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The problem of Glauber coherence(1) in a FEL has been investigated by different groups in several papers.(2-7)

The first analysis was carried out using a group theoretical approach (2,3) in which the symmetry properties of the quantized non-relativistic FEL Hamiltonian was exploited to analyze both the single mode and multi-mode regimes. The main conclusion of these papers was that coherence was maintained. Therefore, provided the electron motion could be treated classically, the FEL interaction "rotates" a Glauber state into another one. This result is reminiscent of the Bloch-Nordsieck Theorem(8). The validity of the assumption of classical electron motion were discussed in Ref.(9).

A quantized dynamical approach to the FEL problem was started in Ref.(4); see also Ref.(7). They considered quantized electrons and electromagnetic fields in the framework of the interaction picture, showing that a FEL amplifier or oscillator is an interesting device that exhibits genuine quantum effects like sub or super-Poissonian photon statistics, and that the radiation field could be in a so-called squeezed state; also they conjectured that Glauber coherence is only preserved by a gain-less process.

A different dynamical method was proposed in Refs.(5,6) in which the Schrodinger equation of a single electron interacting with the combined radiation and wiggler field is solved to first order in the quantum electron recoil. In this way, all

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the quantum effects previously mentioned, were recovered and furthermore we proved in a rigorous manner the conjecture of preservation of coherence to zeroth-order in the electron recoil. All these results were obtained starting from the vacuum state, therefore, we could say that departure from Poisson photon statistics and the presence of squeezed states are closely connected with the radiation field vacuum fluctuations.

In this letter we address the same problem with an arbitrary Glauber coherent state as an initial state. The single mode non-relativistic Hamiltonian describing the FEL process

$$H = \frac{p^2}{2m} + \hbar\omega\left(a_L^\dagger a_L + \frac{1}{2}\right) + \hbar\omega\left(a_U^\dagger a_U + \frac{1}{2}\right) + \hbar\Omega\left(a_L^\dagger a_U\, e^{-2ikz} + a_U^\dagger a_L\, e^{2ikz}\right) \tag{1}$$

The symbols used throughout the paper are summarized in Table 1.; we refer the reader to Refs.(5,10) for a discussion of their physical meaning.

Assuming an initial state consistent of an electron of momentum P_0 and a radiation field in a coherent state with average number of photon $|\alpha_0|^2$ and using the conservation laws of total number of photons and total momentum, we can write the state describing the evolution of the coupled electron and radiation field as

$$|\psi(t)\rangle = e^{i\frac{P_0 t}{2m\hbar} + i\omega t (n_U^0 + 1)} \sum_{n=0}^{\infty} \frac{e^{-\frac{1}{2}|\alpha_0|^2}}{\sqrt{m!}} \alpha_0^n \sum_{\ell=-n}^{\infty} C_\ell(t) |P_0 - 2\ell k\hbar, n+\ell\rangle \tag{2}$$

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where ℓ is the number of exchanged photons during the interaction and $\alpha_o = |\alpha_o| e^{i\omega t}$. We recall that in the FEL process for each exchanged photon, the electron momentum changes by $2\hbar k$.

The coefficient $C_\ell(t)$ which is the amplitude probability for the process satisfy the following differential difference-equation (Raman-Nath equation)[5]

$$i\, C_\ell'(\tau) = \left(-W_o + \epsilon \ell\right)\ell\, C_\ell(\tau) + \Omega_R\left[\sqrt{m+\ell+1}\; C_{\ell+1} + \sqrt{m+\ell}\; C_{\ell-1}\right]$$

$$C_\ell(o) = \delta_{\ell,o} \tag{3}$$

the prime denotes derivative respect to $\tau = \dfrac{c}{L} t \quad (o \leq \tau \leq 1)$ where L is the length of the undulator and $\dfrac{L}{c}$ is the interaction time.

An exact analytical solution of Eq.(3) is not known; however, a perturbative solution in terms of the electron recoil parameter ϵ can be constructed. The zeroth-order solution has been found in Ref.(10) and reads,

$$C_\ell(\tau)\Big|_{\epsilon=0} = \exp\left\{\frac{iW_o}{2}\int_0^\tau d\tau' \, |\alpha(\tau')|^2\right\}\phi_m^\ell(\alpha(\tau)) \tag{4,a}$$

where the orthonormal functions ϕ_m^ℓ are,

$$\phi_m^\ell(\alpha(\tau)) = \sqrt{\frac{m!}{(n+\ell)!}}\; e^{-\frac{1}{2}|\alpha|^2}\left(\alpha(\tau)\right)^\ell \; L_m^\ell(|\alpha|^2) \tag{4,b}$$

and

$$\alpha(\tau) = (-i)\, e^{i W_o \frac{\tau}{2}}\, \Omega_R\left(\frac{\sin W_o \frac{\tau}{2}}{W_o/2}\right)$$

L_{-m}^ℓ are the Laguerre associated polynomials. Replacing the

expression of the coefficient $C_\ell(\tau)$ into Eq. 2 we obtain the zeroth-order state evolution of the FEL system. In Ref. 5 and 6 it was shown that $\left|\psi(\tau)\right\rangle_{\epsilon=0}$ is a coherent state in the Glauber sense under the assumption that the initial state is the vacuum.

We introduce creation and annhilation operators of the form

$$A = a_L\, e^{2ikz} \quad , \quad A^\dagger = e^{-2ikz}\, a_L^\dagger$$

with $[A, A^\dagger] = 1$.

The action of them on the state $\left|\psi\right\rangle$ is

$$A\left|\psi\right\rangle = \left(\alpha(\tau) + \alpha_o\, e^{2ikz}\right)\left|\psi\right\rangle \tag{5}$$

where we have used the relation $L_m^\ell(x) = L_{n-1}^\ell(x) + L_m^{\ell-1}(x)$. From Eq.(5) we cannot conclude that $\left|\psi\right\rangle$ is a coherent state; it is clear that this comes from the presence of an average number of photons $\left|\alpha_o\right|^2$ in the initial state.

The average number of photons in the state $\left|\psi\right\rangle$ is

$$\left\langle\psi\right|A^\dagger A\left|\psi\right\rangle = \left\langle m+\ell\right\rangle_{\epsilon=0} = \left|\alpha_o\right|^2 + \left|\alpha\right|^2 \tag{6}$$

Next, we wish to compute the gain of this device to first order in the electron recoil and investigate the photon statistics of the final state of the radiation field.

The solution of our basic differential-difference equation (R.N.) has been derived in Ref.(10) and reads,

$$\left|\psi(\tau)\right\rangle = exp\left\{ i\frac{W_o}{2}\int_0^\tau d\tau' \left|\alpha(\tau')\right|^2 + i\,\tau\omega(m_U^o + 1)\frac{L}{c} + \frac{iP_o L\tau}{2m\hbar c}\right\} \times \tag{7}$$

$$\times \sum_{m=0}^{\infty} \frac{e^{-\frac{1}{2}|\alpha_0|^2}}{\sqrt{m!}} \alpha_0^n \sum_{\ell=-n}^{\infty} \sqrt{\frac{n!}{(n+\ell)!}} \; e^{-\frac{1}{2}|\alpha(\tau)|^2} (\alpha(\tau))^\ell \left[A_{\ell,n} + i D_{\ell,n} \right] |P_0 - 2\ell k\hbar, n+\ell$$

The coefficients $A_{\ell,n}$ and $D_{\ell,n}$ are lengthly expressions; in the small signal regime, i.e. first order in Ω_R they are given by,

$$A_{\ell,n} \simeq L_m^\ell(\cdot) + \frac{\epsilon}{|\alpha|} \frac{\partial |\alpha|^2}{\partial W_0} \left\{ (2\ell+1)|\alpha|^2 L_m^{\ell+1}(\cdot) - (2\ell-1)(n+\ell) L_n^{\ell-1}(\cdot) \right\} -$$

$$- \frac{\epsilon}{|\alpha|} \frac{\partial |\alpha|^2}{\partial W_0} \left\{ |\alpha|^4 L_m^{\ell+2}(\cdot) - (n+\ell)(n+\ell-1) L_n^{\ell-2}(\cdot) \right\} + \ldots$$

and

$$D_{\ell,n} \simeq + \frac{1}{2} \tau \epsilon \left\{ (2\ell+1)|\alpha|^2 L_n^{\ell+1}(\cdot) + (2\ell-1)(n+\ell) L_n^{\ell-1}(\cdot) \right\} +$$

$$- \frac{\tau}{2} \epsilon \left\{ |\alpha|^4 L_n^{\ell+2}(\cdot) + (n+\ell)(n+\ell-1) L_n^{\ell-2}(\cdot) \right\} + \ldots$$

the argument of the Laguerre polynomials $L_n^\ell(\cdot)$ is $|\alpha(\tau)|^2$. From the above expression for the state vector $|\psi\rangle$ we can evaluate the average value of photons in the final state,

$$\langle n+\ell \rangle_{\epsilon \neq 0} = \sum_{m=0}^{\infty} \frac{e^{-|\alpha_0|^2}}{m!} |\alpha_0|^{2n} \sum_{\ell=-n}^{\infty} \frac{m!}{(n+\ell)!} \; e^{-|\alpha|^2} |\alpha|^{2\ell} (n+\ell) \left[A_{\ell,n}^2 + D_{\ell,n}^2 \right] \quad (8)$$

Using recurrence relations for the Laguerre polynomials and after simple but tedious manipulations we can write the coefficients $A_{\ell,n}$ and $D_{\ell,n}$ to first order in the recoil ϵ

$$D_{\ell,n}^2 \simeq 0$$

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$$A_{\ell,n}^2 \simeq \left(L_n^\ell(\cdot)\right)^2 + \frac{2\epsilon}{|\alpha|}\frac{\partial|\alpha|^2}{\partial W_o}\left\{\left(L_n^\ell(\cdot)\right)^2\left[\ell^2 + |\alpha|^2 - |\alpha|^4\right]\right.$$

$$\left. - 2 L_n^\ell(\cdot) L_n^{\ell-1}(\cdot)\left[\ell^2 + \ell\left(n - |\alpha|^2\right) - n|\alpha|^2\right]\right\} .$$

The photon distribution function for the radiation field is given by

$$W(n,\ell,\epsilon) = \frac{e^{-|\alpha_o|^2}}{n!}|\alpha_o|^{2n} P_n(\ell,\epsilon)$$

where

$$P_n(\ell,\epsilon) = \frac{n!}{(n+\ell)!} e^{-|\alpha(\tau)|^2}|\alpha(\tau)|^{2\ell} A_{\ell,n}^2$$

which satisfy

$$\sum_{n=0}^\infty \sum_{\ell=-n}^\infty W(n,\ell,\epsilon) = 1 \quad ;$$

to show this we make use of the generating function(11)

$$\sum_{\ell=-n}^\infty \lambda^\ell P_n(\ell,\epsilon) = e^{(\lambda-1)|\alpha|^2} L_n\left(-|\alpha|^2\frac{(\lambda-1)^2}{\lambda}\right)$$

and the explicit definition of the Laguerre polynomials.

Along the same lines the average value of photons in the final state for an arbitrary initial coherent state, reads

$$\langle n+\ell\rangle_{\epsilon\neq 0} = |\alpha_o|^2 + |\alpha|^2 - \epsilon\frac{\partial|\alpha|^2}{\partial W_o}\left(1 + 2|\alpha_o|^2\right) \qquad (9)$$

The $|\alpha|^2$ term in the right-hand-side of the above equation corresponds to spontaneous emission; the second term (gain) consists of the sum of two contributions: stimulated quantum vacuum field fluctuations and a classical stimulated one, which

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is proportional to the average number of photons in the input laser wave.

Similarly, the second moment of the distribution $W(n,\ell,\epsilon)$ is given by,

$$\left\langle (m+\ell)^2 \right\rangle_{\epsilon \neq 0} = \left\langle (m+\ell)^2 \right\rangle_{\epsilon = 0} -$$
$$- 2\epsilon \frac{\partial |\alpha|^2}{\partial W_o} \left\{ 2|\alpha_o|^4 + |\alpha_o|^2 \left(3 + 4|\alpha|^2 \right) + 2|\alpha|^2 + \frac{1}{2} \right\} \quad (10)$$

A radiation state in which the fluctuation of the number of photons $\left\langle \Delta N^2 \right\rangle$ is larger (smaller) than the average value of photons $\left\langle N \right\rangle$, corresponds to a photon distribution super (sub)-Poissonian (12). The sub-Poissonian behavior of the photon distribution is an effect which reveals the quantum properties of the radiation field and cannot be explained if we use a classical description of it. Hence, we evaluate the normalized second factorial moment

$$\left\langle \Delta (m+\ell)^2 \right\rangle_{\epsilon \neq 0} - \left\langle m+\ell \right\rangle_{\epsilon \neq 0} = - 2\epsilon \frac{\partial |\alpha|^2}{\partial W_o} \left\{ |\alpha_o|^2 \left(2|\alpha|^2 + 1 \right) + |\alpha|^2 \right\} \quad (11)$$

This expression reduces to the known result of Refs. (4,5) for an initial vacuum state, i.e.

$$\left\langle \Delta \ell^2 \right\rangle_{\epsilon \neq 0} - \left\langle \ell \right\rangle_{\epsilon \neq 0} = - 2\epsilon |\alpha|^2 \frac{\partial |\alpha|^2}{\partial W_o}$$

In Fig. 1 we plot Eq.(11) as a function of the resonance parameter W_o for several values of the initial mean number of photons $|\alpha_o|^2$. For an initial vacuum state and for an arbitrary initial coherent state the final stimulated radiation field is super-Poissonian for positive resonance parameter $W_o > 0$ (above resonance), sub-Poissonian for $W_o < 0$ and Poissonian at resonance $W_o = 0$. This effect   could be

detected in an amplification FEL experiment operating in the visible part of the spectrum.

Concerning the problem of squeezed states, since we know the expression of the amplitude probability in terms of $A_{i,n}$ and $B_{i,n}$, we can easily evaluate the quantitites $< \Delta A_1^2 >$ and $< \Delta A_2^2 >$ where

$$A_1 = \frac{A + A^+}{2}$$

$$A_2 = \frac{A - A^+}{2i}$$

see also Refs.(8,9). In this way we could show that, in presence of gain, we have squeezed states for the FEL amplification. Anyway, since A and A^+ are not pure field operators but electron - field ones the interpretation of squeezing is doubtful and we do not dwell on this point.

In summary, we have shown that:

a) Starting from an initial coherent state we cannot conclude that coherence is preserved at zero gain.

b) The FEL amplified radiation field exhibits both squeezing and sub-Poissonian photon statistics[*].These quantum effects could be susceptible to experimental verification.

[*] The terms photon antibunching (bunching) have been used to describe sub-Poissonian (super-Poissonian) photon distributions.

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TABLE 1.

c	speed of light
m	electron mass
$\hbar \; (=n/2\pi)$	Planck constant
P, Z	electron longitudinal linear momentum and coordinate
$a_{L,U}$, $a^{\dagger}_{L,U}$	annihilation and creation operators of laser (L) and undulator (U) photons
$\vec{k}_L = -\vec{k}_U$	laser and undulator wavevectors in the moving frame
$\omega = |k| \, c$	laser frequency
$n_{L,U}$	number of photons, laser (L) and undulator (U)
L/c	Interaction time
$r_o = e^2/mc^2$	classical electron radius
V	interaction volume
$\Omega = \dfrac{2\pi c^2}{\omega} \, r_o /V$	coupling constant
$2\omega P_o \, L /m \, c^2 = W_o$	resonance parameter
$2\hbar \kappa^2 \, L/mc = \epsilon$	electron recoil parameter
$\Omega_R = \quad L/c \; \Omega \sqrt{m^o_U}$	

FIGURE CAPTIONS.

Fig.1. Deviation from a Poisson distribution. $\left\{\left\langle \Delta(n+\ell)^2\right\rangle - \left\langle n+\ell\right\rangle\right\}/\epsilon$ vs. resonance parameter W_o for two different values of the initial number of photons a) $|\alpha_o|^2 = 0$; b) $|\alpha_o|^2 = 10^{14}$. The coupling constant has been set $\Omega_R = 0.25$ (small signal regime)

no. of initial photons = 0

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