TRIPLE-DECK STRUCTURE SOLUTION TO THE PROBLEM OF SEPARATING FLOWS

by

Yang Maozhao

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PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WPAFB, OHIO.

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Within the range of the triple-deck structure theory, we used the inner layer equation to find a solution to the problem of separating flow over backfacing steps and compression corners. We obtained results identical with those of the Navier-Stokes numerical solution and experimental wind-tunnel values. In this paper we use the upwind scheme with third-order accuracy and Gauss-Seidel iteration with the relaxation factor; this permits an increase in the accuracy of the numerical calculations and in the convergence rate.

1. Introduction

The problem of separating flow over backfacing steps and compression corners has been successfully solved with the Navier-Stokes numerical solution method [10,11]. If the scale of disturbance, including the step height $h$ and the angle of the compression corner $a$, is relatively small, using the Reynolds number of the flow, we obtain $h/x_e = O(\varepsilon^5)$, $a = O(\varepsilon^5)$, and for the flow-direction range of the disturbance $L/x_e = O(\varepsilon^5)$, where $\varepsilon = Re^{-1/6}$. Using Stewardson's triple-deck structure method [1], it is possible to solve the problem more simply and economically than with the Navier-Stokes method. Following the reasoning in Stewardson's article [1], flow along the surface of a wall is divided into an inner layer, a main layer, and an outer layer; these three dissimilar regions are described by different flow equations, and they are linked up by matching conditions. The inner layer region, in particular, has a range of $O(\varepsilon^5)$ and is governed by a viscous boundary layer equation. However, the outer fringe conditions are completely different from the formulation for the classical boundary layer problem: in addition to being matched with the non-viscous, swirling main layer, it also necessarily has a relationship with the outer layer. In actual practice, passing through the

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main layer, a pressure displacement relation occurs, which can be regarded as a supplement to the inner layer boundary layer equation. This constitutes the whole inner layer problem. Because independent flow parameters like $M$, $R$, and $T_w/T_i$, are contained in the scale parameter, and the inner layer equation only contains the disturbance scale parameters $b$ and $a$, there is great interchangeability, and application is possible for a number of flow regions with supersonic and hypersonic flow in which separation occurs; the numerical solution is also very convenient and quick. Nevertheless, it generally is effective only in a small disturbance range, and put to expanded use its limitations become apparent. Furthermore, it is not yet applicable to cases of severe turbulence or three-dimensional flow; these problems await investigation.

Using the triple-deck structure method to study the problem of separating flow for small back-facing steps and small-angle corners in surface burnt/etched images, to obtain the distribution of pressure and shearing stress in a small separation area, and to establish at intervals the aerodynamic model of the surface burnt/etched images, is, it appears, a very promising route. Because this kind of burnt/etched image is generally located within the boundary layer's inner layer, where the disturbed flow range is small, and where the inner layer equation numerical value method is easy to implement, and because little computer time is used, it is doubtlessly very beneficial to develop this kind of investigation.

The numerical solution for inner layer problems has been investigated [2,6]; the time-interrelation method is used to obtain numerical results for back-facing steps and compression corners. However, because an upwind scheme with first-order accuracy is used for the flow, when the flow direction step length is enlarged, it is inevitable that a rather large numerical value viscosity is introduced. Based on our experimental calculation on compression corners, using this kind of first-order accuracy scheme makes it very difficult to obtain pressure distribution results identical with those obtained from the Navier-Stokes equation and experimentation. For this reason, we have switched to the use of the third-order accuracy upwind scheme
for the convection term, which improves the accuracy of the entire calculation. After the corresponding scale parameter is used to calculate the physical plane, a wall surface pressure distribution and friction coefficient distribution are obtained that are identical with the results from the Navier-Stokes equation and with experimentation. At the same time, after the inner layer equation is matched with the main layer pressure displacement relation, the equation in fact becomes elliptical; in considering just this peculiarity, we have accelerated the weakening process of the time-interrelation method for the wall value relaxation, using Gauss-Seidel iteration.

Below, we first discuss the numerical calculations for the inner layer problem, and then we consider the use of this method and a comparison of some sets of calculated results. Finally, we provide a concise summary.

2. Numerical Calculation Method for the Inner layer Region Problem

Reference [1] uses an unusual perturbation method to group the derived inner layer momentum equation with the energy equation and non-coupled equation for the non-compressible boundary layer.

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{\partial U}{\partial Y} 
\]

\[
U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{\rho} \frac{\partial^2 T}{\partial Y^2} 
\]

\[
\frac{\partial P}{\partial Y} = 0 
\]

Here the boundary conditions are:

1) the pressure displacement relations matching the main layer:

\[
\lim_{r \to -} (U - Y) = - \int_{-\infty}^{x} P(t) dt 
\]

\[
\lim_{r \to -} (T - Y) = - \int_{-\infty}^{x} P(t) dt 
\]

2) the wall's condition of having no sliding displacement:

\[U = V = 0, \quad T = 0\]

3) the upstream condition of having the disturbance disappear:

\[U \to Y, \quad T \to Y, \quad \text{when } X \to -\infty\]
4) the downstream condition of having the disturbance disappear:

$$U \ (or \ T) \to Y+H, \ for \ backfacing \ steps \ when \ X\to-\infty \ \ \ \ \ (8')$$

$$U \ (or \ T)\to Y-aX, \ for \ compression \ corners \ when \ X\to+\infty$$

In the actual calculation, conditions (7)' and (8)' undergo modification. For backfacing steps, using the boundary region layer inflation solution, considering it the original upstream section, located on the step shoulder region, in order to match it with the main layer pressure value condition, adjustments are made with every iteration. For compression corners, [7]'s asymptotic equation used upstream by derivation gives the following condition:

$$\frac{\partial r}{\partial X} = \kappa (r-1), \ when \ X\to-\infty \ \ \ \ (7)$$

For the downstream condition, using [8]'s algebraic attenuation asymptotic equation, we derive:

$$\frac{\partial r}{\partial X} = -\theta/3 \frac{r-1}{X}, \ when \ X\to+\infty \ \ \ \ (8)$$

Here, $\kappa$ and $\theta$ are constants.

Using wall surface condition (6) in equation (2), it is possible to derive the wall surface pressure tolerance relationship:

$$\frac{dP}{dX} = \left[ \frac{\partial U}{\partial Y} \right]_{r,*} \ \ \ \ (9)$$

Thus, formulas (1)-(9) represent a complete formulation of the inner layer problem. Here the use of the unusual formula (5) causes the inner layer momentum equation (2) to become, in fact, an elliptical equation. Therefore, we have made use of the time-interrelation, wall surface relaxation Gauss-Seidel iteration for a solution; this takes care of the instability of the return flow region calculations, and also accelerates the convergence process.

In simplifying the wall surface boundary condition, the following transformations are introduced:

$$Z = \begin{cases} Y & \text{for } X \leq 0 \\ Y+H & \text{for } X > 0 \end{cases} \ \ \ \ (10)$$

$$Z = Y-aX \ \ \ \ (compression \ corners), \ for \ X > 0$$
\[ W = \begin{cases} V & \text{for } X < 0 \\ V - aU & \text{for } X > 0 \end{cases} \]  \hspace{1cm} (11)

(11) is only used for compression corners. The linkage and momentum equations do not change with the transformation. Afterwards, formula (2) is rewritten in the form of shearing stress \( \tau \)'s conveyance equation; making \( r = \frac{\partial U}{\partial Z} \), we have:

\[ \frac{\partial r}{\partial t} + U \frac{\partial r}{\partial X} + W \frac{\partial r}{\partial Z} = \frac{\partial^2 r}{\partial Z^2}. \]  \hspace{1cm} (12)

For the other corresponding quantities we have:

\[ U = \int_T \tau(X, Z) \, dZ \]  \hspace{1cm} (13)

\[ W = - \frac{\partial}{\partial X} \left[ \int_T U(X, Z) \, dZ \right] \]  \hspace{1cm} (14)

\[ P(X) = \int_T \left( \frac{\partial \tau}{\partial Z} \right)_{x,t} \, dX \]  \hspace{1cm} (15)

For the time derivative, we use the preceding to write equation (12) as follows:

\[ \Delta \tau_{i,j}^{n+1} + \Delta t \left\{ U_{i,j}' \cdot \left[ \frac{\partial}{\partial X} \Delta \tau_{i,j}^{n+1} \right] + W_{i,j}' \cdot \left[ \frac{\partial}{\partial Z} [\Delta \tau_{i,j}^{n+1}] - \frac{\partial^2 \tau_{i,j}^{n+1}}{\partial Z^2} \right] \right\} 
= - \left\{ U_{i,j}' \left( \frac{\partial \tau}{\partial X} \right)_{i,j} + W_{i,j}' \left( \frac{\partial \tau}{\partial Z} \right)_{i,j} - \left( \frac{\partial^2 \tau_{i,j}}{\partial Z^2} \right)_{i,j} \right\} \Delta t \]  \hspace{1cm} (16)

here making

\[ \Delta \tau_{i,j}^{n+1} = \tau_{i,j}^{n+1} - \tau_{i,j}^{n}, \]

As a result

\[ \tau_{i,j}^{n+1} = \Delta \tau_{i,j}^{n+1} + \tau_{i,j}^{n}. \]  \hspace{1cm} (17)

To take care of several insufficiencies of the first-order accuracy upwind scheme, we use the third-order accuracy upwind scheme for the convection term [9].

\[ u \left( \frac{\partial \phi}{\partial X} \right) = u_+ \frac{2 \phi_{i+1} + 3 \phi_{i+2} - 6 \phi_{i+1} + \phi_{i-1}}{6 \Delta X} \]
\[ + u_- \frac{2 \phi_{i-1} + 3 \phi_{i-2} - 6 \phi_{i-1} + \phi_{i+1}}{6 \Delta X} \]  \hspace{1cm} (18)

in which

\[ u_+ = \frac{1}{2} (|u| + u), \quad u_- = \frac{1}{2} (|u| - u) \]

Using (18) in (16), after adjustment, we produce

\[ \Delta \tau_{i,j}^{n+1} + B(j) \Delta \tau_{i,j}^{n+1} + A(j) \Delta \tau_{i,j}^{n+1} = D(j) \]  \hspace{1cm} (19)
Here,

\[ C(j) = 1 + \frac{2\Delta t}{\Delta Z} + \frac{1}{2} \frac{\Delta t}{\Delta Z} |U_i| \]

\[ B(j) = \left[ \frac{1}{2} \frac{\Delta t}{\Delta Z} W_i, -\Delta t / \Delta Z \right] / C(j) \]

\[ A(j) = \left[ -\frac{1}{2} \frac{\Delta t}{\Delta Z} W_i, -\Delta t / \Delta Z \right] / C(j) \]

\[ D(j) = \left\{ -\frac{\Delta t}{6\Delta X} [u^* (2\Delta r_i^*), -6\Delta r_i^* + \Delta r_i^*] \right. \\
+ u^* (2\Delta r_i^*), -6\Delta r_i^* + \Delta r_i^* \right\} / C(j) \]

Formula (19) is a triple diagonal equation group, but the right-most term includes an unknown quantity \( \Delta r_i^* \); we therefore use the Gauss-Seidel iteration method to solve from the acceleration program. The first iteration makes the right-most term's

\( \Delta r_i^* = 0 \)

Afterwards, we substitute the previous iteration's result, until we obtain a satisfactory convergence criterion solution \( \Delta r_i^* \); hereupon, for the internal point we have:

\( r_i^* = \Delta r_i^* + r_i^* \)

Border region conditions (7)-(9) must also be subjected to further refinement, relaxation factor \( \omega \) for wall surface value \( T \) approaching a super relaxation of 1.0. After adding a low relaxation of \( \omega = 0.25 \) to the pressure, and adjusting the initial section value, it is possible to obtain a good result for acceleration of convergence.
3. Calculated Results and Comparisons

As we stated in our introduction, because the inner layer equation does not contain an independent flow parameter, but only contains disturbance scale \( H \) or \( a \), the calculated results, when transformed through scale relations, can give practical values for corresponding independent current conditions. For backfacing steps, we have calculated a set of results for \( H=4.32 \), which are typified on Fig. 1 through Fig. 5.

![Flow coefficient isopleths](image1)

![Velocity sections](image2)

Fig. 1. Flow coefficient isopleths. 
Fig. 2. Velocity sections.

From Fig. 1 and Fig. 2 it is possible to see clearly the flow conditions in the return flow region existing in the inner portion. Figure 3 shows the distribution of shearing force along the inner wall, and shows that the value for \( \tau_\omega \) in the return flow region is negative; that at the rear halt point the value of \( \tau_\omega =0 \); and that it then rises into the positive range. The value of \( X \) at the rear halt point is 2.8. Figures 4 and 5 show the distribution of thermal current and inner wall pressure. The thermal current is greatest at the rear halt point. Within the return flow region the pressure is negative. Leaving the step back wall, the negative pressure rises quickly; after passing through the rear halt point, it again tends to change slowly. Because there has not yet been found a suitable backfacing stair experiment and Navier-Stokes numerical solution result, no calculations have been undertaken for a comparison of the physical plane.
For compression corners, two sets of supersonic flow conditions were calculated with $a=2.2$ and $a=2.76$. After scale relation transformation calculations, a comparison was undertaken with the Navier-Stokes equation numerical results in [10] and [12] and with wind tunnel experimental results. See Fig. 6, 7, and 8.

It was discovered from the comparison that the artificial consumption member introduced from the upwind scheme with third-order accuracy not only had a numerical stabilizing effect, but also had a certain influence on the accuracy of the solution. The broken line shown in Fig. 8 is twice as high as the artificial viscosity value shown by the solid line, showing that the artificial viscosity value is too high, which gives the calculated results a rather larger deviation from the experimental results. In Fig. 6 and 7, the upstream and downstream pressure values and the friction obstruction values are rather far from the Navier-Stokes solution; the reason for this may be that, at the time of calculation, the upper and lower reaches used asymptotic
conditions. The flow line and velocity section of the other corner separation flow region are more clearly reasonable, and show the reliability of the calculations.

4. Conclusion

Within the range of the triple-deck structure theory, using the inner layer equation to solve the problem of two-dimensional backfacing step and compression corner separating flow, it is possible to obtain reasonable results, identical with those obtained by the Navier-Stokes equation and experimental wind tunnel values.

Because the equation is simple and the solution is economical, the required computer time is only ca 1/10 of the...
time required for the complete Navier-Stokes method. Our method can be used for preliminary analysis and study. The numerical value experiment shows that the third-order accuracy upwind scheme and the Gauss-Seidel iteration with wall surface value relaxation factor are useful in raising the accuracy and effectiveness of calculations.

During these studies and calculations, we had many valuable discussions with comrade Wang Mingzhi. We wish to express our gratitude at this time.

Literature

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