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13. ABSTRACT (Maximum 200 words)

This project has centered on formulating and solving mathematical problems that arise in the study of systems of conservation laws that are not of the classical, strictly hyperbolic type. Potential applications for these results are found in models for three-phase flow in porous media, for compressible two-phase flow, and for flow in elastic and elastoplastic materials (including continuum models for granular flow). Modelling of many different flow processes has led to systems of conservation laws in which the classical assumptions break down in a way which leads to distrust of the models. Research in this and allied projects is directed at extending the mathematical theory of conservation laws. The practical goal of this research is to discover which models are well-posed, and, hence, to enable applied scientists to discover which are correct descriptions of various observed instabilities.

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FINAL REPORT ON AFOSR-86-0088
NONSTRICTLY HYPERBOLIC CONSERVATION LAWS

Barbara Lee Keyfitz, University of Houston

I. PUBLICATIONS SUPPORTED BY THE PROJECT

The following papers, which acknowledge support under this grant, have been completed under the project. Their content is described in the summary of results, below.

1. B. L. Keyfitz, "An analytic model for change of type in three-phase flow", in Numerical Simulation in Oil Recovery, (ed M. F. Wheeler), IMA Vol 11 (1988), Springer, New York, 149-160.

2. B. L. Keyfitz, "Change of type in three-phase flow: a simple analogue", Jour Diff Eqns, 80 (1989) 280-305.

3. B. L. Keyfitz and H. C. Kranzer, "A viscous approximation to a system of conservation laws with no classical Riemann solution", to appear in Proceedings of the International Conference on Hyperbolic Problems at Bordeaux, (ed C. Carasso).

4. B. L. Keyfitz, "A criterion for certain wave structures in systems that change type", in Current Progress in Hyperbolic Systems: Riemann Problems and Computations, (ed W. B. Lindquist), Contemporary Mathematics Vol 100, Amer Math Soc, Providence (1989) 203-213.

5. B. L. Keyfitz, "Admissibility conditions for shocks in systems that change type", to appear in Proceedings of GAMM International Conference on Problems Involving Change of Type (ed K. Kirchgassner).

6. B. L. Keyfitz, "Shocks near the sonic line: a comparison between steady and unsteady models for change of type", to appear in Proceedings of an IMA Workshop on Nonlinear Evolution Equations that Change Type (eds Keyfitz and Shearer).

7. H. C. Kranzer and B. L. Keyfitz, "A strictly hyperbolic system of conservation laws admitting singular shocks", to appear in Proceedings of an IMA Workshop on Nonlinear Evolution Equations that Change Type (eds Keyfitz and Shearer).

8. B. L. Keyfitz and G. G. Warnecke, "The existence of viscous profiles and admissibility for transonic shocks", preprint 1990.

9. B. L. Keyfitz, "Change of type in simple models for two-phase flow", UH Research Report UH/MD-66, 1989.

10. B. L. Keyfitz and H. C. Kranzer, "A system of conservation laws with no classical Riemann solution", UH Preprint, 1990.

11. B. L. Keyfitz, "Conservation laws that change type and porous medium flow", in preparation for Proceedings of SIAM Conference on Mathematical and Computational Issues in Geophysical Fluid and Solid Mechanics (eds Fitzgibbon and Wheeler).

II. SUMMARY OF RESULTS

This project has centered on formulating and solving mathematical problems that arise in the study of systems of conservation laws that are not of the classical, strictly hyperbolic type. Potential applications for these results are found in models for three-phase flow in porous media, for compressible two-phase flow, and for flow in elastic and elastoplastic materials (including continuum models for granular flow). Modelling of many different flow processes has led to systems of conservation laws in which the classical assumptions break down in a way which leads to distrust of the models. Research in this and allied projects is directed at extending the mathematical theory of conservation laws. The practical goal of this research is to discover which models are well-posed, and, hence, to enable applied scientists to discover which are correct descriptions of various observed instabilities.

The classical theory of conservation laws considers a system

$$\frac{\partial}{\partial t} U + \sum \frac{\partial}{\partial x_i}(F_i(U)) = 0 \tag{1}$$

for a state variable $U(x,t) = (u_1, \ldots, u_n)$ and flux functions $F_1, \ldots, F_m$. Hyperbolicity means that the eigenvalues, λ_i, of

$$A(\xi,U) = \sum \xi_i dF_i(U) \tag{2}$$

are real, for all states U and directions ξ_i. We are interested in systems in which the system is hyperbolic in a region $\mathcal{H}$ of state space, but a pair of eigenvalues of (2) becomes complex as the state U crosses a boundary $\mathcal{B}$ and enters a region $\mathcal{E}$. For most of the examples, $m = 1$ and $n = 2$, and $\mathcal{E}$ may be called the elliptic region (of state space). This description fits models for dynamic phase boundaries (such as occur in two-phase elasticity, in fluids of van der Waals type and in austenitic-martensitic transitions in shape-memory alloys), and several models for low-speed flow (such as three-phase pressure-driven flow in porous media and two-way traffic flow). The so-called single pressure model for two-phase compressible, nonreacting unsteady flow in one space dimension is of this type with $n = 4$ or 6. Current research on mathematical models for granular flows suggests that change of type which is related to this sort of model may govern dynamic instabilities.

A second phenomenon associated with systems like (1) which describe complex flows is a possible lack of separation of wave speeds. In the dynamics of ideal gases, there is a single speed, the sound speed, which governs the propagation of signals. The subject of nonstrictly hyperbolic conservation laws was motivated, initially, by consideration of elementary systems, such as an elastic string in the plane, in which there are two distinct modes of propagation (longitudinal and transverse), which need not have distinct speeds. This leads to the possibility of nonlinear interactions, resonance, and other phenomena which may change the mathematical, physical and computational nature of the flows. This phenomenon may occur in any flow that is sufficiently complex to admit more than one characteristic speed.

During the period of the AFOSR grant, progress was made in the mathematical study of several model problems, and some underlying principles became clear. One tentative conclusion is that the global failure of separation of wave-speeds may be more significant in determining resonant interference and catastrophic failure of the model than is a local failure of hyperbolicity.

In a series of papers [3, 7, 10], we study a system, (a simple model for elastic shear flow,) in which the wave speeds, $u + 1$ and $u - 1$, are <u>not</u> globally separated:

$$
\begin{aligned}
&u_t + (u^2 - v)_x = 0 \\
&v_t + (\tfrac{1}{3}u^3 - u)_x = 0
\end{aligned}
\qquad (3)
$$

It is strictly hyperbolic and genuinely nonlinear. There is an approximate solution when viscosity is added, but no classical limit exists as the viscosity tends to zero. We find that a well-defined but singular approximation exists.

By contrast, study of a model system which changes type [1, 2], motivated originally by some numerical results in three-phase porous medium flow, found that the Riemann problem, even for initial states that are in the elliptic region, is "well-posed" in a sense that can be made precise, with shock waves that satisfy classical admissibility conditions. The model system analysed was of the form

$$\begin{aligned} u_t + (f(u) - v)_x &= 0 \\ v_t + \quad (g(u))_x &= 0 \end{aligned} \quad , \tag{4}$$

with f and g chosen so that $\lambda_i(u)$ are complex for u between 0 and 1, and so that λ_1 and λ_2 are both increasing functions of u for $u < 0$, and so that λ_1 is decreasing and λ_2 increasing when $u > 1$. In [2], it was shown that these assumptions on (4) result in a solution of the Riemann problem which is much like solutions found numerically for three-phase flow in porous media using a well-known set of assumptions, Stone's model, for the three-phase relative permeabilities. The particular assumptions on the direction of variation of the wave speeds, λ_i, with the state u are related to the global failure of separation of wave speeds discussed in the previous paragraph, although the context, and the implications for existence of solutions to the Riemann problem, are completely different here.

The result reported in [4] gives a criterion for the appearance of the type of Riemann solution which characterizes this model (not for the model itself). Specifically, there is an isolated point, U_1, on the boundary curve $\mathcal{B}$, where the (unique) right eigenvector is tangent to $\mathcal{B}$. Such a point is found, for example, in model equations for three-phase flow in porous media. It does not occur in models, such as the van der Waals model, where the mathematically unstable region corresponds to a physically unstable one; one notes the contrast with models in which the apparent mathematical instability does not have a physical explanation.

We considered a new approach to the question of admissibility for shocks in systems that change type. One widely used criterion is the construction of viscous profiles for different forms of viscosity: that is, solutions of

$$(5) \qquad \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = \varepsilon \; P \frac{\partial^2 U}{\partial x^2}$$

which are functions of $(x - st)/\varepsilon$ (s is the shock speed). Here P is the <u>viscosity matrix</u>. Different conditions on P are appropriate for different interpretations of the results: positive definite or semidefinite symmetric matrices occur in viscous perturbations of fluid dynamic equations; matrices whose eigenvalues have nonnegative real part (typically not symmetric) model the diffusion introduced by a first-order finite-difference scheme. We obtained the following local result [5] using unfolding theory for vector field dynamics: for a state U_0 in $\mathcal{H}$, near $\mathcal{B}$, there are solutions U_1 of the Rankine-Hugoniot relation, $s(U_1 - U_0) = F(U_1) - F(U_0)$, which are in $\mathcal{E}$. They form a curve; states near one end can be joined to U_0 by forward waves (2-shocks); near the other end there are backward profiles (1-shocks); there is an interval in the middle of the curve where no profiles exist. This qualitative picture, and the bifurcations which separate the intervals on the curve, are generic, though the precise location of the points which separate the intervals depends on the coefficients of P. Using this result, one can explain some numerical evidence (noted in [2]) on competition between modes, which may lead to unstable oscillations in some cases.

There are alternative criteria for admissibility: although a convex entropy function will not exist for a system which changes type, nonconvex functions which have some physical significance may offer a characterization of admissible shocks. There is also a comparison with steady transonic flow in two space dimensions: the system

$$(6) \qquad \frac{\partial}{\partial x} \, (F(U)) + \frac{\partial}{\partial y} \, (G(U)) = 0$$

shares many of the mathematical properties of (1) when $\xi dF + \eta dG$ can have both real (supersonic states) and complex (subsonic) eigenvalues, and shocks between the elliptic and hyperbolic regions are well known to occur. Viscous perturbation of this equation leads to a different mathematical picture, which we studied in [8], using both viscosity and entropy criteria. The correct physical admissibility condition (which is quite different from that for the time-dependent system) is obtained using real physical viscosity or some perturbation of this. Vectorfield dynamics again isolates an admissibility criterion for viscous perturbations. Local methods can be used to distinguish between the steady and unsteady cases; in [6] it is shown that classical concepts from linear hyperbolic theory, such as space-like surfaces and time-like covectors, provide the following characterization: in the steady case, the support cone of a fundamental solution (of the linearized problem) expands to a half-plane near $\mathcal{B}$, while for unsteady problems, it shrinks to a singular ray. Thus it may be expected, on purely mathematical grounds, that the type of shock which is stable will differ in the two cases.

A preliminary study of the possible application of these techniques to problems of physical interest is the subject of [9]: the subject is a standard model of two-phase flow which evidences change of type. A simple asymptotic treatment reduces the system to a pair of equations which are of the same form as a classical model for two-way traffic flow due to Bick and Newell (an early example of a system that changes type).

Much of the work on change of type performed under this grant is summarized in an expository paper, now in preparation [11].