POLARIZATION MATRICES OF LITHIUM TETRABORATE

Arthur Ballato
ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY

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**Abstract:**

In analytical treatments of piezoelectric-acoustic transducers, signal processors, and resonators, the electromechanical transduction mechanism is most often expressed in terms of the elements of the piezoelectric $e$ or $d$ matrices. Molecular interpretations of piezoelectricity, and especially electro-optical applications, usually involve polarization as the preferred variable, and consequently the alternative $[a]$ and $[b]$ matrices are of interest. The elements of these latter sets are calculated for lithium tetraborate from measured elastopiezodielectric constants taken from the literature.

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### Subject Terms

- Piezoelectric resonators; piezoelectric transducers; lithium tetraborate; acousto-optics
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INTRODUCTION

Electromechanical transduction taking place via the piezoelectric effect is characterized phenomenologically by constitutive equations that relate the elastic and electric variables. These equations take a variety of forms, depending upon the choice of independent and dependent variables; the choice is normally dictated by the application. For example, piezoelectric resonators in the form of thickness mode plates are most easily treated using the isagric elastic stiffnesses \([cE]\), the piezoelectric stress constants \([e]\), and the dielectric permittivities at constant strain \([(\varepsilon)S]\).

Various measurement techniques yield values for the elements of a particular coefficient set more directly than those of another. The coefficients appearing in the different equation sets are, however, interrelated, so that once any one complete set is available, all the other sets of elements may be found. The most accurate and precise experimental results to date have been from plate resonator (resonance) and pulse-echo (transit-time) measurements. From the \([cE]\), \([e]\), and \([(\varepsilon)S]\) matrices determined therefrom, those matrices representing material properties expressed in the other alternative forms may be calculated.

Electrooptical applications are becoming increasingly important. So also are treatments of piezoelectric and ferroelectric phenomena from the standpoint of molecular interactions. In both of these cases the constitutive equations using polarization as the independent electrical variable, rather than either electric intensity or displacement, assume greater importance than the sets traditionally used for transducer, signal processing, and resonator applications.

In this report we give the complete sets of linear constitutive equations relating elastic and electric fields. For each equation set the numerical values are computed for lithium tetraborate from the measured \([sE]\), \([d]\), and \([(\varepsilon)T]\) values of Shiosaki, et al. (Ref.1). Coupling to the thermal field is neglected. Rationalized mks units are used throughout.

CONSTITUTIVE EQUATION SETS

Symbols and units for the quantities employed are given in Table 1. In terms of these, six constitutive equation sets are used. Of these, electric intensity, dielectric displacement, and polarization each appear in two sets as an independent variable. The sets are, in compressed matrix notation, as follows. A prime denotes transpose; \([I]\) is the unit matrix.

**I. The Piezoelectric Stress Constant Set**

\[
\begin{align*}
[T] &= [cE] [S] - [e]' [E] \\
[D] &= [e] [S] + [(\varepsilon)S] [E]
\end{align*}
\]  
(1)  
(2)
<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>UNIT</th>
<th>SYMBOL/DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic stress</td>
<td>N/m²</td>
<td>[T]</td>
</tr>
<tr>
<td>Elastic strain</td>
<td>--------</td>
<td>[S]</td>
</tr>
<tr>
<td>Electric intensity</td>
<td>V/m</td>
<td>[E]</td>
</tr>
<tr>
<td>Dielectric displacement</td>
<td>C/m²</td>
<td>[D]</td>
</tr>
<tr>
<td>Dielectric polarization</td>
<td>C/m²</td>
<td>[P]</td>
</tr>
<tr>
<td>Elastic compliance at constant [E], [D], [P]</td>
<td>m²/N</td>
<td>[cE], [cD], [cP]</td>
</tr>
<tr>
<td>Elastic stiffness at constant [E], [D], [P]</td>
<td>N/m²</td>
<td>[sE], [sD], [sP]</td>
</tr>
<tr>
<td>Dielectric permittivity at constant [T], [S]</td>
<td>F/m</td>
<td>[(εs)T], [(εs)S]</td>
</tr>
<tr>
<td>Dielectric constant, relative, at constant [T], [S]</td>
<td>--------</td>
<td>([K]T), ([K]S)</td>
</tr>
<tr>
<td>Dielectric impermeability at constant [T], [S]</td>
<td>m/F</td>
<td>[(εt)T], [(εt)S]</td>
</tr>
<tr>
<td>Dielectric impermeability, relative, at constant [T], [S]</td>
<td>--------</td>
<td>([εt]T), ([εt]S)</td>
</tr>
<tr>
<td>Dielectric susceptibility at constant [T], [S]</td>
<td>F/m</td>
<td>[(εh)T], [(εh)S]</td>
</tr>
<tr>
<td>Dielectric susceptibility, relative, at constant [T], [S]</td>
<td>--------</td>
<td>([εh]T), ([εh]S)</td>
</tr>
<tr>
<td>Reciprocal dielectric susceptibility at constant [T], [S]</td>
<td>m/F</td>
<td>[(εh)T], [(εh)S]</td>
</tr>
<tr>
<td>Reciprocal dielectric susceptibility, relative, at constant [T], [S]</td>
<td>--------</td>
<td>([εh]T), ([εh]S)</td>
</tr>
<tr>
<td>Piezoelectric stress constant</td>
<td>C/m²</td>
<td>[e]</td>
</tr>
</tbody>
</table>
### TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS. (continued)

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>UNIT</th>
<th>SYMBOL/DEFINITION</th>
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<tr>
<td>Piezoelectric strain coefficient</td>
<td>m/V = C/N</td>
<td>[d]</td>
</tr>
<tr>
<td>Piezoelectric stress modulus</td>
<td>N/C = V/m</td>
<td>[h]</td>
</tr>
<tr>
<td>Piezoelectric strain constant</td>
<td>m2/C</td>
<td>[g]</td>
</tr>
<tr>
<td>Piezoelectric polarization modulus</td>
<td>V/m = N/C</td>
<td>[a]</td>
</tr>
<tr>
<td>Piezoelectric polarization constant</td>
<td>m2/C</td>
<td>[b]</td>
</tr>
</tbody>
</table>

Note: Square brackets, sic: [ ], denote matrices.
II. The Piezoelectric Strain Coefficient Set

\[ [S] = [sE] [T] + [d]' [E] \quad (3) \]
\[ [D] = [d] [T] + [(\epsilon_T)E] [E] \quad (4) \]

III. The Piezoelectric Stress Modulus Set

\[ [T] = [cD] [S] - [h]' [D] \quad (5) \]
\[ [E] = -[ h] [S] + [(\beta_T)S] [D] \quad (6) \]

IV. The Piezoelectric Strain Constant Set

\[ [S] = [sD] [T] + [g]' [D] \quad (7) \]
\[ [E] = -[ g] [T] + [(\beta_T)D] [D] \quad (8) \]

V. The Piezoelectric Polarization Modulus Set

\[ [T] = [cP] [S] - [a]' [P] \quad (9) \]
\[ [E] = -[ a] [S] + [(\zeta_T)S] [P] \quad (10) \]

VI. The Piezoelectric Polarization Constant Set

\[ [S] = [sP] [T] + [b]' [P] \quad (11) \]
\[ [E] = -[ b] [T] + [(\zeta_T)T] [P] \quad (12) \]

The electric variables are connected by the relation

\[ [D] = (\epsilon_0) [E] + [P] \quad (13) \]

where \((\epsilon_0)\) is the permittivity of free space, defined by

\[ (\epsilon_0) * (\mu_0) * (c) * (c) = 1 ; \quad (14) \]

\((\mu_0)\) is the permeability of free space, equal, by definition, to \(4 \pi \times 10^{-7}\), and \((c)\) is the velocity of light in vacuo and, also by definition, is equal exactly to \(2.99792458 \times 10^8\) m/s.

From (13) the expressions for the remaining electric variables associated, respectively, with the six equation sets (1) to (12) may be found:

\[ [P] = [e] [S] + [(\chi_S)S] [E] \quad (15) \]
\[ [P] = [d] [T] + [(\chi_T)T] [E] \quad (16) \]
\[ [P] = (\epsilon_0) [h] [S] + [I - (\epsilon_0) (\beta_S)] [D] \quad (17) \]
\[ [P] = (\epsilon_0) [g] [T] + [I - (\epsilon_0) (\beta_T)] [D] \quad (18) \]
RELATIONS AMONG MATERIAL CONSTANTS

The material constants are interrelated by the following formulas:

\[
[cX][sX] = [(\varepsilon)Y] [(\beta)Y] = [I] \\
((\chi)Y) [(\zeta)Y] = [(K_Y - (\chi)r_Y) = [I]
\]

In (21) and (22), \(X = E, D, \) or \(P\) and \(Y = T\) or \(S.\)

\[
[cD] - [cE] = [h]' [e] = [e]' [h] \\
= [h]' [(\varepsilon)S] [h] = [e]' [(\beta)S] [e] \\
= [a]' [e - h * (\varepsilon)o] = [e - h * (\varepsilon)o]' [a]
\]

\[
[cP] - [cD] = [h]' [a] * (\varepsilon)o = [a]' [h] * (\varepsilon)o \\
= [h]' [(\varepsilon)S] [(\zeta)S] [h] * (\varepsilon)o \\
= [a]' [(\beta)S] [(\chi)S] [a] * (\varepsilon)o \\
= [a - h]' [e] = [e]' [a - h]
\]

\[
[cP] - [cE] = [a]' [e] = [e]' [a] \\
= [a]' [(\chi)S] [a] = [e]' [(\zeta)S] [e] \\
= [h]' [e + a * (\varepsilon)o] = [e + a * (\varepsilon)o]' [h]
\]

\[
[sE] - [sD] = [d]' [g] = [g]' [d] \\
= [d]' [(\beta)t] [d] = [g]' [(\varepsilon)t] [g] \\
= [b]' [d - g * (\varepsilon)o] = [d - g * (\varepsilon)o]' [b]
\]

\[
[sD] - [sP] = [b]' [g] * (\varepsilon)o = [g]' [b] * (\varepsilon)o \\
= [g]' [(\varepsilon)T] [(\zeta)T] [g] * (\varepsilon)o \\
= [b]' [(\beta)T] [(\chi)T] [b] * (\varepsilon)o \\
= [b - g]' [d] = [d]' [b - g]
\]

\[
[sE] - [sP] = [b]' [d] = [d]' [b] \\
= [b]' [(\chi)T] [b] = [d]' [(\zeta)T] [d] \\
= [g]' [d + b * (\varepsilon)o] = [d + b * (\varepsilon)o]' [g]
\]
\begin{align*}
((\zeta)S) - ((\zeta)T) &= [b] [a]' = [a] [b]' \\
&= [b] [cP] [b]' = [a] [sP] [a]' \\
((\chi)T) - ((\chi)S) &= ((\epsilon)T) - ((\epsilon)S) \\
&= [e] [d]' = [d] [e]' \\
&= [d] [cE] [d]' = [e] [sE] [e]' \\
((\beta)S) - ((\beta)T) &= [h] [g]' = [g] [h]' \\
&= [g] [cD] [g]' = [h] [sD] [h]' \\
[e] &= [d] [cE] = ((\epsilon)S) [h] = ((\chi)S) [a] \\
[d] &= [e] [sE] = ((\epsilon)T) [g] = ((\chi)T) [b] \\
[h] &= [g] [cD] = ((\beta)S) [e] = ((\chi)S) ((\beta)S) [a] \\
&= [I - (\beta)S * (\epsilon)o] [a] \\
[g] &= [h] [sD] = ((\beta)T) [d] = ((\chi)T) ((\beta)T) [b] \\
&= [I - (\beta)T * (\epsilon)o] [b] \\
[a] &= [b] [cP] = ((\zeta)S) [e] = ((\epsilon)S) ((\zeta)S) [h] \\
&= [I + (\zeta)S * (\epsilon)o] [h] \\
[b] &= [a] [sP] = ((\zeta)T) [d] = ((\epsilon)T) ((\zeta)T) [g] \\
&= [I + (\zeta)T * (\epsilon)o] [g]
\end{align*}

Some alternative relations are the following:
\begin{align*}
[a - h] &= ((\zeta)S) [h] * (\epsilon)o \\
&= ((\beta)S) [a] * (\epsilon)o \\
[b - g] &= ((\zeta)T) [g] * (\epsilon)o \\
&= ((\beta)T) [b] * (\epsilon)o \\
[e + a * (\epsilon)o] &= ((\epsilon)S) [a] \\
[d + b * (\epsilon)o] &= ((\epsilon)T) [b] \\
[e - h * (\epsilon)o] &= ((\chi)S) [h] \\
[d - g * (\epsilon)o] &= ((\chi)T) [g]
\end{align*}

Equations (21) to (43) result from equating like dependent variables pairs selected from equations (1) to (12) and (15) to (20).
Each pair yields one equation in three variables, one mechanical and two electrical, or vice versa. Two other equations exist, again from (1) to (12) and (15) to (20), that contain the same three variables found in each paired equation. One of these auxiliary equations is used to eliminate one of the two variables of the same kind; the result is one equation in two variables, one electrical and one mechanical. These are now independent variables, so the coefficients must vanish; two relations between the material coefficients result. As an example, (3) and (7) both have $[S]$ as dependent variable. Equating them produces one relation in $[T]$, $[E]$, and $[D]$; one of the electrical variables must be eliminated. This is done by using either (4) or (8); each contains the same three variables. If (8) is used to eliminate $[E]$, one obtains $[sE - d'g - sD][T] = [d'(bet)T - g'][D]$. Therefore, $[sE] - [sD] = [d]'[g]$ and $[g] = [(bet)T][d]$. Use of (4) instead of (8) leads to the equations $[sE] - [sD] = [g]'[d]$ and $[d] = [(eps)T][g]$. There are 36 pairs, six each equating $[S]$ and $[T]$, and eight each equating $[E]$, $[D]$, and $[P]$. The 72 relations contain many redundancies. Relations between the elastic, piezoelectric, and dielectric constants are shown schematically in Tables 2 and 3.

**CALCULATION SEQUENCE**

Using as input $[sE]$, $[d]$, and $[(eps)T]$, one may compute the remaining quantities in a variety of ways. The following sequence is typical:

\[
[cE] = [sE]^{-1}
\]
\[
[(bet)T] = [(eps)T]^{-1}
\]
\[
[e] = [d][cE]
\]
\[
[(eps)T] - [(eps)S] = [e][d]'
\]
\[
[(eps)S] = [(eps)T] - [e][d]'
\]
\[
[(bet)S] = [(eps)S]^{-1}
\]
\[
[h] = [(bet)S][e]
\]
\[
[cD] - [cE] = [e]'[h]
\]
\[
[cD] = [cE] + [e]'[h]
\]
\[
[g] = [(bet)T][d]
\]
\[
[sE] - [sD] = [d]'[g]
\]
\[
[sD] = [sE] - [d]'[g]
\]
\[
[(betr)S] = [(bet)S] * (eps)o
\]
\[
[(zetr)S] = [(betr)S][I - (betr)S]^{-1}
\]
TABLE 2. RELATIONS AMONG MATERIAL CONSTANTS.

\[
\begin{align*}
\begin{bmatrix} c \end{bmatrix} & \Delta \varepsilon = \Delta \chi & \begin{bmatrix} d \end{bmatrix} \\
\begin{bmatrix} a \end{bmatrix} & \Delta \xi & \begin{bmatrix} b \end{bmatrix} \\
\Delta c_{PE} & \Delta c_{PD} & \Delta \beta & \Delta s_{ED} & \Delta s_{EP} \\
\frac{\Delta c}{\varepsilon_0} & \frac{\Delta s}{\varepsilon_0}
\end{align*}
\]
\[(zet)S = [(zet)T] / (\epsilon o) \quad (58)\]
\[(bet)T = [(bet)T] * (\epsilon o) \quad (59)\]
\[(zet)T = [(bet)T] [I - (bet)T] (-1) \quad (60)\]
\[(zet)T = [(zet)T] / (\epsilon o) \quad (61)\]
\[(chi)S = [(zet)S] (-1) \quad (62)\]
\[(chi)T = [(zet)T] (-1) \quad (63)\]
\[a = [(zet)S] [e] \quad (64)\]
\[b = [(zet)T] [d] \quad (65)\]
\[[cP] - [cE] = [e]' [a] \quad (66)\]
\[[cP] = [cE] + [e]' [a] \quad (67)\]
\[[cP] - [cD] = [a]' [h] * (\epsilon o) \quad (68)\]
\[[sE] - [sP] = [d]' [b] \quad (69)\]
\[[sP] = [sE] - [d]' [b] \quad (70)\]
\[[sD] - [sP] = [g]' [b] * (\epsilon o) \quad (71)\]
\[[bet)S] - [(bet)T] = [h] [g]' \quad (72)\]
\[[zet)S] - [(zet)T] = [a] [b]' \quad (74)\]

A number of these relations are used as checks. For example, \([(bet)S]\) and \([(bet)T]\) are known from (45) and (49), but the difference is recomputed in (72).

**EXPLICIT FORMULAS FOR POINT GROUP 4mm**

**Elastic:**

The 6x6 elastic constant portion of Table 4 partitions into 4x4 and 2x2 submatrices. The 4x4 elastic stiffness and compliance submatrices are interrelated by formulas (75) to (93). The elastopiezoelectric matrix for class 4mm is found in Cady (Ref. 2). Other references to lithium tetraborate are given in Refs. 3 to 26.

\[A = s33 * (s11 + s12) - 2 * s13 * s13 \quad (75)\]
\[B = (s11 - s12) \quad (76)\]
\[c11 = +(s11 * s33 - s13 * s13) / (A * B) \quad (77)\]
TABLE 4. ELASTO PIEZODIELECTRIC MATRICES FOR POINT GROUP 4mm.

| 11 12 13 00 00 00 | 00 00 31 cE | e' |
| 12 11 13 00 00 00 | 00 00 31 e | (eps)S |
| 13 13 33 00 00 00 | 00 00 33 |
| 00 00 00 44 00 00 | 00 15 00 cD | h' |
| 00 00 00 00 44 00 | 15 00 00 h | (bet)S |
| 00 00 00 00 00 66 | 00 00 00 |

00 00 00 00 15 00 11 00 00 cP | a' |
00 00 00 15 00 00 00 11 00 a | (zet)S |
31 31 33 00 00 00 00 00 33 |

Matrix entries show only subscripts.
\[c_{12} = - (s_{12} * s_{33} - s_{13} * s_{13}) / (A * B)\]  \hfill (78)
\[c_{13} = - s_{13} / A\] \hfill (79)
\[c_{33} = (s_{11} + s_{12}) / A\] \hfill (80)
\[c_{44} = 1 / s_{44}\] \hfill (81)
\[c_{66} = (c_{11} - c_{12}) / 2 = s_{44} / (2 * B)\] \hfill (82)
\[K = c_{33} * (c_{11} + c_{12}) - 2 * c_{13} * c_{13}\] \hfill (83)
\[L = (c_{11} - c_{12})\] \hfill (84)
\[s_{11} = +(c_{11} * c_{33} - c_{13} * c_{13}) / (K * L)\] \hfill (85)
\[s_{12} = -(c_{12} * c_{33} - c_{13} * c_{13}) / (K * L)\] \hfill (86)
\[s_{13} = - c_{13} / K\] \hfill (87)
\[s_{33} = (c_{11} + c_{12}) / K\] \hfill (88)
\[s_{44} = 1 / c_{44}\] \hfill (89)
\[s_{66} = 1 / c_{66}\] \hfill (90)
\[\text{det (3x3) [s]} = A * B\] \hfill (91)
\[\text{det (3x3) [c]} = K * L\] \hfill (92)
\[A * K = B * L = A * B * K * L = 1\] \hfill (93)

Formulas (75) to (93) hold for each set of constant electrical conditions: either E, D, or P constant.

\[[c_{D}] - [c_{E}] = [\text{del } c_{DE}] = [e]' [h] = [h]' [e]\]  \hfill (23)
\[\text{del } c_{DE11} = + e_{31} h_{31}\] \hfill (94)
\[\text{del } c_{DE12} = + e_{31} h_{31}\] \hfill (95)
\[\text{del } c_{DE13} = + e_{31} h_{33} = + h_{31} e_{33}\] \hfill (96)
\[\text{del } c_{DE33} = + e_{33} h_{33}\] \hfill (97)
\[\text{del } c_{DE44} = + e_{15} h_{15}\] \hfill (98)
\[\text{del } c_{DE66} = 0\] \hfill (99)

\[[c_{P}] - [c_{D}] = [\text{del } c_{PD}] = [a]' [h] * (\text{eps}) = [h]' [a] * (\text{eps})\]  \hfill (24)
\[ \text{del cPD}_{11} = ( + a_{31} h_{31} ) * (\varepsilon)_{o} \]
(100)
\[ \text{del cPD}_{12} = ( + a_{31} h_{31} ) * (\varepsilon)_{o} \]
(101)
\[ \text{del cPD}_{13} = ( + a_{31} h_{33} ) * (\varepsilon)_{o} \]
\[ = ( + h_{31} a_{33} ) * (\varepsilon)_{o} \]
(102)
\[ \text{del cPD}_{33} = ( + a_{33} h_{33} ) * (\varepsilon)_{o} \]
(103)
\[ \text{del cPD}_{44} = ( + a_{15} h_{15} ) * (\varepsilon)_{o} \]
(104)
\[ \text{del cPD}_{66} = 0 \]
(105)

\[ [\text{CP}] - [\text{CE}] = [\text{del cPE}] = [e]' [a] = [a]' [e] \]
(25)
\[ \text{del cPE}_{11} = + e_{31} a_{31} \]
(106)
\[ \text{del cPE}_{12} = + e_{31} a_{31} \]
(107)
\[ \text{del cPE}_{13} = + e_{31} a_{33} = + a_{31} e_{33} \]
(108)
\[ \text{del cPE}_{33} = + e_{33} a_{33} \]
(109)
\[ \text{del cPE}_{44} = + e_{15} a_{15} \]
(110)
\[ \text{del cPE}_{66} = 0 \]
(111)

From the del c13 entries we have the ratios
\[ e_{31} / e_{33} = h_{31} / h_{33} = a_{31} / a_{33}. \]
(112)

\[ [\text{SE}] - [\text{SD}] = [\text{del sED}] = [d]' [g] = [g]' [d] \]
(26)
\[ \text{del sED}_{11} = + d_{31} g_{31} \]
(113)
\[ \text{del sED}_{12} = + d_{31} g_{31} \]
(114)
\[ \text{del sED}_{13} = + d_{31} g_{33} = + g_{31} d_{33} \]
(115)
\[ \text{del sED}_{33} = + d_{33} g_{33} \]
(116)
\[ \text{del sED}_{44} = + d_{15} g_{15} \]
(117)
\[ \text{del sED}_{66} = 0 \]
(118)

\[ [\text{SD}] - [\text{SP}] = [g]' [b] * (\varepsilon)_{o} \]
\[ = [b]' [g] * (\varepsilon)_{o} \]
(27)
\[ \text{del sDP}_{11} = ( + g_{31} b_{31} ) * (\varepsilon)_{o} \]
(119)
\[
\begin{align*}
\text{del } s_{DP12} &= ( + g_{31} b_{31} ) * (\varepsilon)_o \\
\text{del } s_{DP13} &= ( + g_{31} b_{33} ) * (\varepsilon)_o \\
&= ( + b_{31} g_{33} ) * (\varepsilon)_o \\
\text{del } s_{DP33} &= ( + g_{33} b_{33} ) * (\varepsilon)_o \\
\text{del } s_{DP44} &= ( + g_{15} b_{15} ) * (\varepsilon)_o \\
\text{del } s_{DP66} &= 0 \\
[sE] - [SP] &= [\text{del } s_{EP}] = [b]' [d] = [d]' [b] \\
\text{del } s_{EP11} &= + d_{31} b_{31} \\
\text{del } s_{EP12} &= + d_{31} b_{31} \\
\text{del } s_{EP13} &= + d_{31} b_{33} = + b_{31} d_{33} \\
\text{del } s_{EP33} &= + d_{33} b_{33} \\
\text{del } s_{EP44} &= + d_{15} b_{15} \\
\text{del } s_{EP66} &= 0
\end{align*}
\]

From the del s13 entries we have the ratios
\[
d_{31} / d_{33} = g_{31} / g_{33} = b_{31} / b_{33}.
\]

Piezoelectric:
\[
\begin{align*}
[e] &= [d] [cE] \\
e_{15} &= + d_{15} cE_{44} \\
e_{31} &= + d_{31} (cE_{11} + cE_{12}) + d_{33} cE_{13} \\
e_{33} &= + d_{33} cE_{33} + d_{13} cE_{13} * 2 \\
[h] &= [(\text{bet})S] [e] \\
h_{15} &= (\text{bet})S_{11} e_{15} \\
h_{31} &= (\text{bet})S_{33} e_{31} \\
h_{33} &= (\text{bet})S_{33} e_{33} \\
[g] &= [(\text{bet})T] [d] \\
g_{15} &= (\text{bet})T_{11} d_{15} \\
g_{31} &= (\text{bet})T_{33} d_{31} \\
g_{33} &= (\text{bet})T_{33} d_{33}
\end{align*}
\]
[a] = [(zet)S] [e] \hspace{1cm} (36)
a_{15} = (zet)S_{11} \text{ } e_{15} \hspace{1cm} (141)
a_{31} = (zet)S_{33} \text{ } e_{31} \hspace{1cm} (142)
a_{33} = (zet)S_{33} \text{ } e_{31} \hspace{1cm} (143)
[b] = [(zet)T] [d] \hspace{1cm} (37)
b_{15} = (zet)T_{11} \text{ } d_{15} \hspace{1cm} (144)
b_{31} = (zet)T_{33} \text{ } d_{31} \hspace{1cm} (145)
b_{33} = (zet)T_{33} \text{ } d_{33} \hspace{1cm} (146)

Dielectric:

[ (bet)Y ] = [ (eps)Y ]^{(-1)} \hspace{1cm} (21)
(bet)Y_{11} = 1 / (eps)Y_{11} \hspace{1cm} (147)
(bet)Y_{33} = 1 / (eps)Y_{33} \hspace{1cm} (148)

[(zetr)Y] = [(betr)Y] [I - (betr)Y]^{(-1)} \hspace{1cm} (149)
(zet)Y_{11} = 1 / ((eps)Y_{11} - (eps)o) \hspace{1cm} (150)
(zet)Y_{33} = 1 / ((eps)Y_{33} - (eps)o) \hspace{1cm} (151)

[(eps)T - (eps)S] = [del (eps)] = [e] [d]' = \hspace{1cm} (30)
[(chi)T - (chi)S] = [del (chi)] = [d] [e]' \hspace{1cm} (30)
del (eps)_{11} = del (chi)_{11} = + e_{15} d_{15} \hspace{1cm} (152)
del (eps)_{33} = del (chi)_{33} = + e_{33} d_{33} + e_{31} d_{31} * 2 \hspace{1cm} (153)

[(bet)S - (bet)T] = [h] [g]' = [g] [h]' \hspace{1cm} (31)
del (bet)_{11} = + h_{15} g_{15} \hspace{1cm} (154)
del (bet)_{33} = + h_{33} g_{33} + h_{31} g_{31} * 2 \hspace{1cm} (155)

[(zet)S - (zet)T] = [del (zet)] = [a] [b]' = [b] [a]' \hspace{1cm} (156)
del (zet)_{11} = + a_{15} b_{15} \hspace{1cm} (157)
del (zet)_{33} = + a_{33} b_{33} + a_{31} b_{31} * 2 \hspace{1cm} (158)
The values measured by Shiosaki, et al. (Ref. 1) are as follows:

**TABLE 5. ISAGRIC ELASTIC COMPLIANCES.**

<table>
<thead>
<tr>
<th>sE11</th>
<th>sE12</th>
<th>sE13</th>
<th>cE33</th>
<th>sE44</th>
<th>sE66</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.81</td>
<td>1.23</td>
<td>-5.92</td>
<td>24.6</td>
<td>17.1</td>
<td>21.4</td>
</tr>
</tbody>
</table>

Units: \(10^{-12}\) m/N.

**TABLE 6. PIEZOELECTRIC STRAIN COEFFICIENTS.**

<table>
<thead>
<tr>
<th>d15</th>
<th>d31</th>
<th>d33</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.07</td>
<td>-2.58</td>
<td>19.4</td>
</tr>
</tbody>
</table>

Units: \(10^{-12}\) m/v.

**TABLE 7. DIELECTRIC PERMITTIVITIES AT CONSTANT STRESS.**

<table>
<thead>
<tr>
<th>(eps)T11</th>
<th>(eps)T33</th>
<th>(eps)T11/(eps)o</th>
<th>(eps)T33/(eps)o</th>
</tr>
</thead>
<tbody>
<tr>
<td>82.61</td>
<td>87.92</td>
<td>9.33</td>
<td>9.93</td>
</tr>
</tbody>
</table>

Units: \(10^{-12}\) F/m.

The input values from Tables 5, 6, and 7 were used to compute the remaining elastic, piezoelectric, and dielectric quantities for lithium tetraborate in the manner discussed in prior sections of this report. The results are given in Tables 8 to 15.
TABLE 8. ELASTIC STIFFNESSES.

<table>
<thead>
<tr>
<th></th>
<th>cE</th>
<th>cD</th>
<th>cP</th>
<th>del cDE</th>
<th>del cPE</th>
<th>del cPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>135.5</td>
<td>136.7</td>
<td>136.8</td>
<td>1.18</td>
<td>1.35</td>
<td>0.167</td>
</tr>
<tr>
<td>12</td>
<td>3.57</td>
<td>4.75</td>
<td>4.92</td>
<td>1.18</td>
<td>1.35</td>
<td>0.167</td>
</tr>
<tr>
<td>13</td>
<td>33.47</td>
<td>37.24</td>
<td>37.78</td>
<td>3.78</td>
<td>4.31</td>
<td>0.535</td>
</tr>
<tr>
<td>33</td>
<td>56.76</td>
<td>68.83</td>
<td>70.54</td>
<td>12.07</td>
<td>13.78</td>
<td>1.71</td>
</tr>
<tr>
<td>44</td>
<td>58.48</td>
<td>61.31</td>
<td>61.66</td>
<td>2.83</td>
<td>3.18</td>
<td>0.358</td>
</tr>
<tr>
<td>66</td>
<td>46.73</td>
<td>46.73</td>
<td>46.73</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Units: $10^9$ N/m².

TABLE 9. ELASTIC COMPLIANCES.

<table>
<thead>
<tr>
<th></th>
<th>sE</th>
<th>sD</th>
<th>sP</th>
<th>del sED</th>
<th>del sEP</th>
<th>del sDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>8.81</td>
<td>8.73</td>
<td>8.73</td>
<td>0.0757</td>
<td>0.0842</td>
<td>0.00848</td>
</tr>
<tr>
<td>12</td>
<td>1.23</td>
<td>1.15</td>
<td>1.15</td>
<td>0.0757</td>
<td>0.0842</td>
<td>0.00848</td>
</tr>
<tr>
<td>13</td>
<td>-5.92</td>
<td>-5.30</td>
<td>-5.29</td>
<td>-0.569</td>
<td>-0.633</td>
<td>-0.0637</td>
</tr>
<tr>
<td>33</td>
<td>24.6</td>
<td>20.3</td>
<td>19.8</td>
<td>4.28</td>
<td>4.76</td>
<td>0.479</td>
</tr>
<tr>
<td>44</td>
<td>17.1</td>
<td>16.3</td>
<td>16.2</td>
<td>0.788</td>
<td>0.883</td>
<td>0.0946</td>
</tr>
<tr>
<td>66</td>
<td>24.4</td>
<td>21.4</td>
<td>21.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Units: $10^{-12}$ m²/N.

TABLE 10. PIEZOELECTRIC [e], [h], AND [a] VALUES.

<table>
<thead>
<tr>
<th></th>
<th>e</th>
<th>h</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.472</td>
<td>5.99</td>
<td>6.75</td>
</tr>
<tr>
<td>31</td>
<td>0.290</td>
<td>4.07</td>
<td>4.64</td>
</tr>
<tr>
<td>33</td>
<td>0.928</td>
<td>13.00</td>
<td>14.84</td>
</tr>
</tbody>
</table>

Units: e: C/m²; h and a: $10^9$ V/m.
### TABLE 11. PIEZOELECTRIC \( [d], [g], \) AND \([b]\) VALUES.

<table>
<thead>
<tr>
<th></th>
<th>(d)</th>
<th>(g)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>8.07</td>
<td>97.7</td>
<td>109.4</td>
</tr>
<tr>
<td>31</td>
<td>-2.58</td>
<td>-29.3</td>
<td>-32.6</td>
</tr>
<tr>
<td>33</td>
<td>19.4</td>
<td>220.6</td>
<td>245.4</td>
</tr>
</tbody>
</table>

Units: \(d: 10^{-12}\) m/V; \(g\) and \(b: 10^{-3}\) m²/C.

---

### TABLE 12. DIELECTRIC (\(\epsilon\)) VALUES.

<table>
<thead>
<tr>
<th></th>
<th>(\epsilon_S)</th>
<th>(\epsilon_T)</th>
<th>(\Delta \epsilon_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>78.80</td>
<td>82.61</td>
<td>3.81</td>
</tr>
<tr>
<td>33</td>
<td>71.41</td>
<td>87.92</td>
<td>16.51</td>
</tr>
</tbody>
</table>

Units: \(10^{-12}\) F/m.

\(\Delta \epsilon_T\) = \(\Delta \epsilon_S\)

---

### TABLE 13. DIELECTRIC (\(\chi\)) VALUES.

<table>
<thead>
<tr>
<th></th>
<th>(\chi_S)</th>
<th>(\chi_T)</th>
<th>(\Delta \chi_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>69.95</td>
<td>73.76</td>
<td>3.81</td>
</tr>
<tr>
<td>33</td>
<td>62.56</td>
<td>79.07</td>
<td>16.51</td>
</tr>
</tbody>
</table>

Units: \(10^{-12}\) F/m.

\(\Delta \chi_T\) = \(\Delta \epsilon_T\)

---

### TABLE 14. DIELECTRIC (\(\beta\)) VALUES.

<table>
<thead>
<tr>
<th></th>
<th>(\beta_S)</th>
<th>(\beta_T)</th>
<th>(\Delta \beta_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12.69</td>
<td>12.11</td>
<td>-0.585</td>
</tr>
<tr>
<td>33</td>
<td>14.00</td>
<td>11.37</td>
<td>-2.63</td>
</tr>
</tbody>
</table>

Units: \(10^9\) m/F.
### TABLE 15. DIELECTRIC (zet) VALUES.

<table>
<thead>
<tr>
<th></th>
<th>(zet)S</th>
<th>(zet)T</th>
<th>del (zet)TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>14.30</td>
<td>13.56</td>
<td>-0.738</td>
</tr>
<tr>
<td>33</td>
<td>15.99</td>
<td>12.65</td>
<td>-3.34</td>
</tr>
</tbody>
</table>

Units: $10^9$ m/F.

---

### CONCLUSIONS

This report provides formulas interrelating the coefficients that appear in the several alternative sets of constitutive equations involving the elastic, piezoelectric, and dielectric properties of crystals. These are then specialized for crystals of class 4mm; using measured values reported for lithium tetraborate, numerical values of the elements of the polarization matrices are calculated.
REFERENCES


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