Technical Document 1641
September 1989

Specification for HF
Maximum Usable
Frequency (MUF) Model

T. N. Roy

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This report documents a high-frequency (HF) maximum usable frequency (MUF) model called MINIMUF-85. Developed at the Naval Ocean Systems Center, this semiempirical model permits simplified HF propagation MUF predictions to be performed in near-real-time. The model predicts the MUF for arbitrary input values of receiver/transmitter position, date, time (UT), and solar sunspot number. Several algorithms are contained in the model to describe the dependence of the MUF on solar activity, geomagnetic latitude, time of year, and time of day. An appendix contains the computer FORTRAN 77 code required to implement the fundamental features of this model.
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1.0 INTRODUCTION

Any communication system operating in the high-frequency (HF) spectrum between 2 and 32 MHz is subject to certain physical limitations on the propagation of its signal. These limitations determine propagation boundaries which are unique and definable for any given point in time and over any path. The upper boundary is called the maximum usable frequency (MUF). The instantaneous MUF is a direct function of ionospheric electron density and is an absolute frequency limit, since the ionosphere is not capable of supporting propagation by the refraction of higher frequencies for that path. Ionospheric electron density is driven both by solar activity and by seasonal and diurnal variations. A model developed to predict MUF must be able to deal with these variations.

In 1978 a semiempirical model for MUF prediction, called MINIMUF-3.5, was developed at the Naval Ocean Systems Center (NOSC). It was used on small mobile propagation forecast (PROPHET) terminals (Ref. 1 and 2). Values for the parameters used in the model were determined by using HF oblique sounder propagation data. A data base of HF oblique sounder data was used in 1981 to evaluate the accuracy of this model (Ref. 3).

In 1986 MINIMUF-3.5 was improved. Called MINIMUF-85, the new model contained improvements in the \( f_0F2 \) algorithms, high sunspot number predictions, an \( M \)-factor algorithm, and a polar region algorithm (Ref. 4). The accuracy of this improved model was evaluated in 1987 by using an expanded data base of HF oblique sounder data (Ref. 5). During this period several user's manuals were also published. These manuals contain specific information on how to use the HF propagation prediction models developed at NOSC (Ref. 6 and 7).
2.0 MUF MODEL INPUT AND OUTPUT PARAMETERS

MUF model input and output parameters and parameter limits are listed in Table 2-1. Monthly median sunspot values can be found in the Solar Indices Bulletin, published by the National Geophysical Data Center, Solar-Terrestrial Physics Division, located in Boulder, Colorado.

Table 2-1. MUF Model Input and Output Parameters.

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Parameter Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INPUT</strong></td>
<td></td>
</tr>
<tr>
<td>Transmitter Latitude</td>
<td>$\frac{-\pi}{2}$ to $\frac{\pi}{2}$ radians</td>
</tr>
<tr>
<td>Transmitter West Longitude</td>
<td>0 to $2\pi$ radians</td>
</tr>
<tr>
<td>Receiver Latitude</td>
<td>$\frac{-\pi}{2}$ to $\frac{\pi}{2}$ radians</td>
</tr>
<tr>
<td>Receiver West Longitude</td>
<td>0 to $2\pi$ radians</td>
</tr>
<tr>
<td>Month</td>
<td>1 to 12</td>
</tr>
<tr>
<td>Day</td>
<td>1 to 31</td>
</tr>
<tr>
<td>Hour</td>
<td>0 to 23 (Universal time)</td>
</tr>
<tr>
<td>Minute</td>
<td>0 to 59 (Universal time)</td>
</tr>
<tr>
<td>Julian Day</td>
<td>1 to 366</td>
</tr>
<tr>
<td>Year</td>
<td>Example: 89 for 1989</td>
</tr>
<tr>
<td>Monthly Median Sunspot Number</td>
<td>0 to 300</td>
</tr>
<tr>
<td><strong>OUTPUT</strong></td>
<td></td>
</tr>
<tr>
<td>Maximum Usable Frequency</td>
<td>2 to 50 MHz</td>
</tr>
<tr>
<td>Distance Between Transmitter and Receiver</td>
<td>0 to $\pi$ radians</td>
</tr>
<tr>
<td>Latitude of the Path Midpoint</td>
<td>$\frac{-\pi}{2}$ to $\frac{\pi}{2}$ radians</td>
</tr>
<tr>
<td>West Longitude of the Path Midpoint</td>
<td>0 to $2\pi$ radians</td>
</tr>
<tr>
<td>Latitude of a Point 1000 km from Receiver</td>
<td>$\frac{-\pi}{2}$ to $\frac{\pi}{2}$ radians</td>
</tr>
<tr>
<td>West Longitude of a Point 1000 km from Receiver</td>
<td>0 to $2\pi$ radians</td>
</tr>
<tr>
<td>Latitude of a Point 1000 km from Transmitter</td>
<td>$\frac{-\pi}{2}$ to $\frac{\pi}{2}$ radians</td>
</tr>
<tr>
<td>West Longitude of a Point 1000 km from Transmitter</td>
<td>0 to $2\pi$ radians</td>
</tr>
<tr>
<td>Azimuth from Receiver to Transmitter</td>
<td>0 to $2\pi$ radians</td>
</tr>
</tbody>
</table>
3.0 MUF MODEL COMPUTER PROGRAM, SUBROUTINES, AND FUNCTIONS

A program called CMUF passes input parameters to the subroutine MUF85 and receives the calculated MUF and geographic path information from this subroutine. The MUF85 subroutine calls the subroutines PATH and RAZGC and the functions FOF2 and SYGN.

The subroutine PATH computes eight values of geographic path information for a given propagation path. The computation assumes a spherical earth with a radius of 6371 km. Required inputs for this routine are transmitter and receiver latitude and west longitude in radians. The following information, in radians, is returned by this subroutine: distance between transmitter and receiver, midpoint latitude, midpoint west longitude, latitude of a point 1000 km from the transmitter and receiver, west longitude of a point 1000 km from the transmitter and receiver, and azimuth from receiver to transmitter. Subroutines GCRAZ and RAZGC are called by PATH.

Subroutine GCRAZ computes the great circle range and azimuth between two points on the earth's surface in radians. Latitude and west longitude of the two points, in radians, are required as input. This computation assumes a spherical earth and recognizes the degenerate cases of a point at the North or South Pole and coincident points.

Subroutine RAZGC computes the latitude and west longitude in radians of a point a specified distance and azimuth from a given point on the earth's surface. Latitude, west longitude, distance, and azimuth, in radians, of the given point to the new point are required as inputs. This computation assumes a spherical earth and recognizes the degenerate cases of the given point at the North or South Pole and when distance is zero.

Function FOF2 provides a correction to the calculation of the F2-layer critical frequency for polar latitudes by using the Chiu polar model (Ref. 8). Inputs to this function are the critical frequency in megahertz, local mean time at the control point, month, day, hour (UT), minute (UT), Julian day and year, geographic latitude and west longitude of the control point in radians, magnetic latitude of the control point in radians, and sunspot number (SSN). This function calculates the corrected F2-layer critical frequency, in megahertz, which is passed to the MUF85 subroutine.

Function SYGN returns the value of zero if the input value is zero, -1 if the input value is less than zero, and +1 if the input value is greater than zero.
4.0 MUF MODEL ALGORITHMS

The expression for the MUF used in MINIMUF-85 is given by

\[
\text{MUF} = A_2 (\text{SSN}) \cdot A_3 (\text{month}) \cdot A_4 (\text{time}) \cdot M \cdot f_0 F2
\]  

(1)

where \( M \) is the obliquity, or \( M \)-factor, which reflects the dependence of the MUF on the transmission path length. The parameter \( f_0 F2 \) is the critical or penetration frequency at vertical incidence for the \( F2 \) layer. The parameters \( A_2, A_3, \) and \( A_4 \) contain the sunspot dependence, the seasonal dependence, and the time dependence in the \( M \)-factor.

The equation for the \( A_2 \) parameter is

\[
A_2 (\text{SSN}) = 1.3022 - (0.00156) R
\]  

(2)

where \( R \) is the monthly median sunspot number. The \( A_2 \) parameters provide a linear decrease as a function of sunspot number.

The equation for the \( A_3 \) parameter is

\[
A_3 (\text{month}) = 0.9925 + 0.011 \sin (m) + 0.087 \cos (m)
\]

\[
- 0.043 \sin (2m) + 0.003 \cos (2m)
\]

\[
- 0.013 \sin (3m) - 0.022 \cos (3m)
\]

\[
+ 0.003 \sin (4m) + 0.005 \sin (5m)
\]

\[
+ 0.018 \cos (6m)
\]  

(3)

where \( m = 2\pi \cdot (\text{month})/12 \) and month = 1, 2, 3... 12. This seasonally dependent factor attempts to account for unusually high electron density during winter months, which allows higher frequencies to propagate on a given transmission path.

The equation for the \( A_4 \) parameter is

\[
A_4 (\text{time}) = 1.11 - 0.01 T_{\text{local}}
\]  

(4)

which adequately fits daytime data and for night

\[
A_4 (\text{time}) = 1.0195 - 0.06 \sin (2T) - 0.037 \cos (2T)
\]

\[
+ 0.018 \sin (4T) - 0.003 \cos (4T)
\]

\[
+ 0.025 \sin (6T) + 0.018 \cos (6T)
\]

\[
+ 0.007 \sin (8T) - 0.005 \cos (8T)
\]

\[
+ 0.006 \sin (10T) + 0.017 \cos (10T)
\]

\[
- 0.009 \sin (12T) - 0.004 \cos (12T)
\]  

(5)
where $T = T_{local} - T_{sunset}$, which represents elapsed time in hours after sunset.

The expression for the $M$-factor is given by

$$M = \left[1 + 2.5[\sin (2.5 \psi k)]^{1/2}\right] \cdot G_1 (\Delta T) \cdot G_2 (L_o) \cdot G_3 (L_1, L_2)$$  \hspace{1cm} (6)$$

where $\psi$ is the great-circle arc length, in radians, of the transmission path, $L_o$ is the latitude of the path midpoint, $L_1$ and $L_2$ are the transmitter and receiver latitudes, and $\Delta T$ is the duration of daylight at the path midpoint in hours. The $k$ parameter is equal to 1.0 for path lengths less than or equal to 4000 km and 0.5 for greater path lengths. The value of 2.5 $\psi k$ is limited to a maximum of $\pi/2$ for path lengths greater than 8000 km.

The first factor of Eq. 6 contains the range dependence of the $M$-factor and was obtained by curve-fitting to exact results for a parabolic layer at a height of 290 km with a ratio of semithickness to base height equal to 0.4. The $G_1$ factor gives recognition to the approximately 50% increase in $F_2$ layer heights observed at high (northern) latitudes during the summer at or near “midnight sun” conditions. The equation for the $G_1$ parameter is

$$G_1 = 1 - 0.1 \exp \left[\frac{(\Delta T - 24)}{3}\right]$$  \hspace{1cm} (7)$$

which has the effect of a 10% reduction in MUF under full midnight sun conditions, with $G_1$ recovering rapidly as the midpath latitude moves toward the equator.

The $G_2$ parameter is designed to produce further, seasonally independent, reductions in MUF for high latitude paths. Since propagation data indicate a fairly abrupt onset of this reduction for absolute values of latitude greater than or equal to 45 degrees, a step function is used which is equal to 1 for absolute values of latitude less than 45 degrees and to 0.8 (20% reduction) for absolute values of latitude greater than or equal to 45 degrees.

The $G_3$ parameter is a correction factor for transequatorial paths to approximate the well-known MUF increases on such paths. The factor applies another 20% correction as a step function equal to 1.0 if the transmitter and receiver latitudes are the same sign and equal to 1.2 if the transmitter and receiver are of opposite sign.

The expression for the critical frequency, $f_o F_2$, is given by

$$f_o F_2 = [A_o + A_1 (SSN) \cdot (\cos x_{eff})^{1/2}]^{1/2}$$  \hspace{1cm} (8)$$

where $A_o$ is equal to 6 and

$$A_1 (SSN) = 22.23 + (0.814)R$$  \hspace{1cm} (9)$$

where $R$ is the monthly median sunspot number. The $A_1$ parameter provides a saturation effect in the behavior of the critical frequency as a function of sunspot number.

In Eq. 8, $x_{eff}$ is an “effective” solar zenith angle. $\cos x_{eff}$ is modeled as the lagged response of a dynamic linear system “driven” by the instantaneous value of $\cos x$. By using an effective value of the zenith angle, recognition is given to the fact that the $F_2$ layer, unlike the $E$ and $D$ layers, does not show a relatively simple $\cos^n x$ diurnal dependence on $x$. The dynamic behavior of the $F_2$ layer is more complicated because of various other dependencies. In keeping with the uncomplicated nature of the $f_o F_2$ model, defining an effective $x$ allows relatively accurate modeling without explicitly including these other dependencies. To model $\cos x_{eff}$ at night, the following quantity is constructed:
In Eq. 10, \(\tau_n\) is a nighttime relaxation time, equal to 2 hours, which is taken to be a constant independent of season and geographical location. \(T - T_{\text{sun set}}\) is the elapsed time in hours since sunset.

However, during the day, a different relaxation time, \(\tau_D\), is assumed. In contrast to \(\tau_n\), \(\tau_D\) does depend on location and position. The daytime relaxation time is assumed to be a function of the actual noontime solar zenith angle (\(\cos \chi_{\text{noon}}\)) and can be expressed as

\[
\tau_D = \tau_o \left( \cos \chi_{\text{noon}} \right)^{P_2}
\]

where \(\tau_o\) and \(P_2\) are constants, equal to 9.7 and 9.6 hours, respectively, and are independent of season or location. The value of \(\tau_D\) is not allowed to fall below 0.1. Note that during summer at equatorial and moderate latitudes, \(\tau_D = \tau_o\), whereas in the winter at high latitudes, \(\tau_D \ll \tau_o\).

The time dependence of the actual \(\cos \chi\) in terms of its value at noon, \(\cos \chi_{\text{noon}}\), is

\[
\cos \chi = \cos \chi_{\text{noon}} \cdot \sin \left[ \frac{\pi(T - T_{\text{dawn}})}{\Delta T} \right]
\]

where \(\Delta T\) is the daytime duration given by

\[
\Delta T = T_{\text{sun set}} - T_{\text{dawn}}
\]

and noon occurs when \(T = T_{\text{dawn}} + \Delta T/2\).

Next, \(\cos \chi_{\text{eff}}\) is assumed to represent the response of a linear first-order system driven by the actual \(\cos \chi\):

\[
\tau_D \frac{d}{dt} \left( \cos \chi_{\text{eff}} \right) + \cos \chi_{\text{eff}} = \cos \chi
\]

By using Eq. 12 for \(\cos \chi\) in Eq. 14, we obtain the daytime \(\cos \chi_{\text{eff}}\):

\[
(\cos \chi_{\text{eff}})_{\text{day}} = \frac{\cos \chi_{\text{noon}}}{1 + \beta^2} \left\{ \sin \left( \alpha \right) + \beta \left( \exp \left[ -\frac{(T - T_{\text{dawn}})}{\tau_D} \right] - \cos \left( \alpha \right) \right) \right\}
\]

where

\[
\beta = \frac{\pi \tau_D}{\Delta T}
\]

and

\[
\alpha = \frac{\pi(T - T_{\text{dawn}})}{\Delta T}
\]

At sunset \((T - T_{\text{dawn}} = \Delta T)\) and \((\cos \chi_{\text{eff}})_{\text{sun set}}\) is
(\cos \chi_{\text{eff}})_{\text{sunset}} = \frac{\cos \chi_{\text{noon}}}{1 + \beta^2} \left\{ \beta \left[ 1 + \exp \left( \frac{-\Delta T}{\tau_D} \right) \right] \right\} \quad (18)

To avoid discontinuities in \cos \chi_{\text{eff}} just after sunrise, \( (\cos \chi_{\text{eff}})_{\text{day}} \) is not allowed to fall below its value just before sunrise.

\[
(\cos \chi_{\text{eff}})_{\text{day}} = \max \{ (\cos \chi_{\text{eff}})_{\text{sunset}} \cdot \exp \left[ \frac{(\Delta T - 24)}{\tau_D} \right] \}
\] (19)

The basic \( f_o F_2 \) model is therefore given in Eq. 8, 10, and 18 for nighttime (i.e., \( T_{\text{sunset}} < T < T_{\text{dawn}} \)) and Eq. 8, 15, and 19 for daytime (\( T_{\text{dawn}} < T < T_{\text{sunset}} \)).

Simple analytical approximations for the times of local noon, sunrise, and sunset and for the noon value of the solar zenith angle are presented in the following equations. To an acceptable degree of accuracy, for the Universal time of local noon the approximation

\[
T_{\text{noon}} = 12 \left( \frac{W}{\chi} + 1 \right) + 0.13 \left[ \sin (Y_1) + 1.2 \sin (2 Y_1) \right]
\] (20)

where

\[ W = \text{longitude west of Greenwich [0,2}\pi \text{ radians}] \]

\[ Y_1 = 0.0172 (D + 10) \]

\[ D = 30.4 (M1 - 1) + D1 \]

\[ D1 = \text{day values which range from 1 to 31} \]

\[ M1 = \text{month values which range from 1 to 12} \]

In terms of the subsidiary variable \( Y_2 \), defined by

\[ Y_2 = 0.409 \cos (Y_1) \text{ [radians]} \] (21)

and

\[ \cos \chi_{\text{noon}} = |\cos (L + Y_2)| \] (22)

where

\[ L = \text{north latitude in radians (-\pi, 2\pi/2)} \]

The duration of the daytime, \( \Delta T \), is approximated by

\[
\Delta T = \frac{24}{\pi} \arccos \left( -0.26 + \frac{\sin (Y_2) \sin (L)}{\cos (Y_2) \cos (L)} \right)
\] (23)

where the factor -0.26 approximates the difference between sunrise (or sunset) at the surface of the earth and at 2F2 layer heights. From Eq. 23
\[ T_{\text{dawn}} = T_{\text{noon}} - \Delta T/2 \]  
(24)

and

\[ T_{\text{sunset}} = T_{\text{noon}} + \Delta T/2 \]  
(25)

where

\[ T_{\text{noon}} \]  
is given in Eq. 20.

The fact that high-latitude ionospheric behavior differs sharply from that at lower latitudes has been recognized for some time. This sharp difference required the addition of a routine to MINIMUF-85 specifically tailored to model the behavior of the MUF at high latitudes. This is accomplished by using a folding function, \( f \), to merge a polar model with the model of lower latitude behavior.

The folding function determines when polar effects (particle precipitation) become dominant. It is a function of geomagnetic latitude and sunspot number and makes an abrupt transition from 0 to 1 between geomagnetic latitudes of 60 degrees to 75 degrees. When the folding function is near 1, particle precipitation effects will dominate, and when the folding function is near 0, solar zenith angle is the major factor in causing ionization. In between, there exists a narrow transition region where both sources of free electrons are significant. The equation for merging the two models is

\[ N_{\text{total}} = (1 - f) N_{\text{MINIMUF}} + f N_{\text{polar}} \]  
(26)

where \( N \) is the electron density calculated from the expression

\[ f_o F^2 (\text{MHz}) = 2.85N^{1.2} \text{[electrons/cm}^3] \]  
(27)

The total electron density at the control point is then converted back to \( f_o F^2 \) by using Eq. 26, and the MUF is obtained by multiplying the value of \( f_o F^2 \) by the range-dependent portion of the \( M \)-factor.

The equation for the folding function is

\[ f = \exp \left( X^b \right) \]  
(28)

where

\[ X = [2.2 + (0.2 + R \times 1000) \sin (I_1)] \cos (I_1) \]  
(29)

where

\[ R = \text{monthly median sunspot number} \]

and

\[ I_1 = \text{magnetic latitude of the control point in radians.} \]
The equation for the polar model of electron density for northern geomagnetic latitudes is

\[ N_{\text{polar}} = (2.0 + 0.012R)(1.0 + 0.3V)W \]  

(30)

where

\[ V = \sin \left( \frac{\pi \text{month}}{12} \right) \]  

(31)

and

\[ W = \exp \left( -1.2 \left\{ \cos \left[ L_1 - 0.41015 \cos \left( \frac{\pi T}{12} \right) \right] \cos \left( L_1 \right) \right\} \right) \]  

(32)

with

\[ L_1 = \text{magnetic latitude of the control point in radians} \]

and

\[ T = \text{the local mean time at the control point in hours} \]

The expression for electron density for southern geomagnetic latitudes is

\[ N_{\text{polar}} = 2.5 + \frac{R}{50} + U[0.5 + S(1.3 + 0.002R)] \]

\[ + (1.3 + 0.005R) \cos \left( \frac{\pi T}{12} - \pi(1 + B) \right) \]

\[ \cdot [1 + 0.4(1 - V^2)] \exp(-VS) \]  

(33)

where

\[ U = \cos \left( 2\pi \text{month}/12 \right) \]  

(34)

\[ S = \cos^4(D_4, 2 - \pi/20) \]  

(35)

\[ B = V \left[ \sin(D_4, 2) - \sin(D_4, 2) \right] \cdot \sin^4(D_4, 2) \]

\[ \cdot (1 + V)U \sin(D_4, Z) \exp \left[ 4 \sin^2(D_4, 2) \right] \]  

(36)

and

\[ Z = |\sin(D_4)|^{1/2} \]  

(37)

where

\[ D_4 = \sin^{-1}(D_3) \]  

(38)
where

\[ D_3 = \text{the maximum of } D_2 \text{ or } -1.0 \]

and

\[ D_2 = \text{the minimum of } D_1 \text{ or } 1.0 \]

with

\[ D_1 = \cos (L_w) \sin (D, 1.2043) \cos (L_1) \]  \hspace{1cm} (39)

where

\[ L_w = \text{geographic latitude in radians} \]

and

\[ D, = \text{geographic west longitude in radians} \]

The equations in the preceding sections yield \( f_0 F2 \) at a specified latitude, \( L \), and west longitude, \( W \). In an actual application, the latitude and longitude of the receiver and transmitter are given quantities, and \( L \) and \( W \) are to be evaluated at specific control points along the great-circle propagation path.

From spherical trigonometry, one obtains the following expressions for the great-circle arc length, \( \psi \) (in radians), connecting two points defined by the (latitude, longitude) pairs \((L_1, W_1)\) and \((L_2, W_2)\):

\[ \psi = \arccos \left[ \sin (L_1) \sin (L_2) + \cos (L_1) \cos (L_2) \cos (W_2 - W_1) \right] \]  \hspace{1cm} (40)

If one travels a fraction, \( K \), of the distance from point 1 to point 2, the north latitude, \( L_K \), and west longitude, \( W_K \), of this location are determined from

\[ L_K = \arcsin \left\{ \frac{\sin \left[ (1 - K) \arccos \left( \frac{\sin (L_1) \sin (L_2) + \cos (L_1) \cos (L_2) \cos (W_2 - W_1)}{\sin (\psi)} \right) \right]}{\sin (\psi)} \right\} \]  \hspace{1cm} (41)

and

\[ W_K = \arccos \left\{ \frac{\cos (L_1) \cos (W_1) \sin \left[ (1 - K) \arccos \left( \frac{\sin (L_1) \sin (L_2) + \cos (L_1) \cos (L_2) \cos (W_2 - W_1)}{\cos (L_K) \sin (\psi)} \right) \right]}{\cos (L_K) \sin (\psi)} \right\} \]  \hspace{1cm} (42)

The calculated MUF is the minimum value evaluated at specific control points along the great-circle propagation path. In MINIMUF-85 these control points are located at the path midpoint for path lengths less than or equal to 4000 km, 2000 km from either terminus for path lengths greater than 4000 km, but less than or equal to 6000 km, and at \( \frac{1}{4} \), \( \frac{1}{2} \), and \( \frac{3}{4} \) of the path length for paths greater than 6000 km and less than or equal to 8000 km in length. For path lengths greater than 8000 km and less than or equal to 12000 km, the control points are located at \( \frac{5}{6} \) of the path length.
In MINIMUF-85 the geomagnetic latitude dependence accounts for critical frequency separation between ordinary and extraordinary ionospheric propagation by adding one-half the gyrofrequency to the $f_o F2$ for latitudes greater than 55°N geomagnetic. The gyrofrequency for an earth-centered dipole field is given by

$$f_H = 0.3789 \left[ 1 + 3 \sin^2 (\theta) \right]^{1/2} - 0.5 \text{ [MHz]} \quad (43)$$

where

$$\theta = \text{latitude of the midpoint of the propagation path in magnetic coordinates (radians)}$$

The geomagnetic latitude $\theta$ is given by

$$\sin (\theta) = \sin (\phi) \sin (\phi_o) + \cos (\phi) \cos (\phi_o) \cos (\lambda - \lambda_o) \quad (44)$$

where

$$\phi = \text{latitude of the midpoint of the propagation path (radians)}$$

$$\lambda = \text{longitude of the midpoint of the propagation path (radians)}$$

$$\phi_o = \text{latitude of the North Magnetic Pole (1.3666 radians or 78.3°N)}$$

$$\lambda_o = \text{longitude of the North Magnetic Pole (1.2043 radians or 69°W)}.$$

Substituting the values of $\phi_o$ and $\lambda_o$, Eq. 44 becomes

$$\sin (\theta) = 0.9792 \sin (\phi) + 0.2028 \cos (\phi) \cos (\lambda - 1.2043) \quad (45)$$
5.0 MUF MODEL TEST CASES

The following tables of test case results are provided as an aid in determining the proper operation of the MUF model algorithms. Table 5-1 lists the results of exercising the MUF model for the range of season values. A sunspot value of 75 was used to generate these values. Transmitter latitude was 0.5712 radian, transmitter longitude was 2.0450 radians, receiver latitude was 0.5304 radian, and receiver longitude was 1.5645 radians. Seasonal values were calculated for the 15th day of January, April, July, and October.

Table 5-1. MUF model season results (MHz).

<table>
<thead>
<tr>
<th>Time (UT)</th>
<th>Winter (1)</th>
<th>Spring (4)</th>
<th>Summer (7)</th>
<th>Fall (10)</th>
</tr>
</thead>
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<td>22.28</td>
<td>28.08</td>
<td>24.22</td>
<td>26.85</td>
</tr>
<tr>
<td>0800</td>
<td>11.29</td>
<td>17.71</td>
<td>16.71</td>
<td>14.52</td>
</tr>
<tr>
<td>1200</td>
<td>8.94</td>
<td>14.25</td>
<td>13.71</td>
<td>13.29</td>
</tr>
<tr>
<td>1600</td>
<td>28.10</td>
<td>25.68</td>
<td>21.32</td>
<td>32.63</td>
</tr>
<tr>
<td>2000</td>
<td>29.05</td>
<td>29.08</td>
<td>24.21</td>
<td>33.49</td>
</tr>
</tbody>
</table>

Table 5-2 lists the results of exercising the MUF model for a solar cycle. Locations of the transmitter and receiver were the same as those in Table 5-1. The date used was 15 January.

Table 5-2. MUF model solar cycle results (MHz).

<table>
<thead>
<tr>
<th>Solar cycle (sunspot number)</th>
<th>Time (UT)</th>
<th>Minimum (10)</th>
<th>Rise and Decline (45)</th>
<th>Near Maximum (75)</th>
<th>Maximum (105)</th>
<th>High Maximum (150)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0000</td>
<td>16.23</td>
<td>20.01</td>
<td>22.28</td>
<td>23.96</td>
<td>25.66</td>
</tr>
<tr>
<td></td>
<td>0400</td>
<td>10.37</td>
<td>11.00</td>
<td>11.39</td>
<td>11.67</td>
<td>11.89</td>
</tr>
<tr>
<td></td>
<td>0800</td>
<td>11.27</td>
<td>11.31</td>
<td>11.29</td>
<td>11.22</td>
<td>11.05</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>9.37</td>
<td>9.14</td>
<td>8.94</td>
<td>8.72</td>
<td>8.36</td>
</tr>
<tr>
<td></td>
<td>1600</td>
<td>19.96</td>
<td>25.06</td>
<td>28.10</td>
<td>30.34</td>
<td>32.62</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>20.42</td>
<td>25.83</td>
<td>29.05</td>
<td>31.41</td>
<td>33.83</td>
</tr>
</tbody>
</table>

Tables 5-3 through 5-7 list the results of exercising the MUF model for various locations of the transmitter and receiver. The date used for the following tests was 15 January at 1200 UT and the sunspot number was 75.0.
Table 5-3. MUF model results (MHz) for transmitter at 75 degrees north latitude (1.3090 radians) and 150 degrees west longitude (2.6180 radians).

<table>
<thead>
<tr>
<th>Receiver, North Latitude, deg (radians)</th>
<th>Receiver, West Longitude, deg (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>75 (1.3090)</td>
<td>17.40</td>
</tr>
<tr>
<td>35 (0.6109)</td>
<td>18.78</td>
</tr>
<tr>
<td>0 (0.0000)</td>
<td>18.73</td>
</tr>
<tr>
<td>-35 (-0.6109)</td>
<td>18.46</td>
</tr>
<tr>
<td>-75 (-1.3090)</td>
<td>11.04</td>
</tr>
</tbody>
</table>

Table 5-4. MUF model results (MHz) for transmitter at 35 degrees north latitude (0.6109 radian) and 150 degrees west longitude (2.6180 radians).

<table>
<thead>
<tr>
<th>Receiver, North Latitude, deg (radians)</th>
<th>Receiver, West Longitude, deg (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>75 (1.3090)</td>
<td>9.67</td>
</tr>
<tr>
<td>35 (0.6109)</td>
<td>9.62</td>
</tr>
<tr>
<td>0 (0.0000)</td>
<td>10.31</td>
</tr>
<tr>
<td>-35 (-0.6109)</td>
<td>13.40</td>
</tr>
<tr>
<td>-75 (-1.3090)</td>
<td>18.17</td>
</tr>
</tbody>
</table>

Table 5-5. MUF model results (MHz) for transmitter at 0 degrees north latitude (0.0 radians) and 150 degrees west longitude (2.6180 radians).

<table>
<thead>
<tr>
<th>Receiver, North Latitude, deg (radians)</th>
<th>Receiver, West Longitude, deg (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>75 (1.3090)</td>
<td>10.57</td>
</tr>
<tr>
<td>35 (0.6109)</td>
<td>9.96</td>
</tr>
<tr>
<td>0 (0.0000)</td>
<td>15.35</td>
</tr>
<tr>
<td>-35 (-0.6109)</td>
<td>17.39</td>
</tr>
<tr>
<td>-75 (-1.3090)</td>
<td>19.95</td>
</tr>
</tbody>
</table>
Table 5-6. MUF model results (MHz) for transmitter at -35 degrees north latitude (-0.6109 radian) and 150 degrees west longitude (2.6180 radians).

<table>
<thead>
<tr>
<th>Receiver, North Latitude, deg (radians)</th>
<th>Receiver, West Longitude, deg (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>75 (1.3090)</td>
<td>11.57</td>
</tr>
<tr>
<td>35 (0.6109)</td>
<td>18.77</td>
</tr>
<tr>
<td>0 (0.0000)</td>
<td>18.21</td>
</tr>
<tr>
<td>-35 (-0.6109)</td>
<td>17.90</td>
</tr>
<tr>
<td>-75 (-1.3090)</td>
<td>18.65</td>
</tr>
</tbody>
</table>

Table 5-7. MUF model results (MHz) for transmitter at -75 degrees north latitude (-1.3090 radians) and 150 degrees west longitude (2.6180 radians).

<table>
<thead>
<tr>
<th>Receiver, North Latitude, deg (radians)</th>
<th>Receiver, West Longitude, deg (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>75 (1.3090)</td>
<td>20.95</td>
</tr>
<tr>
<td>35 (0.6109)</td>
<td>25.18</td>
</tr>
<tr>
<td>0 (0.0000)</td>
<td>23.67</td>
</tr>
<tr>
<td>-35 (-0.6109)</td>
<td>22.98</td>
</tr>
<tr>
<td>-75 (-1.3090)</td>
<td>25.91</td>
</tr>
</tbody>
</table>
6.0 REFERENCES


Appendix

FORTRAN 77 PROGRAM AND SUBROUTINE LISTINGS FOR MUF MODEL

The MUF calculation program and subroutines that follow are written in
FORTRAN 77. The parameters passed to the subroutines and those returned are described
in the comments portion of the routines.
program cmuf

This program calculates the classical maximum usable frequency (MUF) given the values specified.

Parameters input:
- \( t_{\text{lat}} \): transmitter latitude (radians), + north, - south
- \( t_{\text{lon}} \): transmitter longitude (radians), + west, - east
- \( r_{\text{lat}} \): receiver latitude (radians), + north, - south
- \( r_{\text{lon}} \): receiver longitude (radians), + west, - east
- \text{itime}(1): month
- \text{itime}(2): day
- \text{itime}(3): hour (UT)
- \text{itime}(4): minute (UT)
- \text{itime}(5): julian day
- \text{itime}(6): year
- \text{ssn}: monthly median sunspot number

Parameters returned:
- \text{muf}: classical MUF value in MHz
- \text{cpnt}: geographic path information in radians

Subroutines used: muf85

real cpnt(8)
real tlat
tlon
real rlat
real rlon
real ssn
real muf
integer itime(6)

Initialize MUF and geographic path information variables prior to MUF85 subroutine call

\[
\text{muf}=0.0
\]
\[
do\ 100 \ i=1,8 
\text{cpnt}(i)=0.0 
100 \ continue
\]

Enter MUF calculation parameters

\[
\text{tlat}=0.37385 
\text{tlon}=2.76024 
\text{rlat}=0.57125 
\text{rlon}=2.0450 
\text{itime}(1)=1 
\text{itime}(2)=1 
\text{itime}(3)=0 
\text{itime}(4)=0 
\text{itime}(5)=1 
\text{itime}(6)=89 
\text{ssn}=100.0
\]

Calculate classical MUF using MINIMUF-85
call muf35 (tlat, tlon, rlat, rlon, itime, cpnt, ssn, muf)
end
subroutine muf85 (tlat, tlon, rlat, rlon, itime, cpnt, ssn, cmuf)

Updated Nov. 1985
An improved version of muf35 which includes sunspot, season and
diurnal dependence in the M-factor plus an improved FOF2 model
which includes a polar region modification.

This routine computes the classical maximum usable frequency
(cmuf) for a given propagation path. The required input is:

- tlat: transmitter latitude in radians
- tlon: transmitter west longitude in radians
- rlat: receiver latitude in radians
- rlon: receiver west longitude in radians
- itime: six element array containing the (1) month, (2) day
  (3) hour, (4) minute, (5) julian day, and (6) year
- ssn: sunspot number

parameters returned:
- cpnt: geographic path information in radians
- cmuf: classical muf in megahertz.

called by subroutine or function: mufluf

subroutines and functions called: fof2

method: Uses the functional form for the F0F2 and M-factor
described in NOSC TR-186 (MINIMUF-3: A simplified
HF MUF Prediction Algorithm, Rose R.B., J.N. Martin
and P.H. Levine, 1978). Includes improvements as
described in NOSC TR-1121 (MINIMUF-85: An improved
HF MUF Prediction Algorithm, Sailors, D.B., R.A.
Sprague and W.H. Rix).

integer itime(6)
integer khop
integer kkhop
real cpnt(8)
real mfctk
real cplat
real limt
real Itnoon
real daylen
real mfctor
real cpmlat
real sn(6)
real cn(6)
real flat
real tlon
real rlat
real rlon
real ssn
real tday
real qn
real arg
real cfssn
real mfssn
real mfmth
real pthlen
real azim
real pl4000
real cmuf
real cplon
real xkl
real xhop
real akl
real plkhop
real smg
real gyro
real tsol
real soldec
real mfdrnl
real cosxef
real tsunrs
real tsunst
real cnxef
real tdrlx
real tday1
real tnite
real tnite1
real ag
real ag1
real ag2
real ag3
real ag4
real ag5
real ag6
real a14
real a24
real a34
real a44
real beta
real relax
real dayrlx
real alpha
real trnxef
real cpmuf
data pi/3.14159265/,twopi/6.2831853/,halfpi/1.57079632/,
& dtr/0.017453293/,rtd/57.2957795/,r0/6371.1/

c        tday = float( itime(3) ) + float( itime(4) )/60.0

c mfdrl: contains the diurnal dependence of the M-factor

c mfmth: calculated using a 6th order fourier series which is
         a function of month in the new M-factor

c mfssn: a linear function of ssn in the M-factor

c cfssn: a linear function of ssn in the critical frequency expression
M-factor seasonal dependence calculation

do n = 1, 6, 1
qn = float(2*n)
arg = pi*qn*itime(1)/12.0
sn(n) = sin(arg)
cn(n) = cos(arg)
end do
mfmth = .9925+.011*sn(1)+.087*cn(1)-.043*sn(2)
&   +.003*cn(2)-.013*sn(3)-.022*cn(3)
&   +.003*sn(4)+.005*sn(5)+.018*cn(6)

foF2 sunspot number dependence calculation

cfssn = .814*ssn+22.23

M-factor sunspot number dependence calculation

mfssn = 1.3022-.00156*ssn

c control point calculation
call path(tlat,tlon,rlat,rlon,cpnt)
pthlen = cpnt(1)
azim = cpnt(8)
c

c control point changes for muf85
pl4000 = 1.59*cpnt(1)
if(pl4000 .lt. 1.0)pl4000=1.0
mfctk = 1.0/pl4000
if(mfctk .ne. 1.0) mfctk = .5
khop = int(cpnt(1)/.62784)+1
kkhop = khop
if(cpnt(1) .gt. 0.94174)kkhop = 2*khop-1

c cmuf=100.0
do 800 kl = 1,kkhop
c
cplat, cplon = latitude and west longitude of control points
c control point method for muf85
c
if(cpnt(1) .gt. .94174) then
  xkl = k1
  xhop = khop
  akl = xkl/(2.0*xhop)
else
  akl = 1.0/(2.0*pl4000)+float(k1-1)*(.9999-1.0/pl4000)
end if
c
plkhop = pthlen*ak1
c
call razgc( rlat, rlon, plkhop, azim, cplat, cplon )
c
c lmt = local mean time in hours at the control point
c cpmlat = geomagnetic latitude at the control point
c
if ( cplon .ge. 0.0 ) then

23
lmt = cplon
else
  lmt = cplon + twopi
end if
lmt = tday - lmt*rtd/15.0
if ( lmt .lt. 0.0 ) then
  lmt = lmt + 24.0
else if ( lmt .ge. 24.0 ) then
  lmt = lmt - 24.0
end if

C calculation of geomagnetic latitude at the control point

smg = 0.9792*sin(cplat) + 0.2028*cos(cplat)*cos(cplon - 1.2043)
smg = amaxl( aminl( smg, 1.0 ), -1.0 )
cpmlat = asin( smg )

c geomagnetic latitude foF2 dependence calculation
cyro frequency for latitude > 55 degrees
if ( abs( cpmlat ) .lt. 0.95993 ) then
gyro = 0.0
else
gyro = 0.3789*sqrt( 1.0 + 3.0*smg*smg ) - 0.5
end if

c calculation of local noon time
tsol = 2*pi*date/365.25
c soldec = -solar declination
c ltnoon = time of local noon
tsol = 0.0172*(10.0 + float(itime(1)-1)*30.4 + itime(2))
soldec = 0.409*cos(tsol)

ltnoon = 3.82*cplon+12.0+0.13*(sin(tsol)+1.2*sin(2.0*tsol))
if ( ltnoon .gt. 24.0 ) then
  ltnoon = ltnoon - 24.0
else if ( ltnoon .le. 0.0 ) then
  ltnoon = ltnoon + 24.0
end if

M-factor range dependence calculation
mfctor = M-factor = muf/f0F2
mfctor = aminl( 2.5*pthlen*mfctk, halfpi )
mfctor = sin( mfctor )
mfctor = 1.0 + 2.5*mfctor*sqrt( mfctor )

C daylen = length of daylight
tsunrs = time of sunrise
tsunst = time of sunset
tdrlx = daytime relaxation time
night time relaxation time = 2 hours
if ( cos( cplat + soldec ) .gt. -0.26 ) go to 600

no daylight on path at any time during the day

24
cosxef = 0.0
daylen = 0.0
mfdrnl = 1.0
continue

delta T = length of daylight calculation T(sunset)-T(dawn)

daylen = (-0.26+sin(soldec)*sin(cplat))/(cos(soldec)*cos(cplat)
& +1.0e-3)
daylen = amax1( amin1 ( daylen, 1.0 ), -1.0 )
daylen = 12.0 - asin( daylen )*7.6394

T(dawn) calculation

tsunrs = Ltnoon - daylen/2.0
if ( sunrs .lt. 0.0 ) tsunrs = sunrs + 24.0

t(sunset) calculation

tsunst = Ltnoon + daylen/2.0
if ( sunst .gt. 24.0 ) tsunst = sunst - 24.0

calculation of the cosine noon time solar zenith angle

cnxef = abs( cos( cplat + soldec ) )

calculation of the day time relaxation time

tdrlx = 9.7* amax1( cnxef, .1 ) )**9.6
tdrlx = amax1( tdrlx, 0.1 )
tdayl = tday
if (( tsunst .lt. sunrs .and. (tday-tsunst)*(sunrs-tday) .gt. 0.0 ) .or. ( sunst .ge. sunrs .and. (tday-sunrs)*
& (tsunst-tday) .le. 0.0 ))then

night time at control point
M-factor night time dependence calculation
6th order fourier series night time factor for M-factor,

if ( sunst .gt. tWay ) tdayl = tdayl + 24.0
tnite = tdayl - tsunst
tnite = 14.0*tnite/(24.0-daylen)
ag = pi *(tnite1+1.0)/15.0
ag1 = 2.0*ag
ag2 = 4.0*ag
ag3 = 6.0*ag
ag4 = 8.0*ag
ag5 = 10.0*ag
ag6 = 12.0*ag
a14 = 1.0195 -.06*sin(ag1)-.037*cos(ag1)+.018*sin(ag2)
a24 = .003*cos(ag2)+.025*sin(ag3)+.018*cos(ag3)
a34 = .007*sin(ag4)-.005*cos(ag4)+.006*sin(ag5)
a44 = .017*cos(ag5)-.009*sin(ag6)-.004*cos(ag6)
mfdrnl = a14+a24+a34+a44

beta = pi*tdrlx/daylen

25
(T(sunset) - T)/night relaxation time

\[ \text{relax} = \frac{(\text{tsunst} - \text{tdayl})}{2.0} \]
\[ \text{relax} = \min(\max(\text{relax}, -75.0), +75.0) \]

- delta T/day relaxation time

\[ \text{dayrlx} = \frac{-\text{daylen}}{\text{tdrlx}} \]
\[ \text{dayrlx} = \min(\max(\text{dayrlx}, -75.0), +75.0) \]

calculation of sunset cosine effective solar zenith angle

\[ \cos\text{xef} = \text{cnxef}^* (\beta (\exp(\text{dayrlx}) + 1.0) * \exp(\text{relax}) / (1.0 + \beta^2)) \]

else
day time at the control point

if (tunrs .gt. tday) tdayl = tdayl + 24.0

M-factor day time dependence calculation

\[ \text{mdfrnl} = 1.11 - 0.01 \times \text{lmt} \]
\[ \alpha = \pi * (tdayl - tunrs) \times \text{daylen} \]
\[ \beta = \pi \times \text{tdrlx} / \text{daylen} \]

(T(dawn) - T)/day relaxation time

\[ \text{relax} = \frac{(\text{tsunrs} - \text{tdayl})}{\text{tdrlx}} \]
\[ \text{relax} = \min(\max(\text{relax}, -87.0), +87.0) \]
\[ \text{dayrlx} = \frac{-\text{daylen}}{\text{tdrlx}} \]
\[ \text{dayrlx} = \min(\max(\text{dayrlx}, -87.0), +87.0) \]

calculation of day cosine effective solar zenith angle

\[ \cos\text{xef} = \text{cnxef}^* (\sin(\alpha) + \beta (\exp(\text{relax}) - \cos(\alpha))) / (1.0 + \beta^2) \]

c sunrise transition

\[ \text{trnxef} = \text{cnxef}^* (\beta (\exp(\text{dayrlx}) + 1.0)) \]
\[ \times \exp((\text{daylen} - 24.0) / 2.0) / (1.0 + \beta^2) \]
\[ \cos\text{xef} = \max(\cos\text{xef}, \text{trnxef}) \]

end if
cpMUf = MUF at the control point
calculation of f0F2

\[ \text{cpMUf} = \sqrt{6.0 + \text{cfssn} \times \sqrt{\cos\text{xef}}} + \text{gyro} \]

MUF high latitude season dependence calculation

\[ \text{cpMUf} = \text{cpMUf}^* (1.0 - 0.1 \times \exp((\text{daylen} - 24.0) / 3.0)) \]

MUF transsequatorial path dependence calculation

26
CPMUF = CPMUF*( 1.0 + ( 1.0 - SYGN( TLat )*SYGN( RLat ) )*0.1 )

MUF high latitude path dependence calculation

CPMUF = CPMUF*( 1.0 - 0.1*( 1.0 + SYGN( ABS( SIN( CPLAT ) ) )
& - COS( CPLAT ) ) )

FOF2 function corrects for polar region FOF2
result is CPMUF if control point not in polar region

if ( ABS( CPMLAT ) .GE. 0.95993 ) then
    CPMUF = MFCTOR*FOF2( CPMUF, LMT, ITIME, CPLAT, CPLON, CPMLAT, SSN )
else
    CPMUF = CPMUF*MFCTOR
end if

MUF sunspot, season and diurnal dependence

CPMUF = CPMUF*MFSSN*MFMTH*MFDRL
CMUF = AMIN1( CMUF, CPMUF )
     CONTINUE

MUF minimum value 2MHz, maximum value 50 MHz

CMUF = AMIN1( AMAX1( CMUF, 2.0 ), 50.0 )

RETURN
END
subroutine path (tlat, tlon, rlat, rlon, cpnt)

Parameters input:
  tlat: transmitter latitude in radians
  tlon: transmitter west (positive) longitude in radians
  rlat: receiver latitude in radians
  rlon: receiver west (positive) longitude in radians

Parameters returned:
  cpnt(l): distance between the receiver and transmitter in radians
  cpnt(2): latitute of midpoint in radians
  cpnt(3): west longitude in radians
  cpnt(4): latitude of point 1000 km from the receiver in radians
  cpnt(5): west longitude of point 1000 km from receiver in radians
  cpnt(6): latitude of point 1000 km from transmitter in radians
  cpnt(7): west longitude of point 1000 km from transmitter in radians
  cpnt(8): azimuth from receiver to transmitter in radians

* cpnt(4) through cpnt(7) will not be computed for paths less than 1000 km (0.15696 radians) in length.

Subroutines and functions used: gcraz
razgc

Common blocks: none

real cpnt(8)
real rlat
real rlon
real tlat
real tlon
real pi

Get range and azimuth between points 1 and 2

call gcraz( rlat, rlon, tlat, tlon, cpnt(1), cpnt(8) )

Get mid-point coordinates

  pi = cpnt(1)/2.0
  call razgc( rlat, rlon, pi, cpnt(8), cpnt(2), cpnt(3) )

Is path length >= 1000 km?
if ( cpnt(1) .ge. 0.156961231 ) then

Get coordinates of 1000 km points

pl = 0.156961231
call razgc( rlat, rlon, pl, cpnt(8), cpnt(4), cpnt(5) )
pl = cpnt(1) - 0.156961231
call razgc( rlat, rlon, pl, cpnt(8), cpnt(6), cpnt(7) )
return
else
    return
end if
end

end
subroutine razgc(latl, lonl, range, azim, lat2, lon2)

This routine computes the latitude and west (positive) longitude (lat2, lon2) of a point a specified range from a given point on the earth's surface. Also required for input is the azimuth (azim) to the new point in radians. This method assumes a spherical earth (6371.0 km) and recognizes the degenerate cases of the given point being at the north or south pole. For the degenerate cases, azim should be 0 or pi and lon2 is undefined. However, azim is not checked, and lon2 is arbitrarily set equal to lon1. This routine recognizes the degenerate case when range is set to zero. All coordinates are in radians.

Parameters input: lat1, lon1, range, azim

Parameters returned: lat2, lon2

Subroutines and functions used: none

Common blocks: none

Method: Uses law of cosines for sides on spherical triangle defined by (lat1,lon1), north pole and point defined by azim and range.

real lat1
real lon1
real lat2
real lon2
real s1
real c1
real cr
real ca
real cg
real a
real g
real sa

! Test for degenerate cases
1) Given point is north or south pole:
   if ( abs( lat1 - halfpi ) .le. 1.0e-5 ) then
   The given point is the north pole
   lat2 = halfpi - range
   lon2 = lon1
   return
   else
if ( abs( lat1 + halfpi ) .le. 1.0e-5 ) then
  The given point is the south pole
  lat2 = range - halfpi
  lon2 = lon1
  return
end if
end if

2) Coincident points:

if ( range .eq. 0.0 ) then
  Point 2 coincident with point 1
  lat2 = lat1
  lon2 = lon1
  return
end if

General case

sl = sin( lat1 )
c1 = cos( lat1 )
cr = cos( range )
ca = sl*cr + c1*sin( range )*cos( azim )
a = amin1( amax1( ca, -1.0 ), +1.0 )

Test if destination ends up on the poles

if( abs(a).le.1.0e-5 ) then
  lat2 = halfpi
  lon2 = lon1
  return
else
  if( abs(a-pi) .le. 1.0e-5 ) then
    lat2 = -halfpi
    lon2 = lon1
    return
  end if
end if

Get destination coordinates

cg = ( cr - sl*ca )/( c1*sin( a ) )
cg = amin1( amax1( cg, -1.0 ), +1.0 )
g = acos( cg )
lat2 = halfpi - a
sa = sin( azim )
if ( sa .ge. 0.0 ) lon2 = amod( lon1 - g, twopi )
if ( sa .lt. 0.0 ) lon2 = amod( lon1 + g, twopi )
return
end
subroutine gcraz( latl, lon1, lat2, lon2, range, azim )

This routine computes the great circle range and azimuth between two points on the earth's surface. lat1 and lon1 are the coordinates of point 1, lat2 and lon2 are the coordinates of point 2. Both longitudes are west longitudes. West longitudes are positive throughout the Muf85 algorithm. Latitudes are positive if north and negative if south. The output is range, the distance between the two points in radians and azim, the azimuth from one point to the other in radians. This method assumes a spherical earth and recognizes the degenerate cases of point 1 at the north or south pole or points 1 and 2 coincident. All coordinates are in radians.

Parameters input: lat1, lon1, lat2, lon2

Parameters returned: range, azim

Subroutines and functions used: none

Common blocks: none

Method: Uses law of cosines for sides on spherical triangle defined by (lat1,lon1),(lat2,lon2) and north pole.

real lat1
real lon1
real lat2
real lon2
real range
real azim
real s1
real c1
real s2
real c2
real cr
real ca

data pi/3.141592654/, twopi/6.283185308/, halfpi/1.570796327/, &
     dtr/0.017453293/, rtd/57.29577951/

test for degenerate cases
1) Point 1 at north or south pole:
if ( abs( lat1 - halfpi ) .le. 1.0e-5 ) then
  Point 1 is at the north pole
  range = halfpi - lat2
  azim = pi
  return
else
  if ( abs( lat1 + halfpi ) .le. 1.0e-5 ) then
    c c Point 1 is at the south pole
    range = halfpi + lat2
    azim = 0.0
    return
  end if
end if

2) Coincident points:

if ( abs( lat1 - lat2 ) .le. 1.0e-5 .and. 
    abs( lon1 - lon2 ) .le. 1.0e-5 ) then
  c c Points 1 and 2 are coincident
  range = 1.0e-8
  azim = 0.0
  return
end if

General case

s1 = sin( lat1 )
c1 = cos( lat1 )
s2 = sin( lat2 )
c2 = cos( lat2 )
cr = s1*s2 + c1*c2*cos( lon1 - lon2 )
cr = amin1( amax1( cr, -1.0 ), +1.0 )
range = acos( cr )
ca = ( s2 - s1*cr )/( c1*sin( range ) )
ca = amin1( amax1( ca, -1.0 ), +1.0 )
azim = acos( ca )
if ( sin( lon1 - lon2 ) .lt. 0.0 ) azim = twopi - azim
return
end
function fof2( ff2, lmt, itime, lat, lon, mlat, ssn )

This function corrects the f2-layer critical frequency computed by muf85 for polar latitudes using the Chiu model.

Parameters input:
- ff2: critical frequency from muf35 in mhz
- lmt: local mean time at lat,lon in hours
- itime: integer array containing month, day, hour, minute, julian day, and year
- lat: geographic latitude in radians
- lon: geographic west longitude in radians
- mlat: magnetic latitude in radians
- ssn: sunspot number

Parameters returned:
- fof2: the f2-layer critical frequency in mhz - real

Subroutines and functions called: none

Common blocks referenced: none

Method: uses the high latitude electron density model developed by B.K. Ching and Y.T. Chiu (Ching, B.K., Y.T. Chiu, J. Atmos. Terr. Phys., 35, 1615, (1973)) and later improved by Chiu (Chiu Y.T., J. Atmos. Terr. Phys., 37, 1563, (1975)). The peak electron density is then converted to foF2 and used to correct the value input from muf85. A folding function is used in transition latitudes to provide continuity of the transition.

integer itime(6)
real lat
real lmt
real lon
real mlat
real mlon
real phi
real tmo
real cmlat
real x
real ssn
real ff
real gg
real t
real v
real y
real ys
real z
real w
real plr
real u
data
pi /3.1415926/

phi = lmt*pi/12.0
tmo = itime(1) + ( itime(2) + itime(3)/24.0 + itime(4)/1440.0 )/30.0
& - 0.5
cmlat = cos( mlat )
mlon = cos( lat )*sin( lon - 1.2043 )/cmlat
mlon = amax1( amin1( mlon, 1.0 ), -1.0 )
mlon = asin( mlon )
x = ( 2.2 + ( 0.2 + ssn/1000.0 )*sin( mlat ) )*cmlat
ff = exp( -( x**6 ) )
gg = 1.0 - ff
t = pi*tmo/12.0
v = sin( t )
if ( mlat .ge. 0.0 ) then
  w = exp( -1.2*( cos( mlat - 0.41015*cos( phi ) ) - cmlat ) )
  plr = ( 2.0 + 0.012*ssn )*w*( 1.0 + 0.3*v )
else
  u = cos( t+t )
y = sin( mlon/2.0 )
ys = cos( mlon/2.0 - pi/20.0 )
z = sin( mlon )
za = sqrt( abs( z ) )
am = 1.0 + v
b = v*( ( y - z )/2.0 - y**8 ) - am*u*( z/za )*exp( -4.0*y*y )
ys4 = ys**4
  plr = (2.5 + ssn/50.0 + u*( 0.5 + ( 1.3 + 0.002*ssn )*ys4 )
& + ( 1.3 + 0.005*ssn )*cos( phi - pi*( 1.0 + b ) ))
& + ( 1.0 + 0.4*( 1.0 - v*v ) )*exp( -v**ys4 )
end if

fof2 = gg*ff2*ff2/8.12 + 0.66*ff*plr
if ( fof2 .gt. 0.0 ) then
  fof2 = 2.85*sqrt(fof2)
else
  fof2 = ff2
end if
return
end
function sygn( y )
  real function sygn
  x= sygn( y )
  This function returns the value 0 if y=0, -1 if y is less than 0
  or 1 if y is greater than 0.
  Subroutines and functions used: none
  Common blocks: none
  Parameters input: y
  real y
  if(y)100,200,300
  100 sygn=-1.0
      go to 1000
  200 sygn=0.0
      go to 1000
  300 sygn=1.0
  1000 return
  end