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Dwell Time and Average Local Speed in a Resonant Tunneling Structure

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Dwell Time and Average Local Speed in a Resonant Tunneling Structure

L. N. Pandey, D. Sahu and Thomas F. George

We show that the dwell times and the average local speeds of an electron in a resonant tunneling structure depend sensitively on the matching parameter at the interfaces. We point out that there is a need to carry out microscopic calculations to find out which matching parameter is appropriate for a given structure.
We show that the dwell times and the average local speeds of an electron in a resonant tunneling structure depend sensitively on the matching parameter at the interfaces. We point out that there is a need to carry out microscopic calculations to find out which matching parameter is appropriate for a given structure.
Recently resonant tunneling diodes (RTDs) have generated considerable experimental and theoretical interest because of possible device applications. In addition, they have highlighted the need to understand the fundamental physics involved in these and other quantum confined structures. A question which has attracted much attention and controversy concerns the time scales involved in the RTDs. In this work we focus attention on the dwell time (see later for a definition) which is perhaps the least controversial and the most well-accepted quantity. We study the dwell time and its first derivative with respect to position for both symmetric and asymmetric double barrier structures. These are physical quantities and can be obtained from stationary-state solutions of Schrödinger’s equation. Since time-dependent studies of Schrödinger’s equation are computationally time consuming, our studies should provide qualitative guidelines in pursuing such studies. References 8-11 should provide the interested reader with some indications regarding the temporal aspects of resonance tunneling structures.

We consider an unbiased RTD with barrier heights $V_i$ ($i = 1, 2$), barrier widths $a_i$ ($i = 1, 2$) and electron effective masses in the barriers $m_i^*$ ($i = 1, 2$) (in units of the free electron mass). The potential well has width $d$, and the effective mass inside the well and the contact regions are assumed to be $m^*$. Typically the barriers consist of $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ and the well consists of GaAs, so that $m_1^* = m_2^* = 0.09$ and $m^* = 0.067$. The stationary-state properties of the resonant tunneling structure in the effective mass approximation are obtained by solving the time-independent Schrödinger's equation for the envelope function $\psi(z)$ along the growth direction $z$:  

\[ \psi(z) = \begin{cases} e^{-|z|} & \text{for } z < 0 \\ \frac{1}{\sqrt{d}} & \text{for } 0 < z < d \\ e^{z} & \text{for } z > d \end{cases} \]
where $V(z)$ is the conduction band potential energy profile, $E$ is the incident energy, $m$ is the electron mass, and $a, b$ are constants. The importance of writing the kinetic energy term in a form similar to the first term of Eq. (1a) was pointed out by Bastard. The constants $a$ and $b$ in Eq. (1) must appear in the form given in order to make the Hamiltonian Hermitian. The above form of the kinetic energy operator is due to Morrow and Brownstein and is a generalization of Bastard’s form. At the heterojunction interfaces, which we have assumed to be abrupt, the kinetic energy operator dictates the matching conditions on $\psi(z)$ and its spatial derivative. We demand that $m\psi$ and $(a+b)(d\psi/dz)$ be continuous across an interface, implying the physical result that the current density $j \propto \psi (d\psi/dz)/m = m\psi (a+b)(d\psi/dz)$ be continuous. However, in general, the charge density $\rho \propto \psi^2 \psi$ need not be continuous across an interface. For the special case of $a = 0$ and $b = -1$ we obtain, in addition, the continuity of charge density.

For a symmetric double barrier, the transmission coefficient is unity at resonance. For an asymmetric double barrier, the transmission at resonance is less than unity, being equal to $T = T_{\text{low}}/T_{\text{high}}$, where $T_{\text{low}}$ ($T_{\text{high}}$) is the smaller (larger) of the transmission coefficients of the two barriers. In general, for $E < V_1$ and $V_2$, the extremal condition for the transmission coefficient for a RTD satisfying Eq. (1) is

$$\frac{1}{2} \xi_i \tanh \kappa_i a_i (1 - 1,2) \text{ with } \delta_i = \left[ \left( \frac{m^*}{m} \right)^a \kappa_i / k - \left( \frac{m^*}{m} \right)^b \kappa / \kappa_i \right] (1 - 1,2), \text{ where } a = \ldots$$
(2m*E/M^2)^1/4 and κ_i = [2m_i^*(V_i - E)/M^2]^1/4, (i = 1, 2). For E > V_1 or V_2, the hyperbolic functions should be replaced by appropriate circular functions, and κ_i should be imaginary. If the RTD is symmetric, with V_1 = V_2 = V, a_1 = a_2 and m_1^* = m_2^* = m^*, we recover Hauge et. al's result

\[(\cot kd)_r = -\frac{1}{2}(κ_1/k - k/κ_1)\tanh κ_1a_1. \tag{3}\]

In Table 1 we show the lowest resonance energy of a symmetric (E_r) and an asymmetric (E^a_r) RTD as a function of the parameter b [Eq. (1)].

Table 1. Dependence of the lowest resonance energy on the parameter b of Eq. (1). E_r and E_0.5 are, respectively, the resonance energy and energy for a transmission coefficient of 0.5 for a symmetric structure with V_1 = V_2 = 200 meV, d = 100 Å, a_1 = a_2 = 50 Å, m^* = 0.067 and m_1^* = m_2^* = 0.09. E^a_r is the resonance energy for an asymmetric double barrier with V_1 = 100 meV, V_2 = 200 meV, d = a_1 = a_2 = 50 Å, m^* = 0.067 and m_1^* = m_2^* = 0.09.

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<tr>
<th>b</th>
<th>E_r(meV)</th>
<th>E_0.5(meV)</th>
<th>E^a_r(meV)</th>
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<tr>
<td>-2</td>
<td>24.6738</td>
<td>24.6347</td>
<td>47.9598</td>
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<tr>
<td>-1</td>
<td>28.8350</td>
<td>28.7975</td>
<td>57.3208</td>
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<td>0</td>
<td>32.9650</td>
<td>32.9300</td>
<td>66.9193</td>
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<tr>
<td>1</td>
<td>36.8850</td>
<td>36.8567</td>
<td>76.1235</td>
</tr>
<tr>
<td>2</td>
<td>40.4560</td>
<td>40.4337</td>
<td>84.1560</td>
</tr>
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The resonance energy clearly depends on the parameter b; we cannot, a priori, prefer one value of b over another. We study below the dependence of other physical quantities on the
parameter $b$. The dwell time $r_D$ over the region $0$ to $z_0$ of the structure is defined\textsuperscript{7,15} as the integrated probability density of the electron divided by the incident flux:

$$ r_D = \int_0^{z_0} dz |\psi(z)|^2/(|\mathbf{K}/m^*) \quad . \quad (4) $$

The associated average local speed $v$ at a given point $z$ in the structure is

$$ v^{-1} = \partial r_D/\partial z \quad . \quad (5) $$

We have studied the dwell times and average local speeds of an electron in a double barrier structure for energies in the neighborhood of the resonance energy $E_R$. Figures 1-3 show $r_D$ and $v^{-1}$ for two energies $E_R$ (transmission coefficient $T = 1$) and $E_{0.5}$ ($T = 0.5$) as a function of the parameter $b$. These figures clearly show wide variations in the magnitudes of the above physical quantities as one changes $b$. The discontinuities in the velocities (as long as $b$ is not equal to 1) at the two interfaces arise from the discontinuities of charge at the interfaces, as already mentioned. Note also that the discontinuities could be positive or negative at a given interface. The behavior of $r_D$ and $v^{-1}$ for an asymmetric structure are similar (Figs. 4-6); however, the discontinuities of $v^{-1}$ are more prominent in this case.

We have thus shown that the parameter associated with matching condition at the interfaces has a profound effect on the characteristics of the system such as resonance energy, dwell time and the average local speed. It has been shown,\textsuperscript{16,17} in a different context, that for two semi-finite heterostructures, the matching conditions at the interface not only involve the effective masses but also certain other parameters which are microscopic in origin, having no macroscopic analogs. We therefore believe that the arbitrariness in the choice of the parameter $b$ [Eq. (1)] above can
be fixed through microscopic calculations.

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References

14. Following Ref. 6, we have derived Eq. (2) from the condition \( dT/d'd' = 0 \), where \( T \) is the square of the transmission amplitude and \( 'd' \) is the well width. Conventionally, the resonance condition is obtained from \( dT/dE = 0 \). For a symmetric structure, the two definitions are equivalent. For an asymmetric structure, the direction along which the derivative is evaluated makes a difference; it is believed, however, that the difference between the two is exponentially small.
Figure Captions

Fig. 1. Dwell time (top panel) and inverse of the average local speed (bottom panel) as a function of position $z$ relative to left most edge of the structure. The parameters are: $b = -2$, $V_1 = V_2 = 200$ meV, $d_x = 100$ Å, $a_1 = a_2 = 50$ Å, $m = 0.067$ and $m_1 = m_2 = 0.09$. The solid line corresponds to resonance and the dashed line to a transmission coefficient of 0.5.

Fig. 2. Dwell time (top panel) and inverse of the average local speed (bottom panel) as a function of position $z$ relative to leftmost edge of the structure. The parameters are: $b = 0$, $V_1 = V_2 = 200$ meV, $d_x = 100$ Å, $a_1 = a_2 = 50$ Å, $m = 0.067$ and $m_1 = m_2 = 0.09$. The solid line corresponds to resonance and the dashed line to a transmission coefficient of 0.5.

Fig. 3. Dwell time (top panel) and inverse of the average local speed (bottom panel) as a function of position $z$ relative to leftmost edge of the structure. The parameters are: $b = -2$, $V_1 = V_2 = 200$ meV, $d_x = 100$ Å, $a_1 = a_2 = 50$ Å, $m = 0.067$ and $m_1 = m_2 = 0.09$. The solid line corresponds to resonance and the dashed line to a transmission coefficient of 0.5.

Fig. 4. Dwell time (top panel) and inverse of the average local speed (bottom panel) as a function of position $z$ relative to the edge of the smaller of the two asymmetric barriers. The parameters are: $b = -2$, $V_1 = 100$ meV, $V_2 = 200$ meV, $d = a_1 = a_2 = 50$ Å, $m = 0.067$ and $m_1 = m_2 = 0.09$.

Fig. 5. Dwell time (top panel) and inverse of the average local speed (bottom panel) as a function of position $z$ relative to the edge of the smaller of the two asymmetric barriers. The parameters are: $b = 1$, $V_1 = 100$ meV, $V_2 = 200$ meV, $d = a_1 = a_2 = 50$ Å, $m = 0.067$ and $m_1 = m_2 = 0.09$.

Fig. 6. Dwell time (top panel) and inverse of the average local speed (bottom panel) as a function of position $z$ relative to the edge of the smaller of the two asymmetric barriers.
The parameters are: $b = 2$, $V_1 = 100$ meV, $V_2 = 200$ meV, $a_1 - a_2 = 50$ Å, $m = 0.067$ and $m_1 = m_2 = 0.09$. 
Fig. 2

\begin{align*}
\text{dwell time (10^{-12} s)} & \\
0 & 100 & 200 \\
0 & 10 & 20 \\
\end{align*}

\begin{align*}
\frac{1}{v} \text{ (10^{-12} s/A)} & \\
0 & 100 & 200 \\
0.00 & 0.10 & 0.20 \\
\end{align*}
Fig. 3

- Top graph:
  - x-axis: $z$ (Å)
  - y-axis: dwell time ($10^{-12}$ s)
  - Graphs for $b=2$

- Bottom graph:
  - x-axis: $z$ (Å)
  - y-axis: $1/v$ ($10^{-12}$ s/Å)
  - Graphs for $b=2$
Fig. 4

Dwell time (10^{-12} s)

![Graph showing dwell time as a function of z (Å).]

\[ b = -2 \]

\[
\begin{align*}
\text{dwell time (10^{-12} s)} & \\
0 & 0.7 \\
0 & 1.4 \\
0 & 75 \\
0 & 150 \\
\end{align*}
\]

\[
\begin{align*}
\text{\( z (\text{Å}) \)} & \\
0 & 75 \\
0 & 150 \\
\end{align*}
\]

Reciprocal velocity (10^{-12} s/Å)

![Graph showing reciprocal velocity as a function of z (Å).]

\[ b = -2 \]

\[
\begin{align*}
\text{1/\( v \) (10^{-12} s/Å)} & \\
0.000 & 0.015 \\
0.030 & 1.4 \\
0 & 75 \\
0 & 150 \\
\end{align*}
\]
Fig. 5

- Top graph: Dwell time (10^-12 s) vs. z (Å)
- Bottom graph: 1/v (10^-12 s/Å) vs. z (Å)
Fig. 6

- Top graph: dwell time (10^-12 s) vs. z (Å) for b = 2.
- Bottom graph: 1/v (10^-12 s/Å) vs. z (Å) for b = 2.
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