(2)

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ANALYSIS OF THE ACCURACY OF A
PROPOSSED
TARGET MOTION ANALYSIS PROCEDURE

by

Bernabe Carrero Cuberos

1989 September

Thesis Advisor	James N. Eagle
Co-Advisor	Donald P. Gaver

Approved for public release; distribution is unlimited

Unclassified
security classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified	1b Restrictive Markings
2a Security Classification Authority	3 Distribution/Availability of Report
2b Declassification Downgrading Schedule	Approved for public release; distribution is unlimited.
4 Performing Organization Report Number(s)	5 Monitoring Organization Report Number(s)
6a Name of Performing Organization Naval Postgraduate School; 6b Office Symbol (if applicable) 55	7a Name of Monitoring Organization Naval Postgraduate School
6c Address (city, state, and ZIP code) Monterey, CA 93943-5000	7b Address (city, state, and ZIP code) Monterey, CA 93943-5000
8a Name of Funding Sponsoring Organization; 8b Office Symbol (if applicable)	9 Procurement Instrument Identification Number
8c Address (city, state, and ZIP code)	10 Source of Funding Numbers: Program Element No / Project No / Task No / Work Unit Accession No

11 Title (include security classification) ANALYSIS OF THE ACCURACY OF A PROPUSSED TARGET MOTION ANALYSIS PROCEDURE

12 Personal Author(s) Bernabe Carrero Cuberos

13a Type of Report	13b Time Covered	14 Date of Report (year, month, day)	15 Page Count
Master's Thesis	From To	1989 September	66

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes Field	Group	Subgroup	18 Subject Terms (continue on reverse if necessary and identify by block number)
			TMA, Passive bearings-only.

19 Abstract (continue on reverse if necessary and identify by block number)

This thesis investigates the accuracy of a recently proposed passive bearings-only Target Motion Analysis (TMA) procedure. The primary method of analysis is to compare computer generated positions of a Target that is moving with a constant course and speed, with the procedurally derived estimated positions.

A computer model was developed which simulated several possible interactions between the Target and Own Ship. Estimated parameters of the Target track were computed using the procedure under analysis. These values were compared to the "true" values generated by the simulation.

An analysis of the TMA procedure as originally proposed showed that it failed to accurately estimate the target track parameters. However with some modifications, the accuracy improved significantly and it is felt that the procedure can accurately estimate target range (but not necessarily course and speed) for some target geometries. Keywords: target detection; target position indicators; Theses; (KT)

20 Distribution Availability of Abstract ☒ unclassified unlimited ☐ same as report ☐ DTIC users	21 Abstract Security Classification Unclassified	
22a Name of Responsible Individual James N. Eagle	22b Telephone (include Area code) (408) 646-2654	22c Office Symbol 55Er

DD FORM 1473,84 MAR 83 APR edition may be used until exhausted. All other editions are obsolete. security classification of this page

Unclassified

Analysis of the Accuracy of a Propossed Target Motion Analysis Procedure

by

Bernabe Carrero Cuberos
Commander, Venezuelan Navy
B.S., Venezuelan Naval School., 1985

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
1989 September

Author: ______________________
Bernabe Carrero Cuberos

Approved by: ______________________
James N. Eagle, Thesis Advisor

____
Donald P. Gaver, Co-Advisor

____
Peter Purdue, Chairman,
Department of Operations Research

ABSTRACT

This thesis investigates the accuracy of a recently proposed passive bearings-only Target Motion Analysis (TMA) procedure. The primary method of analysis is to compare computer generated positions of a Target that is moving with a constant course and speed, with the procedurally derived estimated positions.

A computer model was developed which simulated several possible interactions between the Target and Own Ship. Estimated parameters of the Target track were computed using the procedure under analysis. These values were compared to the "true" values generated by the simulation.

An analysis of the TMA procedure as originally proposed showed that it failed to accurately estimate the target track parameters. However with some modifications, the accuracy improved significantly and it is felt that the procedure can accurately estimate target range (but not necessarily course and speed) for some target geometries.

TABLE OF CONTENTS

ACKNOLEDGEMENTS

I would like to express my gratitude to Prof. James N. Eagle and to Prof. Donald P. Gaver for their assistance, guidance, and encouragement which they provided during the pursuit of this work.

A special note of thanks is extended to LCDR. William Walsh, CPT. Charles H. Shaw, III, and LT. Scott Negus for their help in the revision of the formal English needed for this work.

I also want to dedicate my work to my lovely wife Tahamara and to my sons Sandy Gabriel, Daniel Antonio, and Jenny Coralis for their patience, understanding and fortitude given to me during my studies at the Naval Postgraduate School.

I. INTRODUCTION TO THE PASSIVE RANGING PROBLEM

A. BRIEF HISTORY

The importance of doing Target Motion Analysis (TMA)[1] by a passive bearings-only tracking technique has had the attention of many tacticians over the last 45 years, mainly because the submarine's objective is to remain covert (undetected) during the tracking maneuver. This guarantees the advantage of surprise if an attack is to be performed.

In the early part of WWII, TMA by passive bearings-only methods began to be used, and the following drawn from *Naval Tactical Decision Aids* by D. H. Wagner, 1989, provides a brief history of its development:

> During WWII LT. F. C. Linch developed the Linch Plot, which uses a pivotal relationship among bearings, bearing rate, and Target relative motion. Based on that, he developed a graphical method that was used until the late 1960's. It was abandoned because it was useful only for the short ranges contemplated by Linch at that time.
>
> During the 1950's, various human plot methods and nomographs were used for TMA. One of the oldest of these was the Strip Plot, later called Geographical Plot. This method has often been used to provide upper and lower bounds on Target range, based on bounds on speed. An additional much used nomographic device has been the Bearing Rate Slide Rule. An early TMA computer was the Position Keeper, which was a carryover from WWII, and used as an aid to approach and attack surface ships.
>
> In 1953, F. N. Spiess published a graphical procedure requiring four bearings and a maneuver by Own Ship. This method is known as Spiess Plot. It remains in use in surface ship TMA.
>
> The 1954 command thesis of LT. J. F. Fagan derived a four-bearing TMA solution, which was a transcendental system of three equations with three unknowns. To reduce to computability by slide rule, he assumed that Own Ship motion during the first three bearings was approximately zero which was probably satisfactory for diesel operations.
>
> In 1957, LT. J. J. Ekelund devised one of the most famous TMA methods, now called Ekelund Ranging.
>
> Between 1967 and 1969, D. C. Bossard with a group of USN officers, made an analysis of the fundamentals of bearings-only ranging and the effect of Own Ship

---

1 Determination of Target range, course, and speed

maneuvers on ranging accuracy. Through this work and trials at sea, they observed that ranging errors could be eliminated by judicious choice of the time for which the range was estimated. Also, by selecting some particular maneuvers, this best time could be controlled to be in the past, present, or future; then they applied this method to various forms of bearings-only ranging, especially to Ekelund and Spiess methods. The general name for these procedures is Time Correction.

In August 1968, a series of exercises was conducted aboard the USS Pargo to compare the different methods in use at that time. The Passive Ranging Manual evolved in three volumes from the report of the analysis of these exercises and was published later as an NWP. This is a basic reference today for TMA.

In the late 1960's the CHURN method appeared, which uses a large number of bearing observations and regression methods to do TMA. This method was developed by General Dynamics' Electric Boat and Librascope.

In 1969, H. W. Headle, J. Di Russo, and E. Messere developed the Manual Adaptive TMA Evaluator (MATE). This method begins with an input estimate of Target course, speed and range, and from there estimated bearings are computed and compared against observed bearings. Differences between these bearing values are plotted against time. If these differences are distributed roughly symmetrically as white noise about zero, while Own Ship changes course or speed, then the trial solution is a good one; otherwise the MATE operator adjusts the input Target parameters.

In the early 1970's, the application of Kalman filtering appeared for the first time in TMA. It was a done by J. S. Davis of NUSC, based on the Ph. D. thesis of D. J. Murphy at Northeastern University. Later versions were developed by IBM.

In the mid 1970's, a new TMA method known as FLIT was devised by ENS. L. Anderson which has seen considerable effective operational use. Its methodology remains classified.

In the mid and later 1970's, ranging and tracking at long range on a sphere were developed by D. C. Bossard, J. B. Behrla, and L. K. Graves of Daniel H. Wagner, Associates. This was motivated by over-the-horizon targeting needs.

In the late 1970's and during the 1980's, further developments were made by using Kalman filtering and also stochastic differential equations. The Maneuvering Target Statistical Tracker (MTST) is a method devised from this basis by W. H. Barker of Daniel H.Wagner, Associates.

From 1985 to 1987 the Generical Statistical Tracker (GST) was developed at Daniel H. Wagner, Associates. This is a PC version of MTST.

B. PURPOSE OF THE THESIS

In late 1988, LCDR P. K. Peppe of the USS La Jolla (SSN 701) proposed a bearing-only Target Motion Analysis (TMA) procedure (here called the Bearings Extrapolation Procedure) for use in submarines [Ref. 1]. It is the purpose of this thesis to test the accuracy of this procedure against a nonmaneuvering Target.

C. PROBLEM DESCRIPTION

A submarine (Own Ship) is trying to determine the range, course, and speed of a ship (Target) that is moving with a fixed course and speed. The data collected by Own Ship consists of passive bearings. Since the bearing data is obtained by a single tracker, a unique tracking solution may only be obtained by a maneuvering Own Ship [Ref. 2]. A TMA method which uses a two-leg maneuver can fulfill this requirement.

The two-leg TMA procedure used in this study is depicted in Figure 1. Own Ship begins by receiving bearings to the Target each 20 seconds for 4 minutes. In Figure 1, t_i is the time at which bearing i is received. After time t_{11}, the bearings stop and Own Ship begins a course change. At time t_{30} Own Ship is on the second leg and bearings resume. Bearings continue on the second leg, the last one being received at time t_{41}. Estimates of Target course, speed and range will be derived from these 24 observed bearings.

The central idea of the Bearings Extrapolation Procedure is that given three bearings observed at three times on a single leg, all future and past bearings can theoretically be determined. (This idea was known by Spiess [Ref. 3] and may predate him.) So, referring to Figure 1, the estimated Target position at time t_0 can be obtained by crossing the true bearing (beginning at point t_0) with the extrapolated bearing from leg two (beginning at point t_0').

The same as above can be done for all time points that have bearings between t_0 and t_{41}. Then regressing linearly on these estimated Target positions, the Target track parameters can be determined.

Although this procedure should give exact answers, in practice large errors can be generated. The reason for these inaccuracies is that bearings to the Target cannot be measured exactly, so the extrapolated bearings will also be in error.

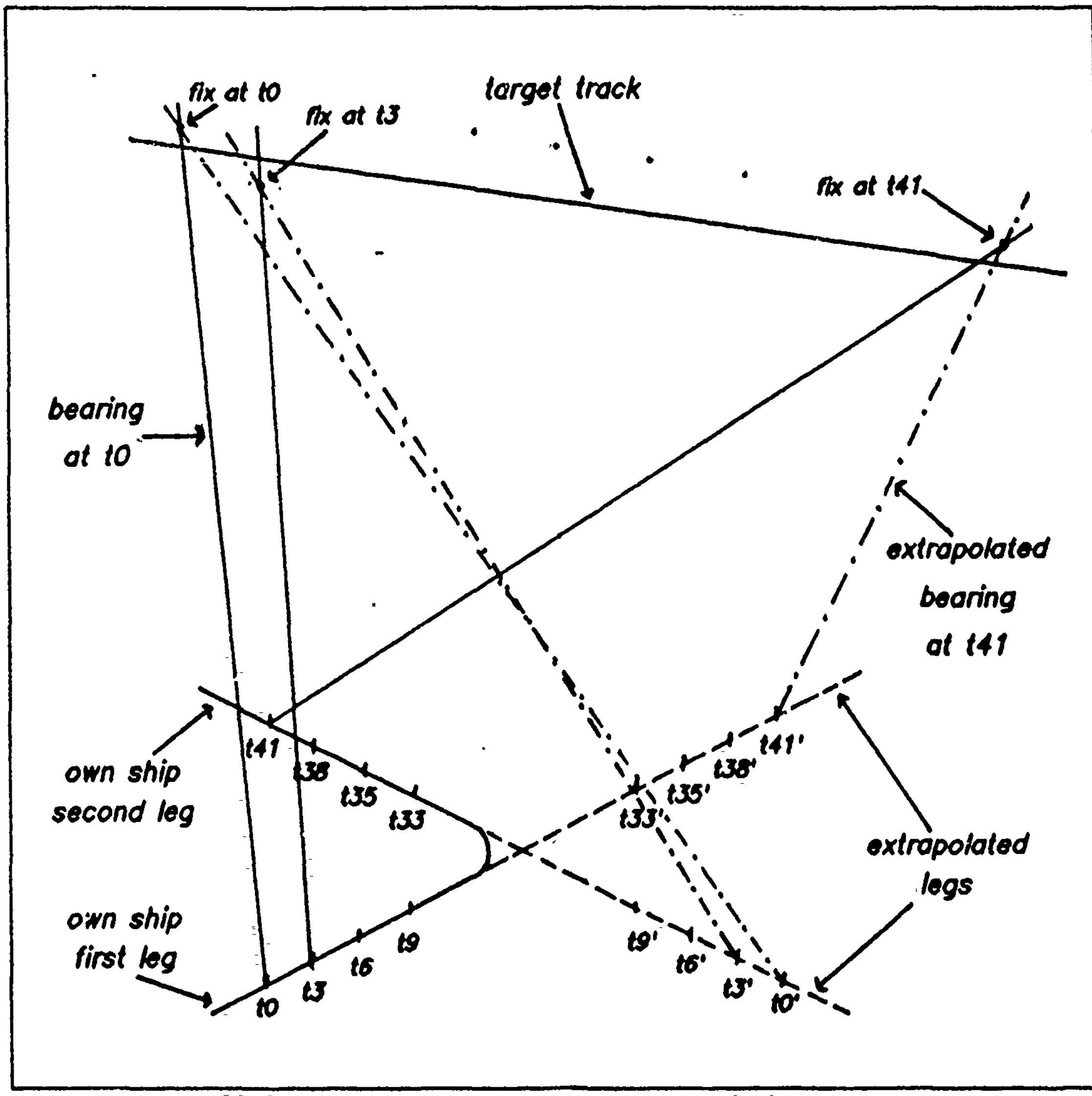

Figure 1. Graphical Representation of the Bearings Extrapolation Procedure.

Exactly how bearings that contain errors are extrapolated, with as much accuracy as possible, forward and backward in time is part of what this thesis examines. The most straightforward method (and the first to be analyzed) is simply to use linear regression on the (B_t , t) data, where B_t is the bearing at time t. Alternate methods include regressing the (B_t, t) data by using quadratic or arctangent models. Each of these procedures were tried and accuracies assessed.

The Spiess Graphical Method, Ekelund Ranging, and the CHURN Method are closely related to the procedure proposed by Peppe, and for that reason they are further explained in Appendix A

D. ASSUMPTIONS FOR THE SIMULATION

The computer simulation used in this study uses the following assumptions:

1. The Target moves at a constant course and speed throughout the tracking maneuver.
2. Differences in depth for the Target and Own Ship are disregarded.
3. Errors in position for Own Ship are disregarded.
4. Errors in bearings to the Target are independent and identically distributed normal random variables with mean zero and standard deviation not larger than 1.5°. In addition, this standard deviation is constant for each simulation experiment.
5. Bearings to the Target are obtained each 20 seconds.
6. Initial course, speed and range for the Target and initial course and speed for Own Ship are as described in Chapter II.

E. TERMINOLOGY

1. Lead-leg. An Own Ship course such that Own Ship speed across the line of sound is in the same direction as the Target speed across the line of sound at the start of the leg.
2. Lag-leg. An Own Ship course such that Own Ship speed across the line of sound is in the opposite direction to the Target speed across the line of sound at the start of the leg.
3. Leg length. The distance along either leg where bearings are received (does not include Own Ship turn).
4. Raw bearing. An unsmoothed bearing to the Target.
5. Faired bearing. A bearing obtained after smoothing of the raw bearings.
6. Extrapolated positions. Positions of Own Ship obtained by extending the first leg forward in time or the second leg backward in time.
7. Extrapolated bearings. Bearings obtained by extrapolating actual bearings forward in time for the first leg or backward in time for the second leg.
8. Lead-leg estimation. The estimated target track obtained by linear regression of estimated Target positions. These estimated Target positions are obtained from the intersection of faired bearings from the lead-leg and extrapolated bearings from the lag-leg.
9. Lag-leg estimation. The estimated target track obtained by linear regression of estimated Target positions. These estimated Target positions are obtained from the intersection of faired bearings from the lag-leg and extrapolated bearings from the lead-leg.

10. Lead angle. Angle between the line of sound and the course of Own Ship when it is leading the Target.

11. Lag angle. The angle between the line of sound and the course of Own Ship when it is lagging the Target.

12. Angle on the bow. The angle between the line of sound and the course of the Target.

II. TESTING THE BEARING EXTRAPOLATION PROCEDURE

A. INITIAL CONDITIONS

In order to perform a thorough analysis of the Bearing Extrapolation Procedure, 5,184 separate simulation experiments were conducted. Each experiment used one of the possible combinations of the following parameters:

1. The initial Target positions were all due north from Own Ship (i. e., 000°) at one of three different initial distances: large (= 60,000 yds), medium (= 30,000 yds), and short (= 10,000 yds).
2. Six courses for the Target were used: 030°, 060°, 090°, 120°, 150°, and 175°.
3. Three speeds for the Target: 25, 15, and 5 knots.
4. Four courses for Own Ship in the lead-leg: 060°, 070°, 080°, and 090°.
5. Four courses for Own Ship in the lag-leg: 270°, 280°, 290°, and 300°.
6. Three speeds for Own Ship: 5, 10, and 15 knots.
7. Two maneuvers by Own Ship: lead-lag and lag-lead.
8. Four bearing error standard deviations: 0°, 0.5°, 1°, and 1.5°.

It is important to mention that because of problem symmetry and the fact that both lead-lag and lag-lead maneuvers are simulated, there was no need for simulating courses for the Target from 180° to 360°.

The randomness in the simulation was introduced only through errors in the received bearings.

B. DESCRIPTION OF THE SIMULATION

The simulation was written in Fortran 77 [Ref. 4] and executed on an IBM 3033 mainframe. Appendix B has a complete listing of all variables of the simulation program. Appendix C is a listing of the simulation program code.

For each combination of initial conditions, the simulation was repeated 100 times with different seeds for pseudorandom-generation of bearing error. For each of the repetitions, the following steps were conducted:

1. Target positions were simulated every 20 seconds for the entire maneuver.
2. For the first leg the following data was generated:
    a. Own Ship position every 20 seconds.
    b. Bearing to the Target every 20 seconds (normal errors are added).

c. Own Ship track extended beyond the turn.

3. For the first leg the following data was then computed:

   a. Coefficients for linear and quadratic fits for bearings versus time.

   b. Faired bearings and extrapolated bearings using the results from linear or quadratic fit.

4. Own Ship course was determined for the second leg; based on the required second leg, lead or lag angle, and the line of sound specified by the extrapolated bearing at the middle of the maneuver (t_{20}).

5. Simulations and computations for the second leg were completed in the same fashion.

6. Estimated positions of the Target in two coordinates (X and Y) were computed by determining the point of intersection of faired bearings with their corresponding extrapolated bearings.

7. Linear and orthogonal regression is conducted on the estimated Target positions (in X versus Y coordinates), to determine the Target course.

8. Range to the Target at the end of the maneuver was obtained by computing the distance between Own Ship's actual position at the end of the second leg and the estimated Target position.

9. Finally, Target speed was obtained by computing the distance between the estimated Target position at time t_0 and the estimated Target position at the end of the second leg (t_{41}), on the fitted Target track, and then dividing this distance by $t_{41} - t_0$.

The simulation of the bearing errors was done using the Linear Congruential Method for uniform pseudorandom numbers, combined with the Box and Muller Method to produce normal variates [Refs. 5,6]. These procedures were selected to allow the simulation to be performed on a personal computer.

C. TESTING THE BEARING EXTRAPOLATION PROCEDURE UNDER IDEAL CONDITIONS

Here bearings to the Target were considered without error. Also the bearing rates immediately before the turn (used to extrapolate bearings forward in time) and immediately after the turn (used to extrapolate bearings backward in time) were computed exactly with no errors introduced. The only source of error in this case was the linear bearing extrapolation. This case is intended to be an optimistic bound on the accuracy of the bearing extrapolation procedure when linear regression is used.

The measure of effectiveness (MOE) for the accuracy of this procedure was the percentage of times that the estimated error was within 10% for range, 20° for course, and 10% for speed from the actual parameters of the Target track. These results are presented in Table 1 and Table 2 for lead-lag and lag-lead maneuvers respectively. Each number in these Tables resulted from the examination of 18 simulation experiments (six Target courses and three Target speeds). For example, when the initial distance was large, lead and lag angles 70°, and Own Ship speed 15 knts, then five of the 18 simulations experiments (or 27.8%) resulted in the calculated Target speed being within 10% of the actual Target speed. Note that without bearing error, the simulation experiments are deterministic.

Table 1. ACCURACY IN IDEAL CONDITIONS FOR BEARINGS EXTRAPOLATION PROCEDURE (LEAD-LAG MANEUVER)

Own Ship speed (knts)	Lead and lag angle (°)	Initial distance	Percentage of times the absolute error is smaller than 10% of actual for range and speed or 20° of actual for course.		
			Range (10%)	Course (20°)	Speed (10%)
15	70	Large	100	50.0	27.8
		Medium	100	50.0	27.8
		Short	38.9	50.0	22.2
	80	Large	100	50.0	33.3
		Medium	100	55.6	33.3
		Short	50.0	50.0	27.8
	90	Large	100	55.6	27.8
		Medium	100	55.6	27.8
		Short	50.0	66.7	33.3
10	70	Large	100	44.4	44.4
		Medium	94.4	38.9	33.3
		Short	55.6	38.9	22.2
	80	Large	100	44.4	33.3
		Medium	94.4	44.4	33.3
		Short	55.6	44.4	27.8
	90	Large	100	55.6	33.3
		Medium	94.4	50.0	33.3
		Short	61.1	50.0	27.8
5	70	Large	83.3	44.4	22.2
		Medium	72.2	44.4	27.8
		Short	44.4	44.4	27.8
	80	Large	88.9	33.3	27.8
		Medium	77.8	33.3	27.8
		Short	44.4	44.4	22.2
	90	Large	88.9	33.3	27.8
		Medium	77.8	33.3	22.2
		Short	44.4	33.3	16.7

Table 2. ACCURACY IN IDEAL CONDITIONS FOR BEARINGS EXTRAPOLATION PROCEDURE (LAG-LEAD MANEUVER)

Own Ship speed (knts)	Lead and lag angle (°)	Initial distance	Percentage of times the absolute error is smaller than 10% of actual for range and speed or 20° of actual for course.		
			Range (10%)	Course (20°)	Speed (10%)
15	70	Large	88.9	77.8	27.8
		Medium	66.7	77.8	27.8
		Short	50.0	50.0	33.3
	80	Large	88.9	94.4	38.9
		Medium	72.2	88.9	50.0
		Short	50.0	33.3	33.3
	90	Large	94.4	94.4	44.4
		Medium	72.2	88.9	44.4
		Short	38.9	27.8	27.8
10	70	Large	83.3	88.9	22.2
		Medium	66.7	94.4	22.2
		Short	44.4	50.0	27.8
	80	Large	83.3	94.4	33.3
		Medium	61.1	88.8	33.3
		Short	50.0	44.4	22.2
	90	Large	77.8	94.4	38.9
		Medium	61.1	88.9	38.9
		Short	50.0	38.9	16.7
5	70	Large	72.2	77.8	33.3
		Medium	50.0	77.8	27.8
		Short	38.9	66.7	27.8
	80	Large	72.2	77.8	33.3
		Medium	50.0	77.8	27.8
		Short	44.4	66.7	27.8
	90	Large	72.2	77.8	38.9
		Medium	50.0	77.8	33.3
		Short	44.4	66.7	22.2

As can be seen in Table 1 and Table 2, the following results were obtained under ideal conditions:

1. Range estimation is good, but only for medium or large initial distances.
2. Accuracy improves with increases in Own Ship speed.
3. Course and speed estimation is less accurate than range estimation.
4. Accuracy improves with increases in lead and lag angle.
5. Lead-lag maneuvers give better range estimates than do lag-lead, but not necessarily better course and speed estimates.

It is felt that the procedure performed poorly for short initial distances, in part because the bearing versus time curve is more nonlinear in this case.

D. TESTING WITH BEARING ERRORS

For these experiments, the program was run with bearing errors added to the simulated bearings. Also the bearing rate for each leg was computed using a linear least squares technique.

Only lead or lag angles of 90° were considered, since it was determined in the previous step that this is the best course for Own Ship to follow. Accuracy in course and speed were not considered since the errors for these parameters were too large, even in ideal conditions. An Own Ship speed of 5 knots was not considered for the same reason.

The MOE for these experiments was the percentage of times (runs) that the estimated range at time t_{41} was within 5%, 10%, and 20% of the actual range. Each number in Table 3 and Table 4 results from 1,800 trials (six Target courses, three Target speeds and 100 replications per (course, speed) combination).

Table 3. ACCURACY IN REAL CONDITIONS FOR BEARING EXTRAPOLATION PROCEDURE (LEAD-LAG MANEUVER)

Own Ship speed (knts)	St. Dev. bearing error	Initial distance	Percentage of times the absolute error is smaller than 5%, 10%, and 20% of actual for range.		
			5%	10%	20%
15	1.5°	large	6.2	11.8	25.5
		medium	12.3	24.4	45.4
		short	15.9	30.6	53.7
	1.0°	large	8.9	18.4	36.6
		medium	15.8	37.8	60.3
		short	18.7	38.3	60.4
	0.5°	large	16.9	33.7	61.9
		medium	26.5	52.5	82.2
		short	19.4	39.2	62.8
10	1.5°	large	4.2	8.8	18.2
		medium	8.4	17.0	33.9
		short	12.6	24.4	51.2
	1.0°	large	5.8	12.3	26.4
		medium	10.7	23.5	45.3
		short	15.1	30.2	59.7
	0.5°	large	10.8	22.9	45.8
		medium	19.0	38.9	68.7
		short	20.3	38.8	69.4

Table 4. ACCURACY IN REAL CONDITIONS FOR BEARING EXTRAPOLATION PROCEDURE (LAG-LEAD MANEUVER)

Own Ship speed (knts)	St. Dev. bearing error	Initial distance	Percentage of times the absolute error is smaller than 5%, 10%, and 20% of actual for range.		
			5%	10%	20%
15	1.5°	large	4.7	11.3	23.2
		medium	10.3	19.7	36.5
		short	6.7	12.3	23.9
	1.0°	large	7.11	15.9	34.3
		medium	11.7	24.2	46.6
		short	6.7	12.2	22.2
	0.5°	large	16.2	31.8	56.1
		medium	17.1	30.9	54.9
		short	5.3	11.6	21.4
10	1.5°	large	4.0	7.4	18.1
		medium	7.3	13.7	26.9
		short	5.7	11.7	21.9
	1.0°	large	5.4	10.8	23.4
		medium	9.0	17.3	35.9
		short	5.0	10.8	23.2
	0.5°	large	11.0	22.2	42.9
		medium	12.4	25.1	46.2
		short	3.8	9.7	23.6

From Table 3 and Table 4, the following results are obtained:

1. The higher the Own Ship speed, the better the accuracy.
2. The best accuracy is obtained for medium initial distances.
3. The smaller the bearing error, the better the accuracy.
4. A lead-lag maneuver gives better accuracy than lag-lead maneuver.

It is noted that orthogonal regression was also tried on the estimated Target positions, but with results similar to those obtained with linear regression. Subsequent experiments used only linear regression.

E. ANALYSIS OF THE RESULTS

It appears that even in the absence of bearing errors, the Bearing Extrapolation Procedure gives poor estimates of Target course and speed. And when bearing errors are introduced, the range errors also become unacceptable. It is the purpose of this section to determine why these poor results were obtained.

1. Differences Between Lead-Leg and Lag-Leg-Estimation

Figures 2 to 5 show actual and estimated Target positions for four encounter geometries with no error in bearing. So the only errors introduced are due to the linear extrapolation of bearings. It is clear that lag-leg estimation is much more accurate than the lead-leg-estimation. It is also clear that any attempt to regress estimated Target positions over both legs can result in large errors.

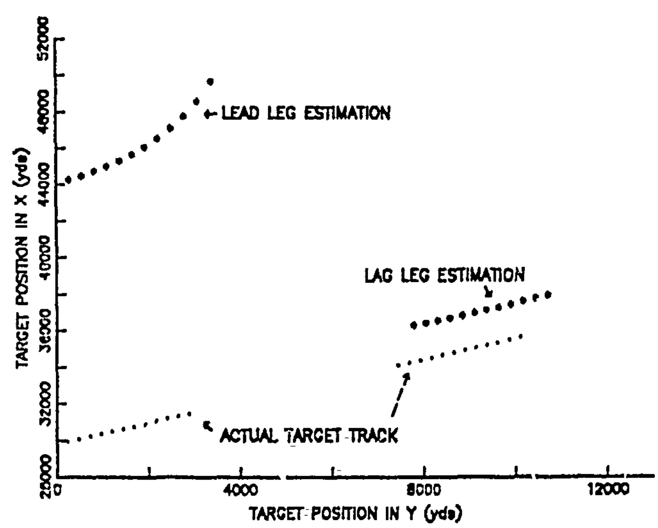

Figure 2. Actual (.) and Estimated (*) Target Track for Lead-Lag Maneuver. (Target Course = 060°)

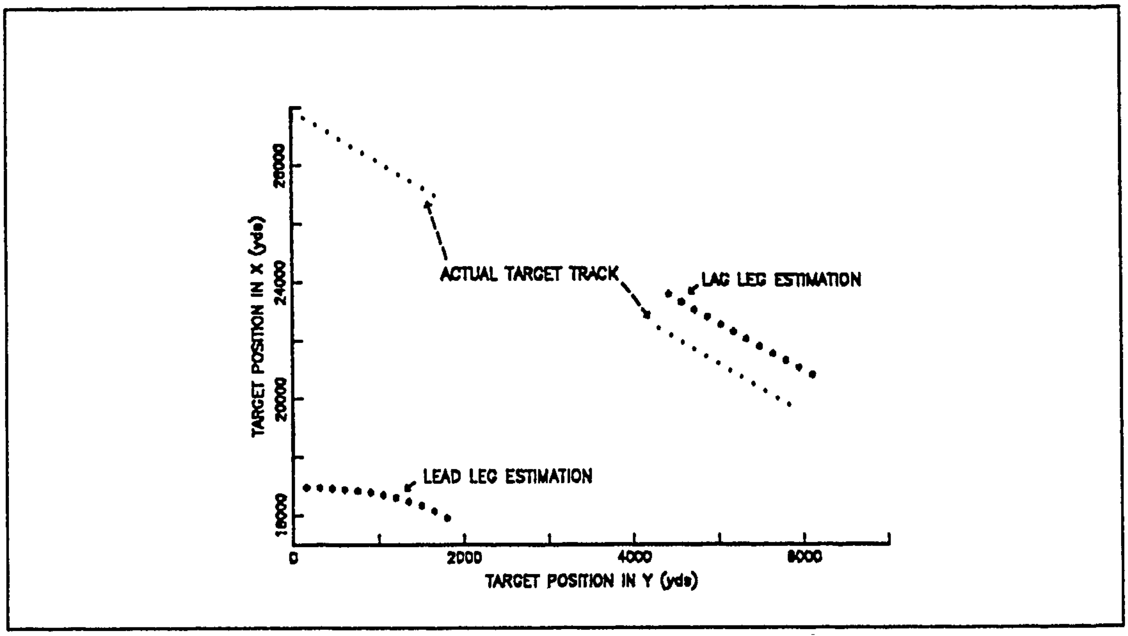

Figure 3. Actual (.) and Estimated (*) Target Track for Lead-Lag Maneuver. (Target Course = 150°)

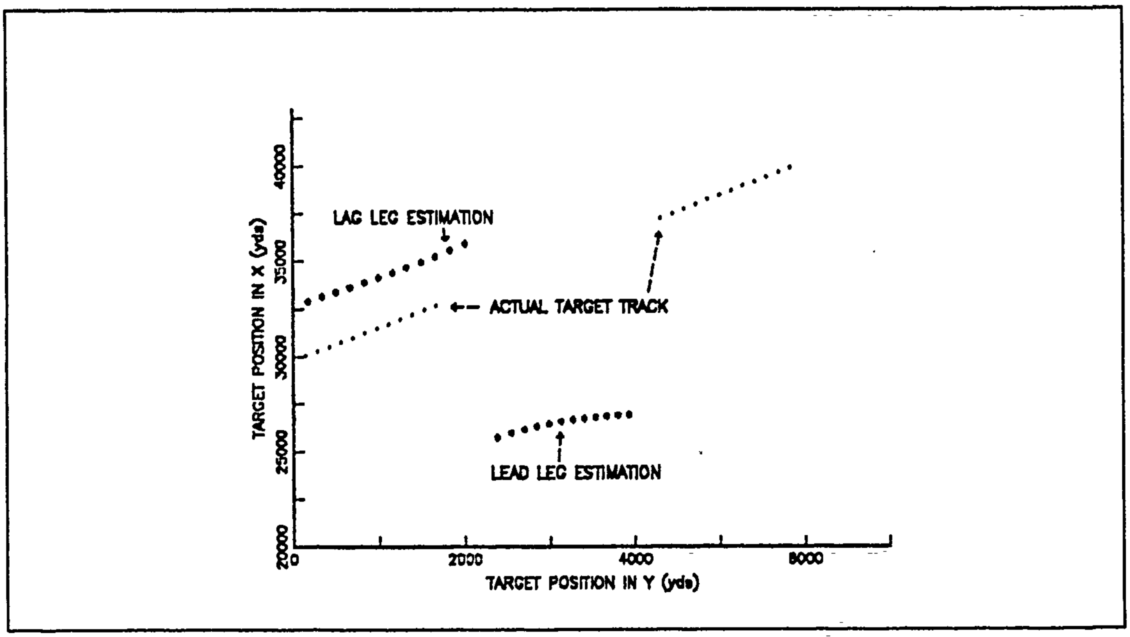

Figure 4. Actual (.) and Estimated (*) Target Track for Lag-Lead Maneuver. (Target Course = 030°)

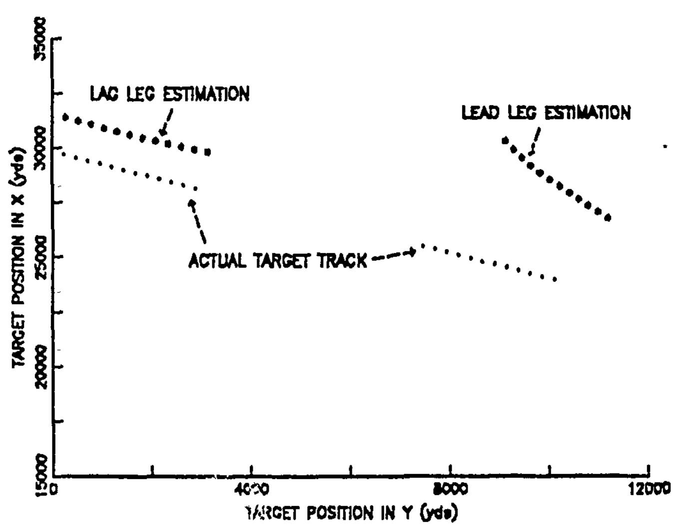

Figure 5. Actual (.) and Estimated (*) Target Track for Lag-Lead Maneuver. (Target Course = 120°)

2. Explanation of Why Lag-Leg Estimation is Better than Lead-Leg Estimation

To help explain why the lag-leg estimation was better than lead-leg estimation, scatter plots of bearings versus time for numerous encounter geometries were obtained. Figures 6, 7, and 8 are typical. In Figure 6, a lead-lag encounter is represented. It was observed that during the lead-leg (first leg), the bearing rate was generally smaller than that of the lag-leg (second leg). Also, and perhaps more importantly, the change in bearing rate (bearing acceleration) was also smaller in the lead-leg. When this happens, it is possible to use linear regression to extrapolate the lead-leg bearings accurately into the future. This makes the lag-leg estimation more accurate. It is clear from Figure 6 that if linear regression were used to extrapolate the lag-leg bearings into the past, significant errors would result, and these errors would likely cause the lead-leg estimation to be less accurate.

In the same fashion, Figure 7 shows a plot of bearings versus time for a typical lag-lead encounter. Here also, the lead-leg (second leg) has a smaller bearing rate and bearing acceleration. So linear regression can be used to extrapolate accurately the lead-leg bearings backward in time, again making the lag-leg estimation most accurate.

Unfortunately, however, there are encounter geometries where neither the lead nor the lag-legs show small bearing acceleration. For example, Figure 8 shows a case where the Target is at high speed, short initial distance, and on course 160°. Here both lead and lag-leg bearings are nonlinear, making the Bearing Extrapolation Procedure unusable. Extensive experimentation indicates that when all the following conditions hold:

Target course $\geq$ 150 °,

Target speed $\geq$ 5 knots,

Target range $\geq$ 40,000 yds;

then the lead-leg bearings (and the lag-leg bearings) are not significantly linear to be accurately extrapolated, and as a result, lag-leg estimation (as well as lead-leg estimation) is poor.

Finally, it is noted that for very long initial distances (> 40,000 yds), both the lead and lag-legs have approximately a linear change in the bearings, making both lead and lag-leg estimations reasonably accurate.

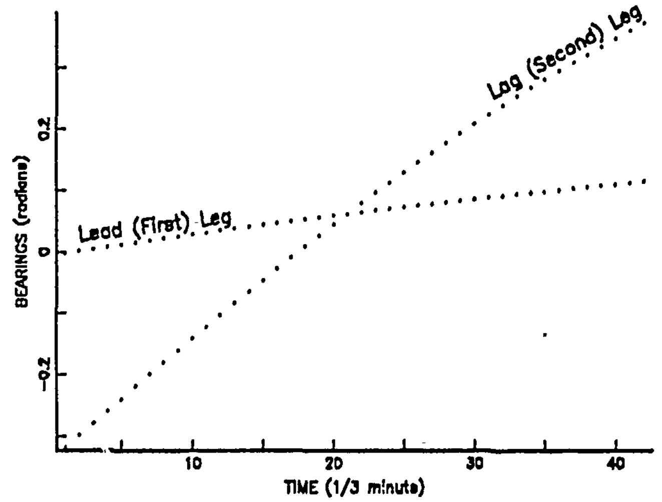

Figure 6. Bearings in Both Legs for Lead-Lag Maneuver (Target course = 060°)

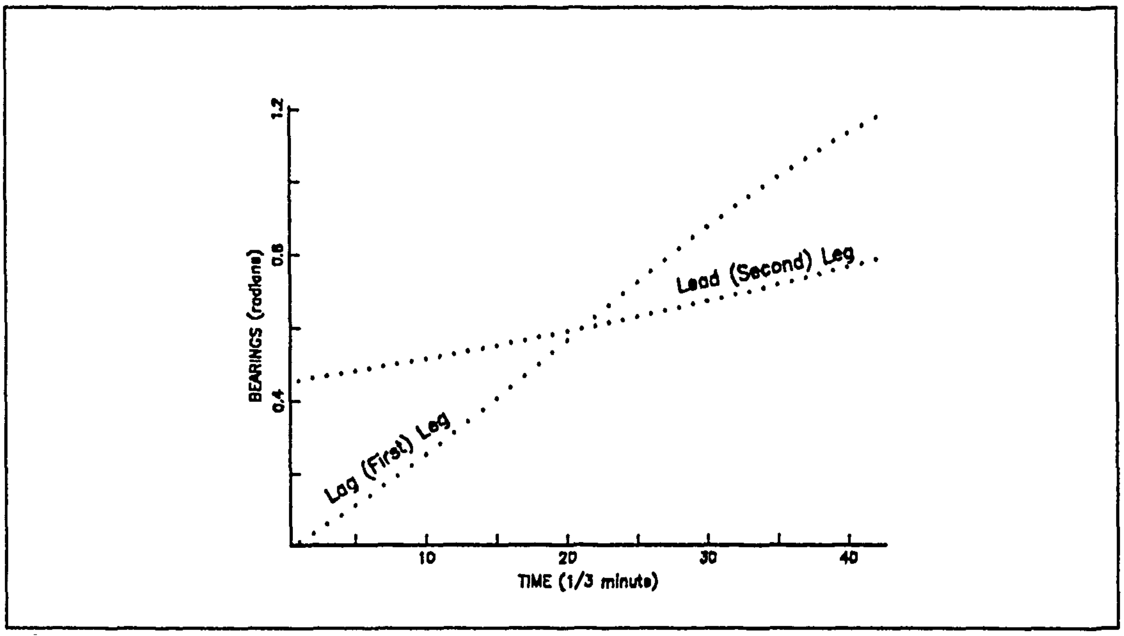

Figure 7. Bearings in Both Legs for Lag-Lead Maneuver (Target Course = 140°)

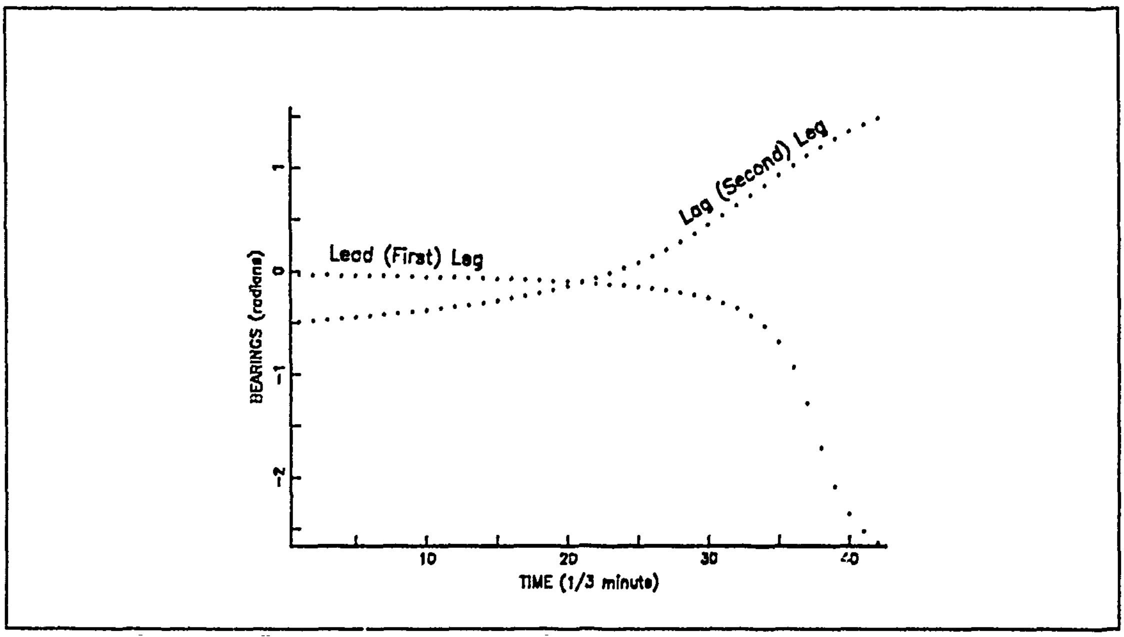

Figure 8. Bearings in Both Legs for Lead-lag Maneuver at Short Distance and Closing Target.

F. CONCLUSIONS

1. Even when bearings were without error, the Bearing Extrapolation Procedure was unsuccessful in estimating Target course and speed. However, range estimates were reasonable.
2. When bearing errors were introduced, range estimates also became inaccurate.
3. For most encounter geometries, lag-leg estimation gave more accurate results than did lead-leg estimation. However, for some important cases both lag and lead-leg estimations performed poorly.

III. POSSIBLE IMPROVEMENTS, AND ANALYSIS OF THEIR ACCURACIES

The lack of accuracy of the proposed procedure, even under ideal conditions, led to attempts to improve its performance. The following are some suggested minor changes and two suggested major changes that were found to improve the accuracy of the Bearing Extrapolation Procedure.

A. MINOR CHANGES

1. Increase in Leg Length

In an attempt to improve performance, several different leg lengths were tried. It was discovered that leg lengths of 8 to 12 minutes gave the best results. Shorter legs gave too little data and longer legs resulted in nonlinear changes in lead-leg bearings. Based on these tests, ten minutes was selected as the leg length for subsequent simulation experiments.

2. Use of a Quadratic Model to Fair the Lag-Leg

Another important change from the original simulation that was tried was to use a quadratic model to fair the bearings in the lag-leg. This produced better accuracy in the estimated Target track parameters. It is noted that current manual TMA methods (in particular, the Time-Bearing Plot) use linear fairing.

3. Increase in Turn Time

Contrary to Ekelund ranging, if an instantaneous turn is considered, large errors in the estimated Target track parameters are generated. This is because some actual and corresponding extrapolated bearings are too close to each other (i. e., a small baseline). The intersection of those bearings often results in a estimated Target position far away from the true position of the Target.

Based on these considerations, it was determined that the time required for Own Ship to turn should be increased. After many trials, it was found that the best turn duration is ten minutes, if leg length is also ten minutes. For longer times, the bearing rate in the lead-leg does not remain low and constant, reducing the accuracy of the lead-leg bearing extrapolation.

B. ELIMINATION OF THE LEAD-LEG-ESTIMATION

As shown in Chapter II, when linear extrapolation of the bearings from the lag-leg is used, increased errors are generated in the estimation of the Target track parameters. This occurs because the lag-leg generates a high and variable bearing rate compared to the lead-leg. A simple solution to the problem is to drop the estimated Target positions generated by the extrapolation of the lag-leg bearings.

One problem with this idea is that a lead-lag maneuver will then produce more accurate estimates of Target position at the end of the maneuver than will a lag-lead maneuver. This occurs since in the lead-lag case, the estimated Target positions are obtained for times during the second (i.e., the lag) leg. And in the lag-lead case, the estimated Target positions are for times in the first (again, the lag) leg. So to use a lag-lead maneuver to estimate Target positions at the end of the maneuver requires that these positions be extrapolated in time using derived estimates of Target course and speed. This leads to accumulated errors in the final Target position.

To test for the possible improvements that might result when using only lag-leg estimation, simulation experiments were conducted with the following parameters:

1. Speed of Own Ship: 5, 10, and 15 knots.
2. Speed of the Target: 5, 15, and 25 knots.
3. Initial range: large (= 60,000), medium (= 30,000), and short (= 10,000) yds.
4. Only the lead-lag maneuver is considered.
5. Only the initial lead angle of 90° is cosidered.
6. Leg length: 4 minutes.
7. Time between legs: 6 minutes.
8. Standard deviation bearing error: 0°, 0.5°, 1.0°, and 1.5°.
9. Bearings for the first leg were faired and extrapolated linearly.
10. Bearings in the second leg were faired linearly.

It is important to note that other values for lead angle were not considered because, as discussed in Chapter II, it was found that the best results were obtained for a 90° lead angle.

In order to show how the minor changes improve the performance of this major suggested change (i. e., elimination of the lead-leg estimation), the simulation was run for three different cases:

1. Without Minor Changes

The simulation program was run 100 times for each combination of the above parameter values. It was determined that for the ideal case (0° bearing errors) the accuracy of the predicted Target parameters was adequate, but with bearing errors introduced, large errors in all three Target parameters were produced. So it was concluded that simply eliminating the lead-leg estimation was not a sufficient improvement.

2. With Increased Leg Length and Lag-Leg Faired with a Quadratic Model

Here leg length was increased to ten minutes and the lag-leg was faired with a quadratic model. The same simulation experiments were run as before. Only the results for lead-lag maneuvers are presented in Table 5. Lag-lead maneuvers gave uniformly worse results for reasons discussed above. Additionally, and more surprisingly, the accuracy for the lag-lead maneuver was not as good as the lead-lag, even when estimating the Target parameters at the end of the first leg. This was a confirmation that, in general, the lead-lag maneuver gives better results than do lag-lead [Ref. 7: pp. 3].

Results for estimated Target course and speed were very precise only when the bearing error was zero and when Own Ship speed was greater than ten knots. When errors in bearings were included, the accuracy in estimated Target course and speed was tremendously reduced. For example, for large initial distance and 15 knots Own Ship speed the following was obtained:

1. With no bearing error, the error in estimated speed was always less than 10% and the error in course was less than 10° in 83% of the different geometries.
2. With a small bearing error (standard deviation = 0.5°), only 33.4% of the runs had an error in speed of less than 10% . In 15.1% of the runs, the error in course was less than 10°.

One of the conclusions of this study is that the Bearing Extrapolation Procedure, even as modified here, does not produce accurate estimates of Target course and speed. With Target range, however, more success was obtained. Table 5 shows the percentage of simulation runs that produced an estimated Target range at the end of the maneuver within 5%, 10%, and 20% of the true range.

Table 5. ACCURACY IN ESTIMATED RANGE WHEN ONLY LAG-LEG-ESTIMATION IS USED. (LEG LENGTH = 10 MIN).

Own Ship speed (knts)	St. Dev. bearing error	Initial distance	Percentage of times the absolute error is smaller than 5%, 10%, and 20% of actual for range.		
			5%	10%	20%
15	1.5°	large	21.6	41.7	72.1
		medium	28.3	54.6	82.2
		short	10.9	19.7	40.9
	1.0°	large	30.6	57.1	84.6
		medium	35.4	62.9	86.6
		short	10.7	18.6	40.2
	0.5°	large	49.9	78.8	93.7
		medium	43.8	71.1	88.6
		short	10.3	18.9	39.6
	0.0°	large	72.2	88.9	94.4
		medium	55.6	77.8	88.9
		short	11.1	22.2	38.9
10	1.5°	large	15.2	30.6	55.9
		medium	20.2	39.4	70.7
		short	16.4	32.4	51.3
	1.0°	large	21.0	41.2	70.1
		medium	26.2	[illegible]9.5	76.6
		short	16.1	32.7	52.6
	0.5°	large	35.4	62.1	88.7
		medium	35.4	61.3	79.5
		short	15.3	32.3	52.5
	0.0°	large	72.2	77.8	94.4
		medium	38.9	72.2	77.8
		short	16.7	27.8	50.0

From Table 5 it can be seen that the larger the Own Ship speed , the better is the estimate of range. This is shown in Figure 9 which is the plot of the accuracy in estimated range versus standard deviation of bearing errors when the Own Ship speed is 10 and 15 knots. This accuracy is the percentage of times that the absolute error is within 20% of the actual for range at the end of the maneuver.

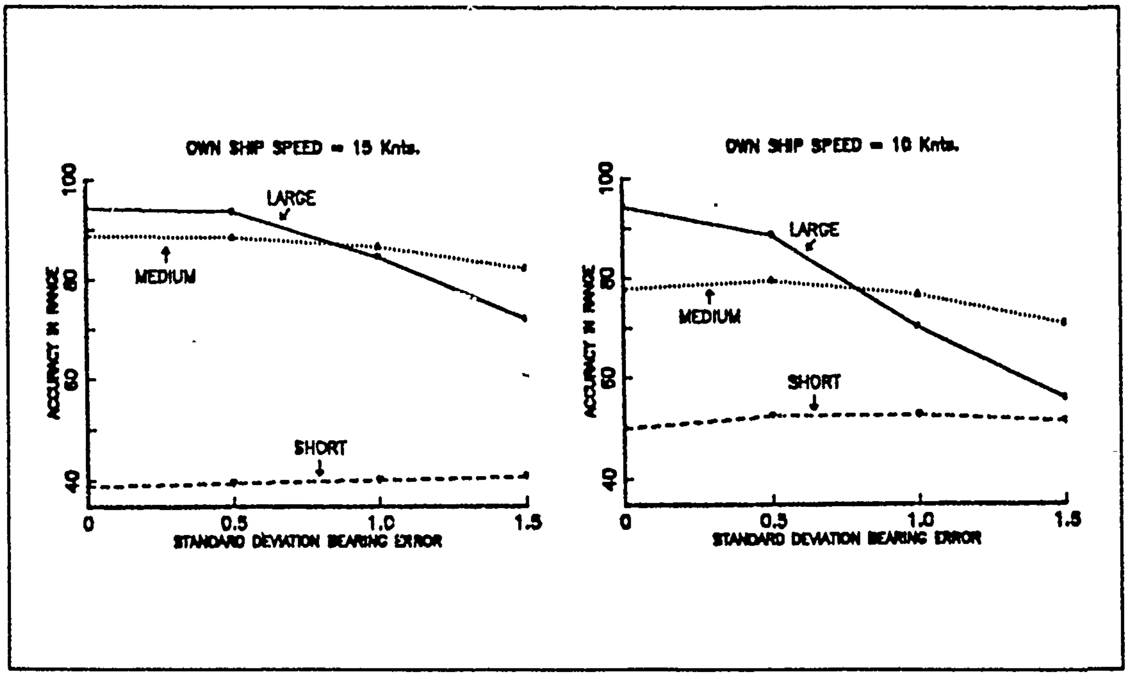

Figure 9. Comparison of Accuracy for Different Own Ship Speeds by Using only the Lag-Leg-Estimation.

Figure 10 shows the comparison of the results of these suggested improvements with the results obtained in Chapter II for the central idea. It appears that for short initial distances the accuracy of the procedure decreases, but for large and medium distances, even with large bearing error (standard deviation = 1.5°), the accuracy increases considerably. Note that the accuracy for short distances was not adequate in either case.

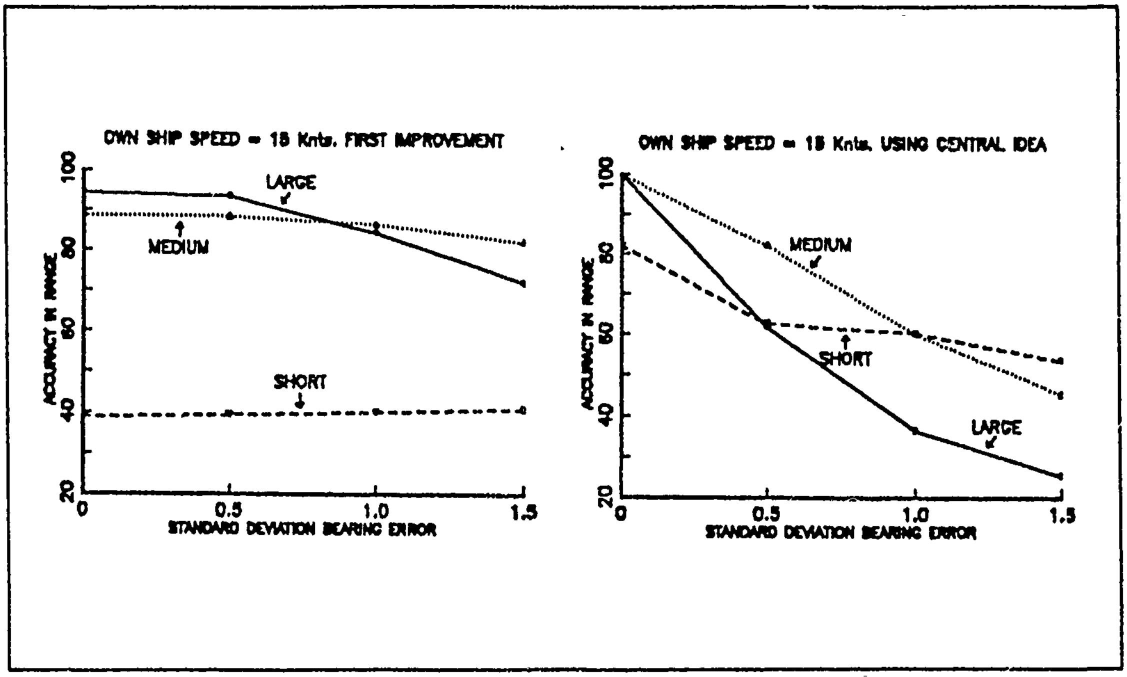

Figure 10. Comparison of the Accuracy Between Results by Using only the Lag-Leg-Estimation and the Results by Using the Central Idea.

3. With Previous Minor Changes Plus an Increase in Turn Duration

In addition to the two minor changes made before, here the turn duration was increased to ten minutes and the simulation experiments were repeated. Table 6 shows the results for this case.

Table 6. ACCURACY IN ESTIMATED RANGE WHEN ONLY LAG-LEG-ESTIMATION IS USED. (LEG LENGTH = 10 MIN. TURN TIME = 10 MIN.)

Own Ship speed (knts)	St. Dev. bearing error	Initial distance	Percentage of times the absolute error is smaller than 5%, 10%, and 20% of actual for range.		
			5%	10%	20%
15	1.5°	large	25.5	47.7	78.3
		medium	30.6	54.3	83.3
		short	12.1	21.7	38.6
	1.0°	large	34.8	62.0	87.8
		medium	37.3	59.9	86.7
		short	13.3	22.9	38.2
	0.5°	large	44.3	73.5	91.3
		medium	40.7	62.1	87.8
		short	14.4	24.2	38.6
	0.0°	large	66.7	88.9	94.4
		medium	50.0	61.1	88.9
		short	16.7	22.2	38.9
10	1.5°	large	18.3	36.2	65.1
		medium	24.6	46.2	72.0
		short	14.7	28.7	50.3
	1.0°	large	25.9	48.2	78.4
		medium	29.6	53.9	75.5
		short	13.3	28.1	50.6
	0.5°	large	31.8	59.0	85.5
		medium	32.2	58.6	76.9
		short	13.2	26.6	50.9
	0.0°	large	66.7	77.8	94.4
		medium	33.3	66.7	77.8
		short	16.7	22.2	44.4

By comparing the results obtained for the accuracy in range in Table 5 (Own ship turn = 6 minutes) and Table 6 (Own ship turn = 10 minutes), it is appears that small improvements in precision are obtained by using ten minutes for the turn. Figure 11 is a graphic representation of this fact.

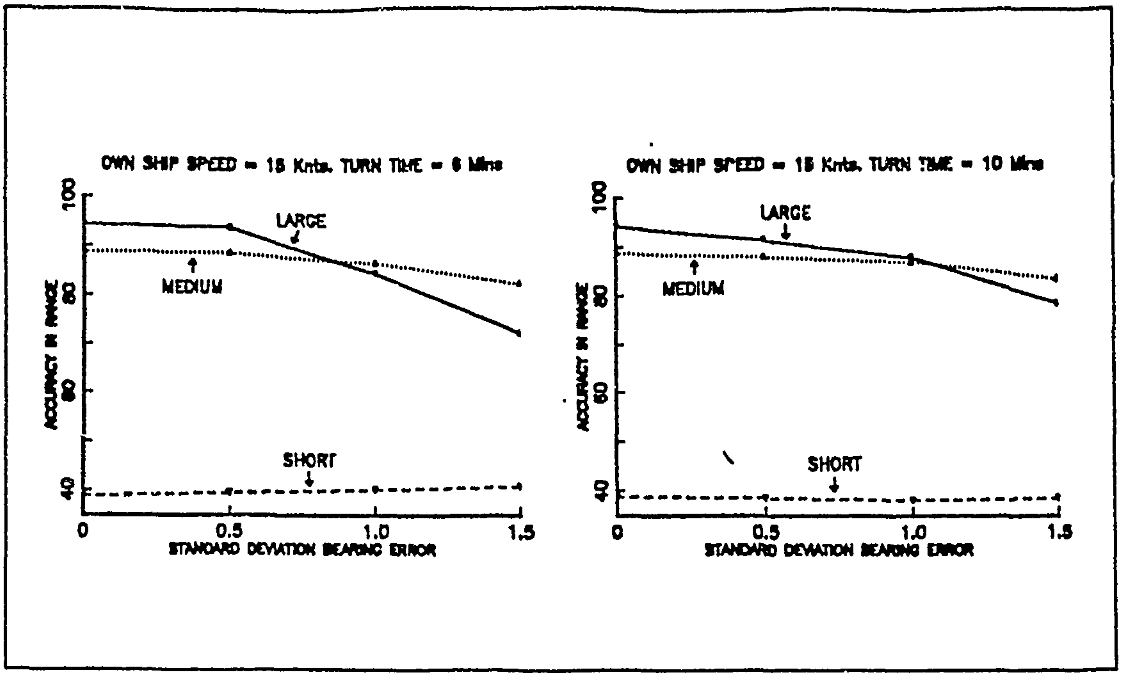

Figure 11. Comparison of Accuracy for Different Own Ship Turn Times by Using only the Lag-Leg-Estimation.

a. Effect of Target Courses Greater than 150° when Lag-Leg Estimation and Minor Changes are Used

It was stated in Chapter II that the bearing rate for the lead-legs does not remain constant when the Target course is greater than 150° (initial angle on the bow less than 30°). This is because range during the maneuver decreases to the point where the lead-leg bearings do not maintain linearity. To determine the effect on the estimation of Target range that this behavior produces, simulations were conducted as before except that results were obtained for 12 separate Target courses between 030° and 175°. Previous results were averaged over Target course, so the effect of a particular Target course was not observed.

The results of one simulation experiment are depicted in Figure 12 and presented in Table 7. The parameters used to obtain these results were the following:

1. Initial distance: 30,000 yds.
2. Target couses: 030°, 050°, 070°, 090°, 110°, 130°, 150°, 155°, 160°, 165°, 170°, and 175°.
3. Target speeds: 25, 15, and 5 knots.

4. Own Ship speed: 15 knots.
5. Standard deviation of bearing error: 1.5°.
6. Leg length: 10 minutes.
7. Turn time: 10 minutes.

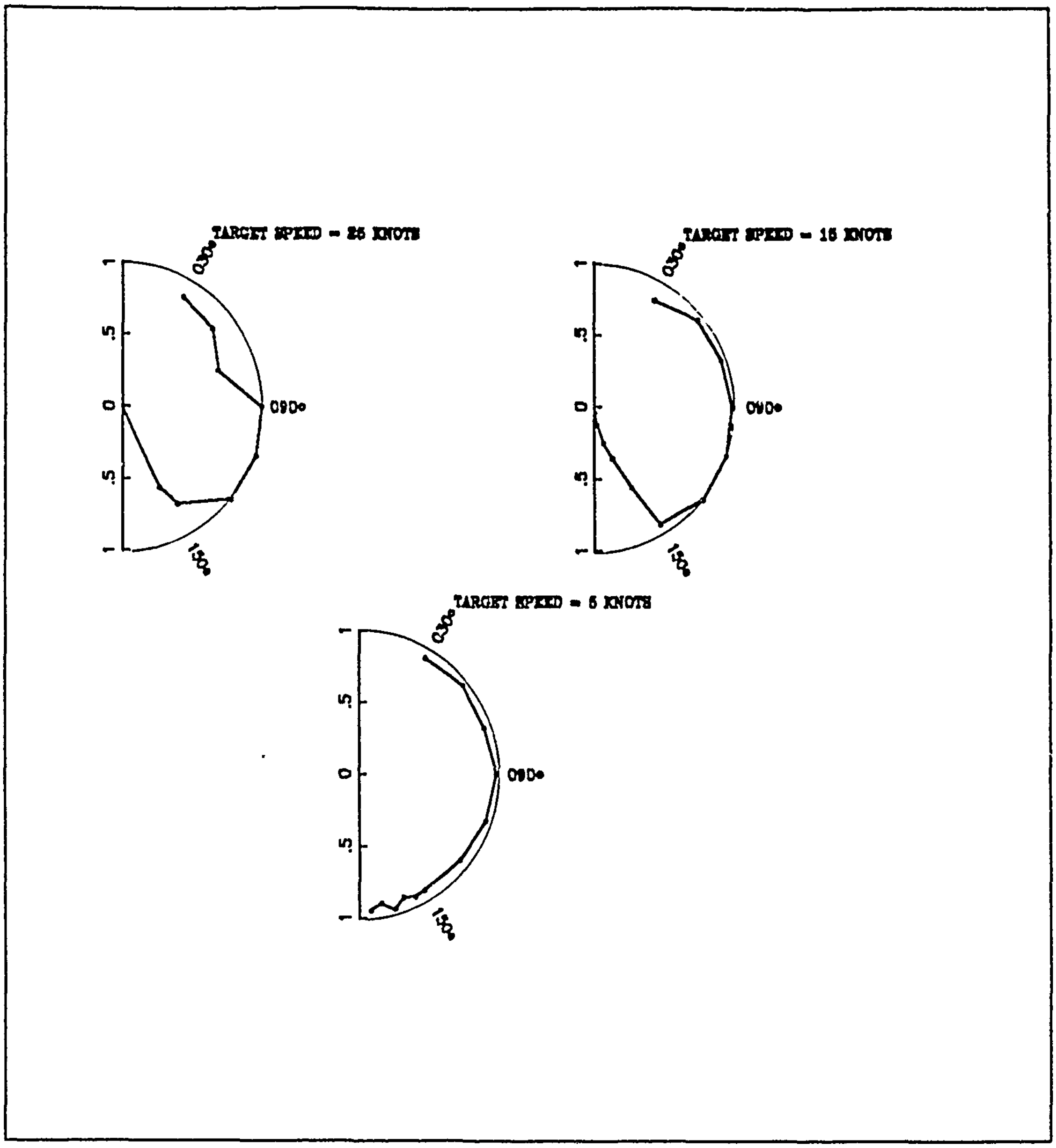

Figure 12. Fraction of Times Absolute Error is within 20% of Actual for Range, for Different Target Courses and Speeds.

Table 7. FRACTION OF TIMES ABSOLUTE ERROR IS WITHIN 20% OF ACTUAL FOR RANGE, FOR DIFFERENT TARGET COURSES AND SPEEDS.

Target speed (knts)	Target course (°)											
	030	050	070	090	110	130	150	155	160	165	170	175
25	0.87	0.83	0.72	0.98	1.00	1.00	0.78	0.62	0.00	0.00	0.00	0.00
15	0.86	0.95	0.95	0.97	0.99	1.00	0.94	0.62	0.38	0.26	0.13	0.05
5	0.94	0.96	0.94	0.97	0.95	0.93	0.93	0.94	0.91	0.97	0.91	0.95

From Figure 12 and Table 7. it can be seen that for a Target speed of 5 knots, approximately equal accuracy was obtained for all Target courses. But for Target speeds of 15 and 25 knots, much less accuracy was obtained when the Target course was between 150° and 175°. For Target courses less than 150°, there was less observed change in the estimation of Target range.

C. USE OF AN ARCTANGENT MODEL TO EXTRAPOLATE THE LAG-LEG BEARINGS

As presented so far, a major problem with the Bearing Extrapolation procedure is that the lag-leg bearings cannot be linearly extrapolated forward or backward in time while maintaining accuracy. The solution to this problem suggested in the previous several sections was simply not to use extrapolated bearings from the lag-leg. The problem with this approach is that tracking information is lost when these extrapolated bearings are not used. In this section, an attempt will be made to recover these extrapolated bearings by using an arctangent, rather than linear, extrapolation model.

It is known that without change in Own Ship course, bearing to a constant course and speed moving Target is of the form

$$B_t = B_0 + \tan^{-1}[\frac{V_0}{R_{cpa}}(t - t_0)], \tag{3.0}$$

where

B_0 is the bearing to the Target at CPA[2],

2 Closest Point of Approach

V_0 is the relative speed between Target and Own Ship,

R_{cpa} is the range to the Target at CPA,

t_0 is the time at which CPA is to occur. [Ref. 8: pp. 2]

Assuming that the extrapolated bearings for the lag-leg follow this arctangent model, two different attempts were made in order to include these bearings in the solution of the problem.

1. Fitting the Arctangent Model by Using Least Squares

An attempt was made to use a least squares technique to find the values of B_0, $\frac{V_0}{R_{cpa}}$, and t_0 in (3.0) which produced the best fit to the observed bearing data. Although this approach was ultimately unsuccessful, there may be another least squares procedure that will prove more robust. To help any subsequent researcher, the details of this attempt are presented to illustrate what has already been examined.

Let

$$\alpha = \frac{V_0}{R_{cpa}}.$$

Now make the change of variable

$$Y_t = \tan(B_t - B_0),$$

or

$$Y_t = \alpha(t - t_0).$$

Then applying least squares, it is necssary to minimize

$$SS = \sum_{t=1}^{n} (Y_t - \alpha t + \alpha t_0)^2.$$

A necessary condition is

$$\frac{\partial SS}{\partial \alpha} = -2\sum_{t=1}^{n}(Y_t - \alpha t + \alpha t_0)t + 2\sum_{t=1}^{n}(Y_t - \alpha t + \alpha t_0)t_0 = 0, \tag{3.1}$$

and

$$\frac{\partial SS}{\partial t_0} = 2\sum_{t=1}^{n}(Y_t - \alpha t + \alpha t_0)\alpha = 0. \tag{3.2}$$

Since α cannot be zero unless relative speed (V_0) is zero, from (3.2),

$$\sum_{t=1}^{n}(Y_t - \alpha t + \alpha t_0) = 0. \tag{3.3}$$

Dividing (3.3) by n and solving for α,

$$\alpha = \frac{\overline{Y}}{\bar{t} - t_0}. \tag{3.4}$$

Now substituting (3.3) in (3.1) gives

$$\sum_{t=1}^{n} tY_t - \alpha\sum_{t=1}^{n} t^2 + \alpha t_0 \sum_{t=1}^{n} t = 0. \tag{3.5}$$

Substituting (3.4) in (3.5) and solving for t_0 yields

$$t_0 = \frac{\bar{t}\sum_{t=1}^{n} tY_t - \overline{Y}\sum_{t=1}^{n} t^2}{\sum_{t=1}^{n} tY_t - \overline{Y}\sum_{t=1}^{n} t}. \tag{3.6}$$

Because the determination of Y_t for all t, requires knowledge of the value of the bearing at CPA (B_0), an iterative procedure was attempted where the first value for B_0 was guessed, then with equations (3.5) and (3.6), α and t_0 were computed. Then by using

$$B_0 = \frac{1}{n}\sum_{t=1}^{n}\{B_t - \tan^{-1}[\alpha(t - t_0)]\},$$

a new value for B_0 was obtained. This iteration was continued in an attempt to converge on the true B_0. Unfortunately for even small errors in bearings, (standard deviation = 0.5°), the procedure would not reliably converge.

2. Estimation of the Parameters for the Arctangent Model by Solving a System of Linear Equations

It is known that with three different bearings to a Target that is moving with a constant course and speed, the unknown parameters of the equation

$$B_t = B_0 + \tan^{-1}[\alpha(t - t_0)]$$

can be determined. [Ref. 7: pp. 2]

Now let B_1, B_2, and B_3 be the bearings obtained at times t_1, t_2, and t_3 respectively. Then it can be written

$$B_1 = B_0 + \tan^{-1}[\alpha(t_1 - t_0)],$$

$$B_2 = B_0 + \tan^{-1}[\alpha(t_2 - t_0)],$$

$$B_3 = B_0 + \tan^{-1}[\alpha(t_3 - t_0)].$$

An attempt will be made to solve for B_0, α, and t_0. Substracting,

$$B_2 - B_1 = \tan^{-1}[\alpha(t_2 - t_0)] - \tan^{-1}[\alpha(t_1 - t_0)],$$

$$B_3 - B_1 = \tan^{-1}[\alpha(t_3 - t_0)] - \tan^{-1}[\alpha(t_1 - t_0)],$$

$$B_3 - B_2 = \tan^{-1}[\alpha(t_3 - t_0)] - \tan^{-1}[\alpha(t_2 - t_0)].$$

Now taking the tangent of both sides and knowing the trigonometric identity for the tangent of the difference of two angles, the following can be obtained:

$$\tan(B_2 - B_1) = \frac{\alpha(t_2 - t_1)}{1 + \alpha^2(t_2 - t_0)(t_1 - t_0)},$$

$$\tan(B_3 - B_1) = \frac{\alpha(t_3 - t_1)}{1 + \alpha^2(t_3 - t_0)(t_1 - t_0)},$$

$$\tan(B_3 - B_2) = \frac{\alpha(t_3 - t_2)}{1 + \alpha^2(t_3 - t_0)(t_2 - t_0)}.$$

From these follows that

$$(t_2 - t_0)(t_1 - t_0)\tan(B_2 - B_1) = \frac{1}{\alpha}(t_2 - t_1) - \frac{\tan(B_2 - B_1)}{\alpha^2}, \tag{3.7}$$

$$(t_3 - t_0)(t_1 - t_0)\tan(B_3 - B_1) = \frac{1}{\alpha}(t_3 - t_1) - \frac{\tan(B_3 - B_1)}{\alpha^2}, \tag{3.8}$$

$$(t_3 - t_0)(t_2 - t_0)\tan(B_3 - B_2) = \frac{1}{\alpha}(t_3 - t_2) - \frac{\tan(B_3 - B_2)}{\alpha^2}. \tag{3.9}$$

Dividing (3.7) by (3.8), (3.7) by (3.9), and (3.8) by (3.9), allows the solution for t_0, α, and B_0.

$$t_0 = \frac{t_2\tan(B_3 - B_2)\tan(B_2 - B_1)[(t_2 - t_3)(t_3 - t_1) - (t_3 - t_1)(t_2 - t_1)] + t_1(t_3 - t_2)^2}{(t_2 - t_3)\tan(B_2 - B_1)[(t_3 - t_1)\tan(B_3 - B_2) - (t_3 - t_2)\tan(B_3 - B_1)]}$$
$$\frac{\tan(B_2 - B_1)\tan(B_3 - B_1) + t_3(t_2 - t_1)^2\tan(B_3 - B_2)\tan(B_3 - B_1)}{-(t_2 - t_1)\tan(B_3 - B_2)[(t_3 - t_1)\tan(B_2 - B_1) - (t_2 - t_1)\tan(B_3 - B_1)]},$$

$$\alpha = \frac{(t_2 - t_3)\tan(B_2 - B_1)\tan(B_3 - B_1)}{(t_2 - t_0)(t_3 - t_1)\tan(B_2 - B_1) + (t_2 - t_1)(t_0 - t_3)\tan(B_3 - B_1)},$$

$$B_0 = B_1 - \tan^{-1}[\alpha(t_1 - t_0)].$$

A subroutine to compute these values was written and the simulation of the problem was done with the following conditions:

1. Speed of Own Ship: 5, 10, and 15 knts.
2. Speed of the Target: 5, 15, and 25 knts.
3. Initial range: 10,000, 30,000, and 60,000 yds.
4. Initial angle in the bow for Own Ship: 90°.
5. Legs length: 10 minutes.
6. Time between legs: 6 minutes.
7. Standard deviation bearing error: 0°, 0.5°, 1.0°, and 1.5°.
8. Only lead-lag maneuver is considered.
9. Bearings for the first leg were faired and extrapolated linearly.
10. Bearings in the second leg were faired by using a quadratic model.
11. The extrapolation of the bearings for the second leg was done by using the formulas obtained above and by using faired bearings from that leg. This was done

because the use of raw bearings produced too much error in the estimation of the parameters of the Target.

Results are presented in Table 8. It is emphasized that these results are only for lead-lag maneuvers.

Table 8. ACCURACY IN ESTIMATED RANGE BY USING THE ARCTANGENT EXTRAPOLATION MODEL FOR THE LAG-LEG (LEAD-LAG MANEUVER).

Own Ship speed (knts)	St. Dev. bearing error	Initial distance	Percentage of times the absolute error is smaller than 5%, 10%, and 20% of actual for range.		
			5%	10%	20%
15	1.5°	large	20.4	40.4	70.6
		medium	26.0	50.9	77.8
		short	11.5	25.2	46.9
	1.0°	large	29.6	54.8	83.5
		medium	31.1	59.2	83.8
		short	11.0	25.2	47.1
	0.5°	large	45.9	75.5	93.4
		medium	37.8	66.8	87.2
		short	17.2	31.8	52.1
	0.0°	large	66.7	88.9	94.4
		medium	55.6	66.7	88.9
		short	11.1	27.8	50.0
10	1.5°	large	16.4	30.9	58.6
		medium	23.0	43.8	77.1
		short	19.6	34.4	59.4
	1.0°	large	22.9	43.0	72.5
		medium	29.8	53.9	83.9
		short	19.5	34.4	60.8
	0.5°	large	34.9	60.3	87.2
		medium	32.9	59.1	78.2
		short	13.2	29.1	56.1
	0.0°	large	72.2	77.8	94.4
		medium	38.9	72.2	77.8
		short	11.1	27.8	55.6

Comparing these results with those on Table 3 on page 13, it is clear that this procedure performs better than the original (unimproved) Bearing Extrapolation Procedure. However, the results are about the same as those in Table 8, indicating that simply not using the lag-leg extrapolated bearings is just about as effective as trying to extrapolate them with the arctangent model.

IV. CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

1. The proposed procedure does not allow estimation, with adequate accuracy, of the Target track parameters when linear bearing extrapolation is used for both legs (central idea). Linear bearing extrapolation is only valid for lead-legs where low and constant bearing rate is obtained. For lag-legs the bearing rate is generally higher and more variable.

2. Adequate accuracy for course and speed is not obtained using any of the examined procedures unless bearing error is zero (ideal conditions).

3. Adequate accuracy in estimated range for large and medium initial distances can be obtained if:

   a. Quadratic faired bearings from the lag-leg are used in combination with linear extrapolated bearings from the lead leg.

   b. Lead-lag maneuver is performed.

   c. Lead angle close to 90° is used.

   d. Leg length is larger than seven minutes and less than twelve minutes.

   e. Own ship turn time is larger than six minutes and less than ten minutes.

   f. Angle on the bow is not smaller than 30°. If it is, then Target speed is not greater than 5 knots.

4. Adequate accuracy in range for short initial distances is obtained only when the Target is not closing the Own Ship.

2. RECOMMENDATIONS

1. If this procedure is implemented, use only extrapolated bearings from the lead leg (low and constant bearing rate) and faired bearings for the lag-leg (high and variable bearing rate).

2. Further research should be done to obtain a model that allows use of the extrapolated bearings obtained from the lag-leg.

APPENDIX A. DISCUSSION OF SELECTED SOLUTION METHODS

A. SPIESS RANGING

Spiess theory establishes that the problem of obtaining satisfactory solutions for a bearings-only approach against targets moving with a constant course and speed, can be solved by using four bearings and at least one change of course or speed by Own Ship. With this in mind, several graphical methods capable of giving quick reliable solutions were developed. [Ref. 3]

From the Passive Ranging Manual, Volume III, the basic graphical method for Spiess ranging is explained as follows:

> Given three bearings to the Target, observed at times t_1, t_2, and t_3 together with a fourth time, t_4, the locus, L, of all possible Target positions at time, t_4, for all possible Target tracks which satisfy the three bearings at t_1, t_2, and t_3, and which maintain a constant course and speed. is a straight line. The actual position of the Target at time, t_4, is determined by the intersection of L with the observed bearing at time, t_4, (provided SSK changes course and speed).
>
> This locus, L, can be determined by picking out two arbitrary Target tracks which satisfy the three-bearing conditions, and plotting the position of the Target on these tracks at time t_4 Then L is the line through these two plotted positions. This is the basis of the Spiess four-bearing TMA. The bearing lines at t_1, t_2, and t_3 are used to find the locus of Target positions at t_4. The intersection of this locus with the bearing line at t_4 determines one point on the track. Another point is determined by using the bearing lines at t_2, t_3, and t_4 to find the locus of Target positions at t_1. The intersection of this locus with the bearing line at t_1 determines another point on the track. The track is then the line joining the two constructed Target positions.

Because bearings are not precise enough to apply this method, faired bearings are usually used.

B. EKELUND RANGING

The basic Ekelund solution for a constant course and speed moving Target is based on a two leg maneuver by Own Ship. During these legs, Own Ship records actual Target bearings. The rate of change of the bearings is computed for each leg by applying linear least squares techniques. This method assumes that during the turn the distance that Own Ship moves and the change of angle in the bow can be disregarded; i.e., assumes an instantaneous turn. Given this assumption, the range to the Target can be computed. Using Figure 13, derivation of range is as follows:

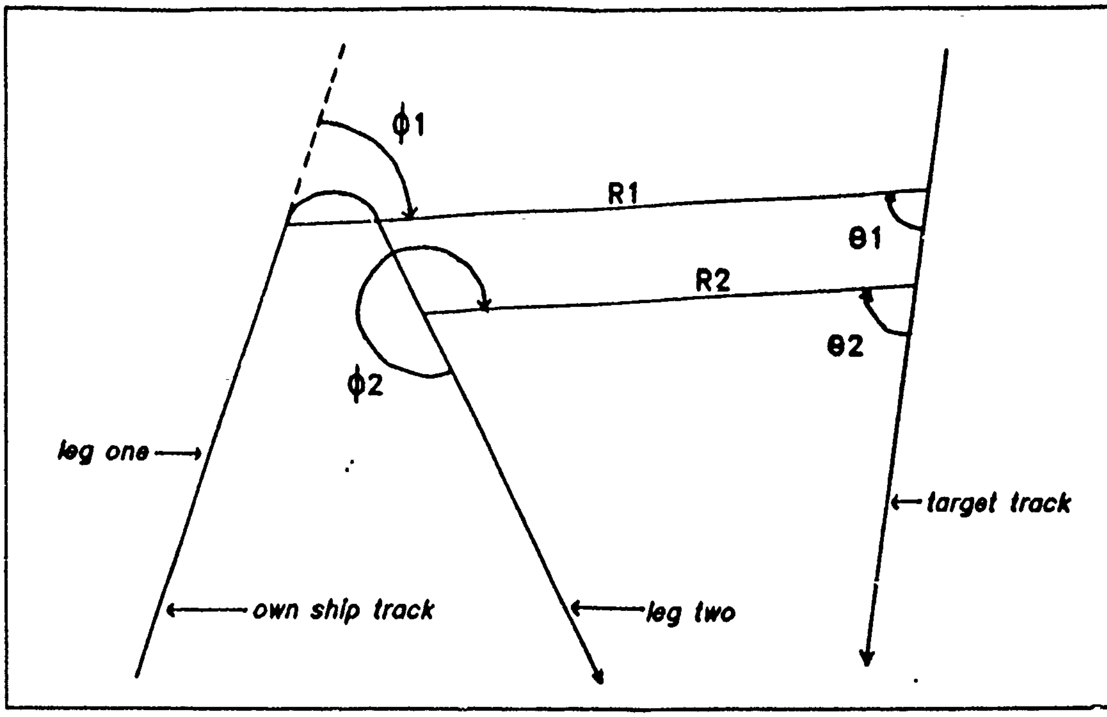

Figure 13. Geometry for Ekelund Ranging Method.

The velocity of the Target across the line of sound for Own Ship's first leg is

$$V_{at1} = V_{t1} \sin \theta_1 = R_1 \frac{\partial \phi_1}{\partial t} = R_1 \dot{B}_{t1} \tag{A.1}$$

and for leg 2

$$V_{at2} = V_{t2} \sin \theta_2 = R_2 \frac{\partial \phi_2}{\partial t} = R_2 \dot{B}_{t2} \tag{A.2}$$

Own ship's velocity across the line of sound for leg 1 can be expressed as

$$V_{ao1} = V_{o1} \sin \phi_1 = R_1 \frac{\partial \theta_1}{\partial t} = R_1 \dot{B}_{o1} \tag{A.3}$$

and for leg 2

$$V_{ao2} = V_{o2} \sin \phi_2 = R_2 \frac{\partial \theta_2}{\partial t} = R_2 \dot{B}_{o2} \tag{A.4}$$

where

$\dot{B}_{ti}$ = Bearing rate of Target in leg i.

R_1 = Range to the Target at the turn.

R_2 = Range to the Target after the turn.

V_{oi} = Velocity of Own Ship in leg i.

V_{ti} = Velocity of the Target in leg i.

Assuming Target velocity remains constant during Own Ship maneuvering; i.e., $V_{t1} = V_{t2} = V_t$ and that the angle in the bow also remains approximately constant; i.e., $\theta_1 = \theta_2 = \theta$, and that the range to the Target does not appreciably changes; i.e., $R_1 = R_2 = R_e$, then equations (A.1), (A.2), (A.3), and (A.4) can be written as

$$V_t \sin \theta_1 = R_e \dot{B}_{t1}, \tag{A.5}$$

$$V_t \sin \theta = R_e \dot{B}_{t2}, \tag{A.6}$$

$$V_{o1} \sin \phi_1 = R_e \dot{B}_{o1}, \tag{A.7}$$

$$V_{o2} \sin \phi = R_e \dot{B}_{o2}. \tag{A.8}$$

and by subtracting (A.7) from (A.5) and (A.8) from (A.6)

$$V_t \sin \theta - V_{o1} \sin \phi_1 = R_e(\dot{B}_{t2} - \dot{B}_{o1}) = R_e \dot{B}_1,$$

$$V_t \sin \theta - V_{o2} \sin \phi_2 = R_e(\dot{B}_{t2} - \dot{B}_{o2}) = R_e \dot{B}_2.$$

Subtracting these last two equations yields

$$R_e = \frac{V_{o1} \sin \phi_1 - V_{o2} \sin \phi_2}{\dot{B}_2 - \dot{B}_1},$$

where, $\dot{B}_i$ = bearing rate of Target relative to Own Ship in leg i.

The problems in accuracy with this method are:

1. It assumes that Own Ship can perform the two legs maneuver with an instantaneous turn [Ref. 9: pp. 1-3].
2. The Ekelund Range Equation is dependent on the ratio of the bearing rates developed on the two legs of the ranging maneuver which are small and not very precise values [Ref. 9: pp. 1-3].

3. In practice, measuring the bearing rate involves a bearing smoothing process which takes place over a portion of each leg and results in bearing rates at times significantly different from the time of the turn [Ref. 9: pp. 3-2].

C. CHURN METHOD

The basic concept of CHURN TMA is similar to that of the strip plot and includes in its solution the Spiess TMA for the particular case where there are only four bearing lines computed [Ref. 9: pp. 7-1, 7-3]. Figure 14, depicts the regular CHURN TMA method.

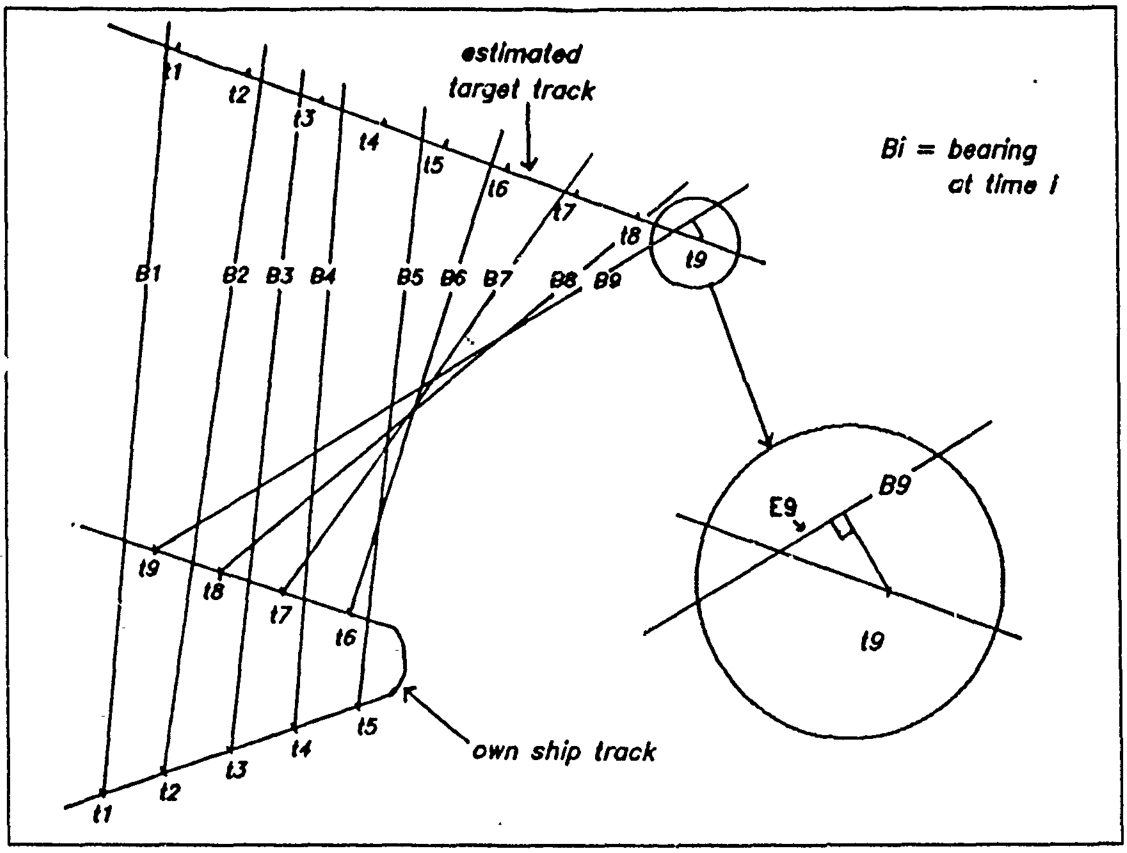

Figure 14. Geometry for CHURN Method.

Own Ship performs a two leg maneuver during which it measures bearings to a Target that is moving with constant course and speed. The CHURN method takes those bearings and fits a Target track to them using the perpendicular distance between a given bearing line and the corresponding Target position on the fitted track as a measure of the goodness of fit. Applying the concept of least squares, the best Target track is that which then minimizes the sum of squares of these distances.[Ref. 9: pp. 7-2]

APPENDIX B. VARIABLES OF THE SIMULATION PROGRAM

1. A,B,C,D,E,F. Entries of the A^{-1} matrix for quadratic regression in the first leg. These values were computed by using APL [Ref. 10]. Then they were introduced as initial data.
2. A0,A1,A2. Coefficients for the quadratic model in the first leg.
3. A01,A11,A22. Coefficients for the quadratic model in the second leg.
4. B22. Temporary variable used to compute the estimated position of the target on the estimated target track at the begining of the maneuver.
5. BEERR. Bearing error.
6. BEEXTR(IN). Vector of extrapolated bearings.
7. BFF. Temporary variable used to compute the estimated position of the target on the estimated target track at the end of the maneuver.
8. BE(IN). Vector of bearings to the target.
9. BE1IN. Y intercept for the linear model of bearings versus time in the first leg.
10. BE1(IN). Temporary vector of bearings to the target.
11. BE2IN. Y intercept for the linear model of bearings versus time in the second leg.
12. BMEAN. Mean of the bearing to the target. Same variable is used for both legs.
13. BO. Bearing to the target at CPA.
14. CC. Counter used in the computation of the precision of the procedure.
15. CINCO. Times that absolute value of the range error is less than 5%.
16. CINCO1. Times that absolute value of the speed error is less than 5%.
17. CINCO2. Times that absolute value of the course error is less than 5°.
18. COFL(2). Vector of own ship courses for the first leg.
19. COSL. Own ship course for the second leg.
20. C5. Percentage of times that error in estimated target course is less than 5°.
21. C10. Percentage of times that error in estimated target course is less than 10°.
22. C20. Percentage of times that error in estimated target course is less than 20°.
23. DEGPI. Pi expressed in degrees.
24. DIEZ. Times that absolute value of the range error is less than 10%.
25. DIEZ1. Times that absolute value of the speed error is less than 10%.
26. DIEZ2. Times that absolute value of the course error is less than 10°.
27. DSEED. Seed for the pseudorandom generation of normal bearing errors.

28. ECTLR. Estimated target course obtained by linear regression on the target fixes.

29. ERANTL. Estimated range to the target at the end of the maneuver.

30. ESLOPE. Temporary variable corresponding to the target course.

31. ESPTL. Estimated speed of the target.

32. EXT(IN). Vector of estimated target positions in the X axis.

33. EYINT. Y intercept of the estimated target course.

34. EYT(IN). Vector of estimated target positions in the Y axis.

35. FBE(IN). Vector of faired target bearings.

36. IN. Time to complete the maneuver in 1/3 of minutes.

37. INFO(IN). Flag used to signal when Estimated X position of the target is equal to the X position of the own ship.

38. IX(3). Vector of initial target distances from own ship in the X axis.

39. IY(3). Vector of initial target distances from own ship in the Y axis.

40. LLL. Counter to determine the amount of fixes used in the regression to determine target track.

41. PER. Percentage error in range computed from the estimated and the simulated range to the target at the end of the maneuver.

42. PER1. Percentage error in speed computed from the estimated and the simulated target speed.

43. PER2. Amount of error in course computed from the estimated and the simulated target course.

44. PI. Pi in radians.

45. R,S,T,U,V,W. Entries of the A^{-1} matrix for quadratic regression in the second leg. These values were computed by using APL [Ref. 10]. Then they introduced as initial data.

46. RANGEF. Simulated range to the target at the end of the maneuver.

47. RCBFL1. Rate of change of bearing during the first leg. This value is computed assuming the linear model for the bearings versus time.

48. RCBSL1. Rate of change of bearing during the Second leg. This value is computed assuming the linear model for the bearings versus time.

49. R5. Percentage of times that error in estimated range to the target is less than 5%.

50. R10. Percentage of times that error in estimated range to the target is less than 10%.

51. R20. Percentage of times that error in estimated range to the target is less than 20%.

52. SCT(6). Vector of simulated target courses.

53. SD. Standard deviation of bearing errors.

54. SLOPE(IN). Vector of faired bearings to the target expressed as slopes in the X-Y plane.

55. SLOPEX(IN). Vector of extrapolated bearings to the target expressed as slopes in X-Y plane.

56. SMALL. Constant that defines equality between simulated Own ship position and estimated target position.

57. SMALLB. Constant used to determine when the faired or extrapolated target bearing correspond to a value of 90° or 270°.

58. SO(3). Vector of simulated own ship speeds.

59. SST(3). Vector of simulated target speeds.

60. SUMB. Summation of the bearing values.

61. SUMB2. Summation of the square of the bearing values.

62. SUMT. Summation of time.

63. SUMTB. Summation of the products of bearing and time.

64. SUMT2. Summation of the squares of the time.

65. SUMT2B. Summation of the products of square of time and bearing.

66. SUMX. Summation of the estimated target positions in the X axis.

67. SUMXY. Summation of the product of estimated target positions in the X axis and the estimated target position in the Y axis.

68. SUMX2. Summation of the square of the estimated target positions in the X axis.

69. SUMY. Summation of the estimated target positions in the Y axis.

70. SUMY2. Summation of the square of the estimated target positions in the Y axis.

71. SXT(IN). Vector of simulated target positions in the X axis.

72. SYT(IN). Vector of simulated target positions in the Y axis.

73. S5. Percentage of times that error in estimated target speed is less than 5%.

74. S10. Percentage of times that error in estimated target speed is less than 10%.

75. S20. Percentage of times that error in estimated target speed is less than 20%.

76. TEL(2). Vector of leg lengths.

77. TEMP. Temporary variable.

78. TMEAN. Mean of the time.

79. TO. Time when CPA occurs.

80. TWO. 2 expressed as double precision constant.

81. VARERB. Variance of bearing error.

82. VEINTE. Times that absolute value of the range error is less than 20%.

83. VEINT1. Times that absolute value of the speed error is less than 20%.

84. VEINT2. Times that absolute value of the course error is less than 20°.

85. VD. Ratio between relative speed of target and own ship and range at CPA.

86. XO(IN). Vector of own ship positions in the X axis.

87. XOEXTR(IN). Vector of extrapolated own ship positions in the X axis.

88. XMEAN. Mean of target positions (fixes) in the X axis.

89. XFF. Estimated target position at the end of the maneuver in the X axis.

90. X22. Estimated target position at the begining of the maneuver in the X axis.

91. YFF. Estimated target position at the end of the maneuver in the Y axis.

92. YMEAN. Mean of target positions (fixes) in the Y axis.

93. YO(IN). Vector of own ship positions in the Y axis.

94. YOEXTR(IN). Vector of extrapolated own ship positions.

95. Y22. Estimated target position at the beggining of the maneuver in the Y axis.

APPENDIX C. SIMULATION PROGRAM CODE

```
      PROGRAM SIMTMA
*******************************************************************************
*                                                                             *
* THIS PROGRAM SIMULATES A PASSIVE BEARINGS-ONLY TARGET MOTION ANALYSIS*
* PROCEDURE WHICH IS PERFORMED BY A SUBMARINE (OWN SHIP) AGAINST A        *
* TARGET WHICH IS MOVING WITH CONSTANT COURSE AND SPEED.                  *
* THE OWN SHIP PERFORMS A TWO LEG MANEUVER. EACH OF THEM WITH CONSTANT *
* COURSE AND SPEED.                                                       *
*                    * *     * *     * *     * *                          *
*                                                                         *
*            WRITTEN BY CMR. CARRERO CUBEROS, BERNABE                     *
*                      VENEZUELAN NAVY                                    *
*                                                                         *
* *****************************************************************************

      PARAMETER(IN = 78)
      INTEGER TEL(2), INFO(IN), LLL, CC, CINCO, DIEZ, VEINTE,
     &CINCO1, DIEZ1, VEINT1, CINCO2, DIEZ2, VEINT2
      DOUBLE PRECISION IX(3), IY(3), DSEED, COFL(2), COSL, SCT(6), SD,
     &SST(3), VARERB, BEERR, EXT(IN), EYT(IN), SO(3), SXT(IN), SYT(IN),
     &XO(IN), YO(IN), BE(IN), SLOPE(IN), BEEXTR(IN), XOEXTR(IN),
     &YOEXTR(IN), SLOPEX(IN), PI, SMALL, DEGPI, TWO, TEMP, SUMX, SUMX2,
     &SUMY2, SUMY, SUMXY, XMEAN, YMEAN, ESLOPE, SMALLB, EYINT, ECTLR,
     &B22, BFF, X22, Y22, XFF, YFF, ESPETL, ERANTL, RANGEF, SUMT, SUMB,
     &SUMT2, SUMB2, SUMTB, TMEAN, BMEAN, RCBFL1, RCBSL1, BE1(IN), BE1IN,
     &BE2IN, FBE(IN), A0, A1, A2, SUMT2B, A, B, C, D, E, F, A01, A11,
     &A21, R, S, T, U, V, W, B0, T0, VD, PER, PER1, PER2, R5, R10, R20,
     &S5, S10, S20, C5, C10, C20
      DATA PI/3.1415926535898D0/, SMALL/0.1D0/, DEGPI/180.D0/, TWO/2.D0/,
     &SCT/153.D0,158.D0,163.D0,167.D0,171.D0,175.D0/, TEL/39,30/,
     &SO/30000.D0,20000.D0,10000.D0/, SST/50000.D0,30000.D0,10000.D0/,
     &COFL/088.D0,272.D0/, SMALLB/.00001D0/, A/118.4785317D0/,
     &B/-3.77185762D0/, C/.02947719689D0/, D/.1205858692D0/,
     &E/-.0009459915779D0/, F/.000007448752582D0/, R/.3438423645D0/,
     &S/-.0450738916D0/, T/.001231527094D0/, U/.007603190052D0/,
     &V/-.00023091133D0/, W/.000007448752582D0/,
     &IY/60000.D0,30000.D0,10000.D0/, IX/0.D0,0.D0,0.D0/
      PRINT*,'ENTER VARIANCE OF BEARING ERROR (SQUARE DEGREES) AND'
      PRINT*,'INITIAL SEED (7 DIGIT INTEGER) TO BEGIN THE SIMULATION'
      READ*,VARERB,DSEED

* TRANSFORMATION OF DATA THAT INVOLVES DEGREES TO RADIANS
      SD = SQRT(VARERB)
      SD = SD*PI/DEGPI
      DO 50 I = 1,2
         COFL(I) = COFL(I)*PI/DEGPI
50    CONTINUE
      DO 100 I = 1,6
         SCT(I) = SCT(I)*PI/DEGPI
100   CONTINUE
```

```
* SELECTION OF TWO DIFFERENT LENGTH LEG
      DO 2000 I = 1,2

* SELECTION FROM THREE DIFFERENT INITIAL DISTANCES BETWEEN TARGET AND
* OWN SHIP
       DO 1999 MMMM = 1,3

* SELECTION FROM THREE DIFFERENT SPEEDS FOR OWN SHIP
        DO 1100 L = 1,3

* SELECTION FROM LEAD-LAG AND LAG-LEAD MANEUVER AND INITIALIZATION
         DO 950 M = 1,1
           CC = 0
           CINCO = 0
           DIEZ = 0
           VEINTE = 0
           CINCO1 = 0
           DIEZ1 = 0
           VEINT1 = 0
           CINCO2 = 0
           DIEZ2 = 0
           VEINT2 = 0

* SELECTION FROM SIX DIFFERENT TARGET COURSES
           DO 900 J = 1,6

* SELECTION FROM THREE TARGET SPEEDS
             DO 850 K = 1,3

* REPETITION OF THE SIMULATION 100 TIMES WITH DIFFERENT SEED
               DO 793 JIJ = 1,100

* INITIALIZATION AND SIMULATION OF TARGET POSITION EVERY 20 SECONDS
                 DO 150 II = 1,2*TEL(I)
                   SXT(II) = 0.D0
                   SYT(II) = 0.D0
                   XO(II) = 0.D0
                   YO(II) = 0.D0
                   BE(II) = 0.D0
                   BE1(II) = 0.D0
                   FBE(II) = 0.D0
                   SLOPE(II) = 0.D0
                   SLOPEX(II) = 0.D0
                   XOEXTR(II) = 0.D0
                   YOEXTR(II) = 0.D0
                   BEEXTR(II) = 0.D0
                   INFO(II) = 0.D0
                   TEMP = (DBLE(II))/180.D0
                   SXT(II) = IX(MMMM) + COS(SCT(J)-PI/TWO)*SST(K)*TEMP
                   SYT(II) = IY(MMMM) - SIN(SCT(J)-PI/TWO)*SST(K)*TEMP
150              CONTINUE

* INITIALIZATION OF REGRESSION TO COMPUTE RATE OF CHANGE OF BEARING IN
* FIRST LEG.
                 SUMT = 0.D0
                 SUMT2 = 0.D0
```

```
      SUMB2 = 0.D0
      SUMB = 0.D0
      SUMTB = 0.D0
      SUMT2B = 0.D0
      DO 750 N = 1,TEL(I)-9

* SIMULATION OF POSITION OF OWN SHIP EVERY 20 SECONDS IN THE FIRST LEG.
        TEMP = (DBLE(N))/180.D0
        XO(N) = COS(COFL(M) - PI/TWO)*SO(L)*TEMP
        YO(N) = SIN(PI/TWO - COFL(M))*SO(L)*TEMP

* SIMULATION OF THE BEARINGS TO THE TARGET IN THE FIRST LEG.
        CALL NORRN(DSEED,BEERR)
        IF(ABS(SYT(N) - YO(N)) .GE. SMALL)THEN
         IF(ABS(XO(N) - SXT(N)) .GE. SMALL)THEN
          IF(YO(N) .LT. SYT(N))THEN
           IF(XO(N) .LT. SXT(N))THEN
            BE(N) = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
           ELSE
            TEMP = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
            BE(N) = TWO*PI + TEMP
           END IF
          ELSE
           IF(XO(N) .LT. SXT(N))THEN
            TEMP = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
            BE(N) = PI + TEMP
           ELSE
            TEMP = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
            BE(N) = PI + TEMP
           END IF
          END IF
          BE(N) = BE(N) + BEERR*SD
          IF(BE(N) .GE. TWO*PI)BE(N) = BE(N) - TWO*PI
         ELSE
          IF(SYT(N) .GT. YO(N))THEN
          BE(N) = 0.D0
         ELSE
          BE(N) = PI
         END IF
         INFO(N) = 99
         END IF
        ELSE
         IF(SXT(N) .GT. XO(N))THEN
          BE(N) = PI/TWO
         ELSE
          BE(N) = 3.D0*PI/TWO
         END IF
        END IF

* COMPUTATION OF COEFFICIENTS FOR QUADRATIC AND LINEAR LEAST SQUARES
* FITS FOR THE BEARINGS IN THE FIRST LEG.
        SUMT = SUMT + DBLE(N)
        SUMT2 = SUMT2 + (DBLE(N))**2
        IF(BE(N) .GT. 3.D0*PI/TWO)THEN
          BE1(N) = BE(N) - TWO*PI
        ELSE
```

```
                 BE1(N) = BE(N)
               END IF
               SUMB = SUMB + BE1(N)
               SUMB2 = SUMB2 + BE1(N)**2
               SUMTB = SUMTB + (DBLE(N))*BE1(N)
               SUMT2B = SUMT2B + (DBLE(N)**2)*BE1(N)
 750         CONTINUE
             TMEAN = SUMT/DBLE(TEL(I)-9)
             BMEAN = SUMB/DBLE(TEL(I)-9)
             RCBFL1 = (SUMTB - SUMT*BMEAN)/(SUMT2 - SUMT*TMEAN)
             BE1IN = BMEAN - RCBFL1*TMEAN
             A01 = R*SUMB + S*SUMTB + T*SUMT2B
             A11 = S*SUMB + U*SUMTB + V*SUMT2B
             A21 = T*SUMB + V*SUMTB + W*SUMT2B

* COMPUTATION OF THE OWN SHIP POSITION AT THE FICTITIOUS TURN
             TEMP = DBLE(TEL(I))/180.D0
             XO(TEL(I)) = COS(COFL(M) - PI/TWO)*SO(L)*TEMP
             YO(TEL(I)) = SIN(PI/TWO - COFL(M))*SO(L)*TEMP

* COMPUTATION OF FAIR BEARINGS DURING THE FIRST LEG USING THE RESULTS
* FROM LINEAR OR QUADRATIC FIT. THIS DEPEND IF THE MANEUVER IS LEAD-LAG
* OR LAG-LEAD.
             DO 755 N = 1,TEL(I)-9
               IF (M .EQ. 1)THEN
                 FBE(N) = RCBFL1*DBLE(N) + BE1IN
               ELSE
                 FBE(N) = A01 + A11*DBLE(N) + A21*(DBLE(N)**2)
               END IF
               IF(FBE(N) .LT. 0.D0)FBE(N) = TWO*PI + FBE(N)
               IF((ABS(FBE(N) - PI/TWO) .LT. SMALLB) .OR.
     &           (ABS(FBE(N) - 3.D0*PI/TWO) .LT. SMALLB))THEN
                 SLOPE(N) = 0.D0
               ELSE
                 SLOPE(N) = 1.D0/TAN(FBE(N))
               END IF
 755         CONTINUE

* IN CASE OF LAG-LEAD MANEUVER (M=2), CALL SUBROUTINE TANGENT TO COMPUTE
* THE COEFFICIENTS THAT RELATES THE BEARINGS TO THE ARCTANGENT MODEL. IN
* CASE OF LEAD-LAG MANEUVER USE THE LINEAR MODEL ALREADY COMPUTED. WITH
* THIS INFORMATION, COMPUTE THE EXTRAPOLATED BEARINGS FROM THE FIRST LEG

             IF(M .EQ. 2)CALL TNGENT(FBE,IN,M,BO,TO,VD)
             DO 760 N = TEL(I)+10,2*TEL(I)
              IF(M .EQ. 1)THEN
                BEEXTR(N) = RCBFL1*DBLE(N) + BE1IN
              ELSE
                BEEXTR(N) = BO + ATAN(VD*(DBLE(N) - TO))
              END IF
              IF(BEEXTR(N) .LT. 0.D0)BEEXTR(N)=TWO*PI+BEEXTR(N)
              IF((ABS(BEEXTR(N) - PI/TWO) .LT. SMALLB) .OR.
     &          (ABS(BEEXTR(N) - 3.D0*PI/TWO) .LT. SMALLB))THEN
                SLOPEX(N) = 0.D0
              ELSE
                SLOPEX(N) = 1.D0/TAN(BEEXTR(N))
```

```
                    END IF
760               CONTINUE

* SIMULATION OF EXTRAPOLATED POSITIONS OF OWN SHIP OUT OF THE FIRST LEG.
                  DO 765 N = TEL(I)+10, 2*TEL(I)
                    TEMP = (DBLE(N))/180.D0
                    XOEXTR(N) = COS(COFL(M) - PI/TWO)*SO(L)*TEMP
                    YOEXTR(N) = SIN(PI/TWO - COFL(M))*SO(L)*TEMP

765               CONTINUE

* SIMULATION OF FAIR BEARING TO THE TARGET AT THE TURN, BY USING THE
* LINEAR MODEL.
                  FBE(TEL(I)) = RCBFL1*DBLE(TEL(I)) + BE1IN
                  IF(FBE(TEL(I)) .LT. 0.D0)FBE(TEL(I)) = TWO*PI +
     &                      FBE(TEL(I))
                  IF(FBE(TEL(I)) .GE. TWO*PI)FBE(TEL(I)) = FBE(TEL(I)) -
     &                      TWO*PI

* DETERMINATION OF OWN SHIP COURSE FOR THE SECOND LEG ACCORDING WITH THE
* LINE OF SIGHT TO THE TARGET AT THE TURN.
                  IF (M .EQ. 1)THEN
                    COSL = FBE(TEL(I)) -  PI*(.48888889D0)
                  ELSE
                    COSL = FBE(TEL(I)) + PI*(.48888889D0)
                  END IF
                  IF(COSL .LT.  0 D0)COSL = TWO*PI + COSL
                  IF(COSL .GT.  TWO*PI)COSL = COSL - TWO*PI

* INITIALIZATION OF REGRESSION TO COMPUTE RATE OF CHANGE OF BEARING IN
* SECOND LEG.
                    SUMT = 0.D0
                    SUMT2 = 0.D0
                    SUMB2 = 0.D0
                    SUMB = 0.D0
                    SUMTB = 0.D0
                    SUMT2B = 0.D0
                    DO 778 N = TEL(I)+10,2*TEL(I)

* SIMULATION OF POSITION OF OWN SHIP EACH 20 SECONDS IN THE SECOND LEG.
                     TEMP = (DBLE(N) - DBLE(TEL(I)))/180.D0
                     XO(N) = XO(TEL(I)) + SO(L)*TEMP*COS(COSL-PI/TWO)
                     YO(N) = YO(TEL(I)) - SO(L)*TEMP*SIN(COSL-PI/TWO)

* SIMULATION OF THE BEARINGS TO THE TARGET IN THE SECOND LEG.
                     CALL NORRN(DSEED,BEERR)
                     IF(ABS(SYT(N) - YO(N)) .GE. SMALL)THEN
                       IF(ABS(SXT(N) - XO(N)) .GE. SMALL)THEN
                        IF(YO(N) .LT. SYT(N))THEN
                         IF(XO(N) .LT. SXT(N))THEN
                          BE(N) = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
                         ELSE
                          TEMP = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
                          BE(N) = TWO*PI + TEMP
                         END IF
                        ELSE
```

```
         IF(XO(N) .LT.  SXT(N))THEN
          TEMP = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
          BE(N) = PI + TEMP
          ELSE
          TEMP = ATAN((SXT(N) - XO(N))/(SYT(N) - YO(N)))
          BE(N) = PI + TEMP
         END IF
       END IF
       BE(N) = BE(N) + BEERR*SD
       IF(BE(N) .LT.  0.D0)BE(N) = TWO*PI + BE(N)
       IF(BE(N) .GE.  TWO*PI)BE(N) = BE(N) - TWO*PI
      ELSE
       IF(SYT(N) .GT.  YO(N))THEN
        BE(N) = 0.D0
       ELSE
        BE(N) = PI
       END IF
        INFO(N) = 99
      END IF
     ELSE
      IF(SXT(N) .GT.  XO(N))THEN
       BE(N) = PI/TWO
      ELSE
       BE(N) = 3.D0*PI/TWO
      END IF
     END IF

* COMPUTATION OF COEFICIENTS FOR LINEAR AND QUADRATIC FITS FOR THE
* BEARINGS IN THE SECOND LEG
      SUMT = SUMT + DBLE(N)
      SUMT2 = SUMT2 + (DBLE(N))**2
      IF(M .EQ.  1)THEN
        IF(BE(N) .GT.  .65D0*PI)THEN
          BE1(N) = BE(N) - TWO*PI
        ELSE
          BE1(N) = BE(N)
        END IF
      ELSE
        BE1(N) = BE(N)
      END IF
      SUMB = SUMB + BE1(N)
      SUMB2 = SUMB2 + BE1(N)**2
      SUMTB = SUMTB + (DBLE(N))*BE1(N)
      SUMT2B = SUMT2B + (DBLE(N)**2)*BE1(N)
778  CONTINUE
     TMEAN = SUMT/DBLE(TEL(I)-9)
     BMEAN = SUMB/DBLE(TEL(I)-9)
     RCBSL1 = (SUMTB - SUMT*BMEAN)/(SUMT2 - SUMT*TMEAN)
     BE2IN = BMEAN - RCBSL1*TMEAN
     A0 = A*SUMB + B*SUMTB + C*SUMT2B
     A1 = B*SUMB + D*SUMTB + E*SUMT2B
     A2 = C*SUMB + E*SUMTB + F*SUMT2B


* COMPUTATION OF FAIR BEARINGS DURING THE SECOND LEG USING LINEAR
* OR QUADRATIC FIT. THIS DEPEND ON THE TYPE OF MANEUVER.
```

```
                  DO 779 N = TEL(I)+10,2*TEL(I)
                    IF(M .EQ. 1)THEN
                      FBE(N) = A0 + A1*DBLE(N) + A2*(DBLE(N)**2)
                    ELSE
                      FBE(N) = RCBSL1*DBLE(N) + BE2IN
                    END IF
                    IF(FBE(N) .LT. 0.D0)FBE(N) = TWO*PI + FBE(N)
                    IF((ABS(FBE(N) - PI/TWO) .LT. SMALLB) .OR.
     &                (ABS(FBE(N) - 3.D0*PI/TWO) .LT. SMALLB))THEN
                      SLOPE(N) = 0.D0
                    ELSE
                      SLOPE(N) = 1.D0/TAN(FBE(N))
                    END IF
779               CONTINUE

* IN CASE OF LEAD-LAG MANEUVER (M=1), CALL SUBROUTINE TANGENT TO COMPUTE
* THE COEFFICIENTS THAT RELATES THE BEARINGS TO THE ARCTANGENT MODEL. IN
* CASE OF LAG-LEAD MANEUVER USE THE LINEAR MODEL ALREADY COMPUTED. WITH
* THIS INFORMATION, COMPUTE THE EXTRAPOLATED BEARINGS FROM THE SECOND LEG
                  IF(M .EQ. 1)CALL TNGENT(FBE,IN,M,B0,T0,VD)
                  DO 781 N = 1,TEL(I)-9
                    IF(M .EQ. 1)THEN
                      BEEXTR(N) = B0 + ATAN(VD*(DBLE(N) - T0))
                    ELSE
                      BEEXTR(N) = RCBSL1*DBLE(N) + BE2IN
                    END IF
*                   WRITE(1,'(F9.6,1X,I3)')BEEXTR(N),N
                    IF(BEEXTR(N) .LT. 0.D0)BEEXTR(N)=TWO*PI+BEEXTR(N)
                    IF((ABS(BEEXTR(N) - PI/TWO) .LT. SMALLB) .OR.
     &                (ABS(BEEXTR(N) - 3.D0*PI/TWO) .LT. SMALLB))THEN
                      SLOPEX(N) = 0.D0
                    ELSE
                      SLOPEX(N) = 1.D0/TAN(BEEXTR(N))
                    END IF
781               CONTINUE

* SIMULATION OF EXTRAPOLATED OWN SHIP POSITIONS OUT OF THE SECOND LEG.
                DO 782 N = 1,TEL(I)-9
                  TEMP = (DBLE(TEL(I)) - DBLE(N))/180.D0
                  XOEXTR(N) = XO(TEL(I)) - COS(COSL - PI/TWO)*
     &                        SO(L)*TEMP
                  YOEXTR(N) = YO(TEL(I)) + SIN(COSL - PI/TWO)*
     &                        SO(L)*TEMP

782             CONTINUE

* INITIALIZATION OF VARIABLES USED IN THE LINEAR REGRESSION TO COMPUTE
* COURSE OF THE TARGET.
                  SUMX = 0.D0
                  SUMX2 = 0.D0
                  SUMY2 = 0.D0
                  SUMY = 0.D0
                  SUMXY = 0.D0
                  LLL = 0

* COMPUTATION OF EXPECTED POSITION (X,Y) OF THE TARGET.
```

```
                    DO 783 N = 1,2*TEL(I)

* ELIMINATION OF 18 CENTRAL ESTIMATED POSITIONS OF THE TARGET TO ALLOW
* THE MANEUVER OF THE OWN SHIP IN ORDER TO CHANGE COURSE FROM FIRST LEG
* TO SECOND LEG (SIX MINUTES)
                     IF((N .GT. TEL(I)-9) .AND. (N .LE. (TEL(I)+9)))
     &                      GO TO 783

                     IF(INFO(N) .EQ. 99)THEN
                       EXT(N) = XO(N)
                       EYT(N) = SLOPEX(N)*(XO(N) - XOEXTR(N)) + YOEXTR(N)
                     ELSE
                       EXT(N) = XOEXTR(N)
                       EYT(N) = SLOPE(N)*(XOEXTR(N) - XO(N)) + YO(N)
                     END IF

* COMPUTATION OF VALUES OF THE VARIABLES NEEDED TO DO LINEAR REGRESSION
* ON THE TARGET FIXES TO DETERMINE ESTIMATED COURSE
                     LLL = LLL + 1
                     SUMX = SUMX + EXT(N)
                     SUMX2 = SUMX2 + (EXT(N))**2
                     SUMY = SUMY + EYT(N)
                     SUMY2 = SUMY2 + (EYT(N))**2
                     SUMXY = SUMXY + EXT(N)*EYT(N)
783                 CONTINUE
                    XMEAN = SUMX/LLL
                    YMEAN = SUMY/LLL

* ESTIMATED SLOPE (ESLOPE) AND Y INTERCEPT (EYINT) OF TARGET TRACK
                    ESLOPE = (SUMXY - SUMX*YMEAN)/(SUMX2 - SUMX*XMEAN)
                    EYINT = YMEAN - ESLOPE*XMEAN

* ESTIMATION OF COURSE,SPEED AND RANGE OF THE TARGET
                    ECTLR = (PI/TWO - ATAN(ESLOPE))*DEGPI/PI
                    B22 = EYT(1) - EYINT - ESLOPE*EXT(1)
                    BFF = EYT(2*TEL(I)) - EYINT - ESLOPE*EXT(2*TEL(I))
                    Y22 = EYT(1) - B22/(1 + ESLOPE**2)
                    YFF = EYT(2*TEL(I)) - BFF/(1 + ESLOPE**2)
                    X22 = (Y22 - EYINT)/ESLOPE
                    XFF = (YFF - EYINT)/ESLOPE
                    ESPETL = SQRT((XFF - X22)**2+(YFF - Y22)**2)/
     &                       DBLE(2*TEL(I)-1)
                    ERANTL = SQRT((XFF - XO(2*TEL(I)))**2 +
     &               (YFF - YO(2*TEL(I)))**2)
                      RANGEF = SQRT((SXT(2*TEL(I)) - XO(2*TEL(I)))**2 +
     &                        (SYT(2*TEL(I)) - YO(2*TEL(I)))**2)

* INITIAL COMPUTATIONS TO DETERMINE THE PRECISION OF THE PROCEDURE
                    CC =CC + 1

* PRECISION FOR ESTIMATED TARGET RANGE
                    PER = ((ERANTL - RANGEF)*100.D0)/RANGEF
                    IF(ABS(PER) .LE. 20.D0)THEN
                       VEINTE = VEINTE + 1
                         IF(ABS(PER) .LE. 10.D0)THEN
                           DIEZ = DIEZ + 1
```

```
                              IF(ABS(PER) .LE. 5.D0)THEN
                                 CINCO = CINCO + 1
                              END IF
                           END IF
                        ELSE
                        END IF

* PRECISION FOR ESTIMATED TARGET SPEED
                        PER1 = ((ESPETL*180D0 - SST(K))*100.D0)/SST(K)
                        IF(ABS(PER1) .LE. 20.D0)THEN
                           VEINT1 = VEINT1 + 1
                             IF(ABS(PER1) .LE. 10.D0)THEN
                               DIEZ1 = DIEZ1 + 1
                                 IF(ABS(PER1) .LE. 5.D0)THEN
                                    CINCO1 = CINCO1 + 1
                                 END IF
                             END IF
                        ELSE
                        END IF

* PRECISION FOR ESTIMATED TARGET COURSE
                        PER2 = ECTLR - SCT(J)*DEGPI/PI
                        IF(ABS(PER2) .LE. 20.D0)THEN
                           VEINT2 = VEINT2 + 1
                             IF(ABS(PER2) .LE. 10.D0)THEN
                               DIEZ2 = DIEZ2 + 1
                                 IF(ABS(PER2) .LE. 5.D0)THEN
                                    CINCO2 = CINCO2 + 1
                                 END IF
                             END IF
                        ELSE
                        END IF

* OUTPUT INDICATING SIMULATED AND ESTIMATED TARGET TRACK PARAMETERS
                        WRITE(1,'(I3,2X,F9.1,1X,F12.1,2X,2F5.1,1X,2F11.1)
     &                  ')M,RANGEF,ERANTL,SCT(J)*DEGPI/PI,ECTLR,SST(K),
     &                  ESPETL*180.D0
793                  CONTINUE
795                CONTINUE
850              CONTINUE
900            CONTINUE

* FINAL COMPUTATIONS TO DETERMINE THE PRECISION OF THE PROCEDURE.
               R5 = (DBLE(CINCO)/DBLE(CC))*100
               R10 = (DBLE(DIEZ)/DBLE(CC))*100
               R20 = (DBLE(VEINTE)/DBLE(CC))*100
               S5 = (DBLE(CINCO1)/DBLE(CC))*100
               S10 = (DBLE(DIEZ1)/DBLE(CC))*100
               S20 = (DBLE(VEINT1)/DBLE(CC))*100
               C5 = (DBLE(CINCO2)/DBLE(CC))*100
               C10 = (DBLE(DIEZ2)/DBLE(CC))*100
               C20 = (DBLE(VEINT2)/DBLE(CC))*100

* OUTPUT THAT SHOWS THE PRECISION OF THE PROCEDURE
               WRITE(1,'(I1,1X,F7.1,1X,F3.1,1X,F8.1,2X,9F5.1)')
     &              M, SO(L), (SD/PI)*DEGPI, IY(MMMM), R5, R10, R20,
```

```
    &              S5, S10, S20, C5, C10, C20
950       CONTINUE
1100     CONTINUE
1999    CONTINUE
2000   CONTINUE
      STOP
      END
*********************************************************************************
* THIS SUBROUTINE GENERATES ONE NORMAL (0,1) NUMBER BY USING THE BOX *
* MULLER METHOD. THIS NUMBER IS USED BY THE MAIN PROGRAM TO CREATE   *
* THE ERRORS IN THE BEARINGS WHICH ARE ASSUMED NORMAL (0,VARERB)     *
*********************************************************************************
      SUBROUTINE NORRN(DSEED,BEERR)
      DOUBLE PRECISION U(2), D31M1, DSEED, TWO, PI, BEERR
      DATA D31M1 /2147483647.D0/, TWO/2.D0/, PI/3.141592654D0/

* LINEAR CONGRUENTIAL METHOD TO OBTAIN A UNIFORM(0,1)
      DO 5 I =1,2
        DSEED = DMOD(16807.D0*DSEED,D31M1)
        U(I) = DSEED/D31M1
5     CONTINUE

* BOX AND MULLER METHOD TO OBTAIN A NORMAL(0,1)
      BEERR = (SQRT( - TWO*LOG(U(1))))*COS(TWO*PI*U(2))
      RETURN
      END
*********************************************************************************
* SUBROUTINE TO COMPUTE THE PARAMETERS OF THE ARCTANGENT MODEL (BO,   *
* TO, AND VD) BY USING FAIRED BEARINGS FROM THE QUADRATIC MODEL.      *
* NOTE: THIS CASE IS USEFUL ONLY FOR TEN MINUTE LEGS.                 *
*********************************************************************************
      SUBROUTINE TNGENT(FBE,IN,M,BO,TO,VD)
      PARAMETER(PI = 3.141592654D0)
      DOUBLE PRECISION BO, TO, VD, XY, YZ, ZA, T1, T2, T3, FBE(IN),
     &DE, EF, FG, GH, SUMTO, SUMVD, SUMBO
      INTEGER AB, BC, CD, M, IN, COUNT, JK, KJ
      SUMTO = 0.D0
      SUMVD = 0.D0
      SUMBO = 0.D0
      COUNT = 0
      IF(M .EQ. 1)THEN
        JK = 49
        KJ = 78
      ELSE
        JK = 1
        KJ = 30
      END IF
      DO 6 III = JK,(KJ-14)
        AB = III
        BC = III + 7
        CD = III + 14
        T1 = DBLE(AB)
        T2 = DBLE(BC)
        T3 = DBLE(CD)
        DE = T2 - T3
        EF = T3 - T1
```

```
      FG = T3 - T2
      GH = T2 - T1
      XY = TAN(FBE(BC) - FBE(AB))
      YZ = TAN(FBE(CD) - FBE(AB))
      ZA = TAN(FBE(CD) - FBE(BC))
      T0 = (XY*YZ*T1*(FG**2) - ZA*XY*T2*(EF**2) + ZA*YZ*T3*(GH**2))/
   &        (DE*XY*(ZA*EF - YZ*FG) - GH*ZA*(XY*EF - YZ*GH))
      VD = (XY*YZ*DE)/(XY*EF*(T2 - T0) + YZ*GH*(T0 - T3))
      B0 = FBE(AB) - ATAN(VD*(T1 - T0))
      IF(B0 .GT. 2.D0*PI)B0 = B0 - 2.D0*PI
      IF(B0 .GT. 3.D0*PI/2.D0)B0 = B0 - 2.D0*PI
      SUMT0 = SUMT0 + T0
      SUMVD = SUMVD + VD
      SUMB0 = SUMB0 + B0
      COUNT = COUNT + 1
6     CONTINUE
      T0 = SUMT0/DBLE(COUNT)
      VD = SUMVD/DBLE(COUNT)
      B0 = SUMB0/DBLE(COUNT)
      RETURN
      END
```

LIST OF REFERENCES

1. Peppe P. K., Synthetic Target Motion Analysis Solver (SYNSOLVE), 1988.

2. Murphy D. J., Noisy Bearing-Only Target Motion Analysis, Dept. Elec. Eng., Northeastern Univ., 1970

3. Spiess, J.N., Complete Solutions of the Bearings Only Approach Problem, Marine Phis. Lab. Scripps Inst. of Oceanography, California Univ., 1953.

4. Nyhoff, L. R., Fortran 77 for Engineers and Scientists, Macmillan Publishing Company, 1985.

5. Lewis, P. A.,Simulation Methodology for Statisticians, Operations Analists, and Engineers Vol I, Wadsworth & Brooks, 1989.

6. Banks, J.,Discrete-Event System Simulation, Prentice-Hall,Inc., 1984.

7. McCabe, B. J., Why Lead-lag Maneuvers Produce More Accurate Range Estimates than Lag-lead, Daniel Wagner Associates.,1980.

8. McCabe, B. J., Accuracy of Bearing-Only Ranging Algorithms and Tactical Implications, Daniel Wagner Associates., Paoli, Pennsylvania, 1980.

9. Passive Ranging Manual, Vol III, USN, 1970.

10. Turner, J.R. and STSC. Inc., APL is Easy, John Wiley & Sons, Inc., 1987.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Comando Naval de Operaciones Comandancia General de La Armada Av. Vollmer, San Bernardino, Caracas. Venezuela	1
4.	Escuela Superior de Guerra Naval Av. Cuyuni, Colinas de Bello Monte, Caracas. Venezuela	1
5.	Comando de La Escuadra Base Naval "CA Agustin Armario" Puerto Cabello, Edo. Carabobo. Venezuela	1
6.	United States Naval Academy Annapolis, MD 21402-5000	
7.	Commander Submarine Force U.S. Pacific Fleet Pearl Harbor, HI 96860	
8.	COMSUBDEVRON TWELVE U.S Naval Submarine Base New London Groton, CT 06340	1
9.	Comando del Escuadron de Submarinos Base Naval "CA Agustin Armario" Puerto Cabello, Edo. Carabobo. Venezuela	1
10.	Professor Donald P. Gaver Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	1

11. Professor James N. Eagle 1
Department of Operations Research
Naval Postgraduate School
Monterey, CA 93943-5000

12. CDR. Bernabe Carrero Cuberos 3
Comandancia General de La Armada
Av. Vollmer, San Bernardino,
Caracas. Venezuela

13. LCDR. P. Kevin Peppe
USS La Jolla (SSN 701) 1
FPO San Francisco, CA 96671-2381