Math and Physics Studies - Multi-Project Support
Entropy-Based Image Restoration: Modifications
and Additional Results

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Entropy-Based Image Restoration, Modifications and Additional Results

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**Abstract:** This report covers the period October 1987 thru September 1988 to the above referenced contract. The contract effort has been towards the development of a data management program for the SDIO Infrared Background Signature Survey, a shuttle-based experiment to be launched in mid 1990, and the super-resolution of special data observations of IRAS satellite sensors images of galaxies. The attached report addresses the latter. A new approach to the application of maximum entropy principles in image restoration is being developed. It is an iterative process which combines a Wiener filter restoration and modifications to the Wiener filter that are Entropy based. The result is an image with well restored frequency content and very little of the spuriousness commonly introduced by inverse filters. The algorithm is fast, stable to convergence, and will accommodate any specifiable disturbing function.

**Keywords:** Super-resolution, IRAS, entropy, image restoration.
ABSTRACT

A new approach to the application of maximum entropy principles in image restoration is being developed. It is an iterative process which combines a Wiener filter restoration and modifications to the Wiener image that are Entropy-based. The result is an image with well-restored frequency content and very little of the spuriousness commonly introduced by inverse filters. The algorithm is fast, stable to convergence, and will accommodate any specifiable distorting function.

1. INTRODUCTION

It is frequently desirable to remove taking-system degradation from images in an attempt to obtain a truer object record. The most conceptually straightforward of these restoration processes are linear inversion techniques, in which the distorting spread function is effectively deconvolved. Often, in practice, this amounts to a frequency domain division. While the simplicity of inversion methods is attractive, inherent in the frequency domain division is the introduction of large spurious artifacts in the regions where the MTF(divisor) is near zero. The resulting restorations are marked by large periodic 'side lobes' and noise that are solely due to the restoration process, and do not represent the object. Such results are usually undesirable.

Another approach to object estimation from images is probabilistic and involves the concept of entropy in the information capacity sense of the word. Maximum entropy (Maxent) restoration methods use the gathered image and any other available information about the object to predict, without making assumptions, that object which most likely gave rise to the recorded image. Since these estimations begin with no assumptions about the object, the first iteration is a flat, uniform-gray field. As the restoration proceeds, a likely object rises out of the flat field consistent with the observed data and any other known information. After this process, the resulting object estimate consists only of information that was present in the image data. No spurious noise or artifacts are introduced by the process.

We present an image restoration algorithm which is a hybrid of a linear inversion technique and the Maxent concept. The results reported here are an extension of research reported in references 1 and 2.

2. DERIVATION

Assume a subject image which is digital and was formed by the process

\[ y(k) = x(k) * p(k) + n(k), \]

where "\(*\)" denotes convolution, \( y \) is the observed data, \( x \) is the object, \( p \) is the imaging system point spread function, and \( n \) is noise. At least \( y \) and \( p \) are known.

The problem is to estimate \( x(k) \) from restoration of \( y(k) \). Once an estimate of \( x \) is arrived at, \( x_e \) is convolved with \( p \) to form an estimated noiseless image, \( g_e(k) \).
From this quantity, the mean square error is formed:

\[ \text{MSE} = \sum (y(k) - g_e(k))^2 \]

The entropy of \( x_e(k) \) is defined as

\[ S = -\sum x_e(k) \ln(x_e(k)) \]

In many Maxent approaches\(^1\), entropy is maximized without letting \( g_e(k) \) depart significantly from the observed data, \( y(k) \). In particular, a quantity, \( Q \), is formed which, by use of a Lagrange multiplier, binds entropy and MSE together such that a trade-off is made between them when an object estimate is formed. To bring the solution to an extrema, differentiate \( Q \), set the result to zero, and solve for \( x_e(k) \). The resulting implicit equation

\[ x_e(k) = \exp(-1 + \lambda[y(k) - g_e(k)] * p(-k)), \]

is the basis for an iterative solution,

\[ x_e^{n+1}(k) = \exp(-1 + \lambda[y(k) - g_e^n(k)] * p(-k)), \]

in which \( x_e^n(k) \) is a constant.

Restorations from this kind of iterative process are smooth, noise-free, and generally result in a 2-4x resolution increase after many tens of iterations. It should be noted that in this form, the solution tends to be unstable, typically requiring the "slow" averaging in of new iterations; e.g.

\[ x_e^n(k) = 0.95(x_e^{n-1}(k)) + 0.05(x_e^n(k)), \]

where \( x_e^n(k) \) is the estimate that would be directly calculated by the algorithm. It is this characteristic that both allows for eventual convergence and leads to the possible need for a hundred or more iterations.

It is desirable to find a solution which results in a greater resolution increase, converges more quickly, is more stable, and less compute-intensive, especially when one is faced with the restoration of a large database. It is well known that inversion techniques do an adequate job of resolution restoration, and that the maximization of entropy in a solution formulation tends toward smooth, flat object estimates. It is a reasonable idea then, to use an inverse filter to do the bulk of the resolution restoration, and apply entropy constraints in an attempt to defeat the inversion-induced artifacts.

If, in the above derivation, after the differentiation of \( Q \), one solves for \( x_e \) by taking the Fourier Transform of both sides instead of exponentiating, a solution with two terms results. The first is a Wiener filter, and the second is an entropy gradient.

Here entropy is defined as \( S = -\sum h(k) \ln(h(k)) \), where \( h(k) = x_e(k)/\sum x_e(k) \).

Differentiate \( Q \) with respect to \( x(k) \) and set the result to zero:

\[ Q = S - v/2 \text{MSE} \]

\[ -S - \log h(k) + v [y(k) - g(k)] * p(-k) = 0 \]

Fourier Transform both sides and solve for \( X \) in the frequency domain:

\[ X(f) = W(f) - v B(f) \]

\( W(f) \) is a Wiener filter:

\[ W(f) = \frac{Y(f) P^*(f)}{|P(f)|^2 + C} \]

and \( B(f) \) is

\[ B(f) = \frac{D(f)}{|D(f)|^2 + C} \]
A small constant, C, is added to the denominator to control the result for very small values of P(f). This is usual in Wiener inversions. C is an expression of noise:signal and is treated as a constant, though it could be functional.

Choice of the initial value of C and the intermediate values of the entropy gradient weighting, v, are important for stability and convergence. As C goes to zero, the starting estimate for the entropy iterations becomes sharper, so one wants to choose C such that the Wiener filter performs the bulk of the possible resolution increase, but without making the starting estimate so oscillatory that the algorithm will not converge to a solution. Currently an attempt is made to choose $C_0$ to minimize noise:signal in the Wiener inversion. This method of defining $C_0$ was chosen because it is based only on the data (no user experience required), it allows C to be some real representation of its theoretical function, and it can be done automatically by the algorithm. This method works well for most, but not all, of the images we have restored, so additional ways to choose $C_0$ are being investigated.

All of the finesse of this restoration algorithm lies in the iterative application of the entropy gradient. The shape of the gradient is given unambiguously as the second term of the solution for $x(k)$ by the derivation. Its weighting, however, remains to be defined at each iteration. Since the algorithm is a spectral subtraction of a weighted entropy gradient from a Wiener image, the weighting must be large enough to be effective against the Wiener-induced artifacts, but not so large as to introduce additional artifacts. Several calculable attributes of each iteration, such as entropy, MSE, noise, and contrast are used at each iteration to choose a weight which will drive the object estimate closer to a convergent solution.

1. RESULTS

A one-dimensional example demonstrates this restoration concept graphically. Figure 1 is a cut through a computer-simulated object. Figure 2 is the same image degraded by 21-pixel linear motion. Figure 3 is a graphical representation of the two terms which comprise the restoration: 3a is the Wiener inversion (notice the periodic side lobes that have been introduced), and 3b is the entropy gradient term for the first iteration. Clearly, the subtraction of 3b from 3a mainly affects (reduces) the spurious side lobes. The effect of the spectral subtraction is shown in Figure 4a. The original object is obviously restored, but still somewhat degraded with noise. Figure 4b shows the entropy gradient for the next iteration. Notice that it is smaller in amplitude, in accordance with the reduced noise in the estimate it must currently act upon.

Restoration of a two-dimensional simulated point source group tests how well the algorithm preserves relative intensities, as well as showing up any characteristic deformations the algorithm might introduce. The object is the four point sources shown in Figure 5. In Figure 6, that object has been degraded by convolution with a real point spread function from the IRAS (Infrared Astronomical Satellite) telescope. The detectors in the IRAS focal plane are rectangular, have asymmetric responses in both dimensions, and are very large compared to most of the celestial objects imaged onto them. The IRAS psf, then, is mainly the detector response profile. The simulated IRAS image in Figure 6 has had a 5% of peak, Gaussian noise added to it. Figure 7 shows the restoration of the point source group after 20 iterations of the FIER (Filtered Entropy Restoration) algorithm. Both the relative and absolute peak intensities have been quite well restored. The residual noise is on the order of 2%.

Among the IRAS-scanned objects is a double galaxy, N2992, which is effectively a pair of point sources relative to IRAS sampling rates. Figure 8 shows data from the 60 μm band IRAS coverage of N2992. The system psf shape is obvious in this data. In Figure 9, the Wiener inversion is shown. The two point sources are immediately resolved by this first step of FIER. However, many artifacts have been introduced. After approximately 20 iterations of application of the entropy gradient, a restored image, or object estimate results which contains the full Wiener resolution restoration in the object region, and is nearly free of side lobes and other spurious noise. The small side lobes which do remain are on the order of 1% of peak intensity. See Figure 10.
4. SUMMARY REMARKS

An image restoration algorithm which combines the most desirable properties of both linear inversion techniques and Entropy-based estimation techniques has been developed and used to restore astronomical images. This filtered entropy algorithm uses a Wiener filter to effect resolution restoration. It then iteratively applies an entropy-based gradient in the frequency domain to defeat the spuriousness generated by the inversion. The algorithm is stable, converges quickly without averaging with previous iterations, and requires no assumptions about either the noise in the observed data, or the shape of the PSF; any specifiable PSF can be used. The resulting restorations, or object estimations, for objects rising out of a low intensity surround represent a 6-7x resolution increase over the observed data, and the process-induced residuals are approximately 1% of peak intensity of the final restored image.

5. REFERENCES


Figure 1. One dimensional slice of simulated object

Figure 2. One dimensional object degraded by 21-pixel linear motion
Figure 3. Inverse (Wiener) filter solution and entropy gradient term

Figure 4. FIER solution after one iteration & current entropy gradient
Figure 5. Two-dimensional simulated point source object

Figure 6. Simulated point source object convolved with IRAS psf

Figure 7. Simulated point source image restored by FIER
Figure 8. IRAS 60um AO image data of N2992

Figure 9. Inverse filter solution for IRAS 60um N2992

Figure 10. IRAS 60um N2992 restored by FIER