REPORT: DOCUMENTATION PAGE

TITe AND SUBTITLE
NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN STOCHASTIC CONTROL THEORY

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ABSTRACT
This is a summary of research completed during the period of this award, 1 October 1985-30 September 1988 by Fleming, Kushner together with associated postdoctoral and graduate student personnel. The research covers a number of problems in many areas of stochastic control, recursive stochastic algorithms, and related areas of analysis. It is part of a continuing research program pursued successfully for a number of years. The program has been motivated both by traditional applications in control and filtering and by newer areas of application arising in queueing/communication networks and production systems. Other research issues addressed include numerical methods for stochastic control and recursive algorithms for distributed and parallel processing and/or control.

The work of Fleming and Kushner will be summarized in turn, with references to research publications supported under this award.

SUBJECT TERMS

DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

DISTRIBUTION CODE
S
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FEB 28 1990

STATEMENT OF SECURITY CLASSIFICATION
UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

SAR

PRICE CODE
7

AD-A218 419
FINAL TECHNICAL REPORT

AFOSR-85-0315

"NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN STOCHASTIC CONTROL THEORY."

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W. H. Fleming

Fleming's research was concerned with optimal stochastic control theory, nonlinear filtering, large deviations for Markov diffusions, and viscosity solutions of nonlinear partial differential equations.

Fleming's work in optimal stochastic control focused on control of Markov diffusion processes, with complete or partial observations. This work is reported in references [F3] [F4] [F5] [F11 - F12]. [F3] [12] provide a solution to a stochastic production planning problem, making use of viscosity solution methods for a system of first-order nonlinear partial differential equations. [F4] [F11] provide an approximate solution, in the form of an asymptotic series in powers of a small parameter measuring the intensity of noise entering the control dynamics. References [F5] [F13] provide an alternative to the usual dynamic programming, seeking maximal smooth subsolutions rather than solutions to the dynamic programming equations and using an abstract duality theorem from convex analysis.

The theory of large deviations is concerned with asymptotic estimates for exponentially small probabilities of rare events associated with stochastic processes. In 1977, Fleming introduced a stochastic control approach to large deviations of nearly deterministic Markov diffusions. This technique was based on a certain logarithmic transformation. Subsequently, simpler analytical methods based on logarithmic transformations and viscosity solution techniques were developed [F1]. These methods also give more accurate
approximations in the form of an asymptotic series [F4][F11]. These analytical methods were adapted to other classes of Markov processes in [F8].

Fleming and Souganidis [F7][F14] developed a theory of value for two-player, zero sum stochastic differential games. This provides an optimization formula for viscosity solutions to a large class of nonlinear second partial differential equations, of degenerate parabolic type.

References [F9][F10][F15] are concerned with nonlinear filtering, in cases when a many-to-one function function of the system state plus low intensity observation noise is observed. The goal is to obtain finite-dimensional, approximately optimal filters.

Research Publications W.H. Fleming

[F1] - [F15]


H.J. Kushner

Kushner's research covered a wide range of topics in stochastic systems theory and applied probability. These including: large deviations with communications applications, stochastic approximations (convergence theorems, large deviations estimates), adaptive filters, distributed parameter stochastic systems, wide band noise approximations, Monte Carlo methods, distributed and communicating stochastic approximation algorithms, singular stochastic control and computational methods in optimal stochastic control.
The work on large deviations and applications is reported in [K4] [K8] [K16] [K17]. Typical communications applications arise in models with rapidly varying noise inputs, for slowly adapting digital systems, and for tracking systems with small noise effects. Among the accomplishments is a "quick simulation" technique, based on a change of probability measure technique. This method relies on the numerical solution of a first-order nonlinear partial differential equation, connected with the action functional being minimized to obtain the large deviation rate.

The papers on stochastic approximation and recursive algorithms [K5] [K10] [K11] [K21] provide limit theorems and large deviations estimates under conditions on dynamics and noise which are broad enough to fit most current applications in control and communication theory. The global behavior of stochastic approximations was studied by Monte Carlo methods in [K7].

Kushner's work on distributed parameter stochastic systems, described by stochastic partial differential equations, is reported in [K2] [K3]. This work is concerned with such questions as stability and near-stationarity for systems with wide-band noise inputs, and with nonlinear filtering applications.

Further work on wide-band noise approximations in filtering and stochastic control is reported in [K6] [K9] [K14] [K15] [K24]. In the part of this work concerned with stochastic control, the problem is to find nearly-optimal control laws for a wide-band noise driven system, based on optimal control laws for an idealized white-noise driven system (for which there is an extensive theory.)

Recent work by Kushner on stochastic approximation [K12] [K13] concerns distributed and communicating systems, in which processors are located at physically distinct sites.

Routing and flow control problems for queues under heavy traffic can be analyzed using methods of singular stochastic control. Such problems, and related problems for wide-band noise driven singular stochastic control systems were analyzed in [K18] [K19][20]

Research Publications - H.J. Kushner


K 4. Large deviations estimates for systems with small noise effects, and applications to stochastic systems theory, (with P. DuPuis) SIAM J. on Control and Optimization, 24, 1986, pp 979-1008.

K 5. The theory of large deviations and asymptotic analysis of recursive algorithms and stochastic approximation (with P. DuPuis), in Advances in Statistical Signal Processing, ed. by H.V. Poor, JAI Press.


K 17. Upper bounds for large deviations for non-smooth stochastic difference eqns., LCDS Dept. 87-8 (Feb. 87) sub. to App. Prob.


K 19. Optimal and approximately optimal control policies for queues in heavy traffic (with K.M. Ramachandran) to appear SIAM J. on Control and Optimization.

