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A Note on the Statistics of Laser Speckle

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Abstract

The effects of statistical fluctuations of intensity, arising from laser speckle, on eye damage, is examined.

Contents

1 STATISTICAL CONSIDERATIONS OF EYE DAMAGE

In considering eye protection we must take into account not only the average energy or average power density on the retina but also random variation, as noted by Fried (1981). Random variations can arise in at least two ways. Most surfaces, either natural or manmade, are extremely rough on the scale of an optical wavelength. Under illumination by coherent light, a wave reflected from a typical rough surface consists of contributions from many scattering points or areas. Images formed at a given point are the combination of amplitudes of spread functions, each arising from a different scattering point on the surface. The spread functions add with different phases, giving complex interference patterns, which are labeled speckle. The eyes will form a random intensity pattern on the retina: subjective speckle.

In a static, nonturbulent atmosphere, a speckle pattern propagates through the atmosphere unchanged; it is statistically stationary. When turbulence is present, it seems to modulate the brightness of each speckle. Turbulence gives rise to a random, moving patchiness in the atmospheric temperature field and thus in the index of refraction. The combined effect of turbulent eddies is to form a screen, which alters the phases of the waves that combined to form speckles.

It is easy to imagine scenarios in which speckles will form under battlefield conditions. For example, a laser beam entering a helicopter cockpit will undergo scattering off the interior surfaces. Airborne lasers will scatter off the ground, buildings, and trees. In such scenarios, the laser beam may undergo several such scatters—the statistical properties of doubly or multiply scattered laser radiation can be expected to differ from those of ordinary speckle.

1.1 Statistics of Speckle

Classical speckle theory has been well-developed in recent years (Goodman, 1985), but much less work has been done on multiply scattered speckle. If a surface is truly rough on the scale of a wavelength, the field associated with any singly linear polarization must be circular, complex Gaussian. The intensity, I, follows a negative exponential distribution, such that the probability of the intensity lying between I and $I + dI$ is

$$\begin{aligned} p(I|\bar{I}) &= \left(\tfrac{1}{\bar{I}}\right) e^{-\frac{I}{\bar{I}}} & I &\geq 0 \\ &= 0 & I &< 0, \end{aligned}$$

given a mean intensity $\bar{I}$. Since the probability density function is a negative exponential, the fluctuations about the mean are pronounced and the distribution has a long fat tail, as compared with a Gaussian distribution. The long tail implies that high intensities, several times the mean, are not all that improbable.

If we define contrast as the ratio of the standard deviation to the mean, then the contrast of a speckle pattern is unity. Because of its high contrast, speckle is highly disturbing to observers. This is particularly true if fine detail is of interest and speckle results in a significant loss in resolution.

The probability, P_T, of exceeding some threshold intensity, I_T, of damage is

$$P_T = \int_{I_T}^{\infty} \bar{I}^{-1} e^{-\frac{I}{\bar{I}}} dI = exp\left(-\frac{I_T}{\bar{I}}\right).$$

For a given threshold probability P_T and intensity damage threshold I_T, the mean intensity $\bar{I}$ at the retina must be

$$\bar{I} = -\frac{I_T}{\ln P_T}$$

or less, a result discussed by Fried (1981). Thus, if P_T is set at 10^{-6}, $\bar{I}$ should be held to 0.0724 I_T or less, in order to insure that subjective speckle is below threshold at a probability of one in a million. This probability is for a single speckle scene. In a battlefield situation, an individual will be exposed to large numbers of such scenes, as both the laser source and the individual move relative to the rough surface giving rise to the speckle. The probability of 10^{-6} carries into the far reaches of the tail of the distribution. It is assumed that the exponential distribution is still valid in this region.

1.2 Statistics of Scattered Speckle

As a model of speckle scattering, consider a laser beam illuminating a rough surface, which then scatters onto a second surface, which is viewed by an observer. Alternatively, a laser beam could be projected through a ground-glass screen onto a rough object, which is then viewed. The condition for subjective speckle is that the observer is close enough to the laser-illuminated object to resolve clearly the details of the objective speckle pattern projected on the object by the ground-glass screen.

The speckle pattern from the first scattering has a nominal instantaneous power density I' and is a random quantity governed by a negative exponential distribution with mean $\bar{I}$. The subjective speckle on the retina will also have a negative exponential distribution. The overall power density on the retina will then have a probability density p_s, given by

$$p_s(I|\bar{I}) = \int_0^\infty p(I|I')p(I'|\bar{I})dI'$$

or

$$p_s(I|\bar{I}) = \bar{I}^{-1}\int_0^\infty I'^{-1}exp\left[-\left(\frac{I}{I'}+\frac{I'}{\bar{I}}\right)\right]dI'.$$

The integral can be evaluated in a straightforward fashion. Instead we consider a somewhat more general problem, in which the calculation of the conditional probability p_s is a special case. Let X be a random variable with a gamma probability density function

$$p_X(x) = \frac{1}{\Gamma(\nu)}\alpha(\alpha x)^{\nu-1}exp(-\alpha x) \qquad \alpha > 0,$$

with scale factor α and index ν; $\Gamma(\nu)$ is the gamma function. We note that a negative exponential distribution for speckle is a gamma distribution with

$$\alpha = \bar{I}^{-1}, \quad \nu = 1.$$

Now consider a second random variable Y, with a gamma density function index μ and a scale factor β. The product of X and Y, Z,

$$Z = XY,$$

has a probability density function

$$\begin{aligned} p_Z(z) &= \int_0^\infty p_X(x) p_Y\left(\frac{z}{x}\right)\frac{dx}{x} \\ &= \frac{1}{\Gamma(\nu)\Gamma(\mu)}\alpha\beta(\beta z)^{\mu-1}\int_0^\infty (\alpha x)^{\nu-1}e^{-\alpha x}(\beta x)^{-(\mu-1)}e^{-\frac{\beta z}{x}}\frac{dx}{x} \\ &= \frac{1}{\Gamma(\nu)\Gamma(\mu)}\alpha^\nu \beta z^{\mu-1}\int_0^\infty x^{\nu-\mu-1} exp\ -\left(\alpha x + \frac{\beta z}{x}\right) dx. \end{aligned}$$

From the theory of Bessel functions we have the equality

$$K_\lambda(y) = \frac{1}{2}\left(\frac{1}{2}y\right)^\lambda \int_0^\infty \tau^{-(\lambda+1)} exp\left(-\tau - \frac{y^2}{4\tau}\right) d\tau,$$

where $K_\lambda(y)$ is a Bessel function of imaginary argument of the second kind, which is sometimes known as a Basset or MacDonald function (Watson, 1944). The probability $p_Z(z)$ is then

$$p_Z(z) = \frac{2\alpha\beta}{\Gamma(\nu)\Gamma(\mu)}\left[(\alpha z)^{\mu-1}(\alpha\beta z)^{\frac{\nu-\mu}{2}} K_{\mu-\nu}\left[2(\alpha\beta z)^{\frac{1}{2}}\right]\right].$$

Thus, the product of two gamma-distributed random variables gives rise to a distribution involving K functions.

Returning to the problem at hand, because we have a product of negative exponentials, the probability density of the intensity of scattered speckle is

$$p_s(I|\bar{I}) = 2\bar{I}^{-1}K_o\left[2\left(\frac{I}{\bar{I}}\right)^{\frac{1}{2}}\right].$$

The threshold probability for scattered speckle is then

$$P_T = \int_{I_T}^\infty p_s(I|\bar{I})dI = 2\bar{I}^{-1}\int_{I_T}^\infty K_o\left[2\left(\frac{I}{\bar{I}}\right)^{\frac{1}{2}}\right] dI.$$

The integral $\int_{I_T}^\infty K_o(x)dx$ is tabulated function (Abramowitz and Stegun, 1965). For a threshold probability $P_T = 10^{-6}$, the allowable average intensity of ordinary speckle is 0.0724 I_T, while that of scattered speckle is 0.017 I_T. The fatter tail of the K_o distribution lowers the allowable average intensity by a factor of 4.3 for scattered speckle.

1.3 Turbulence Modification of Speckle

For a laser beam propagating through the atmosphere at levels of turbulence below saturation, the probability distribution for intensity tends to follow either a lognormal or a Rice-Nakagami distribution of the form

$$p(x) = \left[\frac{1}{\beta}\, exp\; -\left(\frac{\alpha+\beta}{\beta}\right) x\right] I_o \left[\frac{2(\beta x)^{\frac{1}{2}}}{\alpha}\right],$$

where $I_o(x)$ is a Bessel function of the first kind with imaginary argument (Goodman, 1985). We assume for the purpose of this calculation that the Rice-Nakagami distribution is the appropriate distribution for low levels of turbulence. In turn, the Rice-Nakagami distribution can be very closely approximated by a gamma distribution with index M. The index M of the gamma distribution is related to the parameters α and β of the Rice-Nakagami distribution by

$$M = \frac{(\alpha+\beta)^2}{\beta^2 + 2\alpha\beta},$$

and the normalized moments of the gamma distribution are determined by

$$\frac{\bar{x}_n}{(\bar{x})^n} = \frac{\Gamma(n+M)}{M^n \Gamma(M)}.$$

The probability density function for the turbulence-modulated spectrum is

$$p(I|\bar{I}) = \int_0^\infty p_D(I|I')p_Q(I|\bar{I})dI',$$

where p_Q is the negative exponential distribution appropriate to the undisturbed, quiet atmosphere, and p_D is the probability distribution for the turbulent modification of the intensity distribution. We thus have the product of two gamma distributions. At low levels of turbulence, the M distribution is appropriate (Strobehn, Wang and Speck, 1975). At higher levels of turbulence, the intensity distribution associated with turbulence alters the negative exponential distribution of the intensity associated with speckle. A distribution of the form

$$p(x) = \frac{1}{\Gamma(M)} \frac{M^M}{\bar{x}} x^{M-1} exp\; - \frac{Mx}{\bar{x}}$$

results in a distribution for the intensity of

$$p(I|\bar{I}) = 2\left(\frac{M}{\bar{I}}\right)^{\frac{M+1}{2}} I^{\frac{M-1}{2}} K_{M-1}\left[2\left(\frac{M}{\bar{I}}I\right)^{\frac{1}{2}}\right],$$

where $K_M(x)$ is the Bessel function of an imaginary argument of the second kind with index M. The moments of this distribution follow from

$$\int_0^\infty K_\nu(x)x^{\mu-1}dx = 2^{\mu-2}\frac{\Gamma(\mu-\nu)}{2}\frac{\Gamma(\mu+\nu)}{2}$$

(Watson, 1944; p. 388). The normalized moments of $p(I|\bar{I})$ are

$$\frac{\bar{I^n}}{(\bar{I})^n} = \frac{\Gamma(n+M)\Gamma(1+M)}{\Gamma(M)M^n},$$

with a normalized variance of $1 + \frac{2}{M}$.

The probability P_T of exceeding some threshold intensity of damage,

$$P_T = 2\left(\frac{M}{\bar{I}}\right)^{\frac{M+1}{2}} \int_{I_T} I^{\frac{M-1}{2}} K_{M-1}\left[2\left(\frac{M}{\bar{I}}\right)^{\frac{1}{2}}\right] dI,$$

can be obtained by integrating by parts and using the recursion relations

$$\begin{aligned} K_{M-1} - K_{M+1} &= -\frac{2M}{x}K_M \\ K_{M-1} + K_{M+1} &= -2\frac{dK_M(x)}{dx}, \end{aligned}$$

which lead to

$$P_T = \frac{2\,(MI_T)^{\frac{M}{2}}}{\bar{I}}\frac{1}{\Gamma(M)}K_M\left[\frac{2\,(MI_T)^{\frac{1}{2}}}{\bar{I}}\right].$$

Observation of turbulently scattered laser light leads to estimates of M (Holmes, Lee and Kerr, 1980; Phillips and Andrews, 1981). With no turbulence, normalized variance of intensity is unity, which corresponds to a large M. As turbulence increases, the variance rises to a value of about 1.25, corresponding to $M = 8$; at still stronger turbulence the variance again decreases. At the limiting value of infinite M, the K distribution asymptotically approaches the negative exponential distribution. At values of $M \approx 8$ for a given value of P_T, the allowable mean intensity will be less than that for ordinary speckle, because of the long tail of the K distribution.

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