BIT RATE RECOVERY USING DIGITAL LADDER NETWORKS

by

Brian W. Kozminchuk and Lawrence E. Kuhl

Communications Electronic Warfare Section
Electronic Warfare Division

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ABSTRACT

An algorithm that estimates the bit rate of PSK and FSK signals has been developed and its performance simulated. The basis of the algorithms stems from a Digital Ladder Network to detect bit changes in intercepted signals. Random bit changes are reflected in large values of the ladder's so-called "likelihood parameter". The characteristics and trends of this parameter are observed and documented. The bit rate is obtained from an estimate of the likelihood parameter's power spectrum. Simulation results show that the algorithm produces a distinct spectral line at the bit rate for signal-to-noise ratios as low as –1 dB.

RESUME

Un algorithme estimant le débit du bit des signaux à modulation par déplacement de la phase et de ceux à modulation par déplacement de la fréquence est présenté et sa performance est simulée. L'algorithme est basé sur un Réseau à Echelle Digitale qui détecte les changements de bit des signaux interceptés. Des changements de bit aléatoires sont reflétés par des valeurs élevées de ce qu'on appelle le "paramètre de probabilité" de la soi-disante échelle. Les caractéristiques et les tendances de ce paramètre sont observées et documentées. Le débit du bit est obtenu par l'estimation du spectre de la puissance du paramètre de probabilité. Les résultats de la simulation démontrent que l'algorithme produit une ligne spectrale au débit du bit pour un rapport signal sur bruit aussi bas que –1 dB.
EXECUTIVE SUMMARY

One of the tasks of the Communications Electronic Warfare Section at the Defence Research Establishment Ottawa (DREO) is the development of state-of-the-art hardware and associated algorithms to aid in the interception and analysis of enemy communication signals. The goal here is to classify and, if possible, identify these signals in order to support the Department of National Defence (DND) and allied military missions. In part, the process of analysing these signals involves determining certain of their characteristics, such as their modulation types, their bit rates, and any other features they may possess.

Until recently, any measurable signal parameters have been determined by systems requiring a large degree of operator involvement. With over two thousand channels to monitor and analyze in the communication band of interest, and the plethora of signals expected in a Western European conflict, the operator's task will be unwieldy. Reducing operator workload and automating the process of signal analysis have led, and are continuing to lead, DND and other NATO countries to develop new systems which exploit the speed, size, weight, reduced power requirements, and programability of today's digital technology.

This report describes, and presents computer simulation results of, a signal analysis algorithm suitable for such digital processing systems. Given data that in practice would be provided by a digital receiver, the algorithm attempts to estimate the bit rate of a class of communication signals that convey information through abrupt changes in their phase.

The simulation results demonstrate that the algorithm performs well. How well depends on the input conditions. First, the results suggest it performs better when some a priori knowledge of the bit rates of interest is available, which typically would be the case. This knowledge would also allow the user to optimize the algorithm to suit the signal. Second, bit rates up to half the rate at which the data is entering the algorithm are detectable for signals ten times or more stronger than the noise. Finally, for weaker signals, the range of detectable bit rates decreases, while the amount of data processing increases to reduce the larger perturbations caused by noise.

Any computer simulation has its limitations. The next step in evaluating the feasibility of a concept is to test it with real signals and hardware. This is required in order to assess the practicality of the method and performance tradeoffs. For the application considered in this report, these concerns will be governed by the speed of a hardware processor programmed with the algorithm, and the range of signal and noise conditions applied to it.
EXECUTIVE SUMMARY (cont'd)

For this purpose, DREO has equipment capable of generating appropriate test signals for a digital receiver. This receiver transforms the signals into data suitable for processing by an experimental digital processor. Such a processor is currently being designed for a variety of applications. The latest in high speed technology, it will be programmable and have the capability of implementing the above algorithm to further characterize its performance.
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1.0 INTRODUCTION

The objective of the study reported on herein was to investigate the possible use of a recursive least squares algorithm implemented as a Digital Ladder Network (also known as ladder or lattice filters) to estimate a digital signal's bit rate.

It had been shown in [3] that the ladder filter's likelihood parameter could be used to indicate bit changes in a signal. Given this, it is therefore conceivable that, by examining the periodicity of these bit changes, the bit rate could be estimated. This is the basis of this report. It extends the results contained in [3] by:

(a) characterizing the gamma function for different ladder parameters and input signal and noise conditions,

(b) estimating the power spectrum of the gamma function to assess the effects of various ladder parameters and input conditions on the quality of the spectral line at the bit rate frequency,

(c) discussing two possible bit rate detection algorithms based on the results in this report.

2.0 THE DIGITAL LADDER NETWORK

In this section, the components and properties of the Digital Ladder Network are discussed. The algorithm used to simulate the filter is also listed.

The Digital Ladder Network is a chain of identical filter stages as shown in Figure 1. The signal samples, \( Y_T \), to be analysed are connected to both inputs of the first stage. The inputs to all subsequent filter stages are the outputs of the stages immediately preceding. The total number of filter stages in the chain is referred to as the filter order.

The input/output signals passed between filter stages are the forward and backward prediction errors \( c \) and \( r \). The forward prediction error is the difference between \( Y_T \) and \( \hat{Y}_T \), the latter of which is an estimate of \( Y_T \) based on a weighted sum of its past values (i.e. \( Y_{T-1}, Y_{T-2}, \ldots \)). Similarly, the backward prediction error is the difference between \( Y_T \) and \( Y^*_T \), the latter of which is the weighted sum of future values of \( Y_T-N \) (i.e. \( Y_{T-N+1}, Y_{T-N+2}, \ldots \)), where \( N \) refers to the filter stage.

Each filter stage has associated with it a gamma parameter that can take on values from 0.00 to 1.00. The values indicate how unpredictable signal samples entering the ladder are. Gamma is also known as the "likelihood variable" [3].

To explain the concept behind gamma, consider the following. Assume the signal is a pure sinusoid with no additive noise. There may be random phase changes in the signal. A pure sinusoid with no phase changes can be regarded as a highly predictable signal. This means that a current sample can be
predicted very accurately by forming a linear combination of past samples, this being due to the high correlation between present and past samples. (Broadband noise is not very predictable because of the low correlation between current and past samples.) For predictable signals, the variance of the prediction error becomes small, which implies the variance in the likelihood function defined in [2] is small. If the error is close to the mean of the likelihood function, gamma will be low. If a sample arrives that produces a large prediction error, this is reflected in a large value for gamma, which remains high until the current sample leaves the filter stage of interest. This large error has the effect of increasing the overall variance of the likelihood function, which implicitly decreases gamma’s sensitivity to new “events”. Thus new events are less unexpected, and the subsequent values for gamma are lower. However, past events can be weighted or “forgotten” to improve gamma’s sensitivity to later events.

The ladder network has associated with it a parameter called "lambda" which can be specified in the range 0.0 to 1.0. Lambda is called the "memory variable" or the "forgetting factor" since it controls the filter’s memory. The section of signal inside the ladder is compared only to past history that is "remembered" and the amount of memory is determined by lambda. A lambda value of 1.0 implies infinite memory whereas values less than 1 imply exponential memory. The effective memory of the filter is approximately $1/(1-\lambda)$ samples.

The algorithm implemented to simulate the ladder filter is listed on the next page in the scalar version from [1].

3.0 THE GAMMA FUNCTION

In this section, the behaviour of the gamma function is examined through simulation. There are three main processing blocks to be simulated as shown in Figure 2: the signal generator; the Digital Ladder Network; and the bit rate recovery algorithm. To examine the gamma function, only the first two blocks need to be simulated.

Simulation proceeds in the following way. The bit stream is created from a pseudo-random sequence with uniform distribution. Either one of three binary signals is produced, i.e., PSK, FSK or square wave (demodulated PSK). Examples of each scheme are given in Figure 3.

Gaussian noise samples are obtained from a normal distribution with zero mean and unit variance, the latter of which is adjusted according to the desired SNR. Impulsive noise occurs in continuous time following a Poisson arrival scheme. Magnitudes are independently chosen from a pseudo-random sequence with a normal distribution. The effect of the noise impulses is calculated at sample times assuming exponential decay.

For the simulations, the sampling rate has been normalized to 1 Hz. The samples are processed by the ladder algorithm. Various values of lambda can be specified and the gamma function can be observed at any of the filter stages.

Effects due to filter order, lambda, noise, and carrier frequency will now be discussed.
DIGITAL LADDER NETWORK ALGORITHM

Input parameters:
N = maximum order of lattice
YT = data sequence at time T
λ = exponential weighting factor

Variables:
\( R_{(p,T)}^c, R_{(p,T-1)}^e \) = sample covariance of forward/backward errors
\( \Delta_{(p,T)} \) = sample partial correlation coefficient
\( \gamma_{(p,T)}^c = 1 - \gamma_{(p,T)} \) = likelihood variable
\( e_{(p,T)}, f_{(p,T-1)} \) = forward/backward prediction errors
\( K_{(p,T)}^e, K_{(p,T)}^r \) = forward/backward reflection coefficients

These computations will be performed once for every time step (T=0,...,TMAX)

Initialize:
\( e_{(0,T)} = r_{(0,T)} = YT \)
\( R_{(0,T)}^c = R_{(0,T-1)}^e = \lambda R_{(0,T-1)}^e + YT YT \)
\( \gamma_{(-1,T)}^c = 1 \)

Do for \( p=0 \) to \( \min\{N,T\} - 1 \)
\( \Delta_{(p+1,T)} = \lambda \Delta_{(p+1,T-1)} + e_{(p,T)} f_{(p,T-1)}^r \gamma_{(p-1,T-1)}^c \)
\( \gamma_{(p,T)}^c = \gamma_{(p+1,T)}^c - f_{(p,T)}^r R_{(p,T)}^r \) \( \gamma \text{ gamma} \)
\( K_{(p+1,T)}^r = \Delta_{(p+1,T)} R_{(p,T)}^r \)
\( e_{(p+1,T)} = e_{(p,T)}^r - K_{(p+1,T)}^r f_{(p,T-1)} \)
\( R_{(p+1,T)}^c = R_{(p,T)}^c - K_{(p+1,T)}^r \Delta_{(p+1,T)} \)
\( K_{(p+1,T)}^e = \Delta_{(p+1,T)} R_{(p,T)}^e \)
\( r_{(p+1,T)} = r_{(p,T-1)} - K_{(p+1,T)}^e e_{(p,T)} \)
\( R_{(p+1,T)}^f = R_{(p,T-1)}^f - \Delta_{(p+1,T)} K_{(p+1,T)}^e \)

Note: Only the variables \( \Delta, R^c, R^r, \gamma^c, r \) need to be stored from one time step to the other. They are all set initially to zero. The quantities \( R^r, \gamma^c, r \) need to be stored twice to avoid "overwriting".

All variables in the network are initially set to zero.

Lambda is a network parameter set externally.
FIGURE 1: THE DIGITAL LADDER NETWORK
FIGURE 2: THE SIMULATED SYSTEM
FIGURE 3: THREE SIGNAL TYPES EXAMINED ARE THE PSK, FSK AND SQUARE WAVE (DEMODULATED) SIGNALS, RESPECTIVELY.
3.1 Filter Stage

Each filter stage has its own gamma parameter. To examine its behaviour as a function of filter stage, a demodulated PSK signal without any noise is analysed by the ladder filter. The gamma functions of the 3rd, 6th and 9th filter stages are observed and appear in Figure 4. The bit rate is 0.0667 Hz with a carrier frequency of 0.2 Hz. Lambda equals 0.98 in all examples.

The following observations are made:

(1) the width of event spikes varies as a function of filter stage
(2) a given spike is equal in height for all stages
(3) the height of any spike depends on how many bit changes have occurred in the recent past
(4) the gamma function settles to a finite value between event spikes and is unaffected by filter stage.

3.2 The Forgetting Factor - Lambda

To examine the behaviour of gamma as a function of lambda, a PSK signal with no additive noise is analysed using three different values of lambda, 0.99, 0.95, and 0.90. In each case, the gamma function of the 9th filter stage is observed and displayed in Figure 5.

The following observations are made:

(1) As lambda decreases, the height of the event spikes increases.
(2) Gamma’s settled value between event spikes increases as lambda decreases.
(3) Between event spikes, the gamma function follows some decay curve down to a settled value. The lower values of lambda exhibit slower decay rates.

These observations can be summarized by the fact that lambda controls the rate at which the likelihood function's variance decreases after an event, thus de-sensitizing gamma's reaction to future events. For values of lambda less than 1.0, events will be forgotten faster, and the variance will decrease. There is a tradeoff, however. Decreasing lambda has the effect of reducing the overall variance, since the squared prediction errors are weighted out exponentially. This results in an overall rise in gamma as shown in the figures.
FIGURE 4: GAMMA vs FILTER STAGE; N=3, 6, and 9; LAMBDA = 0.98. THE PULSES WIDEN AS THE FILTER ORDER IS INCREASED.
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THE GAMMA FUNCTION RISES AS LAMBDA DECREASES.
3.3 Gamma Reaction to Gaussian Noise

Noise is added to the PSK signal, and the result is input to the ladder. The gamma function from the 9th filter stage is observed and shown in Figure 6.

The following observations are made:

(1) The presence of Gaussian noise causes the height of the event spikes to decrease. The more noise, the greater the decrease.

(2) The addition of Gaussian noise causes "false" event spikes to appear in the gamma function.

These results can be explained as follows. When noise is present in greater quantities, the signal is less predictable, resulting in a larger variance in the likelihood function. New events are therefore less unexpected. There are occasions, however, when the odd noise peak is well outside the range of the likelihood function, which gives rise to false spikes in gamma.

3.4 Gamma Reaction to Impulsive Noise

Impulsive noise is assumed to follow a Poisson arrival scheme. To simulate this, time delays between arrivals are chosen randomly from an exponential distribution with the same mean as the Poisson arrival mean. Arrivals are assumed to occur in continuous time; therefore, they do not necessarily coincide exactly with the discrete signal sampling times.

The magnitudes of the impulses are selected randomly from a normal distribution with mean equal to zero and variance determined by the SNR.

The decay of the noise impulses are assumed to follow an exponential decay curve such that:

\[ N_t = N_T \exp(-a(T-t)) \]

where;

- \( N_t \) is the impulsive noise magnitude at time \( t \)
- \( N_T \) is the impulsive noise magnitude at time \( T \)
- \( a \) is the exponential decay factor (\( a > 0 \))
- \( t \) is the time of interest
- \( T \) is the time the impulse originally occurred

Using this equation, the impulsive noise contribution can be calculated at signal sample times.
FIGURE 6: THE EFFECTS OF GAUSSIAN NOISE.
THE SNR'S ARE INFINITY, 10 dB, AND 5 dB.
To examine the effect that impulsive noise has on the gamma function, a PSK signal is analysed by the ladder network first without, and then with added impulsive noise. The mean arrival time of the noise impulses is taken to be 30 seconds in this example. The gamma function of the 9th filter stage is observed and displayed in Figure 7.

The following observations are made:

1. Larger noise impulses cause larger rises in the gamma function.
2. Impulsive noise causes "false" event spikes
3. Bit changes occurring immediately after impulsive noise result in lower event spikes than if no noise had existed.

3.5 Carrier Frequency Effect

The carrier frequency effect is the phenomenon of ripples occurring in the gamma function at twice the signal's carrier frequency.

To examine this phenomenon, a PSK signal with no additive noise is analysed using three different values of lambda. The gamma functions observed at the 3rd and 9th filter stages are shown in Figure 8.

The following observations are made:

1. The ripple frequency remains constant regardless of lambda or the filter stage.
2. Lower values of lambda cause the magnitude of the ripples to increase.

The carrier frequency effect can be explained by modelling the PSK signal as having two carrier signals (same frequency but 180 degrees out of phase). Just after a bit change, the prediction errors are 180 degrees out of phase from the portion of signal in recent history. The zero crossings will be accurately predicted, but not the peaks or troughs. This results in a prediction error ripple having a frequency twice that of the signal. As the old data gets exponentially weighted out by lambda, the ripples disappear.

3.6 Comparison of PSK, FSK and Square Wave Signals

The focus of research was on PSK signals, but preliminary observations have been made concerning FSK and square wave (de-modulated) signals.

To examine the Digital Ladder Network's reaction to these three signals, the following experiment is run. The sampled signals with no noise are processed by the network. Lambda values of 0.98 and 0.90 are used and gamma is observed at the 6th and 9th filter stages. The results of all simulations are shown in Figures 9 through 11.
FIGURE 7: THE EFFECTS OF IMPULSIVE NOISE. FALSE SPIKES ARE CREATED BY THE NOISE.
FIGURE 8: THE CARRIER FREQUENCY EFFECT. THE PSK SIGNAL AT THE TOP IS FOLLOWED BY THREE GAMMA FUNCTIONS FOR N=9 AND TWO GAMMA FUNCTIONS FOR N=3 FOR THE LAMBDA VALUES SHOWN.
The following observations are made concerning the FSK signal:

1. Event spikes are "tent-shaped".
2. The carrier frequency effect occurs at both carrier frequencies.

The following observations are made concerning the square wave signal:

1. The event spikes are very square with level tops and sharp decline.
2. The height of an event spike is independent of filter stage but dependent on lambda.
3. The width of a spike is dependent on filter stage but independent of lambda.
4. Decreasing lambda decreases the rate of decay following a spike.
5. There is no evidence of the carrier frequency effect, as expected.

The following comparisons can be made between the three methods:

1. The PSK and square wave methods have very similar event spike heights and widths.
2. The FSK event spikes are not as high as the PSK and square wave spikes.
3. The PSK and FSK signals have the carrier frequency effect occurring at one and two frequencies, respectively.

4.0 BIT RATE DETECTION

This section outlines the bit rate detection algorithm and examines its performance under different circumstances. Simulations are run varying the filter stage, lambda, noise level, bit rate, and the number of iterations. Preliminary results of the bit rate detection algorithm are presented. The algorithm is described and the results of the simulations are documented.

4.1 The Algorithm

The bit rate detection algorithm uses the Welch method [4] as an iterative method to approximate gamma's power spectrum. A selection algorithm can then choose the bit rate from the spectrum. The algorithm involves the following steps:

1. Select an independent block of gamma values for this iteration.
2. Window the block of data.
3. Apply an FFT to the result.
FIGURE 9: GAMMA REACTION TO FSK SIGNALS. THE SECOND AND THIRD PLOTS FROM THE TOP REFER TO LAMBDA=0.98 AND N=6,9. THE FOURTH AND FIFTH PLOTS REFER TO LAMBDA=0.90 AND N=6,9
FIGURE 10: GAMMA REACTION TO SQUARE WAVE SIGNALS.
THE PARAMETERS ARE THE SAME AS IN FIGURE 9.
FIGURE 11: GAMMA REACTION TO PSK SIGNALS. THE PARAMETERS ARE THE SAME AS IN FIGURE 9.
(4) Square each frequency coefficient to approximate the spectrum for this iteration.

(5) Average this spectrum with all previous ones to get a better approximation to gamma's true power spectrum.

(6) Go back to step 1 for another iteration.

(7) Apply a "choosing" algorithm to determine the bit rate from the spectral estimate.

Rectangular and Hamming windows are used in the simulations. All simulations are run once with each window, all other parameters being equal. The resulting spectral estimates are always shown beside each other for comparison. Each block contains 2048 samples.

4.2 Iterative Improvement

Since the Welch method is iterative, the progression of the spectral estimates are studied.

To investigate this, a PSK signal with a bit rate of 0.0667 Hz and SNR of 0 dB is analysed. Gamma is observed at the 9th filter stage and its power spectrum is then estimated. Figure 12 shows the spectral estimates after 1, 2, 3, 5, 8, and 12 iterations.

The following observations are made:

(1) The overall effect of increasing the number of iterations is that the noise power is averaged out.

(2) A hump occurs at lower frequencies.

This last observation is due to the basic pulse shape of the gamma function which, for the PSK signal, is rectangular. This will produce a sinc type of feature to the power spectrum. Furthermore, the filter order will determine the broadness of this hump as noted in the next section.

4.3 Filter Stage

Gamma can be observed at different filter stages in the lattice network. The effect of filter order on the power spectrum is now considered.

To investigate this, a PSK signal with a bit rate of 0.0667 Hz and SNR of 0 dB is analysed. Gamma functions are observed at the 3rd, 9th and 20th filter stages. Spectral estimates after twelve iterations are shown in Figure 13.
FIGURE 12: ITERATIVE IMPROVEMENT. A PSK SIGNAL IS ANALYSED BY THE LADDER NETWORK WITH LAMBDA=0.98 AND N=9
The following observations can be made:

(1) Filter stage determines where the nulls of the hump occur. If gamma is observed at the Nth filter stage then the nulls occur at integer multiples of $1/N$ Hz.

(2) When the filter order is chosen greater than the bit duration, the bit rate does not appear in gamma's power spectrum. (In a practical implementation, this translates into a filter order*sampling rate having to be less than the bit duration if the bit rate is to appear in gamma's power spectrum.)

(3) The "carrier frequency effect" is demonstrated in the first example by the huge peak at 0.40 Hz (the carrier is 0.20 Hz).

The first point is related to the fact that gamma consists of a sum of scaled and shifted versions of rectangular pulses and the hump is the power spectrum of one of these pulses which is of the form of a sinc function.

The second point can be explained as follows. Filter order determines the width of event spikes in the gamma function. If the bit duration is less than the width of these spikes, they will merge together. This merging results in a large loss of bit rate power.

For low SNR's (< 5 dB S/N) a rough upper limit of detectable bit rates is $1/N$ Hz. The less noise there is in the signal, however, the more bit rates above $1/N$ Hz which can be detected. (In practical terms, the upper limit of detectable bit rates using this approach would be $R_s/N$, where $R_s$ is the sampling rate.)

4.4 Lambda

The effect of lambda is now considered. The same PSK signal using four different values of lambda is analysed. The spectral estimates after twelve iterations are shown in Figure 14 for lambda values of 0.98, 0.95, 0.90, and 0.80. Results indicate that lambda does not noticeably affect performance.

4.5 Signal-to-Noise Ratio

Three different SNR's were investigated. The same random noise values were used except they were scaled depending on the amount of noise in the signal. Figure 15 shows the power spectra after twelve iterations for SNR's of 1, 0 and -1 decibels.
FIGURE 13: FILTER STAGE AFFECTING POWER SPECTRUM; N=3, 9, AND 20 FROM TOP TO BOTTOM, RESPECTIVELY, AFTER 12 ITERATIONS. BIT RATE = 0.0667 Hz, CARRIER FREQUENCY = 0.20 Hz, SNR = 0 dB, LAMBDA=0.98.
FIGURE 15: NOISE LEVEL AFFECTING POWER SPECTRUM; SNR=1.0 AND -1 dB, RESPECTIVELY, AFTER 12 ITERATIONS.
4.6 Bit Rates

It was observed in section 4.3 that a rough upper limit of detectable bit rates is $1/N$ Hz. This limit holds when there is extreme noise (0 dB) in the signal. Less noise allows more bit rates above this "limit" to be detectable. Since this experiment sets the SNR to be 0 dB, however, only bit rates below $1/N$ Hz will be tested.

The results of the tests are shown in Figure 16. Lambda is set to 0.98 and gamma is observed at the 9th filter stage.

The following observations can be made:

1. Bit rate power decreases as the bit rate is closer to $1/N$ Hz.
2. Harmonics of the bit rate that are above $1/N$ Hz are damped.
3. The more harmonics of the bit rate that appear, the lower the bit rate power.

As far as the first point is concerned, it should be noted that this effect lessens when there is less noise in the signal. For a clean signal, all harmonics of the bit rate appear.

The second observation indicates that even though all bit rates below $1/N$ Hz are detectable, it would be better if the bit rate appears in the upper half of this range. The iterations appear to smooth the hump at the higher frequencies first. This means that the variance of the spectrum will be less in the upper half of the range, thus allowing the bit rate to "stand out" better. The experimental results suggest that the ideal range for the bit rate is between $1/2N$ Hz and $1/N$ Hz.

5.0 BIT RATE SELECTION ALGORITHM

Once an estimate of gamma's power spectrum is available, the bit rate must somehow be selected. None of the following selection algorithms have been tested yet and are presented here as suggestions only.

The most straight-forward technique would be to choose the most dominant frequency (i.e., the most power). This method however, would not choose accurately for low SNR's (e.g., see Figure 16) even though the bit rates are plainly visible. The error is due to the fact that the underlying spectral form is not flat.

Instead, the selection algorithm should choose the frequency that "stands out" from the other frequencies immediately beside it. The amount that a particular frequency "stands out" equals its power minus the power of the frequency immediately before it. This method of analysis is referred to as successive differencing.
Choosing the bit rate to be the frequency with the largest difference will be accurate in most examples in Figure 16. It will choose incorrectly, however, whenever a difference due to variance in the hump is larger than the bit rate's difference. The variance in the form can be reduced by performing more iterations.

6.0 CONCLUSIONS

Gamma can be used to indicate bit changes in PSK, FSK and square wave signals when SNR's are above 10 dB.

Other points worthy of note which were provided by the results of the study are:

1. Gamma detects phase shifts better than frequency shifts.

2. Bit rates within the range 0 to \( R_s / 2 \) Hz, where \( R_s \) is the sampling rate, are detectable when there is little noise in the signal (> 10 dB SNR).

3. A narrower range of bit rates are detectable when the signal has extreme noise (0 dB SNR). It was shown through simulation that a rough upper limit of detectable bit rates would be \( R_s / N \), where \( N \) is the filter order.

4. The bit rate detection algorithm can best be used to verify when a signal with a known bit rate is transmitting. The models can be used to choose the combination of lambda and filter stage that provide the best performance of the algorithm.

5. Gamma can be used to flag the transmission of burst signals if the channel is initially quiet.

More research is needed in the following areas:

1. Simulation of the algorithm's reaction to FSK signals.

2. Investigation of the effects of impulsive noise on the algorithm's ability to detect the bit rate.

3. Simulation of the bit rate selection algorithm to test its accuracy.

Finally, it is recommended that this algorithm be tested with "live" RF signals. This is possible with DREO's digital quadrature receiver (DAR-4000) and the proposed adaptive signal processing test bed.
FIGURE 16: RANGE OF DETECTABLE BIT RATES. FIVE PSK SIGNALS WITH DIFFERENT BIT RATES ARE ANALYSED. THE POWER SPECTRUM ESTIMATES ARE AFTER 12 ITERATIONS.
7.0 BIBLIOGRAPHY


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An algorithm that estimates a signal's bit rate has been developed and simulated. The basis of the algorithm stems from a Digital Ladder Network to detect bit changes in intercepted signals. Random bit changes are reflected in large values of the ladder's so-called "likelihood parameter". The characteristics and trends of this parameter are observed and documented. The bit rate is obtained from an estimate of the likelihood parameter's power spectrum. Simulation results suggest that the algorithm is accurate for signal-to-noise ratios as low as 0 dB.

Keyword, Descriptors or Identifiers:
- Recursive Least Squares
- Least Squares
- Signal Processing
- Bit Rate Detection
- Digital Ladder Networks
- Digital Ladder Networks
- Parameter Estimation