AD-A214 807

MATHMATICS OF WAVE PROPAGATION IN RANDOM MEDIA

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Final

September 1979

unclassified

unclassified

767-5025

unclassified

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Previous editions are obsolete.
Introduction

Since this grant was begun, interim reports have been submitted each year describing the work undertaken during that year. Therefore in the present report, we shall describe the work during the final year 9/30/78-9/30/79.

A number of new studies were started and some research reports were written on this work. In addition some work was published and other work was submitted or accepted for publication. In Appendix A, a list of the last 4 reports written, BR-128 through BR-131 is presented, together with their publication status and that of BR-122. Of them 4 were published during the period covered by this report, and 1 has been submitted, but not yet accepted.

In the next section the contents of reports BR-128 through BR-131 are described.
BR-128. One-dimensional, nonlinear wave motion in an infinitely conducting, electrically neutral, incompressible elastic medium in the presence of a magnetic field is studied. It is assumed that the elastic medium, in the absence of the magnetic field, is isotropic and that the elastic stresses are derivable from an internal energy function which depends solely on the elastic strains and the entropy. The magnetic field manifests itself as a body force per unit deformed volume arising from the action of the magnetic induction on induced currents. All physical quantities are assumed to depend on only one space variable and time; however, no restriction is placed on the orientations of the velocity, magnetic and strain vectors. The analysis is based on the fact that the governing equations constitute a first order hyperbolic system of strict conservation laws. This fact implies, in advance, that incompressible magnetoelastic media possess a theory of simple waves and shocks analogous to that of gas dynamics and magneto-gas-dynamics. There are essentially two types of simple waves and two types of shocks, called slow and fast, as well as contact layers and contact discontinuities. The results are applied to a magnetoelastic "piston" problem in which the motion is initiated by purely magnetic (nonmechanical) means.

BR-129. A survey of the theory of elastic waveguides is presented, with special emphasis on guides of general cross-section. First the theory of low frequency waves on rods is
presented and the results of various authors are compared. The waves near their cutoff frequencies are treated. Next high frequency waves, including Rayleigh waves on helical paths, are presented. Then the analysis of high frequency waves along generators of rods is considered. Finally the ray theory and the asymptotic theory of surface wave propagation are presented.

BR-130. Flows of incompressible inviscid heavy fluids with free or rigid boundary surfaces are considered. For slender streams of fluid, the flow and the free boundaries are represented by a number of different asymptotic expansions in powers of the slenderness ratio. There are three kinds of outer expansions representing respectively jets, which have two free boundaries, wall flows, which have one free and one rigid boundary and channel flows, which have two rigid boundaries. The flow at the junction of two or more outer flows is represented by an inner expansion. Previously we constructed the three outer expansions and the inner expansion at the junction of a wall flow and a jet [J. Fluid Mech. 59, 417-432 (1973)]. Now we construct the inner expansions at the junctions of a channel flow and a jet, a channel flow and a wall flow, and a jet and the two wall flows into which it splits upon hitting a wall. We also match each inner expansion to the adjacent outer expansions. These seven expansions can be combined to solve many problems involving flows of slender streams.
The development of the theory of linear wave propagation is described after a brief sketch of what wave propagation is. First the classical techniques of images and separation of variables are considered, followed by Sommerfeld's extension of the image method to multi-sheeted spaces, and Watson's transformation of series solutions to more rapidly converging forms. Then the Wiener-Hopf method of solving certain integral equations and Schwinger's variational method of calculating scattering parameters are introduced. Next the normal mode theory of propagation and its development by Pekeris, Brekhovskikh and others is described. Then the WKB method and its extensions are presented. This is followed by discussions of ray theory, of the "parabolic equation" method, and of waves in heterogeneous and random media. Finally prospects for the future are considered, with emphasis on the use of computers and methods of calculation.
| BR-122 | A. Bensoussan  
|        | J. L. Lions  
|        | G. C. Papanicolaou  |
|        | Boundary Layers and Homogenization  
|        | of Transport Processes  |
|        | Sciences, Kyoto Univ., Vol. 15,  
| BR-128 | F. Karal  
|        | J. Bazer  |
|        | Nonlinear Magnetoelastic Wave Motion  
|        | in Incompressible Infinitely  
|        | Conducting Solids  |
| BR-129 | J. B. Keller  |
|        | Elastic Waveguides  |
|        | Pub: Modern Problems in Elastic Wave  
|        | Propagation, J. Miklowitz and  
|        | J. D. Achenbach, eds., Wiley,  
| BR-130 | J. Geer  
|        | J. B. Keller  |
|        | Slender Streams  |
|        | Pub: J. Fluid Mech., 93 (1979),  
|        | pp. 97-115.  |
| BR-131 | J. B. Keller  |
|        | Progress and Prospects in the Theory  
|        | of Linear Wave Propagation  |