FORTRAN SUBROUTINES FOR THE EVALUATION OF THE
CONFLUENT HYPERGEOMETRIC FUNCTIONS

WILLIAM GRATG
BENY NETA

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Prepared by:

WILLIAM GRAGG
Professor of Mathematics

Reviewed by:

HAROLD M. FREDRICKSEN
Chairman
Department of Mathematics

Released by:

KNEALE T. MARSHALL
Dean of Information and Policy Sciences
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12 PERSONAL AUTHOR(S):
William Gragg and Beny Neta

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

In this report, we list the Fortran subroutines for evaluating the confluent hypergeometric functions M(a,b;x) and U(a,b;x). These subroutines use the stable recurrence relations given e.g. in Wimp.
Fortran Subroutines for the Evaluation of the Confluent Hypergeometric Functions

W. Gragg
and
B. Neta

Naval Postgraduate School
Department of Mathematics
Monterey, CA  93943
Abstract

In this report we list the Fortran subroutines for evaluating the confluent hypergeometric functions \( M(a,b;x) \) and \( U(a,b;x) \). These subroutines use the stable recurrence relations given e.g. in Wimp.

Key words:
confluent hypergeometric functions
stable algorithm
Fortran subroutine
recurrence relation
Introduction

It is well known that the ordinary differential equation

\[ \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} - ay = 0 \]

has a solution

\[ y(x) = AM(a,1;x) + BU(a,1;x) \]

if \( a \) is not a negative integer.

This problem arises e.g. when solving the linearized shallow water equations with the full linear variation in depth included (see Williams, Staniforth and Neta, [1]).

The computation of the confluent hypergeometric functions is based on the Miller algorithm (see e.g. Wimp. [2]). In general, one has a second order difference equation

\[ z(n) + a(n)z(n+1) + b(n)z(n+2) = 0, \quad n \geq 0, \quad b(n) \neq 0. \]

If \( b(n) = 0 \) for some \( n \), in some cases one can make a change of variable \( Y(n) = \lambda(n)z(n) \) which will produce an equation of the desired type. Let \( w(n) \) be a nontrivial solution and the sum of the normalizing series

\[ S = \sum_{k=0}^{\infty} c(k)w(k) \neq 0 \]
is known. Let $N$ be a large integer and define $z_N(n), 0 \leq n \leq N+1$, by

$$
z_N(n) = \begin{cases} 
0 & n = N+1 \\
1 & n = N 
\end{cases}
$$

and

$$
z_N(n) + a(n)z_N(n+1) + b(n)z_N(n+2) = 0, \quad n = N-1, \ldots, 1, 0.
$$

One can approximate $w(n)$ by $w_N(n)$

$$
w_N(n) = \frac{Sz_N(n)}{S_N}
$$

where

$$
S_N = \sum_{k=0}^{N} c(k)z_N(k).
$$

The algorithm is said to converge if

$$
w_N(n) \rightarrow w(n) \text{ as } N \rightarrow \infty.
$$

The function $M(a, b; x)$ satisfies the recurrence relation

$$
(2n+b+2)(n+a)z(n) - (2n+b+1)\left\{(2a-b) + \frac{(2n+b)(2n+b+2)}{\infty}\right\}z(n+1)
$$

$$
- (2n+b)(n+b+1-a)z(n+2) = 0.
$$

The minimal solution is

$$
w(n) = \frac{\xi(n)}{(b)^{2n}} M(a+n, 2n+b; x)
$$
where

\[(c)_n = \frac{\Gamma(n+c)}{\Gamma(c)}.\]

The normalization relationship used in our subroutine is

\[S = b-1 = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (b-1)(b+2k-1)w(k).\]

An obvious modification must be made if \(b = 1\). The algorithm is not defined if \(b, b+1-a, a\) are negative integers or zero.

The function \(U(a,b;x)\) satisfies the relationship

\[(n+a)(n+a+1-b)z(n) - (n+1)[2(n+a+1)+x-b]z(n+1) + (n+1)(n+2)z(n+2) = 0.\]

The minimal solution is

\[w(n) = \frac{x^n(a)_n(a+1-b)_n}{n!} U(a+n,b;x)\]

for \(|\text{arg } x| < \pi\). A normalization relation is

\[1 = \sum_{k=0}^{\infty} w(k, , ).\]

In the next section we give a listing of the Fortran subroutines.
Subroutine Miller

SUBROUTINE MILLER(N, ALPHA, BETA, X, S, SS, COEFF)
INTEGER N
REAL, 8 ALPHA, BETA, X, SS
REAL, 8 S(0:1000)
EXTERNAL COEFF
C USES THE J.C.P. MILLER ALGORITHM TO COMPUTE
C S(0:N).
C BEGIN
INTEGER NN, K
REAL, 8 T, D, EPS, A, B, C
REAL, 8 OLDS(0:1000)
EPS = 0.000000001
C INITIALIZE OLDS.
DO 1 K = 0, 1000
OLDS(K) = 0
1 CONTINUE
C CHOOSE INITIAL NN.
NN = N + 2
C INITIALIZE K, S AND T.
2 K = NN
S(K+1) = 0
S(K) = 1
CALL COEFF(K, ALPHA, BETA, X, A, B, C)
T = 2*C*S(K)
C TAKE A BACKWARD RECURRENCE STEP AND UPDATE IT.
3 K = K - 1
CALL COEFF(K, ALPHA, BETA, X, A, B, C)
S(K) = A*S(K+1) + B*S(K+2)
C CHECK FOR OVERFLOW AND RESCALE IF NECESSARY.
D = DABS(S(K))
IF (D GT 1.D30) THEN
C BEGIN
CALL SCALE(K, NN, S, T, D)
END IF
IF (K GT 0) THEN
C BEGIN
T = T + 2*C*S(K)
GO TO 3
END IF
T = T + C*S(0)
DO 4 K = 0, N
S(K) = S(K)/T
4 CONTINUE
C TEMPORARY PRINT STATEMENT.
C PRINT*, S(0)
C TEST FOR CONVERGENCE.
D = 0
5 DO 5 K = 0, N
D = D + S(K)**2
5 CONTINUE
D = DSQRT(D)
T = 0
DO 6 K = 0, N
   T = T + (S(K) - OLDS(K))**2
6 CONTINUE
   T = DSQRT(T)
C TAKE ANOTHER STEP IF NO CONVERGENCE.
   IF (T .GT. EPS-D) THEN
C BEGIN
      NN = 2*NN
   DO 7 K = 0, N
      OLDS(K) = S(K)
7 CONTINUE
   IF(NN .LE. 1000) GO TO 2
   PRINT 999, NN, ALPHA, BETA, X, T
999 FORMAT(' ** NO CONVERGENCE ** NN AP CP X T ',15.4E14.7)
   END IF
   SS=S(0)
RETURN
END
SUBROUTINE COEFF(N, ALPHA, BETA, X, A, B, C)
INTEGER N
REAL=8 ALPHA, BETA, X, A, B, C
C COMPUTES COEFFICIENTS USED BY J.C.P. MILLER ALGORITHM FOR
C A CONFLUENT HYPERGEOMETRIC FUNCTION M(a, b: x)
C SEE JET WIMP, COMPUTATION WITH RECURRENCE RELATIONS.
C PITMAN 1984 PP. 61-62
C BEGIN
   INTEGER M, K
   REAL=8 T, U, V, W
   S = 2*ALPHA - BETA
   T = N + ALPHA
   M = 2*N
   U = M + BETA
   V = U + 1
   W = V + 1
   A = (S/W + U/X)*V/T
   B = (N + BETA - ALPHA + 1)*U/T/W
   T = 1
   IF (N .GT. 0) THEN
      C BEGIN
      S = BETA - 1
      DO 1 K = 1, N-1
         T = -T*(1+S/K)
         1 CONTINUE
      T = -T*(1+S/M)
      END IF
      C = T
      RETURN
   END

SUBROUTINE SCALE(K, N, S, T, D)
INTEGER N, K
REAL=8 T, D
REAL=8 S(0:1000)
C BEGIN
   INTEGER J
   T = T/D
   DO 1 J = K, N
   S(J) = S(J)/D
   1 CONTINUE
   RETURN
END
SUBROUTINE COEFU(N, ALPHA, BETA, X, A, B, C)
INTEGER N
REAL*8 ALPHA, BETA, X, A, B, C
C COMPUTES COEFFICIENTS USED BY J.C.P. MILLER ALGORITHM FOR
C A CONFLUENT HYPERGEOMETRIC FUNCTION \( U(a,b;x) \)
C SEE JET WIMP. COMPUTATION WITH RECURRENCE RELATIONS.
C PITMAN 1984 PP. 63-64
C BEGIN
INTEGER M, K
REAL*8 S, T, U, V, W
S = ALPHA + QFLOAT(N)
T = S + 1.0
U = S*(T - BETA)
V = QFLOAT(N + 1)
W = V + 1.0
A = (2*T + X - BETA)*V/U
B = - V*W/U
C = 1
RETURN
END

Remark: The program that calls Miller must supply as a last parameter either COEFF (for M) or COEFU (for U).
The subroutines are available on a diskette from either author upon request. These subroutines were tested extensively for various values of a, b and x.

Remark: If the parameter is a negative integer, the solution of the differential equation is

\[ y = A L_n(x) + B \{ \ln|x| L_n(x) + \sum_{m=0}^{\infty} \beta_m x^m \} \]

where \( n = -a \).

\( L_n(x) \) are Laguerre polynomials whose coefficients \( a_i \) satisfy

\[ a_i = \frac{i-i-1}{i^2} a_{i-1}, \quad i = 2, \ldots, n \]

\[ a_1 = -n \]

The coefficients \( \beta_m \) satisfy

\[ \beta_{m+1} = \frac{(m-n) \beta_n + \left(1 - \frac{2(m-n)}{m+1} a_m \right)}{(m+1)^2}, \quad m = 1, \ldots, n-1 \]

\[ \beta_m = \frac{1}{(n-1)^2} a_n \quad m = n \]

\[ \beta_m = \frac{(m-n-1)}{m^2} \beta_{m-1} \quad m = n+1, n+2, \ldots \]
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References


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</table>
Professor W. Gragg
Code 53Gr
Department of Mathematics
Naval Postgraduate School
Monterey, CA 93943

Professor H. Dean Victory, Jr.
Texas Tech University
Department of Mathematics
Lubbock, TX 79409

Professor Gordon Latta
Code 53Lz
Department of Mathematics
Naval Postgraduate School
Monterey, CA 93943

Professor Arthur Schoenstadt
Code 53Zh
Department of Mathematics
Naval Postgraduate School
Monterey, CA 93943

Professor M.M. Chawla, Head
Department of Mathematics
III/III/B-1, IIT Campus
Hauz Khas, New Delhi 110016
India

Professor R.T. Williams
Code 63Wu
Department of Meteorology
Naval Postgraduate School
Monterey, CA 93943