Optical Detection of Space Debris Using a Large Achromatic Coronagraph

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Abstract:

The threat of space debris seems to increase steadily, accordingly, new methods of detection are needed. We present a theoretical approach of this problem, assuming the use of a new generation large achromatic ground based coronagraph. Few interesting results are presented including the estimation of the probability of being able to systematically observe small size space debris in a large range of altitudes.

Key words:

space debris - detection - new advanced coronagraph - sky brightness
1 - Introduction.

The importance of detecting space debris was recently stressed (Kessler, 1985; Su, 1986; Koutchmy, 1987 for example) and becomes obvious in the context of the development of Space Stations. The passive ground based optical method was proposed (Weber, 1979; Taff & Jonuskis, 1986); however, neither precise calculations exists nor, a fortiori, concrete measurements. We consider here the possibility offered by the use of a large new technology advanced ground based coronagraph which was recently proposed for coronal studies (Smartt, 1987), assuming the instrument is fully achromatic. We find indeed that the proposed method of grazing incidence detection of debris illuminated under very small angle (typically several arcmin) is very perspective due to the large radiative flux available when the illumination is coming from the Sun.

2 - Theoretical prediction for detecting a space debris.

2-1 - Amount of light coming from a space debris.

We consider a space debris orbiting around the Earth and observed from the ground using a low scattered light level solar telescope outside the limb, the Solar disc being properly blocked and the instrument properly apodized.

Let us define $h_d$: the distance to the debris, with

$$h_d = h_s \sin^{-1} \beta$$  \hspace{1cm} (1)

$\beta$ being the debris elevation and $h_s$ the orbital height; let us assume the effective cross-section of the debris is $\sigma$, $d$ being the debris diameter, with $\sigma = \pi d^2 / 4$. Let us neglect the atmospheric refraction, assuming the observations are made at high elevation in the sky.

We define $\eta$, the specular reflectivity of the debris of effective cross-section $\sigma$, with $0 \leq \eta < 1$ and, indeed, $\eta = \eta(i, r, \sigma, \lambda, ...)$, $i$ and $r$ being the incident and reflective angles of the light on the debris, and $\lambda$ the wavelength where observations are considered; we estimate that the debris, which is metallic of origin, should produce specular reflection when flying over the field of view, during at least a fraction of a second. This question needs probably a careful consideration, including measurements; this is beyond the scope of the present analysis.

$F(\lambda)$ is the solar flux spectral distribution, outside the Earth atmosphere at 1 AU.

$E_t = C \sigma$ is the total amount of light received by the debris during 1 sec, with, according to Allen, 1973, $C = 1.36 \times 10^6 \text{ erg cm}^{-2} \text{sec}^{-1}$ being the solar constant or the amount of energy received at 1 AU, assuming:

$$C = \int F(\lambda) d\lambda$$  \hspace{1cm} (2)
The reflected amount of light by the debris each 1 sec of time is:

\[ E_2 = \sigma \int \eta F(\lambda) d\lambda \approx \eta E_1 \]  \hspace{1cm} (3)

We now easily deduce the amount of light \( E_i \) received by a ground based telescope of \( S_i = \pi R_i^2 \) mirror surface. \( \Sigma \) being the smearing area of the light reflected by the debris, then \( \Sigma = \pi L^2 \) with \( L = h_d \sin \alpha_o \), neglecting the center-limb darkening effect (with \( \alpha_o \) being the angular radius of the Sun); assuming \( \eta_o = \eta_o(\lambda, \beta) \) is the effective Earth atmosphere transmission, we obtain:

\[ E_i = \frac{\pi \sigma R_i^2}{h_d^2 \sin^2 \alpha_o} \int \eta \eta_o F(\lambda) d\lambda \]  \hspace{1cm} (4)

The energy of a photon at the wavelength \( \lambda \) is \( E_{ph} = \frac{hc}{\lambda} \) where \( h \) is the Plank constant; then:

\[ N_i = \frac{E_i}{E_{ph}} = \frac{\pi \sigma R_i^2}{hc} \frac{1}{h_d^2 \sin^2 \alpha_o} \int \eta \eta_o F(\lambda) \lambda d\lambda \]  \hspace{1cm} (5)

will be the amount of photons received per second, by the ground based telescope.

**2-2 - Amount of light received from the background.**

Let us now consider the typical background level overlapping "event" produced by the space debris in the focal plane of the telescope. First we have the sky brightness per 1 arcsec² \( B_{sky} \); for convenience, we better consider the relative sky brightness:

\[ \overline{I}(\lambda, \beta) = \sin \beta \frac{B_{sky}(\lambda, \beta)}{B_{x}(\lambda)} \]  \hspace{1cm} (6)

where \( B_{x}(\lambda) \) is the average solar brightness at \( \lambda \) per 1 arcsec² for the elevation \( \beta \). We will adopt a value from conservative consideration; for a good "coronal" sky in a high altitude site we notice that this level is strongly dependent of \( \lambda \) and is especially low if we observe at longer wavelengths, in the near infrared (see, for exemple, Kneizys and al., 1983 or Nitschelm, 1987).

Then the quantity of light received from the sky by the telescope in the same solid angle amounts to:

\[ E_{sky} = \pi \omega_s R_i^2 \sin^{-1} \beta \int \overline{I}(\lambda, \beta) F(\lambda) d\lambda \]  \hspace{1cm} (7)

where \( \omega_s \) is the solid angle corresponding to the effective image size \( \epsilon \) of the debris as
viewed from the ground (smeared by seeing and telescope imperfections).

Further we obtain:

$$N_{\text{sky}} = \frac{\omega \pi R_i^2 \sin^{-1} \beta}{hc} \int \Gamma(\lambda, \beta) F(\lambda) \lambda d\lambda$$

for the amount of photons received by the telescope during one second over the same solid angle.

2-3 - Modulation and amount of photons received from the space debris.

We will assume the combination of the earth atmospheric seeing and the smearing of the telescope of sufficient large aperture (see further) gives an image of this space debris typically concentrated over an effective area of $\frac{\pi}{4} \text{arcsec}^2$; we are not yet taking into account the motion of the space debris.

In consequence, we deduce the ratio of the debris to sky photon contrast:

$$\rho_d = \frac{E_t - N_t}{E_{\text{sky}} - N_{\text{sky}}} = \frac{\sigma \sin^2 \beta}{\omega \pi \sin^2 \alpha} \frac{\int \eta F(\lambda) \lambda d\lambda}{\int \Gamma(\lambda, \beta) F(\lambda) \lambda d\lambda}$$

which is independent of the telescope diameter. We notice that, for actually detecting the debris, we should take into account $S_D(\lambda)$ the combined detector spectral sensibility and instrument spectral transmission:

$$\rho_d = \frac{\sigma \sin^2 \beta}{\omega \pi \sin^2 \alpha} \frac{\int \eta F(\lambda) S_D(\lambda) \lambda d\lambda}{\int \Gamma(\lambda, \beta) F(\lambda) S_D(\lambda) \lambda d\lambda}$$

To get an actual estimation of $N_t$, we have to integrate also the whole spectrum, taking into account $S_D(\lambda)$.

The same is for the sky brightness; additionally the sky brightness is produced by the superposition of several components:

$$B_{\text{sky}} = B_R + B_T + \cdots$$

We identified:

$$B_R = \hat{B}_R \left( \frac{\Lambda}{\lambda_o} \right)^{-4}$$

(12-a)
as the Rayleigh component of the day sky and

\[ B_T \approx \bar{B}_T \left( \frac{\lambda}{\lambda_o} \right)^{-\frac{3}{2}} \]  

(12-b)

as the instrumentally scattered light; we assume \( B_T \ll B_R \) in the optical region; we notice this stationary component can be easily subtracted with modern detectors. Further we neglect the light scattered by aerosols (to be discussed further) which produces a signature definitively different from those coming from space debris.

2-4 Effects produced by the motion of the space debris.

The typical orbital velocity of the debris, at \( h_o=400 \) km, is

\[ v_d = \left( g_o \frac{R_o^2}{R_o + h_o} \right)^{\frac{1}{2}} \]  

(13)

with \( g_o=9.81 \) m sec\(^{-2}\) and \( R_o=6370 \) km. At 400 km, we have \( v_d \approx 7.7 \) km sec\(^{-1}\).

\( \epsilon \) [arcsec], at the focal plane of the telescope, corresponds, at the distance to the debris \( h_d \), to a length of \( \delta=h_d \sin \beta \). We notice that 1 arcsec = 1/206264 rd and thus, for a debris, near the zenith, \( \delta \approx 2 \) m at 400 km.

So the size \( \epsilon \) [arcsec] of the image, recorded over one or more pixels of the detector, will correspond to an effective transit time of:

\[ r = \frac{\delta}{v_d \sin \beta} = h_d \frac{\sin \beta}{\sin^2 \beta} \left( g_o \frac{R_o^2}{R_o + h_o} \right)^{-\frac{1}{2}} \]  

(14)

which corresponds to the effective exposure time for each pixel.

Additionally the image will be smeared by the motion of the debris (assuming, evidently, no tracking of the debris is performed) and also by an amount comparable to the smearing disc.

The corresponding amount of photons will be \( N_r r \).
2.5 - Numerical application

Let us consider the typical values for the different parameters used in this analysis, namely the:

- Debris diameter $d = 1 \text{ mm}$,
- Effective cross-section $\sigma = \pi \frac{d^2}{4} \approx 0.8 \text{ mm}^2$,
- Altitude $h_d = 4 \times 10^5 \text{ m}$,
- Telescope radius $R_t = 1.0 \text{ m}$,
- Relative sky brightness $\bar{F} = 10^{-5}$,
- Solid angle $\omega_s = 0.185 \times 10^{-10} \text{ sr}$,
- Solar apparent radius $\alpha_s = 16 \text{ arcmin}$,
- Reflectivity and transmittance $\eta = 0.1$,
- Mean wavelength $\bar{\lambda} = 0.65 \mu$,
- Zenithal debris observation $\sin \beta = 1$,
- Solar constant $C = 1.36 \times 10^{-8} \text{ erg cm}^{-2} \text{ sec}^{-1}$.

Then we get for the transit time of the debris over one pixel $t \approx \frac{1}{4000} \text{ sec}$;
- For the debris brightness, we get $E_d \approx 10^{-2} \text{ erg sec}^{-1}$ and $N_d \approx 4 \times 10^8 \text{ photons sec}^{-1}$; then $N_d t \approx 10^6 \text{ photons}$;
- For the sky, we get $E_{sky} \approx 2.5 \times 10^{-5} \text{ erg sec}^{-1}$ and $N_{sky} \approx 10^7 \text{ photons sec}^{-1}$ then $N_{sky} t \approx 2.5 \times 10^5 \text{ photons}$.

Finally, for the contrast ratio of the debris to sky photon counts, we get $\rho_d \approx 40$ if images are made at the rate of 4000 images per second, which corresponds to a fairly large contrast. However, it seems difficult to imagine systematic observation at a rate faster than the video rate where $r \approx 2 \times 10^{-2} \text{ sec}$; the contrast will then be substantially reduced. We notice that the debris will then produce a characteristic signature during the integration time and a suitable algorithm could be implemented to increase the S/N ratio.

3 - Conclusion and discussion

To measure a typical $1 \text{ mm}$ size debris orbiting at 400 km height with a ground based large coronagraph of two meters diameter seems to require only a rather limited detector capability as far as the strong forward "scattering" is considered in the case of a solar illumination. Using modern detectors like a CCD camera, such "event" will be easily measured, provided they cross the field of view.

Let us now estimate for practical reason the number of debris which will cross the field of view of the telescope during one hour. Indeed, if $\gamma$ is the angular field of view of the telescope and $A_d$ the total number of debris in the sky, we have:

$$a_{field} = \frac{A_d}{4} \sin^2 \frac{\gamma}{2}$$  (15)
being the instantaneous mean number of debris in the telescope's field of view.

The transit time of the debris across the field of view of the telescope will be:

$$\Delta t = \frac{\delta \gamma}{v_d \sin \beta}$$  \hspace{1cm} (16)

where $\gamma$ is measured in arc second and where $\delta$ is the actual distance performed by the debris flying over 1 arcsec in the sky. We can notice that the whole revolution time of the debris is:

$$T_d = 2\pi R_0 \frac{R_0 + h_0}{v_d}$$  \hspace{1cm} (17)

Then, we get, for the average number of debris observed per second in the field of view:

$$n_{\text{field}} = \frac{A_d v_d \sin \beta}{4 \Delta t} \frac{\sin^2 \gamma}{2}$$  \hspace{1cm} (18)

Assuming a field of 300 arcsec and a total number of debris of $10^{4.5}$, we obtain $n_{\text{field}} = 0.055$ debris/sec, or $\approx 200$ debris/hour. Assuming a spin rotation period of the debris approximatively equal to ten seconds and an effective "time" of reflection towards the telescope of one tenth of this period, we conclude that the new coronagraph will be able to observe $\approx 20$ debris/hour. This result shows the great potential offers for observing space debris with a ground based large coronagraph.

4 - References.


