Improved Propagation Models for Discrete Random Media

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This final report summarizes the most important results obtained from the research supported under the Army Research Office Grant No. DAAL03-87-C-0002. We developed an exact spectral-domain formulation for characterizing wave propagation in continuous random media. The formulation consists of a pair of coupled first-order differential equations for waves propagating in the forward and backward directions. These generalized transmission line equations fully accommodate backscatter, and they impose no restriction on scattering angle. In our first application of these equations, we generalized the models that are commonly used for propagation of light in the atmosphere and radio waves in ionized media to accommodate backscatter. We showed that the backscatter enhancement predicted by the cumulative forward scatter single backscatter approximation does not occur unless the finite correlation between the forward and backward propagating waves is included.

We then extended to model to discrete random media. We were able to derive a similar pair of differential equations that characterize the propagation of vector waves in sparse discrete random media. The equations are exact for small numbers of particles, and can be used to compute the mutual interaction among scattering objects whose free-space scattering characteristics are known. From this model we generalized the effective wave number that is commonly used for coherent wave propagation to include both backscatter and cross polarization.

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terms. When generalized to second order, our equations can be cast in the same form as the equations of vector radiative transport, but they are derived directly from Maxwell's equations. Future work will address the ramifications of this result.
I Statement of the Problem

Distinctly different approaches traditionally have been used for problems involving wave propagation in continuous and discrete random media. In continuous random media, one invariably uses the parabolic or narrow-angle scatter approximation. For most applications, backscatter is neglected as well. With these assumptions and under the Markov approximation it is possible to derive first-order differential equations that characterize the field moments of all orders. Ongoing research has provided a steady stream of improved solutions, particularly for the fourth-order field moments; however, because of the class of physical problems that are of interest, little effort has been devoted to generalizing the parabolic equation model, at least by Western scientists. In discrete random media, both backscatter and wide-angle scatter are important, and one either uses the formalism developed by Foldy, Lax, and Twersky or uses heuristic radiative transport models.

There is obviously more commonality between scattering phenomena in continuous and discrete random media than these approaches betray. In an attempt to exploit this commonality, we originally proposed to generalize the multiple-phase-screen model to accommodate backscatter and, ultimately, to replace the phase screens with more general scattering functions. Based on reviewer’s comments, however, it became clear that a more rigorous defense of our approach was needed. Thus, we expanded the original work statement to include a more rigorous defense of the spectral-domain method that has evolved from the generalized phase-screen model. Our overall objective of developing improved computational methods by computing the interaction of local wave-wave scattering functions remain unchanged.

II Summary of Principal Results

The propagation of scalar waves in continuous random media are governed by the Helmholtz equation

\[ \nabla^2 \psi + k^2 \psi = -k^2 \epsilon \psi, \tag{1} \]

where \( \epsilon \) is the perturbation to the relative permittivity. In [1] we showed that in terms of the two-dimension Fourier transform of the wavefields propagation in the plus \( z \) and minus \( z \) directions, this equation is equivalent to the coupled first-order differential equations

\[ \frac{\partial \psi^+(K; z)}{\partial z} = i k g(K) \psi^+(K; z) + \delta H^\perp \psi^+(K; z) + \delta H^\parallel \psi^-(K; z). \tag{2} \]
and
\[- \frac{\partial \psi^-(K; z)}{\partial z} = ikg(K)\psi^-(K; z) + \delta H^f \psi^-(K; z) + \delta H^b \psi^+(K; z), \tag{3}\]

where
\[\delta H^{f,b}(K, K'; z) = \frac{(ik/2)(K - K'; z)}{g(K)}, \tag{4}\]
and \[g(K) = \sqrt{1 - (K/k)^2}.\] Insofar as we know, the spectral-domain equations have not previously been reported. In a recent private communication from Dr's Ostashev and Tatarski, however, it was shown that the spectral-domain equations can be derived from the backscatter multiplicity method that they developed.

The leading terms on the right hand side of (2) and (3) represent free-space propagation. The remaining terms represent forward and backward scattering. The scattering functions \(\delta H^{f,b}(K, K'; z)\) represent the spectrum of waves \(K\) generated by a locally incident plane wave \(K'\). The integral form of (4) follows from the linearity of the wave equation. In [1] we showed that under the parabolic approximation, but without neglecting backscatter that similar coupled differential equations for the second-order moments can be obtained. Completely general equations under the Markov approximation were actually derived, but it was not possible to obtain analytic solutions. A new finding from our results is that the finite correlation between the positive and negative propagating waves must be included to produce a backscatter enhancement.

In a second paper [2], we extended the spectral-domain formalism to vector wave propagation in discrete random media. The vector equations take the same form as (2) and (3) with the scalar fields replaced by vector fields and the scattering functions replaced by dyadics. The full generality of the vector equations is yet to be established because we could not derive a unrestricted form for the scattering dyadics; however, for sparse media the scattering dyadics can be computed as a simple summation over the single particle contributions. From this result we were able to derive a general form for the effective propagation constant of the coherent wavefield. For a collection of particles that can be separated by planes, the difference-equation form of our equations provides an exact formulation of the mutual interaction problem. In particular, it provides an efficient means of computing the mutual interaction of two objects whose free-space scattering characteristics are known.

In addition to the two publications, the principle investigator has presented results from this work at URSI, SPIE, and at an international working group meeting on wave propagation in random media held in Tallin, USSR. At the Tallin meeting the work was discussed with V. I. Tatarski and V. E. Ostashev who have been pursuing related studies using essentially a diagram method. The Tallin meeting drew attention to the special case of one-dimensional scattering, which is more amenable to computation. We have been able to show that our approximations for the backscattered intensity are in good agreement with results obtained by using invariant imbedding. Under a new grant,
these results will be reported at the PIERS symposium to be held in July 1989 and MIT.

III Personnel

During the course of this research effort, the following Vista employees participated in addition to the principal investigator:

Dr. Keith A. Haycock
Dr. Hoa D. Ngo
References
