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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

During this last year Osher developed a joint project with James Sethian concerning fronts propagating with curvature dependent speed. They devised new algorithms approximating the equations of motion, which resemble Hamilton-Jacobi equations with parabolic right-hand sides, by using techniques from hyperbolic conservation laws. Essentially non-oscillatory schemes are used. These methods accurately capture the formation of sharp gradients and cusps in the moving fronts. The algorithms handle topological merging and breaking naturally, and work in any number of space dimensions. The methods can also be used for more general Hamilton-Jacobi type problems. Applications of the algorithms include crystal growth, solidification of metals and flame propagation.

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by

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During this last year Osher developed a joint project with James Sethian concerning fronts propagating with curvature dependent speed. ([1]). They devised new algorithms approximating the equations of motion, which resemble Hamilton-Jacobi equations with parabolic right-hand sides, by using techniques from hyperbolic conservation laws. Essentially non-oscillatory schemes are used. These methods accurately capture the formation of sharp gradients and cusps in the moving fronts. The algorithms handle topological merging and breaking naturally, and work in any number of space dimensions. The methods can also be used for more general Hamilton-Jacobi type problems. Applications of the algorithms include crystal growth, solidification of metals and flame propagation.

Recently [2], [3], non-oscillatory methods approximating problems with shocks have been developed. These methods are perhaps state-of-the-art, combining high accuracy in smooth regions (including the entropy condition), with the suppression of spurious oscillations.

This year Osher and C. W. Shu ([4]) have used these principles directly on the fluxes-not the cell averages. They have also implemented a relatively simple high order accurate non-oscillatory Runge-Kutta type time discretization. Both of these advances simplify the programming considerably. Test calculations have been quite successful.

Numerical experiments with the new class of high-order accurate Essentially Non-Oscillatory (ENO) schemes showed that the ENO schemes yield highly accurate results in the smooth part of the solution and high resolution of shocks. However, the ENO schemes, like most other schemes, exhibit unsatisfactory resolution of contact discontinuities. (Unlike shocks, contact discontinuities are linear in nature and therefore are sensitive to numerical dissipation).

Recently Harten has developed a new technique to overcome this difficulty which he calls "subcell resolution" [5]. It is based on the observation that cell-averages of a discontinuous function, unlike point values, contain information about the exact location of the discontinuity within the computational cell. Using this

observation he has designed a new ENO reconstruction technique which is exact for the recovery of discontinuous piecewise polynomial functions (of the appropriate degree) from their given cell-averages. He asks the new reconstruction to calculate a correction term to the numerical flux of the ENO scheme by evaluating the flux through the cell boundaries due to (an appropriate) linear advection of the difference between the old and the new ENO reconstructions.

Numerical experiments with a problem of two interacting blast waves (suggested by Woodward and Colella) showed a definite improvement in the quality of the results.

Engquist in [6] with Lotstedt and Sjogreen introduced TVD-filters with applications to multidimensional shock computations.

Difference and particle method approximations of highly oscillatory solutions of hyperbolic problems were studied in [7] and with Hou in [8] and [9]. In this work the practically important but theoretically difficult case when all modes in a solution can not be well resolved on the computational grid were studied.

Osher and Shu [10] improved the ENO formulation further, using contact discontinuity sharpeners, and a flux simplification. The technique was successfully applied to turbulence amplification in shock wave calculations [11].

Additional new results include an ENO code applied to semi-conductor physics problems - namely the hydrodynamic device model using the ballistic diode as an example. Higher order non-oscillatory methods seem to be appropriate here [12].

Also, the front capturing method devised earlier seems to give excellent results for wide variety of problems, e.g. three dimensional compressible Raleigh-Taylor and Kelvin-Helmholtz problem. This new method uses a fixed Eulerian grid and is easy to code [13].

A graduate student, Huanan Yang obtained a new artificial compression method for ENO schemes which sharpens contacts and multi-dimensional slip lines with no loss of high accuracy or the ENO property [14].

During this period Professor Osher obtained a new closed form inequality for the exact solution to the Riemann problem for Hamilton-Jacobi Equations [15].

This is of computational importance for a host of applied problems including differential games, stochastic control and front propagation.

In addition Professor Engquist with a postdoctoral fellow, Dr. Sjogreen, obtained a new method for computing solutions to hyperbolic conservation laws with stiff lower order terms [16]. This method eliminates unphysical numerical shocks which plague the computation of e.g. chemically reacting flows.

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