The reliability function of a structure—the probability that the structure will perform adequately—represents a condition index that is useful in evaluating the structure's condition. A generally applicable, three-step procedure has been developed for calculating structural reliability using existing information about a particular structure and the evaluator's confidence in this information. The structure's margin of safety is approximated by a normal distribution and its moments are evaluated from point estimates. It is suggested that the three-step procedure be used to evaluate the safety
PREFACE

The study reported here was authorized by Headquarters, U.S. Army Corps of Engineers (HQUSACE), as part of the Concrete and Steel Structures Problem Area of the Repair, Evaluation, Maintenance, and Rehabilitation (REMR) Research Program. The work was performed under Work Unit 3301, "Structures Damage Index to Determine Remaining Life and Reliability of Metal Structures." Mr. Frank Kearney was Principal Investigator for this Work Unit. Dr. Tony C. Liu (CECW-ED) was the REMR Technical Monitor for this work.

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This study was conducted by the Structures Division of JAYCOR for the Engineering and Materials Division (EM) of the U.S. Army Construction Engineering Research Laboratory (USACERL), during the period April 1985 to September 1985. Appreciation is expressed for the assistance of Dr. Dawn White, Mr. Barry Hare, and Dr. Anthony Kao (all of USACERL) and Ms. Caroline Cummins, Ms. Susan Lanier, and Mr. William J. Flathau (all of JAYCOR). COL Carl O. Magnell is Commander and Director of USACERL, and Dr. L. R. Shaffer is Technical Director. Dr. Robert Quattrone is chief of USACERL-EM.
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ASSESSING THE RELIABILITY OF STEEL CIVIL WORKS STRUCTURES

PART I: INTRODUCTION

Background

1. A major mission of the U.S. Army Corps of Engineers (USACE) is maintaining the operational efficiency of various navigation, flood control, and hydroelectric power projects. The importance of this mission increases with time as older projects deteriorate and few new projects are authorized. Of particular concern are steel structures. A recent study (JAYCOR 1985) documented that USACE Civil Works projects include many structurally significant steel features such as bridges, outlet gates, spillway gates, lock gates and valves, sheet piles, hydropower gates, and penstock liners (Figure 1). The study further estimated that each year these steel structures experience about 44 different problems to varying degrees.

2. Research has shown that a relatively significant fraction of the problems occur with steel sheet piling. Deterministic analyses of this structural feature have been conducted by the U.S. Army Construction Engineering Research Laboratory (USA-CERL) (Kearney 1986). That study of sheet pile structures identified the elements that could fail (Figure 2).

3. A structural condition index would make it easier to allocate public funds where they are most needed to solve these problems and avert failures. Such an index should indicate the composite structural integrity and the overall operational condition of a particular feature, and reflect the extent, severity, and type of deterioration. It should also take into account the effects of loads imposed on the structure. The reliability function can provide such a condition index. By quantifying changes in resistance and loading, this function indicates the probability that a structure will perform adequately under certain conditions. Also, as used in this report, it provides more information than a deterministic analysis because it shows the gradual deterioration of condition over time.
Figure 2. Structurally critical elements of steel sheet pile structures
PART II: METHODOLOGY

Reliability as an Index of Condition

7. This chapter describes a general procedure for estimating the reliability of existing CE structures. The reliability of a structure is the probability that, when operating under stated environmental conditions, it will perform its intended function adequately for a specified interval of time (Kapur and Lamberson 1977). The goal of reliability analysis is to find the likelihood that the load exceeds the resistance. Thus, each calculation has two distinct parts for each mode of failure: stress (load) analysis and strength (resistance) analysis. This reliability function provides all the characteristics desirable for an index of a structure's present condition. As a mathematical "probability," it is a numerically ordered measure, with unity representing the best condition (best performance) and zero representing the worst condition (no performance). "Stated environmental conditions" explicitly incorporate the effects of imposed service loadings that were not contemplated during design. The "specified interval of time" is necessary for quantifying any deterioration in strength capacity that a structure experiences. Finally, the "intended function" makes it possible to address manageable operation and maintenance problems.

8. Probabilistic methods will be most useful in evaluating existing structures, rather than in designing new ones, because they are better for handling uncertainties. For design, the Corps of Engineers has traditionally used deterministic rather than probabilistic methods. For example, in designing a lock chamber with sheet pile cells as walls, uncertainties will arise about operational loadings and material strengths. A prudent designer uses deterministic, conservative values, possibly overdesigning the feature. The additional cost incurred by overdesigning is usually only a small fraction of the total construction cost. On the other hand, when evaluating a structure's need for rehabilitation, the same uncertainties will arise as in design, along with others arising from the evaluator's inability to make certain measurements and inspect certain parts. However, in rehabilitation, the costs of uncertainty are much greater. Not only is the actual work
Figure 3. Miter gates: horizontally framed typical girder data
considered random variables. For them, a mean and a standard deviation are assigned. Those two parameters are precisely defined in elementary probability texts (Benjamin and Cornell 1970). The mean measures the central tendency of the variable or the value about which scatter can be expected if repeated observations of the phenomenon are made. The standard deviation, which has the same physical units as the mean, measures the dispersion of repeated observations about the mean value.

14. The engineer can draw upon extensive sources of information to assign values for these parameters. In some cases, a sample of repeated observations having useful descriptive statistics may have been collected. Often, data collected for similar problems may apply. An experienced engineer can quantify his or her expert judgment by selecting the mean and the standard deviation for random variables. This step is different from the design situation in which a single conservative value is assigned to uncertain factors. However, in evaluating existing structures, it is important to quantify what is known about variables by using means, and to quantify what is not known about them by using standard deviations.

15. In the example shown in Figure 3, the geometry of the girder is presumed to be known with confidence. Accordingly, the values for these parameters will be deterministically assigned, as given in Table 1. These numbers correspond to an example design (Granade 1980), except for the moment of inertia, I, which is reduced by 30 percent to reflect hypothetical corrosion of the upstream flange. The yield stress of the structural steel, $f_y$, is assumed to be random. It is reasonable to expect that the findings of a general study (American Iron and Steel Institute 1978) of this factor's variability apply to the example outlined here. Accordingly, the mean and standard deviation of $f_y$ are assigned as indicated in Table 1. Note that the mean value is greater than the 36,000 psi normally used in design. The girder loading is also taken to be random. Judgment is used to assign its mean value to be 150 percent of the hydrostatic value, reflecting the effects of ice, mud, and live loads. The uncertainty about this factor is quantified in a standard deviation of 20 percent of the mean.
of these results using Equations 4 and 5. More generally, if MS (X₁, X₂, ... Xₙ) is a function of n independent input variables:

\[
\frac{\bar{MS}}{ms} = \frac{\bar{MS}_1}{ms_1} \cdot \frac{\bar{MS}_2}{ms_2} \cdots \frac{\bar{MS}_n}{ms_n}
\]  

(8)

\[
1 + \left(\frac{\sigma_{MS}}{\bar{MS}}\right)^2 = \left[1 + \left(\frac{\sigma_{MS_1}}{\bar{MS}_1}\right)^2\right] \left[1 + \left(\frac{\sigma_{MS_2}}{\bar{MS}_2}\right)^2\right] \cdots \left[1 + \left(\frac{\sigma_{MS_n}}{\bar{MS}_n}\right)^2\right]
\]  

(9)

where ms is the margin of safety calculated using the mean value of each variable:

\[
ms = MS(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)
\]  

(10)

and \(\bar{MS}_i\) and \(\sigma_{MS_i}\) are computed from Equations 4 and 5 as if \(X_i\) were the only random variable and the others were equal to their mean values. For this general case, \(2n + 1\) design-like evaluations of the margin of safety are required.

18. With these estimates of \(\bar{MS}\) and \(\sigma_{MS}\), a convenient assumption can be made about the form of the MS distribution in order to complete the estimation of reliability. Since the margin of safety is the difference between resistance and load which, in turn, often involves sums of random variables, there is some justification to approximate MS with a normal distribution (Benjamin and Cornell 1970). Since the structural evaluations of operational interest do not involve extremely rare events, in most cases the reliabilities calculated will not be extremely sensitive to the form of this distribution. The assumption of a normal distribution has two other advantages: it is familiar even to those new to probability, and standard tables and approximating functions are available. Finally, it follows that the reliability is given by:

\[
R = \text{Probability} (MS > 0) = 1 - F\left(-\frac{\bar{MS}}{\sigma_{MS}}\right)
\]  

(11)

in which \(F()\) is the standard normal cumulative distribution function (Benjamin and Cornell 1970).
Reliability, which indexes structural condition on a zero to one scale, follows from Equation 11. Note that this sequence is identical irrespective of the deterministic model selected in Step 1. It applies for a comprehensive, finite element stress analysis as well as for a statically determinate equilibrium equation. Appendix A provides a microcomputer program to implement this procedure.

Example Calculation

21. For the example given in Figure 3, \( n = 2 \) variables have been considered random, \( f_y \) and \( w \). Accordingly, five deterministic evaluations of Equation 3 are required, as summarized in Table 2. The estimates of \( \overline{MS} \) and \( \sigma_{MS} \), considering only \( f_y \) to be random, are from Equations 4 and 5 and the second and third rows of Table 2:

\[
\overline{MS}_1 = \frac{MS_+ - MS_-}{2} = \frac{4271 + 13071}{2} = 8671 \text{ psi}
\]

\[
\sigma_{MS}_1 = \frac{MS_+ - MS_-}{2} = \frac{13071 - 4271}{2} = 4400 \text{ psi}
\]

Considering only \( w \) to be random, the last two rows of Table 2 give:

\[
\overline{MS}_2 = \frac{14856 + 2485}{2} = 8671 \text{ psi}
\]

\[
\sigma_{MS}_2 = \frac{14856 - 2485}{2} = 6186 \text{ psi}
\]

Substitution of these results and \( MS \) from the first row of Table 2, into Eq 8 leads to:

\[
\frac{MS}{8671} = (1.000)(1.000)
\]

from which \( MS = 8671 \text{ psi} \). In turn, Equation 9 becomes:

\[
1 + \left( \frac{\sigma_{MS}}{8671} \right)^2 = (1 + 0.56/2)(1 + 0.713^2)
\]

\[
\sigma_{MS} = 8208 \text{ psi}
\]
PART III: O'BRIEN LOCK AND DAM

24. In 1979, an inspection of the O'Brien Lock and Dam, located on the Illinois waterway in Chicago, revealed excessive corrosion of sheet piling. A deterministic analysis of the problem (Kearney 1986) concluded that the structure should have a service life of 83 years. For this study, the O'Brien Lock and Dam has been used as an example of how probabilistic analysis can be employed to calculate service life.

General Information

25. The O'Brien Lock and Dam, a cellular cofferdam, presents a complex problem because the factors affecting its service life are many and subtle. Cellular cofferdams are double-wall retaining structures constructed of interlocking steel sheet piles. The piles form adjacent cells that are filled with soil or rock fragments (Lacroix, Esrig, and Luscher 1970). In analyzing cofferdams, one needs to consider not only the structure's individual characteristics, but also the interactions among soil, rock, water, weather, steel, and concrete. A dam's shape and location also make it an individual system of unique characteristics: cofferdams may be shaped in a circle, diaphragm, or cloverleaf; sheet piling walls can be constructed on dry land, in fresh water, or in seawater. However, all cofferdams have one factor in common: any interlock in a single sheet pile cell is a possible failure location, and unlike most conventional structures, there is little redundancy in the system.

26. The following types of cofferdam failures have been observed and reported:

a. Sliding on the base.
b. Bank sliding.
c. Flooding of a cell.
d. Erosion in streams—scour and loss of cell fill.
e. Boiling of land cofferdams.
f. Shearing of silt and clay fill.
g. Impact from a barge.
h. Shear rupture of cross wall webs.
Figure 5. Sheet pile interlocks

(a) Free body diagram of interlocking forces

(b) Exaggerated illustration of tensile yielding of the shanks
The variability in $N_u$, interlock strength, is shown by the histogram in Figure 6, which is based on a large number of tests by Kay (1975). It is obvious that the histogram is skewed. The truncation at the upper end is associated with web failures rather than interlock failures, and it is likely that a greater degree of symmetry would have been produced had there been no web failures. There is some justification for fitting a normal distribution to an interlock histogram (Kay 1975). Kay has indicated that a normal distribution with a coefficient of variation (ratio of standard deviation to mean) of 0.11 will provide a good representation of the interlock strength. Therefore, a deterministic analysis of strength, as shown above, results in the mean value of the interlock strength; then a value of 0.11 can be taken for the coefficient of variation.

![Figure 6. Measured distribution of interlock strength (Kay 1975)](image)
Figure 7. Cross section of cofferdam
using the technique described in Part II, is presented in Figure 8. This figure also presents reliability as a function of time, $t$, using Lacroix's assumption that $\frac{\varepsilon}{t} = 0.0025$ in./yr (Lacroix, Esrig, and Luscher 1970).

Shear Rupture in Cross Wall

39. A cross wall may become so corroded that it will rupture in shear. Since the hoop tension is transmitted to the Y-pile at the junctions between the cylindrical walls and the cross walls, horizontal tension will be developed in the cross walls (Figures 9a and 9b). The combination of a cross wall and the segments of the corresponding cylindrical walls act as an I-beam to

![Figure 8](image-url)
resist the shear developed by the lateral pressures on both sides of the land wall (Figure 9c). Consequently, an element of the cross wall near the junction with the Y-pile is subjected to shear, $S$, and tension, $N'$ (Figure 9d). By the Mohr's Circle, the maximum shear (the "load") in the cross wall is:

$$ S_{\text{max}} = \sqrt{S^2 + \frac{1}{4} N'^2} $$  \hspace{1cm} (19)

$N'$ is the tension transmitted to the cross wall and is expressed as:

$$ N' = 2N \cos \alpha $$  \hspace{1cm} (20)

where $N$ is the hoop tension and $\alpha$ is as shown in Figure 9a.

40. This shear is not carried by the cross wall alone; however, due to the lack of a reliable theory to estimate how much of the shear is resisted by the fill, it is assumed that all of it is carried by the cross wall. For the I-beams with heavy flanges, the shearing stress is nearly constant on a cross section of the web of an I-beam:

$$ S = \frac{P}{b} $$  \hspace{1cm} (21)

where $b$ is shown in Figure 9c, and the definition of $P$ follows.

41. The land wall of the lock carries greater shear than the river wall because of the pressure of the backfill. The largest total shear, $P$, occurs at the bottom of the lock chamber and is determined from:

$$ P = L \int_{d}^{H} p \, dx $$  \hspace{1cm} (22)

where $p$ is the net lateral pressure acting on the crosswall. At the bottom of the chamber, the following expression can be substituted for $p$:

$$ p = K_A \gamma (H - s) + (K_A \gamma' + \gamma_w) (s - x), \quad d < x < h $$  \hspace{1cm} (23)
and the reliability of this structural element is:

\[ R(\epsilon) = P(MS > 0). \]  \hspace{1cm} (27)

Figure 10 illustrates this reliability function, developed for the shear strength of cross walls.

Tension Rupture of Tie Rod

44. A guide wall is formed by interlocking Z-shaped sheet piles driven into clay and glacial till. The wall is partially supported by a system consisting of wales and tie rods (Figures 11 and 12). The tie rods are

![Graph showing the reliability over time](image)

**Figure 10.** Estimated condition of cross wall shear resistance, O'Brien Lock and Dam
several yards apart. One bay of the wall between successive tie rods is considered in Equation 28. The tension, \( P \), in a tie rod must balance the forces on one bay of the wall. This force has been derived from Equation 25 in Kearney (1986) and is defined as:

\[
P = \frac{L}{2} \left[ K_A Y (H^2 - s^2) + (K_A Y' + Y_w) s^2 - Y_w h^2 - \bar{K}_Y d^2 \right]
\]  

(28)

The remainder of the variables are defined as follows:

\( L \) = distance between two adjacent rods (84 in.)

\( K_A \) = coefficient of lateral earth pressure (variable)

\( H \) = elevation of top of wall (552 in.)

\( s \) = elevation of backfill saturated water (variable)

\( h \) = elevation of water in river (variable)

\( a \) = elevation of wale (444 in.)

\( \bar{K} \) is as described in Equation 29.

\[
-K_A Y (h^3 - s^3) - (K_A Y' + Y_w) s^3 + Y_w h^3 + \bar{K}_Y d^3 + 3K_A Y a (H^2 - s^2)
\]

\[
+ 3(K_A Y + Y_w) s a^2 - 3Y_w a h^2 - 3\bar{K}_Y d a^2 = 0
\]  

(29)

The rod's strength is calculated from:

\[
P_u = A \sigma_u = \frac{\pi (D - \epsilon)^2}{4} \sigma_u
\]  

(30)

where \( D \) is the diameter of the rod (\( D = 3 \) in.), \( \epsilon \) is the amount of corrosion, and \( \sigma_u \) is the ultimate tensile strength of the steel. The reliability function is defined as:

\[
R(\epsilon) = P(MS > 0) = P(P_u - P > 0)
\]  

(31)

Table 4 lists the random variables affecting the reliability function of the tie rods. Figure 13 illustrates the reliability function. Although the tie rods corroded less than the sheet piles themselves (Kearney 1986), the lower
stability is considered. A deterministic formulation has been developed for this problem (Kearney 1986). The loading is defined as:

\[ M_L = P(a - x) - \frac{1}{6} (K_A \gamma' + \gamma_w - K_A \gamma)(s - x)^3 - \frac{1}{6} K_A \gamma(H - x)^3 \]

\[ + \frac{1}{6} \gamma_w(h - x)^3 + \frac{1}{6} \gamma_Y'(d - x)^3 \]  

(32)

where \( x \) is an arbitrary elevation, \( P \) is computed from Eq 28, and \( K \) is computed from Eq 29. When solved for the point of maximum moment, \( x \) is as follows:

\[ x_{\text{max}} = 2[-(K_A \gamma' + \gamma_w - K_A \gamma)s - K_A \gamma H - \gamma_w h + \gamma_Y' d]/(K - K_A) \gamma' \]  

(33)

On the other hand, the resisting moment is determined by:

\[ M_R = \sigma_y 1 - \frac{\sigma_y^2}{25E(t - \epsilon)^2} \frac{I - Ke}{C} \]  

(34)

where:

- \( \sigma_y \) = yield point of the steel
- \( \gamma_w \) = 18 in. (Figure 11)
- \( E \) = modulus of elasticity
- \( t \) = initial thickness (0.5 in.)
- \( I \) = moment of inertia (18.37 cu in.)
- \( Ke \) = the effect of corrosion on the moment of inertia (55 cu in.)
- \( C \) = 5.5 in. (Figure 11)

Defining the reliability function as:

\[ R(\epsilon) = P(M_R - M_{\text{max}} > 0) \]  

(35)

and using Table 5 for the values of random variables yields Figure 14, which shows the effect of corrosion on the stability of the sheet piles.
would be needed to understand the nature of this problem and its conse-
quences. These findings are consistent with the deterministic service life
assessment in Kearney (1986). However, the probabilistic findings are
believed to be more useful since they estimate gradual deterioration as an
explicit function of corrosion and time.
Load Analysis

Sheet pile bulkheads are one of the most popular types of water-soil supporting systems, although corrosion of the sheet piles seriously affects the bulkheads' bending moment capacity. However, the key question is how the actual soil load distribution is developed. Figure 16 illustrates the lateral earth pressure diagram proposed for the flexible anchored bulkheads in clean sand, and corresponds to the results of the model test with flexible anchored bulkheads conducted at Princeton University (Tschebotarioff 1955). This experiment showed that full restraint of the lower portion of the bulkhead was effective when the ratio D/H (depth [buried] to height) equaled 0.43, where
the tensile strength of the sand layer that has been saturated by the capillary action above the water level. Because of these considerations and the large degree of uncertainty associated with the backfill material (it is not a clean sand), in these calculations $K_A$ has a mean value of 0.4 and a standard deviation of 0.125.

51. The one common factor for all sheet pile bulkhead structures is that the maximum bending moment occurs in the underwater zone. Considering this factor and measuring down from the water level (on the land side), the location of zero shear, $z$, is determined from:

$$A_p + \frac{1}{2} \gamma_w a^2 + \gamma_w az - \frac{1}{2} K_A \gamma h^2 - K_A \gamma h z - \frac{1}{2} K_A \gamma' z^2 = 0 \quad (38)$$

or

$$\frac{1}{2} K_A \gamma' z^2 + (K_A \gamma h - \gamma_w a) z + \frac{1}{2} K_A \gamma h^2 - \frac{1}{2} \gamma_w a^2 - A_p = 0 \quad (39)$$
at that time is $t_o - \epsilon$, where $t_o$ is the initial thickness. The mean section modulus of the pile per unit width is then:

$$S = S_o - K\epsilon$$

(42)

where $S$ is the section modulus of the sheet pile, $S_o$ is the initial value of the section modulus, and the reduction factor, $K$, is a constant which can be determined exactly from the shape of the cross section. $K$ can also be calculated approximately from the properties of the cross section. The result is:

$$K = 28.6 \text{ sq in.}$$

(43)

Consequently, the mean bending capacity, $M_R$, of this sheet pile is determined from:

$$M_R = \sigma_u (10.725 - 28.6\epsilon)$$

(44)

Due to the approximation involved in determining $K$, a variation of 0.2 is chosen for $M_R$, while a mean value of 24,000 psi with a standard deviation of 480 psi is used for $\sigma_u$. The mean value of $M_R$ is calculated from Equation 41.

### Reliability of Sheet Pile Bulkhead

53. The reliability of the sheet pile bulkheads is investigated by considering a margin of safety, $MS$, which is defined as:

$$MS = M_R - M_L$$

(45)

Failure occurs when this margin of safety becomes negative. $MS$ is a random variable because both $M_R$ and $M$ are random variables. The reliability function is defined as:

$$R(\epsilon) = P[MS > 0]$$

(46)
Corrosion of Steel Sheet Piles

55. Buslov (1983) has developed and proposed an analysis of the remaining service life of sheet piles, applying his approach to the port structures of Israel that were inspected for corrosion between 1976 and 1981. Unlike conventional design practice, which recommends the use of a corrosion allowance based only on structural considerations, he offers the addition of a detailed analysis of the actual character of corrosion to the design process. In his approach, the sheet pile section is chosen according to the maximum bending load and resistance; however, this section must contain some corrosion allowance. In the area of active corrosion (mostly in the tidal and splash zone), the existing service life is determined by considering three factors: rate of corrosion, actual bending moments, and flange thickness, which had already been determined based on the value of the maximum bending moment. To illustrate his technique, he considers a corrosion allowance of 2.25 mm (0.09 in.) based on a 30-year life span, or in general, corrosion of 0.075 mm/yr. This general corrosion, plus the maximum bending moment, were used during design to choose the sheet pile section. He then considers the upper portions of the section where the corrosion is rather high (Figure 19) and determines the potential service life for several corrosion rates. Figure 20 contains the result of his calculations for the sheet pile bulkhead at Kishon (Larssen IVn Section; $t_0 = 0.58$ in.).

Probabilistic Analysis

56. The current research considers the same sheet pile bulkhead in Kishon, which consists of a concrete deck on concrete piles, fronting a sheet pile retaining wall (Buslov 1983). Two cases are considered: corrosion of the sheet pile under the water (corrosion is relatively low but moments are rather large) and corrosion in the upper sections of the sheet piles (corrosion is usually high but bending moments are rather small). The sheet pile cross section is a Larssen IVn. The corrosion rate is considered to be at least 0.4
Figure 20. Service life of Larssen IVn sheet piles, $t = 0.58$ in. (14.8 mm), with various rates of corrosion (after Buslov 1983)
indicates that the tidal zone is more likely to be the location of maintenance problems, even though the underwater zone has a larger load. This conclusion is consistent with that of Buslov. The reliability levels of these figures quantify the conservatism of the deterministic method he used.

Figure 21. Reliability of sheet piles; \( M_{\text{max}} \), underwater zone, Kishon
59. The reliability function of an existing structure serves as an index of its structural condition. It explicitly reflects changes in resistance that have accumulated during service as well as changes in loading imposed by operational conditions.

60. This report has documented the development of a straightforward, generally applicable, three-step procedure for evaluating structural reliability. The margin of safety is assumed to be normally distributed, and point estimates are used to calculate the moments; these are standard probabilistic methods. Its computational details resemble familiar design procedures and are thus attractive to practicing civil engineers.

61. The procedure has been applied to three different types of sheet pile structures representative of those maintained by the Corps. Emphasis has been placed on using the available and familiar formulation but in a probabilistic manner. The results indicate that rehabilitative cost avoidances may be achieved if the procedure is used in lieu of traditional deterministic practices.

62. The three-step procedure developed herein is recommended for use in evaluating the safety of existing sheet pile and other metal structures operated and maintained by the Corps of Engineers. The reliability of structures as calculated by this procedure should be used as a component of the condition index of the information system under development to manage REMR activities in Corps field offices.

63. The procedure could be adapted to the particular characteristics of concrete structures to establish a consistent evaluative condition index for these features.


### Table 3
Random Variables Affecting Reliability of Interlocking System

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_A$</td>
<td>0.45</td>
<td>0.125</td>
</tr>
<tr>
<td>$s$, in.</td>
<td>468</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma$, lb/cu in.</td>
<td>0.0723</td>
<td>0.08 $\gamma_w = 0.0029$</td>
</tr>
</tbody>
</table>

### Table 4
Random Variables Affecting Reliability of Tie Rods

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_A$</td>
<td>0.45</td>
<td>0.129</td>
</tr>
<tr>
<td>$h$, in.</td>
<td>444</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma$, lb/cu in.</td>
<td>0.0723</td>
<td>0.0029</td>
</tr>
<tr>
<td>$s$, in.</td>
<td>520</td>
<td>20</td>
</tr>
<tr>
<td>$R$</td>
<td>1.4</td>
<td>0.333</td>
</tr>
<tr>
<td>$\sigma_u$, psi</td>
<td>60,000</td>
<td>6000</td>
</tr>
</tbody>
</table>

### Table 5
Random Variables Affecting Reliability of Sheet Piles Against Buckling

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$, psi</td>
<td>30,000,000</td>
<td>3,000,000</td>
</tr>
<tr>
<td>$\gamma$, psi</td>
<td>36,000</td>
<td>3,600</td>
</tr>
<tr>
<td>$\gamma$, lb/cu ft</td>
<td>0.0723</td>
<td>0.0029</td>
</tr>
<tr>
<td>$K_A$</td>
<td>0.45</td>
<td>0.129</td>
</tr>
<tr>
<td>$R$</td>
<td>1.4</td>
<td>0.333</td>
</tr>
<tr>
<td>$s$, in.</td>
<td>520</td>
<td>20</td>
</tr>
<tr>
<td>$h$, in.</td>
<td>444</td>
<td>20</td>
</tr>
</tbody>
</table>
This appendix provides a microcomputer program for estimating the reliability of existing structures. The program is written in BASIC for an IBM PC or compatible computer. Translation onto other hardware or into other software is straightforward. The main program in lines 10 through 500 implements the general procedure. The subroutine in lines 1000 through 1130 specifically addresses the safety margin for the tutorial miter girder example. The data statements in lines 2000 through 2020 contain the input values for the example. Other problems can be solved by replacing the subroutine and the data statements. The execution of the program for the tutorial example follows the program listing.
1000 SUBROUTINE RETURNS *=MS FOR MITER GIRDER EXAMPLE
1010 A=41.52
1020 AR=78.81
1030 D=7.88
1040 IN=3170
1050 L=740
1060 FA=1.88
1070 CM=5
1080 FY=100
1090 W=15.12
1100 FA=AW/(L/2*C0+1)
1110 FB=W*(T-A)/2/(IN+L/4-L*A*C0*(T-A)*2-A*A)
1120 Y=1.7-F-A-FB
1130 RETURN
2000 DATA "Miter Girder Example",2,"MS"
2010 DATA "fy psi",29660,4400
2020 DATA "w lb/in",1000,200

A3
APPENDIX B: VARIABLE LIST

\[ f_a \] axial

\[ f_{bl} \] flexural stress

\[ a \] area of girder

\[ I \] moment of inertia of girder

\[ w, L, \theta, t \] Figure 3

\[ MS \] margin of safety

\[ x \] arbitrary elevation

\[ X \] generic input random variable (Eq 4 through 10)

\[ \bar{X} \] mean of \( X \)

\[ \sigma_X \] standard deviation of \( X \)

\[ MS \] mean of margin of safety

\[ \sigma_{MS} \] standard deviation of margin of safety

\[ ms \] margin of safety calculated using mean value of each variable

\[ R \] reliability (Eq 11)

radius of circular cell (Eq 15) (O'Brien: 316 in.)

\[ N_u \] interlock strength
\( H \)  
- elevation of the top of the fill (O'Brien: 504 in.)  
- elevation of top of wall (O'Brien: 552 in.)  

\( \gamma \)  
- total unit weight of the cell fill (variable)

\( \gamma' \)  
- submerged unit weight of the cell fill \( = \gamma - \gamma_w \)

\( \gamma_w \)  
- unit weight of water (0.0361 lb/cu in.)

\( K_A \)  
- coefficient of horizontal earth pressure (variable)

\( c \)  
- the amount of corrosion

\( t \)  
- time

\( S \)  
- shearing stress

\( S_{\text{max}} \)  
- maximum shearing stress

\( N' \)  
- tension transmitted to the cross wall

\( \alpha \)  
- Figure 9a

\( F \)  
- Shear force in a cross wall bottom of a chamber  
  probability (Eq 18)

\( P \)  
- tension in a tie rod (Eq 28)

\( b \)  
- Figure 9c  
- distance top of bulk head to point of tie back  
  attachment (Eq 37)

\( p \)  
- net lateral pressure acting on the cross wall
the effect of corrosion on moment of inertia
O'Brien: 55 cu in.)

Figure 11 (O'Brien: 5.5 in.)

anchor pull

Figure 17 (Eq 37)

effect of wall friction on reduction of active
earth pressure

uncertainty in relative importance of passive earth
pressure above anchor, and tensile strength of sand
saturated by capillary action above water level.

location of zero shear

maximum moment

section modulus

initial value of section modulus

reduction factor

reflects the effects of corrosion on section modulus
APPENDIX C: GLOSSARY OF STATISTICAL TERMS

CUMULATIVE DISTRIBUTION FUNCTION: The cumulative distribution function is denoted $F(x)$, and has the properties:

1. $\lim_{x \to -\infty} F(x) = 1$;
2. $\lim_{x \to +\infty} F(x) = 0$
3. $F(x)$ is a nondecreasing function.

MEAN: The sample mean or average of a set of $n$ measurements $x_1, x_2, ..., x_n$ is the sum of these measurements divided by $n$. The mean is denoted by $\bar{x}$, which is expressed operationally

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

NORMAL DISTRIBUTION: A normal distribution has a bell-shaped density

$$f(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

with a mean $= \mu$ and a standard deviation $= \sigma$.

The probability of the interval extending

1. one standard deviation each side of the mean: $P[\mu - \sigma < x < \mu + \sigma] = 0.683$
2. two standard deviations on each side of the mean: $P[\mu - 2\sigma < x < \mu + 2\sigma] = 0.954$
3. three standard deviations on each side of the mean: $P[\mu - 3\sigma < x < \mu + 3\sigma] = 0.997$
VARIANCE: The sample variance, $s$, is the variation of individual data points about the mean. The dispersion of a set of $n$ measurements $x_1, x_2, \ldots, x_n$ is defined as

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$