha_2}{\alpha} \right]^{2.17} \]

and with \( \varepsilon_0 \) the initial porosity

\[ \varepsilon_0 = 1 - \frac{C}{\rho_p V_0(\varepsilon_p)}. \]

Then

\[ f_s = \begin{cases} \frac{2.5\lambda}{REN^{0.01}} f_s & \varepsilon < \varepsilon_0 \\ \frac{2.5\lambda}{REN^{0.01}} \left( \frac{(1-\varepsilon)}{(1-\varepsilon_0)} \right)^{0.45} f_s & \varepsilon \geq \varepsilon_0 \end{cases} \]

Defining

\[ \Phi = \phi - \rho A_x L/C \] \hspace{1cm} (R.10)

and

\[ \frac{d^2 \ln \rho}{dt^2} = C_1(t) - \frac{\Phi A_x^2 P}{V_M \rho} + \frac{\Phi A_y^2 P_{rel}}{M_p V_F}, \] \hspace{1cm} (R.11)
where
\[ C(t) = \dot{m} \left( \frac{1}{m} - \frac{1}{\rho_p V_p} \right) + \frac{V_p^2}{V^2} - \frac{\dot{m}^2}{m^2} \]  
(R.12)

and
\[ V_p = V_o + A_S (z_p - z_p) - \frac{C}{\rho_p} + \frac{m}{\rho_p}, \]
\[ \dot{m} = \rho_p \cdot S \frac{dx}{dt}, \]
\[ \ddot{m} = \rho_p \cdot S \frac{dx}{dt} + \rho_p \cdot S \frac{d^2x}{dt^2}, \]  
(R.13)

or
\[ \ddot{m} = \rho_p \cdot \frac{dS}{dx} \left( \frac{dx}{dt} \right)^2 + \rho_p \cdot S \frac{d^2x}{dt^2} \]

with
\[ \frac{dx}{dt} = aP_m^n \]  
(R.14)

and
\[ \frac{d^2x}{dt^2} = anP_m^{n-1} \frac{dP_m}{dt} \]  
(R.15)

where \( dP_m/dt \) is determined numerically, or from
\[ \frac{dP_m}{dt} = \frac{aRT}{m} - P_m \dot{V} + \frac{nRT}{m} \]

and
\[ \dot{V}_p = \frac{8A_s}{M_p} (P_b - P_m). \]  
(R.16)

Then
\[ P(z) = P_{br} + (a_1(t) + a_2(t)P_b) J_1(z) + b(t)J_2(z) \]  
(R.17)

in the mixture region, where
\[ a_1(t) = \frac{CA_S}{g_S V^2(z_b)} \left( \frac{\phi_S A_S}{V(z_b)} - \frac{\dot{\psi}_1 + D - \frac{e_{br} A_S}{\rho_f V^2(z_b)} P_m}{1 - \frac{e_{br} C}{\rho_f V^2(z_b)}} \right) \]  
(R.18)
and

\[
\begin{align*}
-a_2(t) &= -\frac{CE\phi_2 A_2^2 P_{\infty}}{V^2(z_0)\varepsilon M_f} \\
1 - \frac{\phi_2 C}{\rho_f V(z_0)} \\
\end{align*}
\]

(R.19)

and

\[
\begin{align*}
b(t) &= -\frac{C\phi_2 A_2^2}{2g_z V^3(z_0)} \\
\end{align*}
\]

(R.20)

\[
J_1(z) = \int_0^z \frac{V(x)}{A(z)} \, dx,
\]

(R.21)

\[
J_2(z) = \frac{V^2(z)}{A^2(z)},
\]

\[
\begin{align*}
\phi'_1 &= \phi_u U_s - \phi U_p - \frac{\phi \dot{e}}{e^2} \left[ V_p + L \frac{d\ln \rho}{dt} - U_p \right] + \frac{L \phi_u}{e} \frac{d\ln \rho}{dt} \\
&+ \frac{2A_2 \phi U_s}{V(z_0)} [U_s - U_p] + \phi_2 \frac{\rho}{\rho_f D_p} (U_s - U_p)^2 f_\kappa \\
\end{align*}
\]

(R.21a)

and

\[
\begin{align*}
\phi_2 &= 1 - \phi - \phi_u \frac{(1-\varepsilon)}{\varepsilon}, \\
\phi_3 &= \phi_u U_s^2 + (1-\phi) U_p^2, \\
D &= \frac{L \phi_u}{e} C_1(t), \\
E &= 1 - \frac{LA_2}{V_f}.
\end{align*}
\]

(R.22)

Also in the ullage region

\[
\frac{\partial P}{\partial z} = -\frac{\rho}{g_0} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right]
\]

(R.23)

12
and
\[ P(z) = P(z_b) - \frac{P}{g_o} \left[ (V_p + z_p \Delta) (z - z_b) - \frac{z^2 - z_b^2}{2} \right], \tag{R.24} \]

where
\[ \Delta = \frac{d^2 \ln p}{dt^2} - \left( \frac{d \ln p}{dt} \right)^2. \]

Defining the mean pressure as
\[ P_m = \frac{\int_{z_b}^z P(z) A(z) dz}{\int_{z_b}^z A(z) dz} \tag{R.25} \]

and substituting for \( V_p \) and \( \frac{d^2 \ln p}{dt^2} \),

then
\[ P_{b'} = P_b \left( 1 - a_i(t) J_i(z_b) + \frac{\rho L A_b}{M_p} - \frac{\rho L^2 A_b^2}{2 V_p M_p} \right) \]
\[ + \frac{\rho L^2}{2 g_o} \left( C_i(t) - \left( \frac{d \ln p}{dt} \right)^2 + \frac{g_o A_b^2 P_{\text{ref}}}{M_p V_p} \right) \]
\[ - a_i(t) J_i(z_b) - b(t) J_b(z_b) - \frac{A_b \rho L P_{\text{ref}}}{M_p} \tag{R.26} \]

and
\[ P_b = \left[ P_m - \frac{\rho L^2}{2 g_o} \left( C_i(t) - \left( \frac{d \ln p}{dt} \right)^2 + \frac{g_o A_b^2 P_{\text{ref}}}{M_p V_p} \right) \right] \]
\[ - \frac{\rho A_b L^2}{3 g_o V(z_p)} \left( C_i(t) + \frac{g_o A_b^2 P_{\text{ref}}}{M_p V_p} - \left( \frac{d \ln p}{dt} \right)^2 \right) \]
\[ + \frac{b(t) J_i(z_b)}{V(z_p)} + \frac{A_b L a_i(t) J_i(z_b)}{V(z_p)} + \frac{A_b L b(t) J_b(z_b)}{V(z_p)} + \frac{\rho A_b^3 L^2 P_{\text{ref}}}{2 V(z_p) M_p} \]
\[ - a_i(t) J_i(z_b) - b(t) J_b(z_b) - \frac{A_b \rho L P_{\text{ref}}}{M_p} \right] \]
\[ \left( 1 + \frac{A_b L a_i(t) J_i(z_b)}{V(z_p)} - \frac{\rho A_b^3 L^2}{2 V(z_p) M_p} + \frac{a_i(t) J_i(z_b)}{V(z_p)} - a_i(t) J_i(z_b) \right) \]
\[ + \frac{\rho L A_b}{M_p} - \frac{\rho L^2 A_b^2}{2 V_p M_p} + \frac{\rho A_b^3 L^2}{3 V(z_p) V_p M_p} \tag{R.27} \]
where

\[ J_3(z_b) = \int_0^{z_b} A(z)J_1(z)dz \]  

(R.28)

and

\[ J_4(z_b) = \int_0^{z_b} \frac{V^2(z)}{A^2(z)} dz. \]  

(R.29)

The kinetic energy is

\[
KE = \left(1 - \frac{\varepsilon}{2g_o}\right) \frac{U^2 A^2 \rho_0 J_4(z_b)}{V(z_b)^2} + \frac{\varepsilon}{2g_o} \frac{U^2 A^2 \rho J_4(z_b)}{V(z_b)^2} \\
+ \frac{\rho A_b L}{6g_o} \left(3V_p^2 + 3V_p \rho L \frac{d\ln \rho}{dt} + L^2 \left(\frac{d\ln \rho}{dt}\right)^2\right). 
\]  

(R.30)

The mean pressure over the mixture region is

\[
P_{\text{mix}} = P_{Bp} + \frac{(a(t) + a(t)P_p)}{V(z_b)} J_3(z_b) + \frac{b(t)J_4(z_b)}{V(z_b)}. \]  

(R.31)

2. CALCULATIONS

A number of propellant charges were simulated with XKTC to probe the influence of chambrage and velocity lag. The same charges were simulated with IBRGA (input description and listing given in Appendices 4 and 5) using data bases as consistent with those of XKTC as the physical scope of the lumped-parameter model would permit, and using each of the various pressure gradient models. The resultant maximum breech pressures, velocities, and histories of pressure-time and mean-pressure-to-base-pressure curves are compared.

The calculations performed with XKTC involved data bases with evenly distributed seven-perforated propellant having an initial porosity of 0.4, zero barrel resistance, and with all the propellant ignited at the initial instant. All calculations were performed for a flat-based projectile and nominal heat loss.

The parameters used in the computer codes were:

Large chamber simulations:
- Bore diameter: 127 mm
- Volume: 9,832.2 cm³
- Travel: 4.572 m
- Propellant mass: 9.8 kg
- Projectile mass: 2.45, 9.8, 39.2 kg
Small chamber simulations:

- Bore diameter: 28.65 mm
- Volume: 98.322 cm³
- Travel: 1.880 m
- Propellant mass: .098 kg
- Projectile mass: .0245, .098, .392 kg

Propellant characteristics:
- Impetus: 1,136 J/g
- Covolume: .976 cm³/g
- Gamma: 1.23
- Flame temperature: 3,141 K
- Molecular weight: 23.0 g/gmole
- Density: 1.66 g/cm³
- Burning rate: $1.10519p^{10}$ mm/s (p is in MPa)

Three maximum breech pressures were studied, nominally 517 MPa, 345 MPa, and 172 MPa. The maximum pressures were achieved by allowing the web to vary, with the grain length kept between two to three times the outer diameter. Calculations were performed for both bore diameter chambers and right circular cylindrical chambers with a larger diameter than the bore. Chambrage (defined as the distance over which the chamber tapers down to the bore area) was 76.2 mm for the large volume chamber, with the chamber length being changed from 776.22 mm (straight chamber) to 541.02 mm. For the smaller volume chamber, we had 25.4 mm of chambrage with the chamber length being changed from 152.4 mm (straight chamber) to 101.6 mm.

3. RESULTS

3.1 Chambrage. The effect of chambrage was assessed at a nominal pressure of 345 MPa using XKTC. Calculations were performed for a straight chamber to determine grain dimensions corresponding to a maximum breech pressure of 345 MPa. These grain dimensions were then used with a chamber with the same volume but having chambrage. The presence of chambrage was found to reduce the maximum breech pressure substantially. Finally, XKTC was run with chambrage and the dimensions of the grains changed to restore the maximum breech pressure to 345 MPa. These sets of grain dimensions were then used with a version of IBRGA into which the chambrage gradient equation had been encoded. Equivalent computer runs on IBRGA with the Lagrange gradient are also given in Table 1.

Adding chambrage to the gradient equation in the lumped-parameter code captures the large drop in maximum breech pressure and the concomitant drop in muzzle velocity seen when chambrage is added to XKTC. The gradient equation with chambrage also gives the proper amount of increase in maximum breech pressure and muzzle velocity, as seen in XKTC calculations, when the grain dimensions are changed to
TABLE 1. Chambrage Calculations.

<table>
<thead>
<tr>
<th>Chamber volume, cm³</th>
<th>Chambrage, cm</th>
<th>XKTC Chambrage gradient</th>
<th>Chambrage gradient</th>
<th>Lagrange gradient</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>Maximum breech pressure, MPa</td>
<td>Muzzle velocity, m/s</td>
<td>Maximum breech pressure, MPa</td>
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16
restore the maximum breech pressure to 345 MPa. If the Lagrange gradient equation were to be used, a
difference from XKTC of approximately 10% at a charge mass to projectile mass ratio (c/m) of one and
approximately 30% at a c/m of four would occur in the predicted values of maximum breech pressure.

The chambrage gradient equation gives calculated maximum breech pressures, which are as much as
7% high and muzzle velocities low by 3 to 4% when compared to XKTC calculations. The resultant gradient
structure is illustrated, in Figures 1-6, with representative plots of the ratio of the mean pressure to the base
pressure for both XKTC and the analytic chambrage gradient equation. The figures also include histories
of breech and base pressure.

3.2 Multiphase Flow. The effects of multiphase flow were examined using XKTC. Calculations
were based on a straight chamber; nominal maximum breech pressures of 172 MPa, 345 MPa, and 517 MPa;
large and small chamber volumes; and values of c/m equal to .25, 1, and 4. The grain dimensions from the
XKTC calculations were used to define equivalent data bases for IBRGA into which the two-phase gradient
equation had been encoded.

The pressure at which the propellant burns is taken as the mean pressure over the mixture region
instead of the mean pressure over the entire volume behind the projectile, as is conventionally assumed.
Comparable Lagrange gradient calculations are also supplied in Table 2.

The calculated maximum breech pressures and velocities for both the two-phase and Lagrange gradient
for c/m of .25 are in very close agreement with XKTC at all pressures. For calculations at c/m’s of 1 and
4, the Lagrange and two phase gradients tend to give close maximum breech pressures, at worst about 6%
different from XKTC.

The history of the ratio of space mean to base pressures for representative two-phase gradient
calculations and for corresponding XKTC calculations is plotted in Figures 7-12, together with associated
breech-pressure and base-pressure histories. The Lagrange gradient would be a straight line with a ratio
value of 1.083 for a c/m of 0.25; 1.333 for a c/m of 1; and 2.333 for a c/m of 4. The two-phase gradient
equation has the same shape and magnitude of the first peak in the mean pressure to base pressure ratio as
XKTC, with the maximum occurring earlier in the analytic two-phase gradient calculations and the recovery
to the Lagrange ratio value not being as fast as XKTC. The comparison of shape and magnitude of the
pressure ratio histories strongly suggests that the dominant physical processes are captured by the two-phase
gradient equation. In particular, the undulatory character of the curve is seen to be attributable, at least in
part, to the two-phase aspects of the flow, rather than being due to transient or wave propagation phenomena
as might at first be thought.
Figure 1. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage.

Figure 2. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage.

Figure 3. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage.

Figure 4. IBRGA Calculation With Chambrage Gradient With Figure 1 Values.

Figure 5. IBRGA Calculation With Chambrage Gradient With Figure 2 Values.

Figure 6. IBRGA Calculation With Chambrage Gradient With Figure 3 Values.
<table>
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<tr>
<th>Chamber volume, cm³</th>
<th>c/m</th>
<th>XKTC Maximum breech pressure, MPA</th>
<th>XKTC Muzzle velocity, m/s</th>
<th>Two-Phase gradient Maximum breech pressure, MPA</th>
<th>Two-Phase gradient Muzzle velocity, m/s</th>
<th>Lagrange gradient Maximum breech pressure, MPA</th>
<th>Lagrange gradient Muzzle velocity, m/s</th>
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Figure 7. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and No Chambrage.

Figure 8. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and No Chambrage.

Figure 9. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and No Chambrage.

Figure 10. IBRGA Calculation With Two-Phase Gradient With Figure 7 Values.

Figure 11. IBRGA Calculation With Two-Phase Gradient With Figure 8 Values.

Figure 12. IBRGA Calculation With Two-Phase Gradient With Figure 9 Values.
The drop off of the pressure ratio at the slivering event seen in IBRGA calculations may cause numerical problems. (This event is smoothed over in XKTC since all the grains do not sliver at the same time because of the different pressure fields seen by the grains, as well as the finite time required for the transfer of information.) This problem is not felt to affect the calculated velocity significantly, but a smoothing routine has been incorporated into IBRGA to circumvent any difficulty.

The two-phase gradient equation also has, as input, a multiplier to the friction factor. By making this value small, an approximation to stick propellant behavior can be captured.

3.3 Combined. Table 1 is reproduced in Table 3 with the chambrage gradient equation replaced by the RGA gradient equation with the appropriate kinetic energy term. As with the two-phase gradient equation, the burning rate is calculated from mean pressure over the mixture region.

The shapes of the ratio and pressure-time plots, Figures 13-18, are qualitatively similar to those produced by XKTC. This reinforces the idea that both two phase and chambrage effects are important factors in respect to the time-dependent behavior of the pressure gradient. The result of the RGA gradient equation calculations are in general slightly better, when compared to XKTC, than the corresponding calculations with the chambrage gradient equation.

One of the RGA gradient equation inputs is a multiplier to the friction factor. By making this value small, an approximation to stick propellant behavior can be captured.

4. CONCLUSIONS

It is concluded that incorporating chambrage into a gradient equation is important and can make as much as a 20 to 30% difference in predicted maximum breech pressure at c/m of 4. The inclusion of two-phase effects into the gradient equation seems to be well motivated in that a large portion of the structure of the gradient history is captured.

At low c/m's (0.25), any of the gradient equations will give good results; but as the c/m gets larger (>1), the correction for chambrage is required.

The use of the RGA gradient equation offers a natural way to account for stick propellant phenomena having to do with stick propellant motion and the pressure history in which it burns.

The use of the proper kinetic energy term for the gradient equation, as well as the calculation of the mean pressure over the mixture region, should be included in any gradient model when applicable.
### TABLE 3. RGA Calculations.

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<th>Chamber volume, cm$^3$</th>
<th>Chambrage, c/m</th>
<th>XKTC</th>
<th>Maximum breach pressure, MPa</th>
<th>Muzzle velocity, m/s</th>
<th>RGA gradient</th>
<th>Maximum breach pressure, MPa</th>
<th>Muzzle velocity, m/s</th>
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Figure 13. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage.

Figure 14. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage.

Figure 15. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage.

Figure 16. IBRGA Calculation With RGA Gradient With Figure 13 Values.

Figure 17. IBRGA Calculation With RGA Gradient With Figure 14 Values.

Figure 18. IBRGA Calculation With RGA Gradient With Figure 15 Values.
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5. REFERENCES


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6. LIST OF SYMBOLS

- $\varepsilon$, porosity defined as the volume of gas/total volume
- $\lambda$, particle shape factor for gas flow resistance
- $\mu$, viscosity of gas
- $\rho$, density
- $\rho_p$, propellant density
- $\phi$, mass fraction burned
- $A(z)$, area at position $z$
- $A_B$, tube area
- $C$, total charge mass
- $D_p$, effective diameter of the propellant grain, i.e., $6^* \text{volume of propellant grain/surface area of a propellant grain}$
- $f_s$, interphase drag
- $f_{so}$, baseline friction factor
- $f_{sc}$, dimensionless friction factor, a function of Reynolds number and particle shape
- $GD$, grain diameter
- $GL$, grain length
- $g_o$, a constant to reconcile dimensions
- $L$, the distance between the leading edge of the propellant bed and the base of the projectile, i.e., $L = (z_p - z_b)$
- $m(t)$, rate of combustion of propellant
- $m$, mass of propellant burned
- $M_p$, mass of projectile
- $m_u$, molecular weight of propellant gas
- $P$, pressure
- $P_b$, projectile base pressure
- $P_{br}$, breech pressure
- $P_m$, mean pressure
- $P_{res}$, resistive pressure
- $R$, universal gas constant
- $\text{REN}$, particle reynolds number
- $S$, surface area of the propellant
- $T$, gas temperature
- $t$, time
- $u$, velocity
\( U_p \)  
\( U_p^* \)  
\( u_p \)  
\( U_p \)  
\( V(z) \)  
\( V_f \)  
\( V_0 \)  
\( V_p \)  
\( x \)  
\( z \)  
\( z_p \)  
\( z_{0p} \)  
\( \cdot \)  
\( \cdot\cdot \)
APPENDIX 1:
INFLUENCE OF CHAMBRAGE
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The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

**Influence of Chambrage.** For the effects of chambrage on the form of the gradient equation, the propellant is assumed to be uniformly distributed between the breech and the base of the projectile, and all variations in tube area are confined to the chamber.

The continuity and momentum equations for unsteady flow of a homogeneous inviscid substance through a tube with variable area are

\[
\frac{\partial \rho}{\partial t} + \frac{1}{A} \left( \frac{\partial A u}{\partial z} \right) = 0 \quad (C.1)
\]

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + g \frac{\partial \rho}{\partial z} = 0. \quad (C.2)
\]

Making the Lagrange assumption

\[
\frac{\partial \rho}{\partial z} = 0, \quad (C.3)
\]

then (C.1) becomes

\[
\frac{\partial \rho}{\partial t} = - \frac{\rho}{A} \left( \frac{\partial A u}{\partial z} \right). \quad (C.4)
\]

The density may be written as

\[
\rho = \frac{C}{V(\tau_p)}. \quad (C.5)
\]

Differentiating (C.5), we get

\[
\frac{d\rho}{dt} = - \left( \frac{C}{V^2(\tau_p)} \right) \left( \frac{dV(\tau_p)}{dt} \right). \quad (C.6)
\]

But,

\[
V(\tau_p) = V(\tau_{p0}) + (\tau_p - \tau_{p0}) A_s \quad (C.7)
\]

and

\[
\frac{dV(\tau_p)}{dt} = A_s \dot{\tau}_p = A_s V_p \quad (C.8)
\]
therefore,
\[
\frac{dp}{dt} = - \frac{C A_p V_p}{V(z_p)}.
\]  
(C.9)

But,
\[
\rho = \frac{C}{V(z_p)}
\]

therefore,
\[
\frac{dp}{dt} = - \frac{A_p \rho V_p}{V(z_p)}.
\]  
(C.10)

Substituting (C.10) into (C.4), we get
\[
- \frac{A_p \rho V_p}{V(z_p)} = - \frac{\rho}{A} \left( \frac{\partial A u}{\partial z} \right)
\]  
(C.11)
or
\[
\frac{A_p A V_p}{V(z_p)} = \frac{\partial A u}{\partial z}.
\]  
(C.12)

Integrating (C.12) with \( u(0) = 0 \) and noting that \( V_p \) and \( V(z_p) \) are functions of time only,
\[
A(z) u(z) = A u = \frac{A_p V_p}{V(z_p)} \int_0^z A(z) dz = \frac{A_p V_p V(z)}{V(z_p)}.
\]  
(C.13)

Therefore, the velocity \( u \) at a given time is given by:
\[
u = \left( \frac{A_p V_p}{V(z_p)} \right) \left( \frac{V(z)}{A(z)} \right).
\]  
(C.14)

Note that if \( A = A_p \), then \( V(z) = A_p z \), then \( u = (z/z_p) \ V_p \) as for the Lagrange gradient.  
(C.15)

The pressure distribution follows from the momentum equation (C.2)
\[
\delta_0 \frac{\partial p}{\partial z} = - \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right).
\]  
(C.16)

Differentiating (C.14), we get \( \partial u/\partial t \) and \( \partial u/\partial z \) for substitution into (C.16).
To get \( \partial u/\partial t \) we differentiate (C.14), noting that only \( V(z) \) and \( V(z_p) \) are functions of time and

\[
\frac{\partial V(z_p)}{\partial t} = \frac{\partial (V(z_p))}{\partial t} + \frac{(z_p - z_{p0})}{\partial t} A_0 = A_0 z_p = A_0 V(z_p),
\]  
(C.17)

then

\[
\frac{\partial u}{\partial t} = \frac{A_0 V_p V(z)}{A(z)V(z_p)} - \left( \frac{A_0 V_p V(z)}{A(z)V^2(z_p)} \right) \left( \frac{\partial V(z_p)}{\partial t} \right) = \frac{A_0 V_p V(z)}{A(z)V(z_p)} - \frac{A_0^2 V^2(z)}{A(z)V(z_p)}.
\]  
(C.18)

To get \( \partial u/\partial z \), we differentiate (C.14), noting that only \( V(z) \) and \( A(z) \) are functions of \( z \) and

\[
\frac{\partial V(z)}{\partial z} = \frac{\partial}{\partial z} \left( A(z)dz \right) = A(z),
\]  
(C.19)

then

\[
\frac{\partial u}{\partial z} = \frac{A_0 V_p A(z)}{V(z_p)A(z)} - \left( \frac{A_0 V_p V(z)}{A^2(z)V(z_p)} \right) \left( \frac{dA}{dz} \right) = \frac{A_0 V_p}{V(z_p)} - \left( \frac{A_0 V_p V(z)}{A^2(z)V(z_p)} \right) \left( \frac{dA}{dz} \right).
\]  
(C.20)

Substituting (C.5), (C.14), (C.18), and (C.20) into (C.16) yields

\[
\rho_0 \frac{\partial P}{\partial z} = - \frac{CA_0 V(z)V_p}{A(z)V^2(z_p)} + \left( \frac{CV^2(z)V^2 A_0^2}{A^3(z)V(z_p)} \right) \left( \frac{dA}{dz} \right).
\]  
(C.21)

Integrating (C.21), and noting \( V_p \) and \( V(z_p) \) are functions only of time, then

\[
\rho_0 \int_0^t \partial P = \rho_0 (P(t) - P(0)) = - \frac{CA_0 V_p}{V^2(z_p)} \int_0^t \frac{V(z)dz}{A(z)}
\]  
\[
+ \frac{CV^2 A_0^2}{V^3(z_p)} \int_0^t \left( \frac{V^2(z)}{A^3(z)} \right) \left( \frac{dA}{dz} \right) dz.
\]  
(C.22)
Noting
\[ \int_0^r \left( \frac{V'(z)}{A(z)} \right) \left( \frac{\partial A}{\partial z} \right) dz = \int_0^r \frac{V(z)}{A(z)} dz - \frac{V^2(z)}{2A^2(z)} \]

(from integration by parts), then (C.22), the pressure distribution behind the projectile is

\[ g \cdot (P(z) - P(0)) = \left( \frac{CA^2_V^2}{V^3(z_p)} - \frac{CA_V^2 \dot{V}_p}{V^2(z_p)} \right) J_1(z) - \frac{CV^2 A^2_B}{2V^3(z_p)} J_2(z), \quad \text{(C.23)} \]

where

\[ J_1(z) = \int_0^r \frac{V(z)}{A(z)} dz, \quad \text{(C.24)} \]

which can be evaluated algebraically or numerically if \( V(z) \) and \( A(z) \) are known, and

\[ J_2(z) = \frac{V^3(z)}{A^2(z)}. \quad \text{(C.25)} \]

Rewriting (C.23) in the form

\[ P(z) = P(0) + a(t)J_1(z) + b(t)J_2(z) \quad \text{(C.26)} \]

gives

\[ a(t) = \frac{1}{g} \left( \frac{CA^2_V^2}{V^3(z_p)} - \frac{CA_V^2 \dot{V}_p}{V^2(z_p)} \right) \quad \text{(C.27)} \]

and

\[ b(t) = -\frac{CV^2 A^2_B}{2g V^3(z_p)}. \quad \text{(C.28)} \]

Defining the mean pressure in the chamber to be

\[ P_m = \frac{\int_0^r P(z)A(z)dz}{\int_0^r A(z)dz}, \quad \text{(C.29)} \]
then using (C.26),

\[
P_m = \frac{\int_0^{z_p} [P(0) + a(t)J_1(z) + b(t)J_2(z)] A(z)\,dz}{\int_0^{z_p} A(z)\,dz}
\]  

(C.30)

\[
P_m = \frac{P(0) \int_0^{z_p} A(z)\,dz}{V(z_p)} + \frac{a(t) \int_0^{z_p} J_1(z)A(z)\,dz}{V(z_p)} + \frac{b(t) \int_0^{z_p} J_2(z)A(z)\,dz}{V(z_p)}
\]  

(C.31)

or

\[
P_m = P(0) + \frac{a(t)}{V(z_p)} J_3(z_p) + \frac{b(t)}{V(z_p)} J_4(z_p)
\]  

(C.32)

where

\[
J_3(z_p) = \int_0^{z_p} A(z)J_1(z)\,dz = \int_0^{z_p} \left( \int_0^z \frac{V(x)}{A(x)} \,dx \right) A(z)\,dz
\]  

(C.33)

and

\[
J_4(z_p) = \int_0^{z_p} A(z)J_2(z)\,dz = \int_0^{z_p} \frac{V^2(z)}{A(z)} \,dz
\]  

(C.34)

and where, if \(A(z)\) and \(V(z)\) are known, \(J_3(z_p)\) and \(J_4(z_p)\) may be evaluated algebraically or numerically. The acceleration of the projectile is given by

\[
\psi_p = \frac{g \cdot A_s}{M_p} [P_s - P_{re}].
\]  

(C.35)

Substitution into (C.27) gives

\[
a(t) = a_1(t) + a_2(t)P_s
\]  

(C.36)
where

\[ a_1(t) = \frac{C A_B}{g_0 V^2(z_p)} \left( \frac{A_B V_p^2}{V(z_p)} + \frac{g_0 A_B P_{cm}}{M_p} \right) \]  (C.37)

and

\[ a_2(t) = -\frac{C A_B^2}{M_p V(z_p)^2} . \]  (C.38)

Substituting (C.36) into (C.26) and (C.32) and noting \( P_B = P(0) \),

then

\[ P_B = \left[ 1 - a_2(t) J_1(z_p) \right] P_B - a_1(t) J_1(z_p) - b(t) J_2(z_p) \]  (C.39)

and

\[ P_m = P_B + \frac{[a_1(t) + a_2(t) P_B]}{V(z_p)} J_1(z_p) + \frac{b(t)}{V(z_p)} J_2(z_p). \]  (C.40)

Substituting (C.39) into (C.40) and rearranging terms,

\[ P_m = \left( 1 - a_2(t) J_1(z_p) + \frac{a_2(t) J_2(z_p)}{V(z_p)} \right) P_B - a_1(t) J_1(z_p) \]

\[ - b(t) J_2(z_p) + \frac{a_1(t)}{V(z_p)} J_1(z_p) + \frac{b(t)}{V(z_p)} J_2(z_p). \]  (C.41)

Solving (C.41), we get \( P_B \) in terms of \( P_m \)

\[ P_B = \frac{P_m + a_1(t) J_1(z_p) + b(t) J_2(z_p) - \frac{a_1(t) J_2(z_p)}{V(z_p)} - \frac{b(t) J_1(z_p)}{V(z_p)}}{1 - a_2(t) J_1(z_p) + \frac{a_2(t) J_2(z_p)}{V(z_p)}} \]  (C.42)

Therefore, assuming the mean pressure, \( P_m \), can be calculated from an equation of state, then (C.42) will give the projectile base pressure, \( P_B \). The breech pressure, \( P_B \), is calculated from (C.39) knowing \( P_B \). The pressure distribution is given by (C.26).
The evaluation of \( J_1(z_p), J_2(z_p), J_3(z_p), \) and \( J_4(z_p) \) can be simplified by noting that the variation in area is confined to the chamber and, therefore,

\[
V(z) = V(z_{po}) + A_b(z - z_{po}), \quad \text{for } z \geq z_{po}, \tag{C.43}
\]

and

\[
J_1(z_p) = \int_0^{z_p} \frac{V(z)}{A(z)} \, dz = \int_0^{z_{po}} \frac{V(z)}{A(z)} \, dz + \int_{z_{po}}^{z_p} \frac{V(z)}{A(z)} \, dz
\]

\[
= J_1(z_{po}) + \int_{z_{po}}^{z_p} \frac{V(z_{po}) + A_b(z - z_{po})}{A_b} \, dz
\]

\[
= J_1(z_{po}) + \frac{1}{A_b} \left( V(z_{po}) (z_p - z_{po}) + \frac{A_b (z_p - z_{po})^2}{2} \right) \tag{C.44}
\]

\[
J_2(z_p) = \frac{V^2(z)}{A^2(z)} \bigg|_{z_p} = \frac{[V(z_{po}) + A_b(z_p - z_{po})^2]}{A_b^2}, \tag{C.45}
\]

\[
J_3(z_p) = \int_0^{z_p} \left( \int_0^z \frac{V(x)}{A(x)} \, dx \right) A(z) \, dz
\]

\[
= \int_0^{z_{po}} \left( \int_0^z \frac{V(x)}{A(x)} \, dx \right) A(z) \, dz + \int_{z_{po}}^{z_p} \left( \int_0^z \frac{V(x)}{A(x)} \, dx \right) A_b \, dz
\]

\[
= J_3(z_{po}) + \int_{z_{po}}^{z_p} \left( J_1(z_{po}) + \frac{1}{A_b} \left( V(z_{po})(z - z_{po}) + \frac{A_b}{2} (z - z_{po})^2 \right) \right) A_b \, dz
\]

\[
= J_3(z_{po}) + A_b J_1(z_{po}) (z_p - z_{po}) + \frac{V(z_{po}) (z_p - z_{po})^2}{2}
\]

\[
+ \frac{A_b}{6} (z_p - z_{po})^3, \tag{C.46}
\]

\[
J_4(z_p) = \int_0^{z_p} \frac{V^2(z)}{A(z)} \, dz = \int_0^{z_{po}} \frac{V^2(z)}{A(z)} \, dz + \int_{z_{po}}^{z_p} \frac{[V(z_{po}) + A_b(z - z_{po})^2]}{A_b} \, dz
\]

\[
= J_4(z_{po}) + \frac{[V(z_{po}) + A_b(z_p - z_{po})^2] - V^3(z_{po})}{3A_b^2}. \tag{C.47}
\]

Equations (C.44) - (C.47) require the evaluation of the integrals \( J_1(z_{po}) \) - \( J_4(z_{po}) \) only once.
The kinetic energy (KE) of the gas/solid mixture will be required, and since \( \rho \, dV = dm \) and \( dV = A(z) \, dz \), then

\[
\varepsilon_e KE = \frac{1}{2} \int_{\varepsilon}^{\varepsilon_e} u^2 \, dm = \frac{1}{2} \int_{\varepsilon}^{\varepsilon_e} u^2 \rho dV = \frac{1}{2} \int_{\varepsilon}^{\varepsilon_e} u^2 \rho A(z) \, dz \quad (C.48)
\]

and from (C.5),

\[
\rho = \frac{C}{V(\varepsilon_p)}
\]

and from (C.14)

\[
u = \frac{A_2 V_p V(\varepsilon)}{V(\varepsilon_p) A(z)}
\]

therefore,

\[
\varepsilon_e KE = \frac{1}{2} \int_{\varepsilon}^{\varepsilon_e} \frac{CA_2^2 V_p^2 V(\varepsilon)}{A(z) V(\varepsilon_p)} \, dz
\]

\[
= \frac{CA_2^2 V_p^2}{2V^2(\varepsilon_p)} \left( \int_{\varepsilon}^{\varepsilon_e} \frac{V(\varepsilon)}{A(\varepsilon)} \, dz \right),
\]

or

\[
KE = \frac{CA_2^2 V_p^2}{2\varepsilon_e V^2(\varepsilon_p)} J_4(\varepsilon_p). \quad (C.49)
\]
APPENDIX 2:

INFLUENCE OF PROPELLANT VELOCITY LAG
The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

**Influence of Propellant Velocity Lag.** For the effects of the propellant velocity lag on the form of the gradient equation, the propellant is assumed to be uniformly distributed between the breech face and the leading edge of the bed, with the leading edge of the bed being initially at the base of the projectile. The tube diameter is the same as the bore diameter.

The continuity equations for each of the phases in the mixture region are

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = \dot{m}(t). \tag{L.1}
\]

\[
\rho_p \frac{\partial e}{\partial t} - u_p \frac{\partial}{\partial z} \left[\rho_p^2 \rho_p \rho_e \right] \frac{\partial u_p}{\partial t} = \dot{m}(t). \tag{L.2}
\]

Making assumptions analogous to the Lagrange assumption,

\[
\frac{\partial \rho}{\partial z} = 0 \quad (\text{gas density constant throughout the tube}) \tag{L.3}
\]

\[
\frac{\partial e}{\partial z} = 0 \quad (\text{porosity constant throughout the mixture region}) \tag{L.4}
\]

and noting the assumption that \(\dot{m}(t)\) is only a function of time, then from (L.2),

\[
\rho_p \frac{\partial e}{\partial t} - u_p \frac{\partial}{\partial z} \left(1 - e\right) \rho_p - (1 - e) \rho_p \frac{\partial u_p}{\partial z} = \dot{m}(t). \tag{L.5}
\]

Noting (L.4), then

\[
u_p \frac{\partial}{\partial z} \left(1 - e\right) \rho_p = 0. \tag{L.6}
\]

Therefore,

\[
(1 - e) \rho_p \frac{\partial u_p}{\partial z} = \rho_p \frac{\partial e}{\partial t} - \dot{m}(t) = f(t) \tag{L.7}
\]
and

\[ u_p(z) = \int \partial u_p = \frac{\rho_0 \frac{\partial \bar{z}}{\partial z} - \dot{m}(t)}{(1-\epsilon) \rho_p} z + f_1(t). \]  

(L.8)

Since \( u_p(0) = 0 \), then \( f_1(t) = 0 \), and

\[ u_p(z_b) = U_p = \frac{\rho_0 \frac{\partial \bar{z}}{\partial z} - \dot{m}(t)}{(1-\epsilon) \rho_p} z_b, \]

or

\[ \frac{U_p}{z_b} = \frac{\rho_0 \frac{\partial \bar{z}}{\partial z} - \dot{m}(t)}{(1-\epsilon) \rho_p}. \]  

(L.9)

Substituting (L.9) into (L.8) gives

\[ u_p(z) = \frac{U_p}{z} z. \]  

(L.10)

From (L.1),

\[ \frac{\partial \bar{p}}{\partial t} + u \left( \frac{\rho \partial \bar{e}}{\partial z} + \bar{e} \frac{\partial \rho}{\partial z} \right) + \rho \frac{\partial u}{\partial z} = \dot{m}(t), \]

and using (L.3) and (L.4), then

\[ \frac{\partial u}{\partial z} = -\frac{\partial \bar{p}}{\partial z} + \dot{m}(t) \quad (a \ function \ of \ time \ only). \]  

(L.11)

Integrating (L.11) gives

\[ u(z) = \int \partial u = -\frac{\partial \bar{p}}{\partial z} + \frac{\dot{m}(t)}{\epsilon \rho} z + f_2(t). \]  

(L.12)

Since \( u(0) = 0 \), then \( f_2(t) = 0 \), and

\[ u(z_b) = U_\bar{z} = -\frac{\partial \bar{p}}{\partial z} + \frac{\dot{m}(t)}{\epsilon \rho} z_b, \]

or

\[ \frac{U_\bar{z}}{z_b} = -\frac{\partial \bar{p}}{\partial z} + \frac{\dot{m}(t)}{\epsilon \rho}. \]  

(L.13)
Substituting (L.13) into (L.12) gives

\[ u(z) = \frac{U_\epsilon}{Z_b} \cdot z. \]  

(L.14)

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

\[ \varepsilon \cdot (U_\epsilon - U_p) = U_{s+} - U_p. \]  

(L.15)

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption L.3.

Also, in the ullage region, the continuity equation is given by

\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0. \]  

(L.16)

Using (L.3), we get

\[ \frac{\partial u}{\partial z} = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial t} \right) \quad (a \ function \ of \ time \ only). \]  

(L.17)

Integrating (L.17) we get

\[ u(z) = -\frac{1}{\rho} \left( \frac{d\rho}{dt} \right) z + f_\epsilon(t). \]  

(L.18)

Using the boundary condition

\[ u(z_p) = V_p \]  

(L.19)

gives

\[ u(z) = V_p + (z_p - z) \frac{d\ln \rho}{dt}. \]  

(L.20)

Evaluating (L.20) at \( z_b \) gives

\[ U_{s+} = V_p + \frac{L}{\rho} \left( \frac{d\rho}{dt} \right) = V_p + L \frac{d\ln \rho}{dt}. \]  

(L.21)

Eliminating \( U_{s+} \) from (L.15) with (L.21) gives

\[ \varepsilon \cdot (U_\epsilon - U_p) = V_p + L \frac{d\ln \rho}{dt} - U_p. \]  

(L.22)
\[ U_t = U_p + \frac{1}{\varepsilon} \left( V_p + L \frac{d \ln \rho}{dt} - U_p \right) \]  

(L.23)

Differentiating (L.23) yields

\[ \dot{U}_t = - \dot{U}_p \left( \frac{1 - \varepsilon}{\varepsilon} \right) - \frac{\dot{V}_p}{\varepsilon} \left( V_p + L \frac{d \ln \rho}{dt} - U_p \right) \]
\[ + \frac{\dot{V}_p}{\varepsilon} + \frac{L}{\varepsilon} \left( \frac{d^2 \ln \rho}{dt^2} \right) + \frac{L}{\varepsilon} \left( \frac{d \ln \rho}{dt} \right) \]  

(L.24)

We anticipate the need to factor out the base pressure \( P_B \) dependence of both \( \dot{V}_p \) and \( d^2 \ln \rho/dt^2 \).

The projectile acceleration is given by

\[ \dot{V}_p = \frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{ref}}{M_p} \]  

(L.25)

To get the base pressure \( P_B \) dependence associated with \( d^2 \ln \rho/dt^2 \), we start with the definition of the gas density

\[ \rho = \frac{m}{V_F} \]  

(L.26)

where \( m \) = mass of gas burned and \( V_F \) = free volume. Therefore,

\[ V_F = V_o + A_B (z_p - z_{pe}) - \frac{C}{\rho_p} + \frac{m}{\rho_p} \]  

(L.27)

and

\[ V'_F = A_B \dot{z}_p + \frac{\dot{m}}{\rho_p} = A_B V_p + \frac{\dot{m}}{\rho_p} \]  

(L.28)

and

\[ \dot{V}'_F = A_B \dot{V}_p + \frac{\dot{m}}{\rho_p} \]  

(L.29)

where it is understood that \( m/\rho_p, \dot{m}/\rho_p, \) and \( \ddot{m}/\rho_p \) may be in the form \( \sum_{m_i} \sum_{\rho_{p_i}} \sum_{\rho_{p_i}} \), where the index refers to different propellants.
From (L.26) we get
\[ \ln p = \ln m - \ln V_F, \quad (L.30) \]
and
\[ \frac{d \ln p}{dt} = \frac{\dot{m}}{m} - \frac{V_F}{V_F} \quad (L.31) \]
and
\[ \frac{d^2 \ln p}{dt^2} = \frac{\ddot{m}}{m} - \frac{\dot{m}^2}{m^2} - \frac{V_F^2}{V_F^2} + \frac{V_F^2}{V_F^2}. \quad (L.32) \]

Substituting (L.29) and (L.25), we get
\[ \frac{d^2 \ln p}{dt^2} = c(t) - \frac{g_e A_e^2 P_B}{V_F M_p} + \frac{g_e A_e^2 P_M}{V_F M_p}, \quad (L.33) \]
where
\[ c(t) = \ddot{m} \left( \frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{V_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2}. \]

We still require \( \dot{m} \), which we get from
\[ \dot{m} = \rho_p S \frac{dx}{dt}, \quad (L.34) \]
where \( S \) is the surface area of the propellant and \( \frac{dx}{dt} \) is the burning rate of the propellant, where \( \frac{dx}{dt} \) is of the form
\[ \frac{dx}{dt} = a P_m^n. \quad (L.35) \]

Therefore, from (L.34),
\[ \ddot{m} = \rho_p S \frac{dx}{dt} + \rho_p S \frac{d^2 x}{dt^2} \]
\[ = \rho_p \frac{dS}{dx} \left( \frac{dx}{dt} \right)^2 + \rho_p S \frac{d^2 x}{dt^2}, \quad (L.36) \]
and from (L.35),
\[ \frac{d^2 x}{dt^2} = an P_m^{n-1} \frac{dP_m}{dt} \quad (L.37) \]
where \( \frac{dP_m}{dt} \) is determined numerically or from

\[
\frac{dP_m}{dt} = \frac{mRT}{m_o} - P_mV + \frac{mRT}{m_o}.
\]

Also defining \( \phi \) as the mass fraction of propellant burned, then

\[
\rho = \frac{\phi C - \rho A_b L}{\epsilon V(z_b)} = \frac{\phi C}{\epsilon V(z_b)} \quad \quad \text{(L.38)}
\]

where

\[
\phi_* = \phi - \frac{\rho A_b L}{C} \quad \quad \text{(L.39)}
\]

and

\[
\rho_p = \frac{(1 - \phi)C}{(1 - \epsilon)V(z_b)} \quad \quad \text{(L.40)}
\]

Differentiating (L.39), we get

\[
\phi_* = \dot{\phi} - \frac{\rho A_b L}{C} \left( L + L \frac{dln \rho}{dt} \right) \quad \quad \text{(L.41)}
\]

Solving (L.40) for \( \epsilon \), we get

\[
\epsilon = 1 - \frac{(1 - \phi)C}{\rho_p V(z_b)} \quad \quad \text{(L.42)}
\]

Differentiating (L.42) and noting that

\[
V(z_b) = V(z_{po}) + (z_b - z_{po}) A_b \quad \quad \text{(L.43)}
\]

and, therefore,

\[
\dot{V}(z_b) = \dot{z}_b A_b = U_p A_b \quad \quad \text{(L.44)}
\]

then

\[
\dot{\epsilon} = \frac{\phi C}{\rho_p V(z_b)} + \frac{(1 - \phi) C\dot{V}(z_b)}{\rho_p V^2(z_b)} = \frac{\phi C}{\rho_p V(z_b)} + \frac{(1 - \phi) C U_p A_b}{\rho_p V^2(z_b)} \quad \quad \text{(L.45)}
\]

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We now have the quantities which are required for the solution of the momentum equation from which we get the pressure distribution in the mixture region.

Consider the momentum equations for each phase

\[ \frac{\partial \rho u}{\partial t} + \frac{\partial (1-\epsilon) \rho_u u_p}{\partial x} + \frac{\partial \epsilon}{\partial x} + \frac{\partial \rho^2}{\partial x} = -f_s + \dot{m}u_p \]  

\[ \frac{\partial (1-\epsilon) \rho_u u_p}{\partial t} + \frac{\partial (1-\epsilon) \rho_u u_p^2}{\partial x} + (1-\epsilon) \frac{\partial \rho}{\partial x} = f_s - \dot{m}u_p \]

where

\[ f_s = \frac{(1-\epsilon)}{D_p} \rho (u - u_p)^2 f_{sc} \quad \text{(L.48)} \]

and

\[ f_{sc} = \begin{cases} 
\frac{2.5\lambda}{RE^{0.81}} f_{so}, & \epsilon < \epsilon_o \\
\frac{2.5\lambda}{RE^{0.81}} \left( \frac{1-\epsilon}{1-\epsilon_o} \right)^{0.45} f_{so}, & \epsilon \geq \epsilon_o 
\end{cases} \quad \text{(L.49)} \]

where

\[ \lambda = \left( 0.5 + \frac{\epsilon_o}{\epsilon} \right)^{2.17} \quad \text{(L.50)} \]

and

\[ REN = \frac{\rho D_p |U_p - U_l|}{\mu} \]

Adding (L.46) and (L.47), we get

\[ \frac{\partial P}{\partial z} = -\frac{\partial \rho u}{\partial t} - \frac{\partial (1-\epsilon) \rho_u u_p}{\partial t} - \frac{\partial \epsilon u^2}{\partial z} - \frac{\partial (1-\epsilon) \rho_u u_p^2}{\partial z} \quad \text{(L.51)} \]

or

\[ \frac{\partial P}{\partial z} = -\frac{\partial}{\partial t} [\epsilon u + (1-\epsilon) \rho_u u_p] - \frac{\partial}{\partial z} [\epsilon u^2 + (1-\epsilon) \rho_u u_p^2] \quad \text{(L.52)} \]
From (L.10) we get \( u_p(z) = U_p(z/z_b) \), from (L.14) we get \( u(z) = U_p(z/z_b) \), from (L.38) we get

\[
\varepsilon p = \frac{\phi C - pA_pL}{V(z_b)} = \frac{\phi C}{V(z_b)},
\]

and from (L.40) we get

\[
(1 - \varepsilon)p_p = \frac{(1 - \phi)C}{V(z_b)}.
\]

which, when substituted into (L.52), yields

\[
\frac{\partial P}{\partial z} = - \frac{\partial}{\partial t} \left( \frac{\phi C U_p z}{V(z_b)z_b} + \frac{(1 - \phi) C U_p z}{V(z_b)z_b} \right)
- \frac{\partial}{\partial z} \left( \frac{\phi C U_p z^2}{V(z_b)z_b} + \frac{(1 - \phi) C U_p z^2}{V(z_b)z_b} \right).
\]

(L.53)

Differentiating (L.53) and noting \( z_b, V(z_b), \phi, \phi, U_p, \) and \( U_p \) are functions of time only and that

\( z_b = U_p, \ V(z_b) = A_p U_p, \) and \( V(z_b) = A_p z_b, \) we get

\[
\frac{\partial P}{\partial z} = - z C \left( \frac{\phi U_z U_z + \phi U_z U_z + (1 - \phi) U_p - \phi U_p}{z_b V(z_b)} \right)
+ \frac{z C [\phi U_z + (1 - \phi) U_p]}{z_b V(z_b)} \frac{z_b}{z_b}
+ \frac{z C [\phi U_z + (1 - \phi) U_p]}{z_b V(z_b)} \frac{V(z_b)}{z_b}
- \frac{2z C [\phi U_z^2 + (1 - \phi) U_p^2]}{z_b V(z_b)}.
\]

(L.54)

or

\[
\frac{\partial P}{\partial z} = - \frac{z C}{z_b V(z_b)} \left( \phi U_z U_z + \phi U_z U_z + (1 - \phi) U_p - \phi U_p \right)
- \frac{\phi U_z + (1 - \phi) U_p]}{z_b}
- \frac{\phi U_z + (1 - \phi) U_p]}{z_b}
+ 2 \frac{\phi U_z^2 + (1 - \phi) U_p^2}{z_b}.
\]

(L.55)
or
\[
g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_bV(z_b)} \left( \phi_s U_s + \phi_s U_s + (1-\phi) \frac{U_p - \phi U_p}{z_b} \right). \tag{L.56}
\]

To get \( U_p \) for substitution into (L.56), we use another form of the solid phase momentum equation
\[
\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} + \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} = \frac{f_s}{(1-\epsilon)\rho_p}, \tag{L.57}
\]
and noting that at \( z = z_o, \) \( \partial u_p(z_o)/\partial t = U_p, \) and that \( \partial U_p/\partial z = 0 \) (since \( U_p \) is a function of time only), then
\[
U_p + \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \big|_{z_b} = \frac{f_s}{(1-\epsilon)\rho_p}. \tag{L.58}
\]
Substituting (L.48) for \( f_s \) in (L.58), we get
\[
U_p = \frac{\rho}{D_p\rho_p} \frac{(U_s - U_p)^2}{D_p\rho_p} \big|_{z_b} \tag{L.59}
\]
Substituting (L.24) into (L.56), we get
\[
g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_bV(z_b)} \left( \phi_s \frac{V_p}{\epsilon} - \frac{(1-\epsilon)}{\epsilon} \frac{U_p}{\phi_s} \right)
- \frac{\epsilon}{\xi} \frac{\epsilon}{\xi} \left( V_p + L \frac{dlnP}{dt} - U_p \right) + \frac{L\phi_s}{\epsilon} \frac{d^2lnP}{dt^2}
+ \frac{\phi_L}{\epsilon} \frac{dlnP}{dt} + \phi_s U_s + (1-\phi) \frac{U_p - \phi U_p}{z_b}
+ \frac{2\phi_s U_s}{z_b} \left( U_s - U_p \right). \tag{L.60}
\]
Substituting (L.58) into (L.60), we get

\[ g_\circ \frac{\partial P}{\partial z} = - \frac{zC}{z_bV(z_b)} \left( \phi_2 \frac{V_p}{\varepsilon} + \phi_3 U_p - \phi_5 U_p \right) \]

\[ - \frac{\dot{\phi}_2}{\varepsilon^2} \left( V_p + L \frac{d\ln \rho}{dt} - U_p \right) + \frac{\phi_2 L}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} \]

\[ + \frac{\phi_2 L}{\varepsilon} \frac{d\ln \rho}{dt} + \frac{2\phi_4 U_p}{z_b} (U_p - U_p) + \frac{\phi_2 \rho}{D_\rho \rho_p} (U_p - U_p)^2 \]

\[ + \frac{\phi_3 \rho}{\rho_p} \frac{\partial P}{\partial z} \mid_{z_b} \]  

(L.61)

or

\[ g_\circ \frac{\partial P}{\partial z} = - \frac{zC}{z_bV(z_b)} \left( \phi_1 - \phi_2 \frac{\partial P}{\partial z} \mid_{z_b} \right) \]  

(L.62)

where

\[ \phi_1 = \phi_1' + \frac{\phi_2 \dot{V}_p}{\varepsilon} + \frac{L \phi_3}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} \]  

(L.63)

and

\[ \phi_1' = \phi_3 U_p - \phi_4 U_p - \frac{\phi_2 \dot{\varepsilon}}{\varepsilon^2} \left( V_p + L \frac{d\ln \rho}{dt} - U_p \right) \]

\[ + \frac{\phi_2 L}{\varepsilon} \frac{d\ln \rho}{dt} + \frac{2\phi_4 U_p}{z_b} (U_p - U_p) + \frac{\phi_2 \rho}{D_\rho \rho_p} (U_p - U_p)^2 \]

(L.64a)

and

\[ \phi_2 = 1 - \phi - \phi_3 \frac{(1 - \varepsilon)}{\varepsilon} \]  

(L.64b)

To get \( \frac{\partial P}{\partial z} \mid_{z_b} \) for elimination from (L.62), we evaluate (L.62) at \( z_b \) and solve for \( \frac{\partial P}{\partial z} \mid_{z_b} \). (L.62)

evaluated at \( z_b \) is

\[ g_\circ \frac{\partial P}{\partial z} \mid_{z_b} = - \frac{z_b C}{z_bV(z_b)} \left( \phi_1 - \phi_2 \frac{\partial P}{\partial z} \mid_{z_b} \right) \]  

(L.65)

or

\[ \frac{\partial P}{\partial z} \mid_{z_b} = - \frac{C \phi_1}{g_\circ V(z_b) \left( 1 - \frac{\phi_2 \rho}{\rho_p \varepsilon} \right)} \]  

(L.66)
Substituting (L.66) back into (L.62), we get

\[
g_0 \frac{\partial P}{\partial z} = - \frac{zC}{g_0 V(z_b)} \left( \Phi_1 + \frac{\phi_1 \rho \Phi_1}{\rho g_0 V(z_b) \left( 1 - \frac{\rho \Phi_1}{\rho g_0 V(z_b)} \right)} \right) \tag{L.67}
\]

or

\[
\frac{\partial P}{\partial z} = - \frac{zC}{g_0 V(z_b)} \left( \frac{\Phi_1}{1 - \frac{\rho \Phi_1}{\rho g_0 V(z_b)}} \right) \tag{L.68}
\]

or

\[
\frac{\partial P}{\partial z} = - k_1(t)z \tag{L.69}
\]

where

\[
k_1(t) = k_1 = \frac{C}{g_0 V(z_b)} \left( \frac{\Phi_1}{1 - \frac{\rho \Phi_1}{\rho g_0 V(z_b)}} \right) \tag{L.70}
\]

\(k_1\) depends upon the base pressure \(P_B = P(z_B)\) (one of the values we wish to solve for) through \(V_p\) (the acceleration of the projectile) and also through \((d^2 \ln \rho)/(dr^2)\).

Substituting (L.25),

\[
\dot{V}_p = \frac{g_0 A_p P_B}{M_p} - \frac{g_0 A_p P_{res}}{M_p}
\]

and (L.33) into (L.70), we get

\[
k_1 = \frac{C}{g_0 V(z_b)} \left( \Phi_1' + D - EP_{res} + EP_B \right) \tag{L.71}
\]

where

\[
D = \frac{L \Phi c(t)}{\epsilon} \tag{L.72}
\]

and

\[
E = \frac{\Phi_1}{\epsilon} \left( 1 - \frac{LA_B}{V_f} \right) \frac{g_0 A_B}{M_p} \tag{L.73}
\]
or

\[ k_1 = k_{11} P_g + k_{12} \]  \hspace{1cm} (L.74)

where

\[ k_{11} = \frac{CEk_2}{g_s z_b V(z_b)} \]  \hspace{1cm} (L.75)

\[ k_2 = \frac{1}{1-\frac{\gamma C}{\gamma_g V(z_b)}} \]  \hspace{1cm} (L.76)

and

\[ k_{12} = \frac{(\Phi_1' + D) Ck_2}{g_s z_b V(z_b)} - k_{11} P_{ru}. \]  \hspace{1cm} (L.77)

Therefore, (L.69) becomes

\[ \frac{\partial P}{\partial z} = -(k_{11} P_g + k_{12}) z. \]  \hspace{1cm} (L.78)

Integrating (L.78) gives us the pressure distribution in the mixture \(0 < z < z_b\)

\[ P(z) = P(0) - \frac{k_{11} P_g + k_{12}}{2} z^2. \]  \hspace{1cm} (L.79)

We now need the pressure distribution in the ullage region. For the ullage region, the momentum equation for the gas is

\[ \frac{\partial P}{\partial z} = -\frac{\rho}{g} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right). \]  \hspace{1cm} (L.80)

The equation of continuity in the ullage region is

\[ \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial t} = -\frac{\partial \ln \rho}{\partial t}. \]  \hspace{1cm} (L.81)

Integrating (L.81) and noting \((d\ln \rho)/dt\) is a function of time only, we get

\[ u(z) = -\frac{\partial \ln \rho}{\partial t} z + f(t) \]  \hspace{1cm} (L.82)

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with the boundary condition,

\[ u(z_p) = V_p = -z_p \frac{d \ln \rho}{d t} + f(t) \quad (L.83) \]

Therefore

\[ f(t) = V_p + z_p \frac{d \ln \rho}{d t} \quad (L.84) \]

and

\[ u(z) = -\frac{d \ln \rho}{d t} z + V_p + z_p \frac{d \ln \rho}{d t} = V_p + (z_p - z) \frac{d \ln \rho}{d t}. \quad (L.85) \]

Differentiating (L.85) gives

\[ \frac{\partial u}{\partial t} = V_p + (z_p - z) \frac{d^2 \ln \rho}{d t^2} + z_p \frac{d \ln \rho}{d t} \quad (L.86) \]

or

\[ \frac{\partial u}{\partial t} = V_p + (z_p - z) \frac{d^2 \ln \rho}{d t^2} + V_p \frac{d \ln \rho}{d t}. \quad (L.87) \]

Substituting (L.81), (L.85), and (L.87) into (L.80), we get

\[ \frac{\partial P}{\partial z} = -\frac{\rho}{g_o} \left( V_p + (z_p - z) \frac{d^2 \ln \rho}{d t^2} + V_p \frac{d \ln \rho}{d t} \right) \]

\[ - \left( V_p + (z_p - z) \frac{d \ln \rho}{d t} \right) \left( \frac{d \ln \rho}{d t} \right) \quad (L.88) \]

or

\[ \frac{\partial P}{\partial z} = -\frac{\rho}{g_o} \left( V_p + (z_p - z) \frac{d^2 \ln \rho}{d t^2} - (z_p - z) \left( \frac{d \ln \rho}{d t} \right)^2 \right). \quad (L.89) \]

Integrating (L.89) from \( z_b \) to \( z \) and noting \( z_p, V_p, \) and \( \ln \rho \) are functions of time only, we get

\[ P(z) = P(z_b) - \frac{\rho}{g_o} \left( V_p z - \frac{(z_p - z)^2}{2} \left( \frac{d^2 \ln \rho}{d t^2} \right) \right. \]

\[ + \frac{(z_p - z)^2}{2} \left( \frac{d \ln \rho}{d t} \right)^2 \Bigg|_{z_b}^{z}. \quad (L.90) \]
\[ P(z) = P(z_b) - \frac{\rho}{g_0} \left( \frac{V_p z^2 - (z_p - z)^3}{2} \right) + \Delta \]  

(L.91)

where

\[ \Delta = \frac{d^2 \ln \rho}{dt^2} - \left( \frac{d \ln \rho}{dt} \right)^2. \]  

(L.92)

Therefore, we get the pressure distribution in the ullage region as

\[ P(z) = P(z_b) - \frac{\rho}{g_0} \left( \frac{V_p (z - z_b) + [(z_p - z_b)^2 - (z_p - z)^2]}{2} \right) \]  

(L.93)

or

\[ P(z) = P(z_b) - \frac{\rho}{g_0} \left( V_p + z_p \Delta \right) (z - z_b) - \frac{\Delta}{2} \]  

(L.94)

Defining the mean pressure \( P_m \) to be

\[ P_m = \frac{\int_0^P P(z) A(z) \, dz}{\int_0^P A(z) \, dz} \]  

(L.95)

and, since \( A(z) = A_B \) is a constant, then

\[ P_m = \frac{\int_0^{z_p} P(z) \, dz + \int_{z_p}^{z_b} P(z) \, dz}{z_p}. \]  

(L.96)

Substituting the pressure distribution in the mixture region (L.79) and the pressure distribution in the ullage region (L.94) into (L.96) to get the mean pressure, and noting that \( P(0) = P_{Br} \) (the pressure at the breech) we get

\[ P_m = \frac{1}{z_p} \left( \int_0^{z_b} \left( P_{Br} - \frac{(k_{1,2} P_{Br} + K_{1,2})}{2} z^2 \right) \, dz \right) \]

\[ + \int_{z_p}^{z_b} \left( P(z_b) - \frac{\rho}{g_0} \left( V_p + z_p \Delta \right) (z - z_b) - \frac{\Delta}{2} \right) \, dz \]  

(L.97)

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\[ P_m = \left[ P_{br} z_b - \frac{(k_{11} P_b + k_{12})}{6} z^3_b + P(z_b) (z_p - z_b) \right. \]
\[ - \frac{\rho}{2 \sigma_p} \left( (V_p + z_p \Delta) (z_p - z_b)^2 \right. \]
\[ - \frac{\Delta}{3} \left( z_p^2 - z_b^2 \right) + z_b^2 \Delta (z_p - z_b) \left] / z_p \right. \]
\[ (L.98) \]

or, since \( L = z_p - z_b \),

\[ P_m = \frac{z_b}{z_p} - \frac{k_{11} z^2 b P_b}{6z_p} - \frac{k_{12} z^2 b}{6z_p} + \frac{P(z_b) L}{z_p} \]
\[ - \frac{\rho L^2}{2 \sigma_p z_p} \left( \frac{V_p}{z_p} + \frac{2}{3} \Delta \right). \]
\[ (L.99) \]

We need to substitute for \( P(z_b) \). To get \( P(z_b) \), we evaluate (L.79) at \( z_b \) and get

\[ P(z_b) = P_{br} - \frac{k_{11} z^2 b P_b}{2} - \frac{k_{12} z^2 b}{2}. \]
\[ (L.100) \]

Substituting (L.100) into (L.99), we get

\[ P_m = P_{br} - \frac{k_{11} z^2 b P_b}{2} - \frac{k_{12} z^2 b}{2} \]
\[ + \frac{L}{z_p} \left( P_{br} - \frac{k_{11} z^2 b P_b}{2} - \frac{k_{12} z^2 b}{2} \right) \]
\[ - \frac{\rho L^2 V_p}{2 \sigma_p z_p} - \frac{\rho L^3 \Delta}{3 \sigma_p z_p}. \]
\[ (L.101) \]

Substituting (L.25) for \( V_p \) and combining terms, we get

\[ P_m = P_{br} + P_b \left( - \frac{k_{11} z^2 b}{6z_p} - \frac{L k_{11} z^2 b}{2z_p} - \frac{\rho L^3 A_B}{2z_p M_p} \right) \]
\[ + \frac{\rho L^3 A_B P_{ref}}{2z_p M_p} - \frac{k_{12} z^2 b}{6z_p} - \frac{L k_{12} z^2 b}{2z_p} - \frac{\rho L^3 \Delta}{3 \sigma_p z_p}. \]
\[ (L.102) \]

Evaluating (L.94) at \( z_p \), we get

\[ P(z_p) = P(z_b) - \frac{\rho}{\sigma_p} \left[ (V_p + z_p \Delta) (z_p - z_b) - (z_p^2 - z_b^2) \frac{\Delta}{2} \right], \]
\[ (L.103) \]

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and substituting (L.100) into (L.103) and noting $L = z_p - z_o$, we get

$$P(z_p) = P_B = P_{B'} - \frac{k_{11}z_p^2}{2} - \frac{k_{12}z_b^2}{2} - \frac{\rho L \dot{V}_p}{g_o} - \frac{\rho L z_p \Delta}{g_o},$$  

(L.104)

Substituting (L.25) for $V_p$, and collecting $P_B$ terms for (L.104), we get

$$0 = P_{B'} + P_B \left( -1 - \frac{k_{11}z_b^2}{2} - \frac{\rho L A_B}{M_p} \right) - \frac{k_{12}z_b^2}{2} + \frac{\rho L A_B P_{ra}}{M_p}$$

$$- \frac{\rho L z_p \Delta}{g_o} + \frac{\rho \Delta}{2g_o} (z_p^2 - z_b^2),$$

or

$$0 = P_{B'} + P_B \left( -1 - \frac{k_{11}z_b^2}{2} - \frac{\rho L A_B}{M_p} \right) - \frac{k_{12}z_b^2}{2} + \frac{\rho L A_B P_{ra}}{M_p}$$

$$- \frac{\rho L^2 \Delta}{2g_o}.$$

(L.105)

(L.106)

Subtracting (L.106) from (L.102) to eliminate $P_{B'}$ and get $P_B$ in terms of $P_m$, we get

$$P_m = P_B \left( - \frac{k_{11}z_b^2}{6z_p} - \frac{L k_{11}z_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p} + 1 + \frac{k_{12}z_b^2}{2} + \frac{\rho L A_B}{M_p} \right)$$

$$+ \frac{\rho L^2 A_B P_{ra}}{2z_p M_p} - \frac{k_{12}z_b^2}{6z_p} - \frac{L k_{12}z_b^2}{2z_p} + \frac{k_{12}z_b^2}{2}$$

$$- \frac{\rho L A_B P_{ra}}{M_p} + \frac{\rho L^2 \Delta}{2g_o} \left( 1 - \frac{2L}{3z_p} \right).$$

(L.107)

Noting the dependence of $P_B$ in $\Delta$, we use (L.92)

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left( \frac{d \ln \rho}{dt} \right)^2.$$
or, substituting for

\[ \frac{d^2 \ln p}{dt^2} \]

from (L.33) and eliminating \( V'p \) with (L.25), we get

\[ \Delta = c_i(t) - \frac{g_vA_g^2 p_g}{V_p M_p} + \frac{g_v A_B P_{ra}}{M_p V_p} - \left( \frac{d\ln p}{dt} \right)^2. \]  

(L.108)

Substituting (L.108) into (L.107) and collecting \( P_B \) terms, we get

\[ P_m = P_B \left[ - \frac{k_{12} z_b^2}{2z_p} - \frac{Lk_{12} z_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p} + 1 + \frac{k_{12} z_b^2}{2} + \frac{\rho L A_B}{M_p} \right. \]

\[ - \frac{A_g^2 \rho L^2}{2V_p M_p} \left( 1 - \frac{2L}{3z_p} \right) + \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B P_{ra}}{M_p} - \left\{ \frac{k_{12} z_b^2}{2} \left( \frac{z_b}{3} + L \right) \right\} \]

\[ + \frac{\rho L^2 A_B P_{ra}}{2z_p M_p} + \frac{\rho L^2}{2g_o} \left( 1 - \frac{2L}{3z_p} \right) \left( c_i(t) - \left( \frac{d\ln p}{dt} \right)^2 \right) \]

\[ + \left. \frac{A_g^2 \rho L^2}{2M_p V_p} \left( 1 - \frac{2L}{3z_p} \right) P_{ra} \right]. \]  

(L.109)

Substituting (L.108) into (L.106) and collecting \( P_B \) terms, we get

\[ P_{Br} = P_B \left[ 1 + \frac{k_{12} z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_g^2 \rho L^2}{2V_p M_p} \right] + \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B P_{ra}}{M_p} \]

\[ + \frac{\rho L^2}{2g_o} \left( c_i(t) - \left( \frac{d\ln p}{dt} \right)^2 \right) + \frac{A_g^2 \rho L^2}{2V_p M_p} P_{ra}. \]  

(L.110)

Equation (L.109) gives \( P_B \) in terms of \( P_m \) (which is determined using an equation of state) and then (L.110) gives \( P_{Br} \) in terms of \( P_B \).

The kinetic energy (KE) of the gas and solid will be required. The kinetic energy is given by

\[ \frac{g_o}{2} \int_{V} u^2 dm \]

(L.111)
where \( u \) is the velocity and \( dm \) is the differential of mass; and since \( \rho_d V = dm \) and \( dV = A(z) \, dz = A_d \, dz \), then

\[
g_\circ \, KE = \frac{1}{2} \int_{-L}^{L} u^2 \, dm + \frac{1}{2} \int_{-R}^{R} u^2 \, dm
\]

or

\[
g_\circ \, KE = \frac{1}{2} (1-\varepsilon) \int_{-L}^{L} u^2 \rho_p A_d \, dz + \frac{1}{2} \varepsilon \int_{-R}^{R} u^2 \rho A_d \, dz
\]

\[
+ \frac{1}{2} \int_{-R}^{R} u^2 \rho A_d \, dz.
\]

Substituting (L.10) for \( u_p \), and (L.14) for \( u \) in the mixture region, and (L.20) for \( u \) in the ullage region, we get

\[
g_\circ \, KE = \frac{1}{2} (1-\varepsilon) \rho_p A_d \left( \frac{U_p^2}{z_b^2} \right) \int_{-L}^{L} z^2 \, dz + \frac{\varepsilon}{2} \rho A_d U_p^2 \int_{-R}^{R} z^2 \, dz
\]

\[
+ \frac{\rho A_d}{2} \int_{-R}^{R} \left( V + (\varepsilon - z) \frac{\text{dlnp}}{dt} \right)^2 dz,
\]

or

\[
g_\circ \, KE = \frac{A_d z_b}{6} \left( \varepsilon \rho U_p^2 + (1-\varepsilon) \rho_p U_p^2 \right)
\]

\[
- \frac{\rho A_d}{6} \left( \frac{\text{dlnp}}{dt} \right) \left( V + (\varepsilon - z) \frac{\text{dlnp}}{dt} \right)^3 \bigg|_{-R}^{R}
\]

or

\[
KE = \frac{A_d z_b}{6g_\circ} \left[ \varepsilon \rho U_p^2 + (1-\varepsilon) \rho_p U_p^2 \right]
\]

\[
+ \frac{\rho A_d L}{6g_\circ} \left[ 3V^2_p + 3V_p L \frac{\text{dlnp}}{dt} + L^2 \left( \frac{\text{dlnp}}{dt} \right)^2 \right].
\]

It would appear that the pressure used in evaluating the burning rate should be the mean pressure over the region occupied by the propellant.
The mean pressure over the mixture region is

\[ P_{\text{mix}} = \frac{\int_{0}^{b} P(z)A(z)dz}{\int_{0}^{b} A(z)dz}, \quad (L.116) \]

and since \( A(z) = A_B \) and using the pressure distribution in the mixture region given by (L.79), then

\[ P_{\text{mix}} = \frac{A_B \int_{0}^{b} \left[ P_{Br} - \left( \frac{h_{11}r_{11} + k_{12}}{2} \right) z^2 \right] dz}{A_B \int_{0}^{b} dz}, \quad (L.117) \]

or

\[ P_{\text{mix}} = P_{Br} - \frac{(k_{11}P_B + k_{12})}{6} z^2. \quad (L.118) \]
APPENDIX 3:
COMBINED INFLUENCE OF CHAMBRAGE AND
PROPELLANT VELOCITY LAG
The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

For the effects of the propellant velocity lag and chambrage on the form of the gradient equation, the variation in area is assumed confined to the chamber of the gun and the propellant initially uniformly fills the chamber.

The continuity equations for each of the phases in the mixture region are

\[
\frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial \rho \mu A}{\partial z} = \dot{m}(t) \quad (R.1)
\]

\[
\rho_p \frac{\partial \varepsilon}{\partial t} - \frac{1}{A} \frac{\partial}{\partial z} \left( 1 - \varepsilon \right) \frac{\partial \rho_p \mu_p A}{\partial z} = \dot{m}(t) \quad (R.2)
\]

Making assumptions analogous to the Lagrange assumption,

\[
\frac{\partial \rho}{\partial z} = 0 \quad (gas \ density \ constant \ throughout \ the \ tube) \quad (R.3)
\]

\[
\frac{\partial \varepsilon}{\partial z} = 0 \quad (porosity \ constant \ throughout \ the \ mixture \ region) \quad (R.4)
\]

and noting the assumption that \( \dot{m}(t) \) is a function of time, then (R.1) becomes,

\[
\frac{\partial \rho}{\partial t} + \frac{\varepsilon \rho}{A} \frac{\partial u A}{\partial z} = \dot{m}(t), \quad (R.4a)
\]

or

\[
\frac{\partial u A}{\partial z} = \left( \frac{\dot{m}(t) - \frac{\varepsilon \rho}{A}}{\varepsilon \rho} \right) A \quad (R.5)
\]

Integrating R.5 and noting that the right side is a function of time only except for A which is a function of z only, we get

\[
u A = f(t) \int A \, dz + k(t) = f(t) \, V(z) + k(t). \quad (R.6)
\]

Applying the boundary condition \( u(0) = 0 \) implies that \( k(t) = 0 \) since \( V(0) = 0 \).
Also \( u(z_b) = U_s \) implies that
\[
    f(t) = \frac{U_p A(z_b)}{V(z_b)},
\]
(R.7)
and, since \( A(z_b) = A_B \),
\[
    u(z) = \frac{U_p A_B V(z)}{A(z) V(z_b)}.
\]
(R.8)

Also (R.2) becomes after using (R.3) and (R.4)
\[
    \rho_p \frac{\partial V}{\partial t} - \frac{(1-\varepsilon) \rho_p}{A} \frac{\partial u_p A}{\partial z} = \dot{m}(t)
\]
(R.9)
or
\[
    \frac{\partial u_p A}{\partial z} = \frac{\dot{m}(t) - \rho_p \frac{\partial V}{\partial t}}{(1-\varepsilon) \rho_p} A.
\]
(R.10)

Since the right side of (R.10) is a function of time only except for \( A \), which is a function of \( z \) only, on integrating, we get
\[
    u_p A = f_1(t) \int A \, dz + k_1(t) = f_1(t) V(z) + k_1(t).
\]
(R.11)

Applying the boundary condition, \( u_p(0) = 0 \) implies that \( k_1(t) = 0 \) since \( V(0) = 0 \).

Also \( u_p(z_b) = U_p \) implies that
\[
    f_1(t) = \frac{U_p A(z_b)}{V(z_b)},
\]
(R.12)
and since \( A(z_b) = A_B \),
\[
    u_p(z) = \frac{U_p A_B V(z)}{A(z) V(z_b)}.
\]
(R.13)

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form
\[
    \varepsilon (U_s - U_p) = U_s - U_p
\]
(R.14)

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption R.3.
As with the velocity lag derivation, the ullage region is the same.

That is the continuity equation given by

\[
\frac{\partial p}{\partial t} + \frac{\partial \rho u}{\partial z} = 0, \quad (R.15)
\]

and using (R.3),

\[
\frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial t} \quad (R.16)
\]

where the right side of (R.16) is a function of time only.

Therefore,

\[
u = -\frac{1}{\rho} \frac{\partial p}{\partial t} z + f_3(t). \quad (R.17)
\]

Since \( u(z_b) = U_{z+} \), then

\[
U_{z+} = -\frac{1}{\rho} \frac{\partial p}{\partial t} z_b + f_3(t), \quad (R.18)
\]

and since \( u(z_p) = V_p \), then

\[
V_p = -\frac{1}{\rho} \frac{\partial p}{\partial t} z_p + f_3(t). \quad (R.19)
\]

Subtracting (R.18) from (R.19) and noting

\[
\frac{1}{\rho} \frac{\partial p}{\partial t} = \frac{\ln p}{dt}
\]

we get

\[
U_{z+} = \frac{\ln p}{dt} (z_p - z_b) + V_p. \quad (R.20)
\]

Also solving (R.19) for \( f_3(t) \) and substituting into (R.17), we get

\[
u(z) = V_p + (z_p - z) \frac{\ln p}{dt}. \quad (R.21)
\]
Eliminating $U_\epsilon$ from (R.14) with (R.20) gives
\[ \epsilon \left( U_\epsilon - U_p \right) = V_p + L \frac{d\ln p}{dt} - U_p \]  
(R.22)

or
\[ U_\epsilon = U_p + \frac{1}{\epsilon} \left( V_p + L \frac{d\ln p}{dt} - U_p \right). \]  
(R.23)

Differentiating (R.23) yields
\[ \dot{U}_\epsilon = \dot{U}_p \left( \frac{1-\epsilon}{\epsilon} \right) - \frac{\dot{\epsilon}}{\epsilon^2} \left( V_p + L \frac{d\ln p}{dt} - U_p \right) \]
\[ + \frac{V_p}{\epsilon} + \frac{L}{\epsilon} \frac{d^2\ln p}{dt^2} + \frac{L}{\epsilon} \frac{d\ln p}{dt}. \]  
(R.24)

We anticipate the need to factor out the base pressure $P_b$ dependence of both $\dot{V}_p$ and $d^2\ln p/dt^2$.

The projectile acceleration is given by
\[ \dot{V}_p = \frac{g_s A_s P_b}{M_p} - \frac{g_s A_s P_{ba}}{M_p}. \]  
(R.25)

To get the base pressure $P_b$ dependence associated with $d^2\ln p/dt^2$ we start with the definition of the gas density
\[ \rho = \frac{m}{V_F} \]  
(R.26)

where $m =$ mass of gas burnt and $V_F =$ free volume.

Therefore
\[ V_F = V_o + A_b \left( z_p - z_{po} \right) - \frac{C}{\rho_p} + \frac{m}{\rho_p} \]  
(R.27)

and
\[ \dot{V}_F = A_b \dot{z}_p + \frac{m}{\rho_p} = A_b V_p + \frac{m}{\rho_p} \]  
(R.28)
and

\[ V_F = A_F V_F' + \frac{\dot{m}}{\rho_p} \]  \hspace{1cm} (R.29)

where it is understood that

\[ \frac{m}{\rho_p}, \quad \frac{\dot{m}}{\rho_p}, \quad \text{and} \quad \frac{\ddot{m}}{\rho_p} \]

may be in the form

\[ \sum \frac{m_i}{\rho_{p_i}}, \quad \sum \frac{\dot{m}_i}{\rho_{p_i}}, \quad \text{and} \quad \sum \frac{\ddot{m}_i}{\rho_{p_i}} \]

where the \( i \) index refers to different propellants.

From (R.26) we get

\[ \ln \rho = \ln m - \ln V_F' \]  \hspace{1cm} (R.30)

and

\[ \frac{d \ln \rho}{dt} = \frac{\dot{m}}{m} - \frac{V_F'}{V_F} \]  \hspace{1cm} (R.31)

and

\[ \frac{d^2 \ln \rho}{dt^2} = \frac{\ddot{m}}{m} - \frac{\dot{m}^2}{m^2} - \frac{V_F''}{V_F'} + \frac{V_F'^2}{V_F^2} \]  \hspace{1cm} (R.32)

Substituting (R.29) and then (R.25), we get

\[ \frac{d^2 \ln \rho}{dt^2} = C_1(t) - \frac{8 \rho_A^2 P_e}{V_F M_p} + \frac{8 \rho_A^2 P_{rei}}{M_p V_F} \]  \hspace{1cm} (R.33)

where

\[ C_1(t) = \dot{m} \left( \frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{V_F'^2}{V_F^2} - \frac{\dot{m}^2}{m^2} \]

We still require \( \dot{m} \), which we get from

\[ \dot{m} = \rho_p S \frac{dx}{dt} \]  \hspace{1cm} (R.34)
where $S$ is the surface area of the propellant and $dx/dt$ is the burning rate of the propellant where $dx/dt$ is of the form

$$\frac{dx}{dt} = a P_m^n. \quad (R.35)$$

Therefore, from (R.34) we get

$$\dot{m} = \rho_p S \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2}$$

and from (R.35) we get

$$\frac{d^2x}{dt^2} = a n P_m^{n-1} \frac{dP_m}{dt} \quad (R.36)$$

where $dP_m/dt$ is determined numerically or from

$$\frac{dP_m}{dt} = \frac{\alpha P_m}{\gamma} + \frac{T_m}{\gamma} - P_m \frac{\dot{V}}{V} \quad (R.37)$$

Also, defining $\phi$ as the mass fraction of propellant burned, then

$$\varepsilon \rho = \frac{\phi c - \rho A_b L}{V(z_a)} = \frac{\phi c}{V(z_a)} \quad (R.38)$$

where

$$\phi_* = \phi - \frac{\rho A_b L}{C} \quad (R.39)$$

and

$$(1-\varepsilon) \rho_p = \frac{(1-\phi) C}{V(z_a)}. \quad (R.40)$$

Differentiating (R.39), we get

$$\phi_* = \phi - \frac{\rho A_b}{C} \left[ L + \frac{d \ln \rho}{dt} \right]. \quad (R.41)$$
Solving (R.40) for $\epsilon$, we get

$$
\epsilon = 1 - \frac{(1-\phi)C}{\rho_p V(z_b)}.
$$

(R.42)

Differentiating (R.42) and noting that

$$
V(z_b) = V(z_{po}) + (z_b - z_{po})A_B
$$

and therefore

$$
\dot{V}(z_b) = \dot{z}_bA_B = U_p A_B,
$$

then

$$
\dot{\epsilon} = \frac{\phi C}{\rho_p V(z_b)} + \frac{(1-\phi) \dot{C} V(z_b)}{\rho_p V^2(z_b)} = \frac{\phi C}{\rho_p V(z_b)} + \frac{(1-\phi) \dot{C} U_p A_B}{\rho_p V^2(z_b)}.
$$

(R.45)

We now have the quantities which are required for the solution of the momentum equation from which we get the pressure distribution in the mixture region.

Consider the momentum equations for each phase

$$
\frac{1}{A} \left[ \frac{\partial A \rho u}{\partial t} + \frac{\partial A \rho u^2}{\partial z} \right] + \frac{(1-\epsilon) \rho u^2}{\partial z} \frac{\partial \rho}{\partial z} = -f_s + m u_p
$$

(R.46)

and

$$
\frac{1}{A} \left[ \frac{\partial A (1-\epsilon) \rho u}{\partial t} + \frac{\partial A (1-\epsilon) \rho u^2}{\partial z} \right] + (1-\epsilon) \frac{\partial \rho}{\partial z} \frac{\partial \rho}{\partial z} = f_s - m u_p
$$

(R.47)

where

$$
f_s = \frac{(1-\epsilon)}{D_p} \rho (u - u_p)^2 f_{sc}
$$

(R.48)

and

$$
f_{sc} = \begin{cases} 
\frac{2.5\lambda}{REN^{0.61}} f_{so}, & \epsilon < \epsilon_o \\
\frac{2.5\lambda}{REN^{0.61}} \left( \frac{(1-\epsilon)}{\epsilon} \frac{\xi}{\epsilon} \right)^{0.45} f_{so}, & \epsilon \geq \epsilon_o
\end{cases}
$$

(R.49)
where
\[ \lambda = \left(0.5 + \frac{\alpha_l}{\sigma_0}\right)^{2.17} \]  \hspace{1cm} (R.50)

and
\[ REN = \rho D_p \mid U_l - U_p \mid / \mu. \]

Adding (R.46) and (R.47) we get
\[ \varepsilon \frac{\partial \rho}{\partial z} = - \frac{1}{A} \frac{\partial}{\partial t} \left[ \varepsilon \rho u + A(1-\varepsilon)\rho_p u_p \right] \]
\[ - \frac{1}{A} \frac{\partial}{\partial z} \left[ \varepsilon \rho u^2 + A(1-\varepsilon)\rho_p u_p^2 \right]. \]  \hspace{1cm} (R.51)

From (R.8) we get
\[ u(z) = \frac{U_p A_p V(z)}{A(z)V(z_b)}, \]
from (R.13) we get
\[ u_p(z) = \frac{U_p A_p V(z)}{A(z)V(z_b)}, \]
from (R.38) we get
\[ \varepsilon \rho = \frac{\phi C}{V(z_b)}, \]
and from (R.40) we get
\[ (1-\varepsilon) \rho_p = \frac{(1-\phi)C}{V(z_b)} \]
which, when substituted into (R.51), yields
\[ \varepsilon \frac{\partial \rho}{\partial z} = - \frac{1}{A} \frac{\partial}{\partial t} \left[ \frac{\phi C U_p A_p V(z)}{V^2(z_b)} + \frac{(1-\phi)C U_p A_p V(z)}{V^2(z_b)} \right] \]
\[ - \frac{1}{A} \frac{\partial}{\partial z} \left[ \frac{\phi C U_p^2 A_p^2 V^2(z)}{V'(z_b)A(z)} + \frac{(1-\phi)C U_p^2 A_p^2 V^2(z)}{V'(z_b)A(z)} \right], \]  \hspace{1cm} (R.52)
Performing the indicated differentiation in (R.53), we get

\[
\frac{g_0}{\partial z} \frac{\partial P}{\partial t} = - \frac{CA_\delta V(z)}{A(z)} \left[ \frac{\phi_1 U_z}{V^2(z_b)} + \frac{\phi_2 U_\delta}{V^2(z_b)} - 2 \frac{\phi_3 U_\delta V(z_b)}{V^2(z_b)} - \frac{\phi_4 U_\delta}{V^2(z_b)} \right] \\
+ \frac{CA_\delta^2}{A(z)V^2(z_b)} \left[ \phi_1 U_z^2 + (1-\phi)U_\delta^2 \right] \frac{\partial}{\partial z} \left( \frac{V^2(z_b)}{A(z)} \right) \\
- \frac{CA_\delta^2}{A(z)V^2(z_b)} \left[ \phi_1 U_z^2 + (1-\phi)U_\delta^2 \right] \frac{\partial}{\partial z} \left( \frac{V^2(z)}{A(z)} \right).
\]  (R.54)

Noting that, since

\[
V(z_b) = V(z_{bo}) + A_\delta (z_b - z_{bo}),
\]  (R.55)

and therefore

\[
\dot{V}(z_b) = A_\delta \dot{z}_b = A_\delta U_p,
\]  (R.56)

and since \( \partial V(z)/\partial z = A(z) \), and \( A(z) \) and \( V(z) \) are function of \( z \) only,

\[
\frac{\partial}{\partial z} \left( \frac{V^2(z)}{A(z)} \right) = 2V(z) - \frac{V^2(z)}{A^2(z)} \frac{\partial A(z)}{\partial z}, \]  (R.57)

then

\[
\frac{g_0}{\partial z} \frac{\partial P}{\partial t} = - \frac{CA_\delta V(z)}{A(z)V^2(z_b)} \left[ \phi_1 U_z + \phi_2 U_\delta - 2 \frac{\phi_3 U_\delta A_\delta U_p}{V(z_b)} - \phi_4 U_p \right] \\
+ (1-\phi)U_\delta - 2 \frac{(1-\phi)U_\delta A_\delta}{V(z_b)} + 2A_\delta \frac{U_\delta^2}{V(z_b)} \left( \phi_1 U_z^2 + (1-\phi)U_\delta^2 \right) \\
- \frac{A_\delta V(z)}{A^2(z)V(z_b)} \left( \phi_1 U_z^2 + (1-\phi)U_\delta^2 \right) \frac{\partial A(z)}{\partial z} \]  (R.58)
or

\[
\frac{\partial P}{\partial z} = - \frac{CA_b V(z)}{A(z)V^2(z_b)} \left[ \phi \dot{U}_s + (1-\phi)\dot{U}_p + \phi U_s - \phi U_p \right]
+ 2 \frac{A_b \phi U_k}{V(z_b)} (U_s - U_p)
- \frac{A_b V(z)}{A^2(z)V(z_b)} \left( \phi U_s^2 + (1-\phi)U_p^2 \right) \frac{\partial A(z)}{\partial z}.
\]  

(R.59)

Eliminating \( \dot{U}_s \) from (R.59) by using (R.24), we get

\[
\frac{\partial P}{\partial z} = - \frac{CA_b V(z)}{V^2(z_b)A(z)} \left[ \frac{\phi \dot{V}_p}{\varepsilon} + \frac{L \phi}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} \right]
+ \dot{U}_p \left( 1 - \phi - \frac{\phi(1-\varepsilon)}{\varepsilon} \right) - \phi \frac{\dot{\varepsilon}}{\varepsilon^2} \left( V_p + L \frac{d \ln \rho}{dt} - U_p \right)
+ \phi \frac{L}{\varepsilon} \frac{d \ln \rho}{dt} + \phi U_s - \phi U_p + \frac{2A_b \phi U_s}{V(z_b)} (U_s - U_p)
- \frac{A_b V(z)}{V(z_b)A^2(z)} \left( \phi U_s^2 + (1-\phi)U_p^2 \right) \frac{\partial A(z)}{\partial z}.
\]  

(R.60)

We need to eliminate \( \dot{U}_p \).

From (R.2) we get, by multiplying by \( u_p \),

\[
\frac{\partial P}{\partial z} = \frac{u_p}{A} \frac{\partial}{\partial t} \frac{(1-\varepsilon)}{\partial z} \rho_p u_p A = \dot{m}(t) u_p
\]  

(R.61)

which, when substituted into (R.47), gives

\[
1 \frac{\partial A(1-\varepsilon)\rho_p u_p}{\partial t} + 1 \frac{\partial A(1-\varepsilon)\rho_p u_p^2}{\partial z} + (1-\varepsilon) \frac{\partial P}{\partial z}
= f_s - u_p \rho_p \frac{\partial \varepsilon}{\partial t} + \frac{u_p}{A} \frac{\partial (1-\varepsilon)\rho_p u_p A}{\partial z}
\]  

(R.62)
or

\[
(1-\varepsilon)\rho_p \frac{\partial u_p}{\partial t} + \frac{(1-\varepsilon)\rho_p u_p}{A} \frac{\partial A}{\partial t} + 2(1-\varepsilon)\rho_p \frac{\partial u_p}{\partial z} \\
+ \frac{(1-\varepsilon)\rho_p u_p^2}{A} \frac{\partial A}{\partial z} + (1-\varepsilon)g \frac{\partial P}{\partial z} \\
= f_z + u_p (1-\varepsilon)\rho_p \frac{\partial u_p}{\partial z} + \frac{u_p(1-\varepsilon)\rho_p u_p}{A} \frac{\partial A}{\partial z}. \quad (R.63)
\]

Evaluating R.63 at \( z_b \) and noting \( \partial A / \partial t = 0 \),

\[ \frac{\partial U_p}{\partial z} \bigg|_{z_b} = 0 \]

and

\[ \frac{\partial A}{\partial z} \bigg|_{z_b} = 0 \text{ for } z_b \geq z_{po}, \]

then

\[ (1-\varepsilon) \ g \rho \frac{\partial P}{\partial z} + (1-\varepsilon)\rho_p U_p = f_z. \quad (R.63a) \]

Using (R.48) with (R.63a), we get

\[ U_p = \frac{\rho(U_p - U_p')f_{zc}}{\rho_p D_p} - \frac{g}{\rho_p} \frac{\partial P}{\partial z} \bigg|_{z_b}. \quad (R.64) \]
Substituting (R.64) into (R.60) gives

\[
\frac{\partial P}{\partial z} = - \frac{CA_\phi V(z)}{V^2(z_b)A(z)} \left[ \phi \frac{\dot{V}_p}{\varepsilon} + \frac{L \Phi_e}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} \right.
\]

\[
+ \left( 1 - \phi - \frac{\phi(1-\varepsilon)}{\varepsilon} \right) \frac{\rho(U_z - U_p)^2}{\rho_p D_p} f_{\varepsilon}
\]

\[
- \left( 1 - \phi - \frac{\phi(1-\varepsilon)}{\varepsilon} \right) \frac{\partial P}{\partial z} |_{z_b}
\]

\[
- \frac{\partial \Phi_e}{\varepsilon^2} \left( V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L \Phi_e}{\varepsilon} \frac{d \ln \rho}{dt}
\]

\[
+ \phi U_z - \phi U_p + \frac{2A_\phi U_z U_p}{V(z_b)} (U_z - U_p)
\]

\[
- \frac{A_\phi V(z)}{V(z_b)A^2(z)} \left\{ \frac{\partial A(z)}{\partial z} \right\} (\phi U_z^2 + (1-\phi)U_p^2) \quad (R.65)
\]

or

\[
\frac{\partial P}{\partial z} = - \frac{V(z)}{A(z)} \frac{CA_\phi}{V^2(z_b)} \left[ \phi_1 - \phi_2 \frac{\partial P}{\partial z} \right] |_{z_b}
\]

\[
- \frac{\phi_2 A_\phi V(z)}{V(z_b)A^2(z)} \frac{\partial A(z)}{\partial z} \quad (R.66)
\]

and

\[
\phi_1 = \phi_1' + \frac{L \Phi_e}{\varepsilon} \frac{d^2 \ln \rho}{dt^2} + \frac{\phi \dot{V}_p}{\varepsilon} \quad (R.67)
\]

and

\[
\phi_1' = \phi U_z - \phi U_p - \frac{\phi \dot{V}_p}{\varepsilon} \left( V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L \Phi_e}{\varepsilon} \frac{d \ln \rho}{dt}
\]

\[
+ \frac{2A_\phi U_z U_p (U_z - U_p)}{V(z_b)} + \phi_2 \frac{\rho(U_z - U_p)^2 f_{\varepsilon}}{\rho_p D_p} \quad (R.68)
\]

and

\[
\phi_2 = 1 - \phi - \frac{\phi(1-\varepsilon)}{\varepsilon} \quad (R.69)
\]

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and
\[ \phi_1 = \phi_2 U^2 + (1-\phi) U^2. \]  \hspace{1cm} (R.70)

To get
\[ \frac{\partial P}{\partial z} \bigg|_{z_b} \]
for elimination from (R.66), we evaluate (R.66) at \( z_b \) and solve for
\[ \frac{\partial P}{\partial z} \bigg|_{z_b} \]

With (R.66) evaluated at \( z_b \) and noting
\[ \frac{\partial A}{\partial z} \bigg|_{z_b} = 0 \]
and \( A(z_b) = A_0 \), we get
\[ aP \left[ g_0 aP \right] V(z) \]
\[ \text{(R.71)} \]
or
\[ \frac{\partial P}{\partial z} = - \frac{C\phi_1}{g_0 V(z_b)} \left( 1 - \frac{\phi_c}{\phi_c V(z_b)} \right) \]  \hspace{1cm} (R.72)

Substituting (R.72) back into (R.66), we get
\[ g_0 \frac{\partial P}{\partial z} = - \frac{V(z)}{A(z)} \frac{CA_0}{V^2(z_b)} \left[ \phi_1 + \frac{\phi_2 g_0 C\phi_1}{\rho_p g_0 V(z_b) \left( 1 - \frac{\phi_c}{\phi_c V(z_b)} \right)} \right] \]
\[ \frac{\phi_2 A_0 V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z} \]  \hspace{1cm} (R.73)
or
\[ g_0 \frac{\partial P}{\partial z} = - \frac{V(z)}{A(z)} \frac{CA_0}{V^2(z_b)} \left[ \frac{\phi_1}{1 - \frac{\phi_c}{\phi_c V(z_b)}} - \frac{\phi_2 A_0 V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z} \right] \]  \hspace{1cm} (R.74)
\[ \phi_i \text{ depends upon the base pressure } P_b = P(z_b) \text{ (one of the values we wish to solve for) through } \dot{V}_p \text{ (the acceleration of the projectile) and also through } \frac{d^2 \ln p}{dt^2}. \]

Substituting (R.33) into (R.67), we get

\[ \phi_i = \phi_i' + \frac{L \phi_i C_i(t)}{\epsilon} - \frac{L \phi_i A_B}{\epsilon V_p} \dot{V}_p + \frac{\phi_i}{\epsilon} \dot{V}_p \quad (R.75) \]

or

\[ \phi_i = \phi_i' + D + \frac{E \phi_i}{\epsilon} \dot{V}_p \quad (R.76) \]

where

\[ D = \frac{L \phi_i C_i(t)}{\epsilon} \quad (R.77) \]

and

\[ E = \left( 1 - \frac{L A_B}{V_p} \right). \quad (R.78) \]

Substituting (R.25) into (R.76), we get

\[ \phi_i = \phi_i' + D + \frac{E \phi_i}{\epsilon} \left( \frac{g_A A_B P_B}{M_p} - \frac{g_A A_B P_{reb}}{M_p} \right) \quad (R.79) \]

or

\[ \phi_i = \phi_i' + D - \frac{E \phi_i g_A A_B P_{reb}}{\epsilon M_p} + \frac{E \phi_i g_A A_B P_B}{\epsilon M_p}. \quad (R.80) \]

Substituting (R.80) into (R.74), we get

\[
\frac{\partial P}{\partial z} = - \frac{V(z) CA_B}{A(z) g_A V^2(z_b)} \left[ \phi_i' + D - \frac{E \phi_i g_A A_B P_{reb}}{\epsilon M_p} + \frac{E \phi_i g_A A_B P_B}{\epsilon M_p} \right] \\
- \phi_B A_B \frac{V(z)}{V(z_b) A_B(z)} \frac{dA}{dz}
\quad (R.81) \]
or
\[
\frac{\partial P}{\partial z} = \frac{CA_\theta}{g_0 V(z)} \left[ - \phi' + D - \frac{E_\theta A_\theta}{\omega_\theta} + \frac{E_\theta A_\theta P}{\omega_\theta} \frac{V(z)}{A(z)} \\
+ \frac{\phi_A V^2(z)}{V(z) A^2(z)} \frac{\partial A(z)}{\partial z} \right].
\] (R.82)

Integrating (R.82) and noting that \(A(z)\) and \(V(z)\) are functions of \(z\) and only of \(z\) and that
\[
\int \frac{V^2(z)}{A^2(z)} \frac{\partial A}{\partial z} \, dz
\]
(by parts) can be written
\[
- \frac{V^2(z)}{2A^2(z)} + \int \frac{V(z)}{A(z)} \, dz
\]
we get
\[
P(z) - P(0) = \frac{CA_\theta}{g_0 V^2(z)} \left[ - \phi'_\theta + D - \frac{E_\theta A_\theta P}{\omega_\theta} + \frac{E_\theta A_\theta P}{\omega_\theta} \int \frac{V(z)}{A(z)} \, dz \\
- \frac{\phi_A V^2(z)}{2V(z) A^2(z)} + \frac{\phi_A V}{V(z)} \int \frac{V(z)}{A(z)} \, dz \right].
\] (R.83)
Noting \( P(0) = P_B \) and collecting terms, we get

\[
P(z) = P_B + \left[ \frac{C A_B}{g_o V^2(z_b)} \left( \frac{\phi y A_B}{V(z_b)} - \frac{\phi y' + D - \frac{E_{\text{d}} A_y F_{\text{rel}}}{c y_p}}{1 - \frac{e_c}{\rho_y V(z_b)}} \right) \right. \\
- \left. \frac{C y A_B^2}{V^2(z_b)} \frac{e_c}{1 - \frac{e_c}{\rho_y V(z_b)}} \right] \int \frac{V(z)}{A(z)} \, dz - \frac{\phi y C A_B^2 V^2(z)}{2 g_o V^2(z_b) A^2(z)}
\]

or

\[
P(z) = P_B + \left( a_1(t) + a_2(t) \right) P_B + J_1(z) + b(t) J_2(z)
\]

where

\[
a_1(t) = \frac{C A_B}{g_o V^2(z_b)} \left( \frac{\phi y A_B}{V(z_b)} - \frac{\phi y' + D - \frac{E_{\text{d}} A_y F_{\text{rel}}}{c y_p}}{1 - \frac{e_c}{\rho_y V(z_b)}} \right)
\]

and

\[
a_2(t) = - \frac{C y A_B^2}{V^2(z_b)} \frac{e_c}{1 - \frac{e_c}{\rho_y V(z_b)}}
\]

and

\[
b(t) = - \frac{C y A_B^2}{2 g_o V^2(z_b)}
\]

and

\[
J_1(z) = \int_0^1 \frac{V(z)}{A(z)} \, dz
\]

and

\[
J_2(z) = \frac{V^2(z)}{A^2(z)}.
\]

(R.84) is the equation for the pressure distribution in the mixture region. To complete the pressure distribution description, we require the distribution in the ullage region which should be identical to the description in the velocity lag derivation that is in the ullage region. The momentum equation for the gas is

\[
\frac{\partial P}{\partial z} = - \frac{\rho}{g_o} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right).
\]
The equation of continuity in the ullage region is
\[
\frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{d\rho}{dt} = -\frac{d\ln\rho}{dt} \tag{R.91}
\]

Integrating (R.91) and noting \(\frac{d\ln\rho}{dt}\) is a function of time only we get
\[
u(z) = -\frac{d\ln\rho}{dt} z + f(t). \tag{R.92}
\]

With the boundary condition \(u(z_p) = V_p\) we get
\[
V_p = -z_p \frac{d\ln\rho}{dt} + f(t) \tag{R.93}
\]

and therefore,
\[
f(t) = V_p + z_p \frac{d\ln\rho}{dt} \tag{R.94}
\]

and
\[
u(z) = V_p + (z_p - z) \frac{d\ln\rho}{dt}. \tag{R.95}
\]

Differentiating (R.95) and noting \(\dot{z}_p = V_p\) gives
\[
\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z) \frac{d^2\ln\rho}{dt^2} + V_p \frac{d\ln\rho}{dt}. \tag{R.96}
\]

Substituting (R.91), (R.95), and (R.96) into (R.90), we get
\[
\frac{dP}{dz} = -\frac{\rho}{g_o} \left[ \dot{V}_p + (z_p - z) \frac{d^2\ln\rho}{dt^2} + V_p \frac{d\ln\rho}{dt} \right] 
\]
\[
+ \left( V_p + (z_p - z) \frac{d\ln\rho}{dt} \right) \left( -\frac{d\ln\rho}{dt} \right) \tag{R.97}
\]

or
\[
\frac{dP}{dz} = -\frac{\rho}{g_o} \left[ \dot{V}_p + (z_p - z) \frac{d^2\ln\rho}{dt^2} - (z_p - z) \left( \frac{d\ln\rho}{dt} \right)^2 \right]. \tag{R.98}
\]

Integrating (R.98) from \(z_o\) to \(z\) and noting \(z_p, \dot{V}_p,\) and \(\ln\rho\) are functions of time only, we get
\[
P(z) = P(z_o) - \frac{\rho}{g_o} \left[ \dot{V}_p z - \frac{(z_p - z)^2}{2} \frac{d^2\ln\rho}{dt^2} \right.
\]
\[
+ \left. \frac{(z_p - z)^2}{2} \left( \frac{d\ln\rho}{dt} \right)^2 \right|_{z_o} \tag{R.99}
\]
or
\[ P(z) = P(z_b) - \frac{\rho}{g_o} \left[ \dot{V}_p (z - z_b) + \left( \frac{(z_p - z_b)^2}{2} - \frac{(z_p - z)^2}{2} \right) \Delta \right] \]

(R.100)

where
\[ \Delta = \frac{d^2 \ln \rho}{dt^2} - \left( \frac{d \ln \rho}{dt} \right)^2 \]

(R.101)

Therefore, we get the pressure distribution in the ullage region as
\[ P(z) = P(z_b) - \frac{\rho}{g_o} \left[ \dot{V}_p (z - z_b) + \left( \frac{(z_p - z_b)^2}{2} - \frac{(z_p - z)^2}{2} \right) \Delta \right] \]

(R.102)

or
\[ P(z) = P(z_b) - \frac{\rho}{g_o} \left[ (V_p + z_p \Delta) (z - z_b) - \frac{(z^2 - z_b^2)}{2} \right] \]

(R.103)

Defining the mean pressure \( P_m \) to be
\[ P_m = \frac{\int_0^{z_p} P(z) A(z) dz}{\int_0^{z_p} A(z) dz} \]

(R.104)

and since \( \int_0^{z_p} A(z) dz = V(z_p) \) and using (R.84) and (R.103)

\[ P_m = \frac{\int_0^{z_p} \left( P(z) A(z) dz \right)}{V(z_p)} \]

\[ + \frac{\int_0^{z_p} \left( P(z_b) - \frac{\rho}{g_o} \left[ (V_p + z_p \Delta) (z - z_b) - \frac{(z^2 - z_b^2)}{2} \right] \right) A(z) dz}{V(z_p)} \]

(R.105)

or
\[ P_m = \frac{P_{br} V(z_b) + a_1(t) J_1(z_b) + a_2(t) P_b J_2(z_b) + b(t) J_4(z_b)}{V(z_b)} \]

\[ + \frac{A_g L P(z_b)}{V(z_p)} - \frac{A_g P}{g_o} \left( \frac{\dot{V}_p z_p + \dot{z}_p V_p^2}{2} - \frac{\Delta}{2} \left( \frac{V_p^3}{2} - \frac{z_p^3}{2} \right) \right) \]

(R.106)
\[
P_m = P_{br} \frac{V(z_b)}{V(z_p)} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) P_B J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L P(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{re}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_p V(z_p)} \]

where

\[
J_3(z_b) = \int_0^{z_b} A(z) J_1(z) \, dz
\]

and

\[
J_4(z_b) = \int_0^{z_b} \frac{V^2(z)}{A(z)} \, dz.
\]

Substituting in for \( \dot{V}_p \) from (R.25) into (R.107), we get

\[
P_m = P_{br} \frac{V(z_b)}{V(z_p)} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L a_2(t) J_1(z_b) P_B}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{re}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_p V(z_p)}.
\]

To eliminate \( P(z_b) \), we evaluate (R.84) at \( z_b \) and get

\[
P(z_b) = P_{br} + (a_1(t) + a_2(t) P_B) J_3(z_b) + b(t) J_4(z_b)
\]

and substitute into (R.110) to get

\[
P_m = P_{br} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L a_2(t) J_1(z_b) P_B}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{re}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_p V(z_p)}
\]

or

\[
P_m = P_{br} + P_B \left( \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2V(z_p) M_p} + \frac{a_2(t) J_3(z_b)}{V(z_p)} \right) + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{re}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_p V(z_p)}
\]
We now need $P_n$ in terms of $P_B$ to eliminate $P_{nB}$ from (R.113) and give us an equation for $P_B$.

Evaluating (R.103) at $z_p$, we get

$$P(z_p) = P_B = P(z_b) - \frac{P}{g_o} \left[ (\bar{V}_p + z_p\Delta) L - (z_p^2 - z_b^2) \frac{\Delta}{2} \right] \quad (R.114)$$

or, since $z_p^2 - z_b^2 = L(z_p + z_b)$,

$$P_B = P(z_b) - \frac{\rho L}{g_o} \left( \bar{V}_p + \frac{L\Delta}{2} \right), \quad (R.115)$$

and substituting (R.25) for $\bar{V}_p$, we get

$$P_B = P(z_b) - \frac{\rho L A_B P_B}{M_p} + \frac{A_BPL_{rel}}{M_p} - \frac{\rho L^2 \Delta}{2g_o}. \quad (R.116)$$

Substituting (R.111) for $P(z_b)$ into (R.116) and collecting terms, we get

$$0 = P_{nB} + P_B \left[ 1 + a_2(t)J_1(z_b) - \frac{\rho L A_B}{M_p} \right] + a_1(t)J_1(z_b)$$

$$+ b(t)J_2(z_b) + \frac{A_BPL_{rel}}{M_p} - \frac{\rho L^2 \Delta}{2g_o}. \quad (R.117)$$

Subtracting (R.117) from (R.113) eliminates $P_{nB}$ and gives $P_B$ in terms of $P_m$ or

$$P_m = P_B \left[ 1 - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} + \frac{A_BL a_2(t)J_1(z_b)}{V(z_p)} \right.$$

$$- \frac{\rho A_B L^2 \Delta}{2V(z_p)M_p} + \frac{a_2(t)J_2(z_b)}{V(z_p)} \left] + \frac{a_1(t)J_1(z_b)}{V(z_p)} + \frac{b(t)J_2(z_b)}{V(z_p)} \right.$$
Since $\Delta$ has a dependence on $P_B$ through $d^2\ln p/dt^2$ using (R.33) with (R.117), we get

$$P_{Br} = P_B \left( 1 - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_p M_p} \right)$$

$$+ \frac{\rho L^2}{2g_0} \left( C_i(t) - \left( \frac{d\ln p}{dt} \right)^2 + \frac{g_A A_B^2 P_{rel}}{M_p V_p} \right) - a_1(t)J_1(z_b)$$

$$- b(t)J_2(z_b) - \frac{A_B \rho L P_{rel}}{M_p} , \quad (R.119)$$

and using (R.33) with (R.118), we get

$$P_m = P_B \left[ 1 + \frac{A_B L a_2(t)J_1(z_b)}{V(z_p)} - \frac{\rho A_B^3 L^2}{2V(z_p) M_p} + \frac{a_2(t)J_3(z_b)}{V(z_p)} \right]$$

$$- a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_p M_p} + \frac{\rho A_B^3 L^3}{3V(z_p) V_p M_p}$$

$$+ \frac{\rho L^3}{2g_0} \left( C_i(t) - \left( \frac{d\ln p}{dt} \right)^2 + \frac{g_A A_B^2 P_{rel}}{M_p V_p} \right) \left( 1 - \frac{2A_B L}{3V(z_p)} \right)$$

$$+ \frac{a_1(t)J_3(z_b)}{V(z_p)} + \frac{b(t)J_4(z_b)}{V(z_p)} + \frac{A_B L a_2(t)J_1(z_b)}{V(z_p)}$$

$$+ \frac{A_B L b(t)J_2(z_b)}{V(z_p)} + \frac{\rho A_B^3 L^2 P_{rel}}{2V(z_p) M_p}$$

$$- a_1(t)J_1(z_b) - b(t)J_2(z_b) - \frac{A_B \rho L P_{rel}}{M_p} . \quad (R.120)$$

Equation (R.120) gives $P_B$ in terms of $P_m$ (which is determined using an equation of state) and then (R.119) gives $P_{Br}$ from $P_B$.

The evaluation of $J_1(z_b), J_2(z_b), J_3(z_b),$ and $J_4(z_b)$ can be simplified by noting that the variation in area is confined to the chamber and, therefore,

$$V(z) = V(z_{bo}) + A_B(z - z_{bo}), \quad \text{for} \quad z \geq z_{bo}, \quad (R.121)$$

and

$$J_1(z_b) = \int_{z_{bo}}^{z_b} \frac{V(z)}{A(z)} \, dz = \int_{z_{bo}}^{z_b} \frac{V(z)}{A(z)} \, dz + \int_{z_{bo}}^{z_b} \frac{V(z)}{A(z)} \, dz$$

$$= J_1(z_{bo}) + \int_{z_{bo}}^{z_b} \frac{V(z_{bo}) + A_B(z - z_{bo})}{A_B} \, dz$$

$$= J_1(z_{bo}) + \frac{1}{A_B} \left( V(z_{bo}) (z_b - z_{bo}) + \frac{A_B(z_b - z_{bo})^2}{2} \right) . \quad (R.122)$$
\[ J_2(z_b) = \frac{V^2(z)}{A(z)} \bigg|_{z_b} = \frac{[V(z_b) + A_b (z_b - z_{bo})]^2}{A_b^2}, \tag{R.123} \]

\[ J_3(z_b) = \int_{z_b}^{z_b} \left( \int_{z_b}^{z_b} \frac{V(x)}{A(x)} \, dx \right) A(z) \, dz \]

\[ = \int_{z_b}^{z_b} \left( \int_{z_b}^{z_b} \frac{V(x)}{A(x)} \, dx \right) A(z) \, dz + \int_{z_b}^{z_b} \left( \int_{z_b}^{z_b} \frac{V(x)}{A(x)} \, dx \right) A_b \, dz \]

\[ = J_3(z_{bo}) + \int_{z_b}^{z_b} \left[ J_1(z_{bo}) + \frac{1}{A_b} \left[ V(z_{bo}) (z - z_{bo}) + \frac{A_b}{2} (z - z_{bo})^2 \right] \right] A_b \, dz \]

\[ = J_3(z_{bo}) + A_b J_1(z_{bo}) (z_b - z_{bo}) + \frac{V(z_{bo}) (z_b - z_{bo})^2}{2} \]

\[ + \frac{A_b}{6} (z_b - z_{bo})^3, \tag{R.124} \]

\[ J_4(z_b) = \int_{z_b}^{z_b} \frac{V^2(z)}{A(z)} \, dz = \int_{z_b}^{z_b} \frac{V^2(z)}{A(z)} \, dz + \int_{z_b}^{z_b} \frac{[V(z_{bo}) + A_b (z_b - z_{bo})]^2}{A_b} \frac{A_b}{3} \, dz \]

\[ = J_4(z_{bo}) + \frac{[V(z_{bo}) + A_b (z_b - z_{bo})]^3 - V^3 (z_{bo})}{3A_b^2}. \tag{R.125} \]

Equations (R.122) - (R.125) require the evaluation of the integrals \( J_1(z_{bo}) - J_4(z_{bo}) \) only once.

The kinetic energy (KE) of the gas and solid will be required. The kinetic energy is given by

\[ g_o \, KE = \frac{1}{2} \int u^2 \, dm \]

where \( u \) is the velocity and \( dm \) is the differential of mass, and since \( \rho dV = dm \) and \( dV = A(z) \, dz \), then

\[ g_o \, KE = \frac{1}{2} \left( 1 - \varepsilon \right) \int_{z_b}^{z_b} u^2 \rho A(z) \, dz \]

\[ + \frac{1}{2} \varepsilon \int_{z_b}^{z_b} u^2 \rho A(z) \, dz + \frac{1}{2} \int_{z_b}^{z_b} u^2 \rho A_b \, dz. \tag{R.126} \]
We get \( u \) in the mixture region from (R.8) and \( u_p \) from (R.13) and \( u \) in the ullage region from (R.21), and therefore

\[
\begin{align*}
g_o \ KE &= \frac{1}{2} (1 - \epsilon) \int_o^{z_b} \frac{U_p^2 A_{2p}^2 \rho_p V^2(z)}{V^2(z_b) A(z)} \, dz \\
&+ \frac{\rho A_p}{2} \int_o^{z_b} (V_p^2 + \frac{dln \rho}{dt} (z_p - z))^2 \, dz \\
&+ \frac{\rho A_p}{2} \int_o^{z_b} \left( V_p + \frac{dln \rho}{dt} (z_p - z) \right)^2 \, dz \\
&+ \frac{\rho A_p}{2} \int_o^{z_b} \left( V_p + \frac{dln \rho}{dt} (z_p - z) \right)^2 \, dz \\
&+ \frac{\rho A_p L}{6g_o} \left[ 3V_p^2 + 3V_p L \frac{dln \rho}{dt} + L^2 \left( \frac{dln \rho}{dt} \right)^2 \right].
\end{align*}
\tag{R.127}
\]

or

\[
\begin{align*}
KE &= \frac{(1 - \epsilon)}{2g_o V^2(z_b)} U_p^2 A_{2p}^2 \rho_p J_4(z_b) + \frac{\epsilon U_p^2 A_{2p}^2 \rho J_4(z_b)}{2g_o V^2(z_b)} \\
&+ \frac{\rho A_p L}{6g_o} \left[ 3V_p^2 + 3V_p L \frac{dln \rho}{dt} + L^2 \left( \frac{dln \rho}{dt} \right)^2 \right].
\end{align*}
\tag{R.128}
\]

It would appear that the pressure used in evaluating the burning rate should be the mean pressure over the region occupied by the propellant.

The mean pressure over the mixture region is

\[
P_{mix} = \frac{\int_o^{z_b} P(z)A(z) \, dz}{\int_o^{z_b} A(z) \, dz}
\tag{R.129}
\]

and using (R.84), we get

\[
P_{mix} = \int_o^{z_b} \frac{[P_{br} + (a_1(t) + a_2(t)P_b) J_1(z) + b(t)J_2(z)] A(z) \, dz}{V(z_b)}
\tag{R.130}
\]

or

\[
P_{mix} = P_{br} + \frac{(a_1(t) + a_2(t)P_b)}{V(z_b)} J_3(z_b) + \frac{b(t)}{V(z_b)} J_4(z_b).
\tag{R.131}
\]
INTENTIONALLY LEFT BLANK.
APPENDIX 4:
INPUT DESCRIPTION FOR IBRGA
IBRGA relies on an input data base consisting of all numerical parameters essential for running the code. All values are in metric units. Below is a compilation of a typical data base showing the name and location of each parameter. The names for the numerical values are prefixed with an alphabetical designator corresponding to the position at which the data are to appear, that is, from left to right. The data may be separated by blanks or commas. The units are shown to the right of each input.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>-</td>
<td>chamber volume (cm³)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B.</td>
<td>-</td>
<td>groove diameter (cm)</td>
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<td></td>
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<td></td>
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<tr>
<td>C.</td>
<td>-</td>
<td>land diameter (cm)</td>
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<tr>
<td>D.</td>
<td>-</td>
<td>groove/land ratio (none)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>E.</td>
<td>-</td>
<td>twist (turns/caliber)</td>
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<tr>
<td>F.</td>
<td>-</td>
<td>projectile travel (cm)</td>
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</tr>
<tr>
<td>G.</td>
<td>-</td>
<td>gradient switch (none)</td>
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</tr>
<tr>
<td>H.</td>
<td>-</td>
<td>friction factor (none)</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**record 1a** (Read if and only if gradient switch = 2 or 4)
| A. | -  | number of pairs of points to describe chamber geometry, integer I <= 5 (none) |
| B. | -  | initial distance from breech (cm) |
| C. | -  | diameter at initial distance (cm) |
|    |    |    |
|    |    |    |
|    |    |    |
|    |    | -  | Ith distance from breech (cm) |
|    |    | -  | Ith diameter at Ith distance (cm) |

**record 2**
| A. | -  | projectile mass (kg) |
| B. | -  | switch to calculate energy lost to air resistance, integer (none) (0 = no loss, 1= loss) |
| C. | -  | fraction of bore resistance work used to heat the tube (none) (>= 0.0, <=1.0) |
| D. | -  | gas pressure in front of the projectile (Pa) |
record 3
A. - number of pairs of barrel resistance points (none)  
   (integer J <= 10)
B. - bore resistance (MPa)
C. - travel (cm)
   .
   .
   .
   . - Jth bore resistance (MPa)
   . - Jth travel (cm)

record 4
A. - mass of recoiling parts (kg)
B. - number of recoil point pairs (none) (must be an integer = 2)
C. - recoil force (N) (force to overcome before start of recoil - rod preload)
D. - time of rod preload (s) (must be 0.0)
E. - recoil force (N) (constant resistive force after rise time)
F. - rise time (s) (time to go from start of recoil to constant resistive recoil force)

record 5
A. - free convective heat transfer coefficient (W/cm²-K)
B. - chamber wall thickness (cm) (wall depth which is heated uniformly)
C. - heat capacity of chamber wall (J/g-K)
D. - initial temperature of chamber wall (K)
E. - heat loss coefficient (none) (usually 1, but may be set to 0.0 to eliminate heat loss)
F. - density of chamber wall (g/cm³)

record 6
A. - impetus of igniter (J/g)
B. - covolume of igniter (cm³/g)
C. - adiabatic flame temperature of igniter (K)
D. - mass of igniter (kg)
E. - ratio of specific heats of igniter (none)

record 7
A. - number of propellants (none) (integer <= 10)

record 8
A. - impetus of propellant (J/g)
B. - adiabatic flame temperature of propellant (K)
C. - covolume of propellant (cm³/g)
D. - mass of propellant (kg)
E. - density of propellant (g/cm³)
F. - ratio of specific heats of propellant (none)
G. - propellant form function indicator (none)  
   (integer, may be
   0, zero perforated cylindrical grain
   1, one perforated cylindrical grain
   7, seven perforated cylindrical grain
   15, nineteen perforated hexagonal grain
   19, nineteen perforated cylindrical grain)
H. - length of propellant grain (cm)
I. - diameter of perforations in the propellant grains (cm) (value ignored if not required but must be present)
J. - outside diameter of propellant grain (cm)
(for the hexagonal grain, it is the distance between rounded corners)

record 8 repeated for each propellant

record 9
A. - number of burning rate triplet points (none)
   (integer J <= 10)
B. - exponent (none)
C. - coefficient (cm/s-MPa)
D. - pressure (MPa) (upper pressure limit for which the previous exponent and coefficient are usable)

- Jth exponent (none)
- Jth coefficient (cm/s-MPa)
- Jth pressure (MPa) (if pressure exceeds this limit, then this burning rate equation is used)

record 9 repeated for each propellant

record 10
A. - integration time increment (ms)
B. - print increment (ms)
C. - upper limit on integration time to stop calculation (ms)
Intentionally left blank.
APPENDIX 5:
FORTRAN LISTING OF IBRGA
INTENTIONALLY LEFT BLANK.
program ibrga
common nsl,kpr,fracsl(10),dsdxsl(10),surfsl(10),
& nslp(10),tsl(10),pbrch,pbase,pmean,bbr(10),abr(10),
& deltat,y(20),igrad
character outfll*10
character bdflle*10
dimension br(10),trav(10),rp(10),tr(10),forcp(10),tempp(10),covp(10)
&
dimension chwp(10),rhop(10),gmap(10),nperfs(10),glenp(10),pdp(10)
&.gdiap(10),alpha(10,10),beta(10,10),pres(10,10)
dimension a(4),b(4),ak(4),d(20),p(20),z(20),frac(10),surf(10)
&.volp(10),dsdx(10),nbr(10),ibo(10),ibo(10),d2xd2(10),tng(10)
real lambda,j1zp,j2zp,j3zp,j4zp,j1zb,j2zb,j3zb,j4zb
dimension chdist(5),chdiam(10),bint(4)
dimension nsl(10),surfo(10),dsdxn(10)
call gettim(ihr,imin,isec,ihuns)
pi=3.14159
write(*,15)
15 format(' input name of data file to be used as input ')
read(*,10)bdfile
10 format(a10)
open(unit=2,err=999,file=bdflle,status='old',iostat=ios)
rewind 2
write(*,25)
25 format(' input name of output file ')
read(*,10)outfil
open(unit=6,err=998,file=outfil)
write(6,16)bdflle
16 format(' the input file is ','a10)
do 9 i=1,20
p(i)=0.
y(i)=0.
z(i)=0.
d(i)=0.
9 continue
read(2,*,-end=20,err=30)cham,grvc,aland,glr,twst,travp,igrad
&.fs0
if(igrad.gt.1)go to 51
write(6,55)
55 format(1x,'Using chambrage pressure gradient')
igrad=1
go to 52
c define chambrage assumes nchpts=number of points to define
 c chamber > or = 2 < or = 5 ( ?),chdiam(1) defines chamber diameter
 c at chdist(1) chamber distance. chdiam(nchpts) is assumed to be
 c the bore diameter and chdist(i) is assumed to be 0, i.e. at the
 c breech. Assumes truncated cones.
51 if(igrad.eq.3)go to 401
if(igrad.eq.4)go to 434
write(6,47,err=30)
47 format(1x,'Using chambrage pressure gradient')
go to 436

95
write(6,437)
format(1x,'using rga gradient')
go to 436

read(2,*',end=20,err=-30)nchpts,(chdist(I),chdiam(I),I=1,nchpts)
write(6,53,err=-30)(chdist(I),chdiam(I),I=1,nchpts)
format(111,,' chamber distance cm chamber diameter cm'/(5x,e14 &.6x,e14.6))
do 54 I=1,nchpts
chdist(I)=0.01*chdist(I)
chdiam(I)=0.01*chdiam(I)
calculate chamber integrals and volume
if(nchpts.gt.5) write(6,44,err=-30)
format(1x,'use first 5 points')
if(nchpts.gt.5)nchpts=5
bore=chdiam(nchpts)
if(chdist(1).ne.0.0)write(6,45,err=-30)
format(1x,'# points?')
chdist(1)=0.0
pi3=pi/3.0
bl=0.0
b2=0.0
b3=0.0
b4=0.0
points=25.0
points=points+points
step=chdist(nchpts)/points
zz=0.0
bint(1)=0.0
bint(3)=0.0
bint(4)=0.0
bvol=0.0
r2=0.5*chdiam(1)
k=1
j=int(points+0.5)
do 57 I=1,j
zz=zz+step
if(k.eq.nchpts-1)go to 46
do 58 II=k,nchpts-1
if(zz.gt.chdist(I1).and. zz.lt.chdist(II))go to 59
continue
II=nchpts-1
k=I1
46
diam=(zz-chdist(k))/(chdist(k+1)-chdist(k))
diam=chdiam(k)+diam*(chdiam(k+1)-chdiam(k))
r1=0.5*diam
area=pi*(r1+r2)*(r1+r2)/4.
bvol=bvol+step*pi3*(r1*r1+r1*r2+r2*r2)
bint(1)=bint(1)+step*bvol/area
bint(3)=bint(3)+step*area*bint(1)
bint(4)=bint(4)+step*bvol*bvol/area
r2=r1
temp=abs(1.0-b1/bint(1))
if(abs(1.0-b3/bint(3)).gt.temp)temp=abs(1.0-b3/bint(3))
if(abs(1.0-b4/bint(4)).gt.temp)temp=abs(1.0-b4/bint(4))
if(temp.le.0.001)go to 41
b1=bint(1)
b3=bint(3)
b4=bint(4)
go to 56
41 chamb=bvols*1.e6
c write(6,47,err=30)bint(1),bint(3),bint(4)
c format(1x,'bint 1 = ',e14.6,'  bint 3 = ',e14.6,'  bint 4 = ',e14.
c &6)
chmlen=chdist(nchpts)
go to 52
401 write(6,402)
402 format(1x,'using 2 phase gradient equation')
go to 52
52 write(6,40,err=30)cham,grve,aland,grtwst,grad,fs0
40 format(1x,'chamber volume cm**3',e14.6,'  groove diam cm',e14.6/
&' land diam cm',e14.6,'  groove/land ratio',e14.6,'  twist turns
&/ caliber ',e14.6,'  projectile travel cm',e14.6
&/ gradient # ',i3,’/ friction factor ',e14.6///)
chamb=chamb*1.e-6
grve=grve*1.e-2
aland=aland*1.e-2
travp=travp*1.e-2
read(2,*,end=20,err=-30)prv,t,iair,htfr,pgas
write(6,50,err=-30)prv,t,iair,htfr,pgas
50 format(1x,'projectile mass kg',e14.6,' switch to calculate energy
-&y lost to air resistance J',i2,’/ fraction of work against bore u
&sed to heat the tube',e14.6/1x,' gas pressure Pa',e14.6)
read(2,*,end=20,err=30)npts,(br(:,trav(i),i=1,npts)
write(6,60,err=30)npts,(br(i),trav(i),i=1,npts)
60 format(1x,'number barrel resistance points',i2,’/ bore resistance
& MPa - travel cm/(1x,e14.6,e14.6))
write(6,65)
do 62 i=1,npts
br(i)=br(i)*1.e6
trav(i)=trav(i)*1.e-2
62 continue
65 format(1x)
read(2,*,end=20,err=30)rcwt,nrp,(rp(i),tr(i),i=1,nrp)
write(6,70,err=30)rcwt,nrp,(pr(i),tr(i),i=1,nrp)
70 format(1x,’ mass of recoiling parts kg',e14.6,’ number of recoi
&1 point pairs',i2,’/ recoil force N’,‘/ recoil time sec',i1x,e14
&.6,3x,e14.6))
write(6,65)
read(2,*,end=20,err=30)ho,tshl,cshl,twal,hl,rhocs
write(6,75,err=30)ho,tshl,cshl,twal,hl,rhocs
75 format(1x,’ free convective heat transfer coefficient w/cm**2 K’,
&e14.6,’ chamber wall thickness cm’,e14.6,’ heat capacity of st
&eel of chamber wall J/g K',e14.6,’ initial temperature of chambe
&r wall K',e14.6,’ heat loss coefficient',e14.6,’ density of ch
&amber wall steel g/cm**3',e14.6///)
ho=ho/1.e-4
tshl=tshl*1.e-2
cshl=cshl*1.e+3
rhocs=rhocs*1.e-3/1.e-6
read(2,*,end=20,err=30)forcig,covi,tempi,chwi,gamai
write(6,85,err=30)forcig,covi,tempi,chwi,gamai
85 format(1x,' impetus of igniter propellant J/g',e14.6/',' covolume
 & of igniter cm**3/g',e14.6/',' adiabatic flame temperature of igni
 &ter propellant K',e14.6/',' initial mass of igniter kg',e14.6/',' r
 &tio of specific heats for igniter',e14.6/)
forcig=forcig*1.e+3
covi=covi*1.e-6/1.e-3
read(2,*)nprop
write(6,98)nprop
98 format(,' there are ',i2,' propellants')
read(2,*,end=20,err=30)(forcp(i),tempp(i),covp(i),chwp(i),
&rhop(i),gamap(i),nperfs(i),glenp(i),pdp(i),gdiap(i),i=1,nprop)
write(6,95,err=-30)(i,forcp(i),tempp(i),covp(i),chwp(i)
& ,rhop(i),gamap(i),nperfs(i),glenp(i),pdp(i),gdiap(i),i=1,nprop)
95 format(,' for propellant number',i2,' impetus of propellant J/g
 &',e14.6/',' adiabatic temperature of propellant K',e14.6/',' covol
 &ume of propellant cm**3/g',e14.6/',' initial mass of propellant kg'
 &,e14.6/',' density of propellant g/cm**3',e14.6/',' ratio of speci
 &c heats for propellant',e14.6/',' number of perforations of propell
 &ant',i2/',' length of propellant grain cm',e14.6/',' diameter of per
 &ration in propellant grains cm',e14.6/',' outside diameter of pro
 &pellant grain cm',e14.6/)
tmpi=-0.0
do 96 i=1,nprop
forcp(i)=forcp(i)*1.e+3
covp(i)=covp(i)*1.e-6/1.e-3
rhop(i)=rhop(i)*1.e-3/1.e-6
glenp(i)=glenp(i)*0.01
pdp(i)=pdp(i)*0.01
gdiap(i)=gdiap(i)*0.01
tmpi=tmpl+chwp(i)
kpr=i
call prf710(pdp(i),gdiap(i),glenp(i),nperfs(i),0.,
& frac(i),volp(i),surf(i),dsdx(i))
tng(i)=chwp(i)/rhop(i)/volp(i)
surfo(i)=surf(i)
write(6,408)i,tng(i)
408 format(,' for propellant ',i2,' the total number of grains'
 & ',' is ',e14.6)
96 continue
tmpi=tmpl+chwi
do 97 j=1,nprop
read(2,*,end=20,err=30)nbroj(i),(alpha(j,i),beta(j,i),pres(j,i),
&i=1,nbroj(i))
write(6,110,err=-30)nbroj(i),(alpha(j,i),beta(j,i),pres(j,i),
&i=1,nbroj(i))
110 format(1x,'number of burning rate points',i2/3x,' exponent',8x,
 & coefficient',10x,' pressure',7x,' pressure',7x,' cm/sec-MPa**ai',10x,'MP
 &a'/(1x,e14.6,5x,e14.6,15x,e14.6))
do 112 i=1,nbro(j)
beta(j,i)=beta(j,i)*1.e-2
98
pres(j,i)=pres(j,i)*1.e6

continue

continue

write(6,65)
read(2,*,end=20,err=30)deltat,deltap,tstop
write(6,120,err=30)deltat,deltap,tstop

format(1x,'time increment msec',e14.6,' print increment msec',e14
&.6/1x,'time to stop calculation msec ',e14.6)
write(*,130)
deltat=deltat*0.001
deltap=deltap*0.001
tstop=tstop*.001

format(1x,'end input data -- I.B. calculation start')
if(igrad.eq.2.or.igrad.eq.4)go to 131
bore=(glr*grve*grve+aland*aland)/(glr+1.)
bore=sqrt(bore)

areab=pi*bore*bore/4.
lamba=1./((l13.2+4.*log10(100.*bore))**2)
ipeplot=0
pltdt=deltat
pltt=0.
pmaxm=0.0
pmaxbr=0.0
pmaxba=0.0
tpmaxm=0.0
tpmbxbr=0.0
tpmbxba=0.0
tpmax=0.0
a(1)=0.5
a(2)=1.-sqrt(2.)/2.
a(3)=1.+sqrt(2.)/2.
a(4)=1./6.
b(1)=2.
b(2)=1.
b(3)=1.
b(4)=2.
ak(1)=0.5
ak(2)=a(2)
ak(3)=a(3)
ak(4)=0.5
vp0=0.0
nr0=0.0
tcw=0.0
if(igrad.eq.3)chmlen=cham/areab
zb=chmlen
zp=chmlen
grlen=0.
grdiam=0.
egama=0.
do 5 i=1,nprop
grlen=grlen+chwp(i)*glenp(i)
grdiam=grdiam+chwp(i)*gdiap(i)
ibo(i)=0
egama=eama+chwp(i)*gamap(i)

99
\[ nsl(i) = 0 \]
\[ 5 \quad vp_0 = chwp(i)/rhop(i) + vp_0 \]
\[ volg_i = cham - vp_0 - chwi*covi \]
\[ grlen_7 = grlen/(tmpi - chwi) \]
\[ grdiam = grdiam/(tmpi - chwi) \]
\[ egama = (egama + chwi*gamai)/tmpi \]
\[ isn_ = 0 \]
\[ odlnr = 0. \]
\[ vf_0 = cham - vp_0 \]
\[ eps_0 = 1.0 - vp_0/cham \]
\[ eps = eps_0 \]
\[ gasden = chwi/v_0 \]
\[ prden = tmpi/vp_0 \]
\[ ug = 0. \]
\[ up = 0. \]
\[ pmean = forcig*chwi/volgi \]
\[ pbase = pmean \]
\[ pbrch = pmean \]
\[ opbase = pmean \]
\[ volg = volg_ \]
\[ volgi = volg_ + vp_0 \]
\[ wallt = twal \]
\[ tgas = tempi \]
\[ told = 0. \]
\[ tgas = tgas \]
\[ dtgas = 0. \]
\[ covi = covi \]
\[ t = 0. \]
\[ ptime = 0. \]
\[ ibrp = 10 \]
\[ z(3) = 1. \]
\[ ndc = ibrp + nprop \]
\[ write(6, 132) areab, pmean, vp_0, volg_ \]
\[ format(1x, 'area bore mA^2 ', e16.6, '/' pressure from ign pa', e16.6, / \]
\[ &', 'volume of unburnt prop mA^3 ', e16.6, / \]
\[ & init cham vol-cov ign mA^3 ', e16.6) \]
\[ write(6, 6) \]
\[ format(1x, ' time acc vel dis m press \]
\[ & pbase pbrch ') \]
\[ iswl = 0 \]
\[ 19 \]
\[ continue \]
\[ do 11 J = 1, 4 \]
\[ c FIND BARREL RESISTANCE \]
\[ do 201 k = 2, npts \]
\[ if(y(2) + y(7).ge.trav(k)) go to 201 \]
\[ go to 203 \]
\[ 201 \]
\[ continue \]
\[ k = npts \]
\[ 203 \]
\[ resp = (trav(k) - y(2) - y(7))/(trav(k) - trav(k-1)) \]
\[ resp = br(k) - resp*(br(k) - br(k-1)) \]
\[ c FIND MASS FRACTION BURNED \]
\[ do 211 k = 1, nprop \]
\[ kpr = k \]
\[ if(ibo(k).eq.1) goto 211 \]

100
nsll=0

call prf710(pdp(k),gdiap(k),glenp(k),nperfs(k),y(ibrp+k),
&frac(k),volp(k),surf(k),dsdx(k))

nsl(k)=nsll

if(nsl(k).eq.0)goto 212
if(nslp(k).eq.1)go to 212

write(6,2213)k

2213 format(' propellant',i2,' has slivered')

nslp(k)=1
tsl(k)=y(3)
ism=1

212 continue

if(frac(k).lt.9999) go to 211
frac(k)=1.

tbo(k)=y(3)
ibo(k)=1
ism=1

write(6,456)k

456 format(' propellant',i2,' has burned out')

211 continue

Energy Loss to Projectile Translation

\[ \text{elpt} = \text{prwt} \cdot y(1) \cdot y(1) / 2. \]
\[ \text{epdot} = \text{prwt} \cdot y(1) \cdot z(1) \]

Energy Loss Due to Projectile Rotation

\[ \text{elp} = \pi \cdot \pi \cdot \pi \cdot \text{prwt} \cdot y(1) \cdot y(1) / 4. \cdot \text{twst} \cdot \text{twst} \]
\[ \text{epdot} = \pi \cdot \pi \cdot \text{prwt} \cdot y(1) \cdot z(1) / 2. \cdot \text{twst} \cdot \text{twst} \]

Energy Loss Due to Gas and Propellant Motion

if(igrad.eq.1) go to 214
if(igrad.eq.3) goto 217
if(igrad.eq.4) go to 438

pt=y(2)+y(7)
vzp=bval+areab*pt
j4zp=bint(4)+(bval+areab*pt)**3-bval**3)/3./areab/areab
elgpm=mpi*y(1)*y(1)*areab*areab*j4zp/2./vzp/vzp/vzp

go to 216

438 pb=y(7)+y(10)
vzb=bval+areab*pb
j4zb=bint(4)+(vzb**3-bval**3)/3./areab/areab
elgpm=(1.-eps)*up*up*areab**2*prden*j4zb+
& eps*ug*areab**2*gasden*j4zb

elgpm=elgpm/2./vzb/vzb+gasden*areab*ullen/6.*
& (3.*y(1)*y(1)+3.*yllen*llen+ullen**2*dlrho**2)

approximate epdot

epdot=mpi*y(1)*z(1)/3.

go to 216

214 elgpm=mpi*y(1)*y(1)/6.

go to 216

217 elgpm=areab*zb/6.*(eps*gasden*ug+up+(1.-eps)*prden*up*up)
elgpm=elgpm+gasden*areab*ullen/6.*(3.*y(1)*y(1)+
& 3.*y(1)*yllen*dlrho+ullen**2*dlrho**2)

approximate epdot

epdot=mpi*y(1)*z(1)/3.

Energy Loss From Bore Resistance

elbr=y(4)
\[ z(4) = \text{areab} \times \text{resp} \times y(1) \]
\[ \text{erdot} = z(4) \]

c ENERGY LOSS DUE TO RECOIL
\[ \text{elrc} = \text{rcwt} \times y(6)^2 / y(6) \]
\[ \text{erdot} = \text{rcwt} \times y(6) \times z(6) \]

c ENERGY LOSS DUE TO HEAT LOSS
\[ \text{areaw} = \text{cham} / \text{areab} \times \pi \times \text{bore} + 2 \times \text{areab} \times \pi \times \text{bore} \times (y(2) + y(7)) \]
\[ \text{avden} = 0.0 \]
\[ \text{avc} = 0.0 \]
\[ \text{avcp} = 0.0 \]
\[ z18 = 0 \]
\[ z19 = 0 \]

do 213 k = 1, nprop
\[ z18 = \text{forcp}(k) \times \text{gamap}(k) \times \text{chwp}(k) \times \text{frac}(k) / ((\text{gamap}(k) - 1) / \text{tempp}(k) + z18 \]
\[ z19 = \text{chwp}(k) \times \text{frac}(k) + z19 \]
\[ \text{avden} = \text{avden} + \text{chwp}(k) \times \text{frac}(k) \]
\[ continue \]
\[ \text{avcp} = (z18 + \text{forcp}(k) \times \text{gamap}(k) \times \text{chwp}(k) \times \text{frac}(k) / ((\text{gamap}(k) - 1) / \text{tempp}(k) + z19 + \text{chwi}) \]
\[ \text{avden} = (\text{avden} + \text{chwi}) / (\text{volg} + \text{covl}) \]
\[ \text{avvel} = 0.5 \times y(1) \]
\[ \text{htns} = \lambda \ast \text{avcp} \ast \text{avden} \ast \text{avvel} + \text{ho} \]
\[ z(5) = \text{areaw} \ast \text{htns} \ast (t_{\text{gas}} - w_{\text{wall}}) \ast h_{\text{l}} \]
\[ \text{elht} = y(5) \]
\[ \text{ehdot} = z(5) \]
\[ \text{wallt} = (\text{elht} + \text{hfr} + \text{elbr}) / \text{cshl} / \text{rhocs} / \text{areaw} / t_{\text{shl}} + \text{twal} \]

c WRITE(6, *) \lambda \ast \text{avcp}, \text{avden}, \text{avvel}, \text{ho}, \text{areaw}, \text{htns}, \text{tgas}, \text{wallt}, h_{\text{l}}, z(5)

c & elht

c ENERGY LOSS DUE TO AIR RESISTANCE
\[ \text{air} = \text{iair} \]
\[ z(8) = y(1) \ast \text{pgas} \ast \text{air} \]
\[ \text{clar} = \text{areab} \ast y(8) \]
\[ \text{eddot} = z(8) \ast \text{areab} \]

c RECOIL
\[ z(6) = 0.0 \]
if (pbrch.le.rp(1)/areab) go to 221
\[ \text{rfor} = \text{rp}(2) \]
if (y(3)-tr0.ge.tr(2)) go to 222
\[ \text{rfor} = (\text{rp}(2)-(y(3)-tr0)) / (\text{tr}(2)-\text{tr}(1)) \]
\[ \text{rfor} = \text{rp}(2) - \text{rfor} \ast \text{rp}(2) \ast \text{rp}(1) \]
\[ 222 \]
\[ z(6) = \text{areaw} / \text{rcwt} \ast (pbrch - \text{rfor} / \text{areab} - \text{resp}) \]
if (y(6).lt.0.0)y(6) = 0.0
\[ z(7) = y(6) \]
\[ goto 223 \]
221 \[ tr0 = y(3) \]
223 \[ continue \]

c CALCULATE GAS TEMPERATURE
\[ \text{eprop} = 0.0 \]
\[ \text{rprop} = 0.0 \]
\[ \text{dmfogt} = 0.0 \]
\[ \text{dmfog} = 0.0 \]

do 231 k = 1, nprop
\[ \text{eprop} = \text{eprop} + \text{forcp}(k) \ast \text{chwp}(k) \ast \text{frac}(k) / ((\text{gamap}(k) - 1) / \text{tempp}(k) + \text{rprop}) \ast \text{forcp}(k) \ast \text{chwp}(k) \ast \text{frac}(k) / ((\text{gamap}(k) - 1) / \text{tempp}(k) + \text{rfor} \ast \text{rprop} \ast k) \]
\]
& ((gamap(k)-1.)*tempp(k))
dmfog=dmfog+forcp(k)*rhop(k)*tng(k)*surf(k)*z(ibrp+k)/
& (gamap(k)-1.)
231  continue
tenerg=elpt+elpr+elgpm+elbr+elrc+elht+elar
tgas=(eprop+forcig*chwi/(gamai-1.)-elpt-elpr-elgpm-elbr-elrc-
elht)/((rprop+forcig*chwil/(gamai-1.)/tempi)
tedot=epdot+rephdot+edddot+crddot+ehddot+ehgdot
digas=(dmfog-tedot-tgas*dmfot)/(rprop+forcig*chwil/
& (gamai-1.)/tempi)
c  FIND FREE VOLUME
v1=0.0
cov1=0.0
do 241 k=1,nprop
v1=chwp(k)*(1.-frac(k))/rhop(k)
cov1=cov1+chwp(k)*covp(k)*frac(k)
241  continue
volg=volgi+areab*(y(2)+y(7))-v1-cov1
c  CALCULATE MEAN PRESSURE
r1=0.0
do 251 k=1,nprop
r1=r1+forcp(k)*chwp(k)*frac(k)/tempp(k)
251  continue
pmean=tgas/volg*(r1+forcig*chwi/tempi)
259  resp=resp+pgas*air
if(igrad.eq.1)go to 252
if(igrad.eq.2)goto 403
if(igrad.eq.3)go to 404
if(igrad.eq.4)go to 441
403  if(iswl.ne.0)go to 253
pbase=pmean
pbrch=pmean
if(pbase.gt.resp+1.)iswl=1
go to 257
c  USE CHAMBRAGE PRESSURE GRADIENT EQUATION
253  j1zp=bint(1)+(bvol*pt+areab/2.*pt*pt)/areab
j2zp=(bvol+areab*pt)**2/areab/areab
j3zp=bint(3)+areab*bint(1)*pt+bvol*pt*pt/2.+areab*pt*pt/6.
a2t=tmpl*areab*areao/prwt/vzp/vzp
alf=1.-a2t*j1zp
al1=tmpl*areab*(areab*y(1)*y(1)/vzp+areab*resp/prwt)/vzp/vzp
bt=tmpl*y(1)*y(1)*areab/2./vzp/vzp/vzp
bata=-a1t*j1zp-bt*j2zp
gamma=alf+a2t*j3zp/vzp
delta=bata+a1t*j3zp/vzp+bt*j4zp/vzp
c  calculate base pressure
pbase=(pmean+delta)/gamma
  calculate breech pressure
pbrch=alf*pbase+bata
go to 254
c  USE 2 PHASE GRADIENT EQUATION
404  IF(ISW1.NE.0)GOTO 407
pbase=pmean
pbrch=pmean
103
if(pbase.gt.resp+1)isw1=1
    go to 257
407 if(isw1.eq.2)go to 411
    vzp=cham+areab*(y(2)+y(7))
    vzb=cham+areab*(y(10)+y(7))
    phi=0.
    phidot=0.
    dmorho=0.
    dmcov=0.
    dmromw=0.
    rmomw=0.
    vfree=vzp-v1
    do 405 k=1,nprop
       rmomw=rmomw+chwp(k)*frac(k)*forcp(k)/temp(k)
       phi=chwp(k)*frac(k)+phi
    if(ibo(k).eq.1)go to 405
    dmorho=dmorho+tng(k)*surf(k)*z(ibrp+k)
    phidot=phop(k)*tng(k)*surf(k)*z(ibrp+k)+phidot
    dmcov=phop(k)*tng(k)*surf(k)*z(ibrp+k)*covp(k)+dmcov
    dmromw=dmromw+phop(k)*tng(k)*surf(k)*z(ibrp+k)*
       &forcp(k)/temp(k)
405 continue
    rmomw=rmomw+chwi*forcigt*temp
    gasmas=phi+chwi
    gasden=gasmas/vfree
    phi=(phi+chwi)/tmPi
    if (phi.gt.0.999) then
       iswl=2
       rbm=pbase/pmean
       rm=pbrcb/pmean
       if(phil.0.1)go to 411
       endif
    dmdt=phidot
    phidot=phidot/tmpi
    vdotov=(dmorho+areab*y(1))/vfree
    dlnrho=dmdt/gasmas-vdotov
    dvolvt=dmdt/gasmas-vdotov
    effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
    c GET TIME DERIVATIVE OF MEAN PRESSURE
    dpmdt=(dmromw*tgas-pmean*dvolvt+dtgas*rmomw)/volg
    volprp=0.
    effdia=0.
    dmdmdt=0.
    dmdmor=0.
    avelen=0.
    avedia=0.
    do 406 k=1,nprop
    if(ibo(k).eq.1)go to 406
    volprp=volprp+(1.-frac(k))*chwp(k)/rhop(k)
    dmdmdt=dmdmdt+rhop(k)*tng(k)*dsdx(k)*z(ibrp+k)*z(ibrp+k)
    dmdmdt=dmdmdt+rhop(k)*tng(k)*surf(k)*d2xdt2(k)
    dmdmor=dmdmor+(dsdx(k)*z(ibrp+k)**2+surf(k)*d2xdt2(k))*tng(k)
    effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
406 continue
    c1t=dmdmdt/gasmas-dmdmor/vfree+vdotov**2-(dmdt/gasmas)**2
    104
\[ \text{d}2 \ln r = c1 \text{t-areab}^{*2} \text{pbase/vfree/prwt} \]
\[ \text{d}2 \ln r = d2 \ln r + \text{areab}^{*2} \text{resp/vfree/prwt} \]
\[ \text{zp} = \text{chlmen} + y(2) + y(7) \]
\[ \text{zb} = \text{chlmen} + y(10) + y(7) \]
\[ \text{ullen} = \text{zp} - \text{zb} \]
\[ \text{cnow} = \text{tmpi-gasmas} \]
\[ \text{vp} = y(1) \]
\[ \text{effdia} = \text{effdia/cnow} \]
\[ \text{prden} = \text{cnow/volprp} \]
\[ \text{up} = y(9) \]
\[ \text{phistr} = \text{phi-gasden*areab*ullen/tmpi} \]
\[ \text{ulldot} = \text{vp-up} \]
\[ \text{dphist} = \text{phidot-gasden*areab/tmpi*(ulldot+ullen*dlnrho)} \]
\[ \text{eps} = 1. - (1.-\text{phi})/\text{tmpi/prden/vzb} \]
\[ \text{epsdot} = \text{phidot/tmpi/prden/vzb+}(1.-\text{phi})/\text{tmpi*up*areab/} \]
\[ \text{prden/vzb/vzb} \]
\[ \text{ug} = \text{up} + (\text{vp+ullen*dlnrho-up)/eps} \]
\[ \text{alam} = (1.5*\text{grlen/grdiam})*.666666667 \]
\[ \text{alam} = (0.5+\text{grlen/grdiam})/\text{alam} \]
\[ \text{alam} = \text{alam}^{*2.17} \]
\[ \text{c} \]
\[ \text{VIS kg/s/m} \]
\[ \text{vis} = .00007 \]
\[ \text{ren} = \text{gasden/vis*effdia*abs(ug-up)} \]
\[ \text{if}(\text{ren}.lt.1.).text{ren}.=1. \]
\[ \text{fsrg} = 2.5*alam/ren^{*0.081}((1.-\text{eps})/(1.-\text{eps0})/\text{eps0/eps})^{*0.45} \]
\[ \text{fsc} = \text{fsrg}^{*fs0} \]
\[ \text{phi1} = 1.-\text{phi-phistr*(1.-eps)/eps} \]
\[ \text{phi2} = \text{dphist}^{*ug-phidot*up-phistr*epsdot/eps/eps} \]
\[ \text{& *(vp+ullen*dlnrho-up)+phistr*ulldot*dlnrho/eps} \]
\[ \text{& +2.*phistr*ug/vzb*(ug-up)} \]
\[ \text{phi1p} = \text{phi1p+phi2*gasden/effdia/prden*(ug-up)**2*fsc} \]
\[ \text{ak2} = 1./(1.-\text{phi2*tmpi/prden/vzb}) \]
\[ \text{phi1} = \text{phi1p+phistr*z(1)/eps+ullen*phistr*d2lnr/eps} \]
\[ \text{c} \]
\[ \text{ACCELERATION OF FORWARD BOUNDARY OF PROPELLANT BED} \]
\[ z(9) = \text{gasden*(ug-up)**2*fsc/prden/effdia+tmpi*phi1*ak2} \]
\[ \&/vzb/prden \]
\[ z(10) = y(9) \]
\[ e = \text{phistr/eps*(1.-ullen*areab/vfree)*areab/prwt} \]
\[ dd = \text{ullen*phistr*c1lt/eps} \]
\[ \text{ak11} = \text{tmpi}^{*e*ak2/zb/vzb} \]
\[ \text{ak12} = \text{tmpi*ak2*(phi1p+dd)/zb/vzb-ak11*resp} \]
\[ \text{pbase} = \text{pmean-ak12*zb/2.+gasden*ullen*resp*areab/prwt} \]
\[ \text{pbase} = \text{pbase+ak12*zb*zb*(zb/3.+ullen)/2./zp} \]
\[ \text{pbase} = \text{pbase-gasden*ullen**2*areab/resp/2./zp/prwt} \]
\[ \text{pbase} = \text{pbase-gasden*ullen**2/2.*(1.-2.*ullen/3./zp)*} \]
\[ \&(c1t-dlnrho**2) \]
\[ \text{pbase} = \text{pbase-areab**2*gasden*ullen**2*} \]
\[ &(1.-2.*ullen/3./zp)*/resp/prwt/vfree/2. \]
\[ \text{deno} = \text{ak11*zb**3/6./zp-ullen*ak11*zb*zb/2./zp} \]
\[ \text{deno} = \text{deno+gasden*ullen*areab/prwt-areab**2*gasden*ullen**2} \]
\[ &(1.-2.*ullen/3./zp)/2./vfree/prwt \]
\[ \text{deno} = \text{deno-gasden*ullen**2*areab/2./zp/prwt+1.+ak11*zb*zb/2.} \]
\[ \text{pbase} = \text{pbase/deno} \]
\[ \text{if}(\text{ism}.eq.0)\text{goto453} \]
if(ism.eq.1)goto451
goto452

451

ism=2
tss=sqrt(egama*momw/gasmas*tgas)
write(6,*),tss

tss=ullen/(ullen*odlnr+tss)
tso=y(3)
write(6,*),tss,tso

452
coeffbp=(tss+tso-y(3)-deltat)/tss
if(coeffbp.gt.1.)coeffbp=1.
if(coeffbp.le.0.)then
coeffbp=0.
ism=0
endif

pbase=coeffbp*opbase+(1.-coeffbp)*pbase
write(6,*),coeffbp,opbase,pbase,ism

453

odlnr=dlrho
opbase=pbase

pbrch=pbase*(1.+akl11*zb*zb/2.+gasden*ullen*areab/prwt
&-areab**2*gasden*ullen**2/2./(free/prwt)
pbrch=pbrch+akl2*zb*zb/2.-gasden*ullen*areab*resp/prwt
pbrch=pbrch+gasden*ullen**2/2.*(cit-dlnrho**2)
pbrch=pbrch+areab**2*gasden*ullen**2*resp/(free/prwt

C USING RGA GRADIENT

441

if(iswl.ne.0)go to 444

pbase=pmean
pbrch=pmean
if(pbase.gt.resp+1.)iswl=1
go to 257

444

if(iswl.eq.2)go to 411

vz=cham+areab*(y(2)+y(7))
vzb=cham1+areab*(y(10)+y(7))

j1zb=bint(1)+(bvol*pb+areab2.*pb*pb)/areab
j2zb=(bvol+areab*pb)**2/areab
j3zb=bint(3)+areab*bint(1)*pb+bvol*pb*pb/2.+areab/6.*pb**2

phi=0.
phidot=0.
dmorho=0.
dmcorw=0.
dmomw=0.

vfree=vz-v

do 442 k=1,nprop

rmomw=rmomw+chwp(k)*frac(k)*forcp(k)/tempp(k)
phi=chwp(k)*frac(k)+phi
iif(it(k),eq.1)go to 442

dmorho=dmorho+surf(k)*z(ibrp+k)
phidot=hop(k)*surf(k)*z(ibrp+k)+phidot

dmcorw=hop(k)*surf(k)*z(ibrp+k)*covp(k)+dmcorw

dmomw=dmomw+hop(k)*surf(k)*z(ibrp+k)*
& forcp(k)/tempp(k)

442 continue

rmomw=rmomw+chwi*forcig/tempi

106
gasmas=phi+chwi
gasden=gasmas/vfree
phi=(phi+chwi)/tmpi
if (phi.gt.0.99) then
  isw1=2
rbm=pbase/pmean
rbm=pbrch/pmean
endif

if (phi.ge.1.) go to 411
endif
dmdt=phidot
dmdt=phidot/tmpi
vdotov=(dmmor+areab*y(1))/vfree
dlnrho=dmdt/gasmas-vdotov
dvoldt=dmmor+areab*y(1)-dmcov
c get time derivative of mean pressure
dpmdt=(dmmor+areab*rhoc)*dmcov/vfree/volg
c get time derivative of mean pressure
dpmdt=(dmmor+areab*rhoc)*dmcov/vfree/volg
volprp=0.
effdia=0.
dmdm=0.
dmdm=0.
avlen=0.
avledia=0.
do 443 k=1,nprop
  if(ibo(k),eq.1) go to 443
  volprp=volprp+(1.-frac(k))*chwp(k)/rho(k)
  dmdm=dmdm+dmm+rho(k)*tn(k)*dsdx(k)*z(k)*metal(k)+
  dmdm=dmdm+dmm+rho(k)*tn(k)*surf(k)*d2x2(k)+
  dmdm=dmdm+dmm+rho(k)*tn(k)*surf(k)*d2x2(k)+
  effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
443 continue
c1t=dmdm/gasmas-dmdm=volprp**2-(dmdt/gasmas)**2
d21n=1.-areab**2*pbase/vfree/prwt
d21n=d21n+areab*areab*resp/vfree/prwt
zp=chmlen+y(2)+y(7)
zb=chmlen+y(10)+y(7)
ullen=zp-zb
cnow=tmpi-gasmas
vp=y(1)
effdia=effdia/cnow
prden=cnow/volprp
up=y(9)
phistr=phi-gasden*areab*ullen/tmpi
ulldot=vp-up
phistr=phi-gasden*areab*ullen/tmpi
ulldot=vp-up
eps=1.-(1.-phi)*tmpi/prden/vzb
epsdot=phi*tmpi/prden/vzb+(1.0-phi)*tmpi*up*areab/
  & prden/vzb/vzb
ug=up+(vp+ullen*dlnrho-up)/eps
alam=(1.5*grelen/gradiam)**.666666667
alam=(0.5+grelen/gradiam)/alam
alam=alam**2.17
c VIS kg/s/m
vis=.00007
ren=gasden/vis*effdia*abs(ug-up)
if(ren.lt.1.) ren=1.
fsrg=2.5*alam/ren**.081*((1.-eps)/(1.-eps0)*eps0/eps)**.45
fsc=fsrg*fs0
phi2=1.-phi-phiistr*(1.-eps)/eps
phi1p=dpheistr*ug-phi1p*epsdot/eps/eps
& *(v+p+*u1nih*up+*u1do*du*nho/eps)
& +2.*area*phiistr*ug/vzb*(ug-up)
phi1=phi1p+phi2*gasden/effdia/prden*(ug-up)**2*fsc
ak2=1./(1.-phi2*tmpi/prden/vzb)
phi1=phi1p+phiistr*z(1)/eps+ullen*phiistr*d2lnr/eps

**ACCELERATION OF FORWARD BOUNDARY OF PROPELLANT BED**

```
if(iswl1.ne.0) go to 256
if(pmean.gt.resp) resp=pmean
```

c USE LAGRANGE PRESSURE GRADIENT EQUATION

```
254 z(1)=areab*(pbase-resp)/prwt
if(z(1).lt.0.0) go to 257
```

257 if(iswl1.eq.0) z(1)=0.0
258 if(y(1).lt.0.0) y(1)=0.0
z(2)=y(1)

411 phosph=rbm*pmean
pbrch=rbm*pmean
```
c GET BURNING RATE

do 264 m=1,nprop
z(ibrp+m)=0.0
d2xd2(m)=0.0
if(ibo(m).eq.1) goto 264

do 262 k=1,nbr(m)
if(pmean.gt.pres(m,k))go to 262

262 continue
k=nbr(m)

263 pmix=pmean
if(igrad.eq.3)pmix=pbrch-(ak11*pbase+ak12)/6.*zb*zb
if(igrad.eq.4)pmix=pbrch+(a1t+a2t*pbase)*j3zb/vzb+bt*j4zb/vzb
if(pmix.lt.0.99*pmean)pmix=pmean
z(ibrp+m)=beta(m,k)*(pmix*1.e-6)**alpha(m,k)
abr(m)=alpha(m,k)
bbr(m)=beta(m,k)
d2xd2(m)=beta(m,k)*alpha(m,k)*(pmix*1.e-6)**(alpha(m,k)-1.)*dpmdt*1.e-6

264 continue

do 21 i=1,nde
y(i)=deltat*d(i)+y(i)
21 continue

11 continue
t=t+deltat
told=y(3)
if(pmaxm.gt.pmean)go to 281
pmaxm=pmean
tpmaxm=y(3)

281 if(pmaxba.gt.pbase)go to 282
pmaxba=pbase
tpmaxba=y(3)

282 if(pmaxbr.gt.pbrch)go to 283
pmaxbr=pbrch
tpmaxbr=y(3)

283 continue
if(y(3).lt.ptime)go to 272
ptime=ptime+deltap
pjt=y(2)+y(7)
write(6,7)y(3),z(1),y(1),pjt,pmean,pbase,pbrch
7 format(1x,7e11.4)
if(igrad.gt.2)then
pjt=y(2)+y(7)
prt=y(10)+y(7)
write(6,427)prt,pjt
427 format(1x,'prop travel',e11.4,'proj travel',e11.4)
endif

272 continue
if(t.gt.tstop)goto 200
if(y(2)+y(7),gt.travp)go to 200
rmvelo=y(1)
tmvelo=y(3)

109
disto = y(2) + y(7)
go to 19
write(6,311), y(3)
format(1x, 'deltat t', e14.6, ' intg t', e14.6)
write(6,312), pmaxm, tpmxm
format(1x, 'PMAIMENMEAN Pa ', e14.6, ' time at PMAIMENMEAN sec ', e14.6)
write(6,313), pmaxba, tpmxba
format(1x, 'PMAIMBASE Pa ', e14.6, ' time at PMAIMBASE sec ', e14.6)
write(6,314), pmaxbr, tpmxbr
if(y(2) + y(7) .le. travp) go to 303
dfract = (travp - disto) / (y(2) + y(7) - disto)
rmvel = (y(1) - rmvelo) * dfract + rminvelo
tmvel = (y(3) - tmvelo) * dfract + tmvelo
write(6,318), rmvel, tmvel
format(1x, 'muzzle VELOCITY m/s ', e14.6, ' time of muzzle velocity sec ', e14.6)
go to 319
write(6,327), y(1), y(3)
format(1x, 'velocity of projectile m/s ', e14.6, ' at this time msec ', e14.6)
efi = chwi * forci / (gamai - 1.)
efp = 0.0
do 315 i = 1, nprop
    efp = efp + chtwp(i) * fopr(i) / (gamap(i) - 1.0)
continue
tenerg = efi + efp
write(6,317), tenerg
format(1x, 'total initial energy available J = ', e14.6)
tengas = chwi * forci * gas / (gamai - 1.) / tempi
do 135 i = 1, nprop
    tengas = (frac(i) * chwp(i) * forcp(i) * gas / tempp(i) / (gamap(i) - 1.)) + tengas
as
write(6,328), i, frac(i), tbo(i)
format(' FOR PROPELLANT ', i2, ' MASSFRACT BURNT IS ', e14.6 & ', at time in sec ', e14.6)
continue
write(6,136), tengas
format(1x, 'total energy remaining in gas J = ', e14.6)
write(6,320), elpt
format(1x, 'energy loss from projectile translation J = ', e14.6)
write(6,321), elpt
format(1x, 'energy loss from projectile rotation J = ', e14.6)
write(6,322), elgpm
format(1x, 'energy lost to gas and propellant motion J = ', e14.6)
write(6,323), elpr
format(1x, 'energy lost to bore resistance J = ', e14.6)
write(6,324), elbr
format(1x, 'energy lost to recoil J = ', e14.6)
write(6,325), elrc
format(1x, 'energy lost to heat transfer J = ', e14.6)
write(6,326), elht
format(1x, 'energy lost to air resistance J = ', e14.6)
stop
20 write(*,140)
140 format(1x,'end of file encounter')
stop
30 write(*,150)
999 continue
998 continue
150 format(1x,'read or write error')
stop
end

subroutine prf110(pd,gd,gl,np,x,frac,surf,dx)
common nsl,kpr,fracsl(10),dxsls(10),surfls(10),
& nslp(10),tsl(10),pbrch,pbase,pmean,br(10),ab(10),
& depart,yar(20) grad
dimension ts(10),coef(10)
pi=3.141593
nsl=0

C pd=perforation diameter
C gd=OUTER DIA
C gl=GRAIN LENGTH
C NP=NUMBER OF PERFS
C SURF=OUTPUT SURFACE AREA
C frac=OUTPUT MASS FRACTION OF PROPELLANT BURNED
C w = web = distance between perforation edges
C = distance between outside perf edge and edge of grain
C p = distance between perforation centers
C x1 = distance to inner sliver burnout
C x2 = distance to outer sliver burnout (frac=1)

if(np.eq.0) go to 2000
if(np.eq.1) go to 3000
if(np.eq.7) go to 61
if(np.eq.19) go to 4000
if(np.eq.15) go to 5000
60 WRITE(6,90)
90 FORMAT(1X,'UNACCEPTABLE GRANULATION')
STOP
61 w=(gd-3.*pd)/4.
d=w+pd
sqr3=sqrt(3.)
x1=d/sqr3-pd/2.
x2=(14.-3.*sqr3)*d/13.-pd/2.
v0=pi/4.*gl*(gd-gd-7.*pd*pd)
s0=2.*v0/gl+pi*gl*(gd+7.*pd)
if (x.gt.w/2.+0.0000001) goto 20
vol=pi/4.*(gl-2.*x)**2.7.*(pd+2.*x)**2.
surf=2.*vol/(gl-2.*x)+pi*(gl-2.*x)*(gd-2.*x)+
7.*(pd+2.*x))
frac=1.-vol/v0

111
\[
\text{dsdx} = -4\pi (gd + 7 \cdot pd - 3 \cdot gl + 18 \cdot x)
\]
\[
\text{dsdxsl}(kpr) = \text{dsdx}
\]
\[
\text{fracsl}(kpr) = \text{frac}
\]
\[
\text{surfsl}(kpr) = \text{surf}
\]
return

20
\[
\text{nsl} = 1
\]
\[
\text{coef}(kpr) = 0.
\]
if(igrad.eq.1 or igrad.eq.2) go to 726
if(nslp(kpr).eq.1) go to 26
\[
\text{tsl}(kpr) = \text{yar}(3)
\]
\[
\text{ts}(kpr) = \text{w2}/(-1.+(pbch/pmean)**\text{ab}(kpr))/\text{bbr}(kpr)^{*(\text{pbase}^{*1.e-6}**\text{ab}(kpr))}
\]
continue
\[
\text{coef}(kpr) = (\text{tsl}(kpr)+\text{ts}(kpr)-\text{deltat}+\text{yar}(3))/\text{ts}(kpr)
\]
if(coef(kpr).gt.1) coef(kpr) = 1.
if(coef(kpr).lt.0) coef(kpr) = 0.
726
if(x.ge.x2) go to 30
\[
s1 = 0.
\]
\[
s2 = 0.
\]
\[
v1 = 0.
\]
\[
v2 = 0.
\]
\[
ds1dx = 0.
\]
\[
ds2dx = 0.
\]
\[
y = \sqrt{(pd+2.\cdot x)**2-d*d}
\]
\[
\text{theta} = \text{atan}(y/d)
\]
\[
a1 = \text{theta}/4.\cdot(pd+2.*x)**2-2*d/4.*y
\]
if(x.ge.x1) go to 25
\[
v1 = 3/4.\cdot(gl-2.*x)
\]
\[
v1 = v1*(2.*sqr3*d*d-pi*(pd+2.*x)**2+24.*a1)
\]
\[
s1 = 2.*v1/(gl-2.*x)
\]
\[
s1 = s1+3.*(gl-2.*x)*(pi-6.*theta)*(pd+2.*x)
\]
25
\[
y1 = \sqrt{(gd-2.*x)**2-(5.*d-2.*pd)**2-24.*a1)}
\]
\[
\text{chi} = \text{atan}(y1/(5.\cdot d-2.*pd+2.*x)))
\]
\[
y2 = \sqrt{(pd+2.*x)**2-(3.*d-2.*pd+2.*x))**2}
\]
\[
\text{phi} = \text{atan}(y2/(3.\cdot d-2.*pd+2.*x))
\]
\[
a2 = \text{phi}*(pd+2.*x)**2-\text{chi}*(gd-2.*x)**2
\]
\[
a2 = (a2+2.*sqr3*d*d-sqr((3.*d-pd-2.*x)**2))/(pd+2.*x))**2
\]
\[
v2 = 2*pi*(gd-2.*x)**2*(3.*d-pd-2.*x)**2
\]
\[
v2 = (v2+24.*a1+2.*a2)**(gl-2.*x)/4.
\]
\[
s2 = 2.*v2/(gl-2.*x)
\]
\[
s2 = s2+(gl-2.*x)*(pi-6.*chi)*(gd-2.*x)+2.*(2.*pi-3.*phi-3.*theta)
\]
& *(pd+2.*x))
\[
\text{vol} = v1+v2
\]
\[
\text{surf} = s1+s2
\]
\[
\text{frac} = 1.-\text{vol}/\text{vo}
\]
\[
\text{dsdx} = \text{surf}/(x2-x)
\]
\[
\text{dsdx} = \text{coef}(kpr)*\text{dsdxsl}(kpr)+(1.-\text{coef}(kpr))*\text{dsdx}
\]
\[
\text{dsdxsl}(kpr) = \text{dsdx}
\]
\[
\text{frac} = \text{coef}(kpr)*\text{fracsl}(kpr)+(1.-\text{coef}(kpr))*\text{frac}
\]
\[
\text{fracsl}(kpr) = \text{frac}
\]
\[
\text{surf} = \text{coef}(kpr)*\text{surfsl}(kpr)+(1.-\text{coef}(kpr))*\text{surf}
\]
\[
\text{surfsl}(kpr) = \text{surf}
\]
return
30
\[
\text{vol} = 0.
\]
surf=0.
frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
fracsl(kpr)=frac
if(frac.gt.9999) frac=1.
if(frac.gt.9999) return

dsdx=0.
dsdx=dsdxsl(kpr)*coef(kpr)
dsdxsl(kpr)=dsdx
if(abs(dsdx).lt.1.)dsdx=0.
surf=surfsl(kpr)*coef(kpr)
surfsl(kpr)=surf
return

C
C ZERO PERF CALCULATIONS START HERE.
C
2000 if(gd-2.*x.le.0.0) go to 2001
vO=pi*gd*gd/4.*gl
vol=pi*(gd-2.*x)**2/4.**(gl-2.*x)
frac=1.-vol/vO
surf=pi/2.*((gd-2.*x)**2+pi*(gd-2.*x)*(gl-2.*x))
dsdx=2.*pi*(gd+gl-6.*x)
return
2001 surf=0.
frac=1.0
vol=0.
dsdx=0.
nsl=1
return

C C one perf calculation starts here C
C
3000 if(gd-pd-4.*x.le.0.0) goto 3001
vO=pi/4.**(gd*gd-pd*pd)*gl
vol=pi/4.**(gd-2.*x)**2-(pd+2.*x)**2)*(gl-2.*x)
frac=1.-vol/vO
surf=pi/2.*((gd-2.*x)**2-(pd+2.*x)**2)
surf=surf+pi*(gd-2.*x)*(gl-2.*x)
surf=surf+pi*(pd+2.*x)*(gl-2.*x)
dsdx=4.*pi*(gd+pd)
return
3001 surf=0.
frac=1.0
vol=0.
dsdx=0.
nsl=1
return

C C Below is the calculation for the cylindrical 19 perf grain. C
C
C INPUT
C
C P = PERF DIAMETER
C D = GRAIN DIAMETER
C GL = GRAIN LENGTH
X = DISTANCE BURNT

OUTPUT

VOL = THE VOLUME OF ONE GRAIN AT X.

SURF = THE SURFACE AREA OF ONE GRAIN AT X.

FRAC = THE FRACTION OF GRAIN BURNT AT X.

W = WEB

4000 p=pd
d=gd
W=(D-5.*P)/6.
PI=3.141592654
SQRT3=SQRT(3.)
SQRT5=SQRT(5.)
SQRT6=SQRT(6.)
SQRT10=SQRT(10.)

INITIAL VOLUME AND SURFACE AREA

V0=PI/4.*GL*(D*D-19.*P*P)
S0=2.*V0/GL+PI*GL*(D+19.*P)

X1 = DISTANCE TO INNER SLIVER BURNOUT

X2 = DISTANCE TO OUTER SLIVER BURNOUT

DBC = DISTANCE BETWEEN PERFORATION CENTERS

ASSUMES BURNOUT DOES NOT OCCUR IN LONGITUDINAL DIRECTION

W1 = SECONDARY WEB

DBC=W+P
W1=0.5*(D-P-2.*SQRT3*DBC)
X1=DBC/SQRT3-P/2.
X2=0.25*(DBC*(6.-SQRT10)-2.*P)
IF(X.GT.W/2.)GO TO 110

NOT SLIVERED YET

VOL=PI/4.-(GL-2.*X)*((D-2.*X)**2-19.*(P+2.*X)**2)
SURF=2.*VOL/(GL-2.*X)+PI*(GL-2.*X)*(D-2.*X+19.*(P+2.*X))

ddsx=pi*(-4*D+36*GL-76*P-216*x)
FRAC=I.-VOL/V0
ddsxsl(kpr)=dsdx
fracsl(kpr)=frac
surfsl(kpr)=surf
RETURN

V1=TOTAL VOLUME OF INNER SLIVER, V2=TOTAL VOLUME OF OUTER SLIVER

S1=TOTAL SURFACE AREA OF INNER SLIVERS, S2=TOTAL SURFACE AREA OF OUTER SLIVERS

110 V1=0.
V2=0.
S1=0.
S2=0.
DELTA=0.
CHI=0.
NSL=1
coef(kpr)=0.
if(igrad.eq.1.or.igrad.eq.2)go to 727
if(nslp(kpr).eq. 1)goto 728
tsl(kpr)=yar(3)
ts(kpr)=w/2.*(1.+((brch/pmean)**abr(kpr))/
& (bbr(kpr)**(pbase*1.e-6)**abr(kpr))
728 continue
c coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3))/ts(kpr)
if(coef(kpr).ge.1.)coef(kpr)=1.
if(coef(kpr).lt.0.)coef(kpr)=0.
727 A3=0.
IF(X.GE.X2)GO TO 130
THETA=ACOS(DBC/(P+2.*X))
A1=THETA/4.*(P+2.*X)**2-DBC/(P+2.*X)**2-DBC*DBC)
IF(X.GT.X1)GO TO 120
V1=3.*(GL-2.*X)**2*DBC*DBC/4.*SQRTP(P+2.*X)**2*DBC
SI=2.*V1/GL-2.*X+12.*(GL-2.*X)**2*DBC
A2=.125*(PHI*(P+2.*X)**2-XI*(D-2.*X)**2
& +2.*SQRTP5*DBC*DBC/4.*SQRTP6*DBC3*(D-2.*X)**2)
V2=.25*(GL-2.*X)**2*PI*(P+2.*X)**2-7.*PI*(P+2.*X)**2
& -(2.*SQRTP5*DBC*DBC**2+48.*(A1+A2+A3))
S2=2.*V2/(GL-2.*X)+((D-2.*X)**2*(PI-6.*XI+DELTA))
& +(P+2.*X)**2/(PI-6.*XI+DELTA))
RETURN
A3=0.
IF(X.GE.X2)GO TO 130
THETA=ACOS(DBC/(P+2.*X))
A1=THETA/4.*(P+2.*X)**2-DBC/4.*SQRTP*(P+2.*X)**2-DBC*DBC)
IF(X.GT.X1)GO TO 120
V1=3.*(GL-2.*X)**2*DBC*DBC/4.*SQRTP*(P+2.*X)**2-DBC
SI=2.*V1/GL-2.*X+12.*(GL-2.*X)**2*DBC*DBC/4.*SQRTP
A2=.125*(PHI*(P+2.*X)**2-XI*(D-2.*X)**2
& +2.*SQRTP5*DBC*DBC/4.*SQRTP6*DBC3*(D-2.*X)**2)
V2=.25*(GL-2.*X)**2*PI*(P+2.*X)**2-7.*PI*(P+2.*X)**2
& -(2.*SQRTP5*DBC*DBC**2+48.*(A1+A2+A3))
S2=2.*V2/(GL-2.*X)+((D-2.*X)**2*(PI-6.*XI+DELTA))
& +(P+2.*X)**2/(PI-6.*XI+DELTA))
RETURN
A3=0.
IF(X.GE.X2)GO TO 130
THETA=ACOS(DBC/(P+2.*X))
A1=THETA/4.*(P+2.*X)**2-DBC/4.*SQRTP*(P+2.*X)**2-DBC*DBC)
IF(X.GT.X1)GO TO 120
V1=3.*(GL-2.*X)**2*DBC*DBC/4.*SQRTP*(P+2.*X)**2-DBC
SI=2.*V1/GL-2.*X+12.*(GL-2.*X)**2*DBC*DBC/4.*SQRTP
A2=.125*(PHI*(P+2.*X)**2-XI*(D-2.*X)**2
& +2.*SQRTP5*DBC*DBC/4.*SQRTP6*DBC3*(D-2.*X)**2)
V2=.25*(GL-2.*X)**2*PI*(P+2.*X)**2-7.*PI*(P+2.*X)**2
& -(2.*SQRTP5*DBC*DBC**2+48.*(A1+A2+A3))
S2=2.*V2/(GL-2.*X)+((D-2.*X)**2*(PI-6.*XI+DELTA))
& +(P+2.*X)**2/(PI-6.*XI+DELTA))
RETURN
A3=0.
IF(X.GE.X2)GO TO 130
THETA=ACOS(DBC/(P+2.*X))
A1=THETA/4.*(P+2.*X)**2-DBC/4.*SQRTP*(P+2.*X)**2-DBC*DBC)}
if(abs(dsdx).lt.1.)dsdx=0.
surf=surfsl(kpr)*coeff(kpr)
surfsl(kpr)=surf
RETURN

C
C Below is the calculation for the 19 perf hex grain.
C
C
C Translation of the input values.
C  p= perf diameter
C  d= grain diameter
C  gl= grain length
C  x= distance burnt
C
C Translation of the output values.
C  vol= volume of one grain at x.
C  surf= surface area of one grain at x.
C  frac= mass fraction of the grain burnt at x.
C
C Assignment statement for pi.
5000  pi=3.141592654
    sqrt3=sqrt(3.)
p=pd
d=gd

C  d=6w + 5p is the statement for the grain diameter which will be
C used to calculate the web.
C
C To calculate the web.
   w= (d-5.*p)/6.
C
C Below is the equation to calculate the distance between the perf cen-
C ters.
   dpc= p + w
C To calculate the grain diameter between the flats.
   f= 2.*(sqrt3*dpc + p/2. + w)
C
C To calculate the distance burnt.
   X1=dpc/sqrt3-p/2.
   X2= 0.125*(5.0*dpc-4.*p)
C
C To calculate the area.
   A=sqrt3/3.*((w+p/2.)*2)-6.*((w+p/2.)*2)
C To calculate the initial volume of the sharp corner grain.
   Vs=gl/4.*(2.*sqrt3*f**2-19.*pi*p**2)
C
C To calculate the volume that will be removed from the grain.
   Vr=6.*A*gl
C
C To calculate the initial volume for the 19hex grain with rounded
C corners.
   Vo= Vs - Vr
C
C To calculate the initial surface area of the sharp corner grain.
\(S_s = 2.0 V/s + gl* (2.0 \sqrt{3} f + 19.0 \pi p)\)

C To calculate the surface area that will be removed from the grain.

\(S_r = 12.0 A + gl* (w/p + 2.0) * (4.0 \sqrt{3} - 2.0 \pi)\)

C To calculate the initial surface area for the 19hex grain with rounded corners.

\(S_o = S_s - S_r\)

C To calculate the unknows of the grain under the condition \(x \leq 0.5w\).

if \((0.0 \leq x \text{ and } x \leq 0.5w)\) then

\(A = \sqrt{3}/3.0 * (w - 2.0 * x + (p + 2.0 * x)/2.0)^2 - \pi/6.0 *\)

\& \((w - 2.0 * x + (p + 2.0 * x)/2.0)^2\)

C To calculate the volume that will be removed from the sharp corner grain.

\(V_r = 6.0 A * (gl - 2.0 * x)\)

C To calculate the volume for the sharp corner grain at some distance burnt.

\(V_n = 0.25 * (gl - 2.0 * x) * (2.0 * \sqrt{3} * (f - 2.0 * x)^2 - 19.0 \pi * (p + 2.0 * x)^2)\)

C To calculate the volume for the 19hex grain with rounded corners.

\(V = V_n - V_r\)

C To calculate the surface area that will be removed from the sharp corner grain.

\(S_r = 12.0 A + (gl - 2.0 * x) * (w - 2.0 * x + (p + 2.0 * x)/2.0) * (4.0 \sqrt{3} - 2.0 \pi)\)

C To calculate the surface area for the sharp corner grain.

\(S_n = 2.0 V/(gl - 2.0 * x) + (gl - 2.0 * x) * (2.0 \sqrt{3} * (f - 2.0 * x) + 19.0 \pi * (p + 2.0 * x))\)

C To calculate the surface area for 19hex grain with rounded corners.

\(S = S_n - S_r\)

C To calculate the mass fraction.

\(\text{frac} = 1.0 - V/V_o\)

\(\text{dsdx} = 8.0 \sqrt{3} * (f - 2.0 * x) - 76.0 \pi * (p + 2.0 * x) * (-4.0 \sqrt{3} + 38.0) * \pi - 16.0 \sqrt{3} * (w + p/2 - x) - 8.0 \pi * (w + p/2 - x) + (gl - 2.0 * x) * (4.0 \sqrt{3} - 2.0 \pi)\)

\(\text{surf} = S\)

vol = V

dsdxsl(kpr) = dsdx

fracsl(kpr) = frac

surfsl(kpr) = surf

return

dendif

C Due to the cross section at the sliver point \(x = 0.5w\) there will be 24 identical inner slivers, 12 identical side slivers. After slivering the surface area and the volume function become more complex. Each type of sliver will be treated separately and later the volumes will be combined to complete the function.

C To calculate the 12 identical side slivers for the grain \(x = 0.5/w\).

\(\text{nsi} = 1\)

coef(kpr) = 0.

if (igrad.eq.1.or.igrad.eq.2) go to 729
if(nslp(kpr).eq.1)goto 730

tsl(kpr)=yar(3)

ts(kpr)=w/2.*(-1.+(pbrcp/pmean)**ab(kpr))/
    & (bbr(kpr)*((pbase*1.e-6)**ab(kpr))

730 continue

coef(kpr)=(ts(kpr)+tst(kpr)-(deltat+yar(3))/ts(kpr)

if(coef(kpr).gt.1.)coef(kpr)=1.
if(coef(kpr).lt.0.)coef(kpr)=0.

729 if(w/2.lt.x.and.x.lt.XlI.and.x.lt.X2) then

    C To calculate the areas of the grain.
    A=sqrt(3)/2.*(w-2.*x+(p+2.*x)**2)**2-pi/6.*
    &{(w-2.*x)**2-pi/6.*}**2
    theta=acos(dpcl/p+2.*x)
    A1=theta/4.*(p+2.*x)**2-dpc/4.*sqrt((p+2.*x)**2-dpc**2)
    omega=acos(2.*dpc/(p+2.*x)-1.)
    A2=0.125*(p+2.*x)*((p+2.*x)*\(omega+sin(omega))-2.*dpc*sin(omega))

    C To calculate the volumes of the grain.
    V1=3.*(gl.-2.*x)**2-2*pi*(p+2.*x)**2+24.*A1
    V2=6.*(gl.-2.*x)**2-2*pi*(p+2.*x)**2+24.*A2

    C To calculate the surface areas of the grain.
    S1=2.*V1/(gl.-2.*x)+12.*(gl.-2.*x)*\(pi-6.*theta)*(p+2.*x)
    S2=2.*V2/(gl.-2.*x)+12.*(gl.-2.*x)*\(dpc+2.*x)**2

    C To calculate the total volume and total surface area.
    Vf=V1+V2
    Sf=S1+S2

    C To calculate the mass fraction.
    frac=1.-Vf/Vo
    surf=Sa
    dsdx=2*surf/(x2-x)
    vol=Vf
    dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
    dsdxsl(kpr)=dsdx
    frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
    fracsl(kpr)=frac
    surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
    surfsl(kpr)=surf
    return

endif

if(x.gt.X1.and.x.lt.X2)then

    C To calculate the area of the grain.
    A=sqrt(3)/2.*(w-2.*x+(p+2.*x)**2)**2-pi/6.*
    &{(w-2.*x)**2-pi/6.*}**2
    theta=acos(dpcl/p+2.*x)
    A1=theta/4.*(p+2.*x)**2-dpc/4.*sqrt((p+2.*x)**2-dpc**2)
    omega=acos(2.*dpc/(p+2.*x)-1.)
    A2=0.125*(p+2.*x)*((p+2.*x)*\(omega+sin(omega))-2.*dpc*sin(omega))

    C To calculate the volume of the grain.
    V2=6.*(gl.-2.*x)**2-2*pi*(p+2.*x)**2+24.*A1
    &+2.*A1+(4.*A2)

    C To calculate the surface area of the grain.
    S2=2.*V2/(gl.-2.*x)+12.*(gl.-2.*x)*\(dpc+2.*x)**2*(pi/2.-omega

return
&-theta-sin(omega)))
C To calculate the volume and the surface area.
  Vf=V2
  Sf=S2
C To calculate the the mass fraction.
  frac=1-Vf/Vo
  surf=Sf
  dsdx=-surf/(x2-x)
  vol=Vf
  dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
  dsdxsl(kpr)=dsdx
  frac=coef(kpr)*fracs(kpr)+(1.-coef(kpr))*frac
  fracs(kpr)=frac
  surf=coef(kpr)*surfs(kpr)+(1.-coef(kpr))*surf
  surfs(kpr)=surf
  return
endif
if(x.gLX2)then
  dsdx=0.
  surf=0.
  vol=0.
  frac=fracs(kpr)*coef(kpr)+1.-coef(kpr)
  fracs(kpr)=frac
  if(frac.gt..9999) frac=1.
  if(frac.gt..9999)retum
  dsdx=0.
  dsdx=dsdxsl(kpr)*coef(kpr)
  dsdxsl(kpr)=dsdx
  if(abs(dsdx).lt.1.)dsdx=0.
  surf=surfs(kpr)*coef(kpr)
  surfs(kpr)=surf
  return
endif
end
Intentionally left blank.
APPENDIX 6:
INPUT DATA FOR IBRGA TEST
Intentionally left blank.
123
APPENDIX 7:
OUTPUT FROM IBRGA
INTENTIONALLY LEFT BLANK.
the input file is i6315a
using rga gradient

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<th>chamber diameter cm</th>
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</tr>
<tr>
<td>0.541020E+02</td>
<td>0.127000E+02</td>
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| chamber volume cm**3 | 0.982089E+04 |
| groove diam cm       | 0.127000E+02 |
| land diam cm         | 0.127000E+02 |
| groove/land ratio    | 0.100000E+01  |
| twist turns/caliber  | 0.000000E+00  |
| projectile travel cm | 0.457200E+03  |
| gradient #           | 4             |
| friction factor      | 0.100000E+01  |

| projectile mass kg   | 0.979600E+01 |
| switch to calculate energy lost to air resistance J | 0 |
| fraction of work against bore used to heat the tube | 0.000000E+00 |
| gas pressure Pa      | 0.000000E+00 |
| number barrel resistance points | 5 |
| bore resistance MPa - travel cm | |
| mass of recoiling parts kg | 0.100000E+21 |
| number of recoil point pairs | 2 |
| recoil force N       | 0.300000E+05  |
| recoil time sec      | 0.000000E+00  |
| free convective heat transfer coefficient w/cm**2 K | 0.113500E-02 |
| chamber wall thickness cm | 0.114300E-01 |
| heat capacity of steel of chamber wall J/g K | 0.460280E+00 |
| initial temperature of chamber wall K | 0.273000E+03 |
| heat loss coefficient | 0.100000E+01 |
| density of chamber wall steel g/cm**3 | 0.786120E+01 |

| impetus of igniter propellant J/g | 0.106340E+03 |
| covolume of igniter cm**3/g | 0.975500E+00 |
| adiabatic flame temperature of igniter propellant K | 0.294000E+03 |
| initial mass of igniter kg | 0.471200E-02 |
| ratio of specific heats for igniter | 0.140000E+01 |
| there are | 1 |
| impetus of propellant J/g | 0.113584E+04 |
adiabatic temperature of propellant $K \ 0.314275E+04$

covolume of propellant $cm^3/g \ 0.975440E+00$

initial mass of propellant $kg \ 0.979590E+01$

density of propellant $g/cm^3 \ 0.166050E+01$

ratio of specific heats for propellant $0.123000E+01$

number of perforations of propellant $7$

length of propellant grain $cm \ 0.317500E+01$

diameter of perforation in propellant grains $cm \ 0.508000E-01$

outside diameter of propellant grain $cm \ 0.123063E+01$

for propellant 1 the total number of grains is $0.158099E+04$

number of burning rate points $1$

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time increment msec $0.100000E-01$
print increment msec $0.100000E+00$

time to stop calculation msec $0.300000E+02$

area bore $m^2 \ 0.126677E-01$

presure from ign $pa \ 0.127925E+06$

volume of unburnt prop $m^3 \ 0.589937E-02$

init cham vol-cov ign $m^3 \ 0.981629E-02$

time acc vel dis m press pbasc pbrch

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128
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muzzle VELOCITY m/s 0.136949E+04 time of muzzle velocity sec 0.997004E-02

FOR PROPELLANT I MASSFRAC BURNT IS 0.966455E+00 at time in qec
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APPENDIX 8:
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