Mapping of two coplanar waveguides into parallel plate transmission lines is performed using elliptical integral transformations. This mapping provides more physical insight in estimation of capacitive crosstalk between two coplanar waveguides. Predictions of crosstalk levels agree reasonably well with the experimental measurements on the up-scaled coplanar waveguides with various separations. Scaling of coplanar waveguides does not change the characteristic impedance and the coupling capacitance per unit length. However, the total coupling capacitance varies linearly with scaling.
Crosstalk Between Two Coplanar Waveguides

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1. Introduction

Microstrip lines and coplanar waveguides (CPW) are the primary interconnect structures for monolithic microwave circuits and very high speed integrated circuits [1]. In comparing these transmission lines, the CPW offers some advantages over the microstrip line including easier connection to shunt circuit elements and maintenance of constant characteristic impedance when lateral dimensions are scaled. These advantages come about because the signal line and ground plane conductors of the CPW are located on the same side of the insulating substrate.

A price may have to be paid for this convenience, however, since the electric fields associated with the CPW are not as well confined to the immediate vicinity of the transmission line as is the case with the microstrip line. Because of the extended nature of the fields in the CPW, more care may be required in the design of dense circuitry that uses the CPW for interconnection if appreciable crosstalk between closely spaced lines is to be avoided. We present here a calculation of the crosstalk between two CPWs based on a quasi-static TEM approximation using an elliptical integral transformation. Predictions of crosstalk levels are in good agreement with experimental measurements on two CPWs with various separations.

2. Mapping of Two Coplanar Waveguides into Parallel Plate Transmission Lines

The cross-section of two CPWs is shown in Fig. 1. Here 2a is the signal conductor width, 2b is the separation between the ground conductors of a given CPW, 2c is the separation between centers of two signal conductors, and (2c - b) is the fixed width of the other half CPW ground conductor. The second CPW signal conductor is located at (2c - a) ≤ x ≤ (2c + a).

To calculate the capacitive coupling between two CPWs, an elliptical integral transformation similar to that used by Wen [2] is used to convert the planar geometry of the CPW to the geometry of a parallel plate transmission line. This mapping provides more physical insight than the original planar geometry and allows the capacitive coupling to be readily approximated.

\[ z(x) = \int_0^\infty \frac{h \, dx}{\sqrt{(a^2 - x^2)(b^2 - x^2)}}. \]  

The transformation can be expressed in terms of the general elliptical integral [3] as

\[ F(x, \alpha) = \int_0^\infty \frac{d\theta}{\sqrt{1 - \sin^2 x \sin^2 \theta}}. \]

where the angle z for our application is defined as

\[ \alpha = \arcsin(a/b). \]

The relationships between x and \(\theta\) in eqs. (1) and (2) are

\[ x = a \sin \theta, \quad x = b^2 - (b^2 - a^2) \sin^2 \theta, \quad \text{and} \quad x = b/\sin \theta \]

for \(|x| \leq a, \ a \leq |x| \leq b, \ \text{and} \ b \leq |x|\), respectively. Table 1 shows the result of conformal transformation with the sign convention \(\sqrt{(-1)^2} = +1\) and \(\sqrt{-1} = -j\). This mapping is illustrated in Fig. 2 with...
Table 1. The result of conformal transformation using eqs. (1) and (2) to convert the geometry of coplanar waveguide to the geometry of parallel plate transmission line.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-c)</th>
<th>(-b)</th>
<th>(-a)</th>
<th>(0)</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z(x) )</td>
<td>(-j F(\frac{1}{2} - x, \frac{1}{2}))</td>
<td>(-j F(\frac{1}{2} - x, \frac{1}{2}))</td>
<td>(-F(x, \frac{1}{2}))</td>
<td>(o)</td>
<td>(F(x, \frac{1}{2}))</td>
<td>(-F(x, \frac{1}{2}))</td>
</tr>
</tbody>
</table>

The points \(-C', -B, -A, O, A, B, 2C - B, 2C - A, 2C + A, 2C + B, \) and \(2C + C'\) originating from the integration limits \(x = -c', -b, -a, o, a, b, 2c - b, 2c - a, 2c + a, 2c + b,\) and \(2c + c'\), respectively. The signal conductor with \(|x| \leq \alpha\) in Fig. 1 is mapped to the upper plate in Fig. 2, while the other conductor with \(b \leq |x|\) becomes the lower plate. The second CPW signal conductor located at \((c-a) \leq |x| \leq (2c + a)\) is transformed into:

\[
\Delta z = F\left( x, \arcsin \frac{b}{2c - a} \right) - F\left( x, \arcsin \frac{b}{2c + a} \right). \tag{4}
\]

The metal strip between \(2C + A\) and \(2C - A\) in Fig. 2 represents the width \(\Delta z\) in eq. (4). The coupling capacitance \(c\) between the two CPW signal conductors is estimated from Fig. 2 as a parallel plate capacitance with width \(2b\), separation \(F(\frac{1}{2} - a, \frac{1}{2})\), and length \(l\) as

\[
\Delta c l = 1.5(e_r + 1) \varepsilon_0 l.
\]

Electric field lines originate from the signal conductor 1 and terminate at the signal conductor 2 by way of paths through the underlying substrate and through free space in the region above the CPW plane. These two capacitances in parallel are taken into account by the dielectric constant \((\varepsilon_r + 1) \varepsilon_0\) in eq. (5), which is equivalent to \(2(\varepsilon_r + 1) \varepsilon_0 l\) with \((\varepsilon_r + 1) \varepsilon_0 l/2\) being the effective dielectric constant and 2 representing two capacitances in parallel. The factor 1.5 in this expression is an approximate correction for end effects at the edges of signal conductor 2. We have assumed here that half of the field lines over the gaps adjacent to signal conductor 2 terminate on signal conductor 2 and the remaining half terminate on the ground plane. The value of the correction factor applies in particular to this case with \(a = 3\) mm and \(b = 6\) mm, but may be readily modified to suit other CPW geometries.

A simple circuit model for capacitive crosstalk between two CPWs is shown in Fig. 3. Here the coupling capacitance \(\Delta c l\) between CPW signal conductors is treated as a lumped element coupling two terminated transmission lines with characteristic impedance \(Z_0\) as shown in Fig. 4 in our crosstalk experiments. The validity of treating \(\Delta c l\) as a lumped element will be discussed later. According to this circuit model, a sinusoidal signal voltage of frequency \(f\) and amplitude \(V_1\) at the input of CPW 1 will generate a crosstalk voltage \(V_{12}\) given by

\[
j 2\pi f \Delta c l (V_{12} - V_1) Z_0 = V_{12},
\]

and the crosstalk between the two CPWs is obtained as

\[
T = 10 \log \frac{V_{12}}{V_1} = 10 \log \frac{4\pi^2 f^2 \Delta c l^2 Z_0^2}{1 + 4\pi^2 f^2 \Delta c l^2 Z_0^2}. \tag{6}
\]
where \( Z_0 \) may be obtained from Wen's quasi-static calculation [2] of the characteristic impedance of a single isolated CPW:

\[
Z_0 = \frac{30 \pi}{\sqrt{\frac{1}{\varepsilon_r+1} + \frac{\varepsilon_r}{\varepsilon_r+2}}} \quad \text{(7)}
\]

The same characteristic impedance \( Z_0 \) can also be obtained by dividing the phase velocity \( \sqrt{\mu_0 \varepsilon_0 (\varepsilon_r+1)} \) with the parallel plate capacitance length derived from Fig. 2.

3. Experiment

Comparison of this model with experiment was achieved by fabricating two up-scaled structures in copper clad epoxy-glass material. A slab of marble \( (\varepsilon_r = 6.14) \) approximately 13 cm thick simulated the dielectric substrate. Up-scaled dimensions of the coplanar waveguides in Fig. 4 were \( a = 6 \) mm, \( b = 12 \) mm and a total length \( l = 59 \) cm. The spacing \( c \) between CPWs was varied from 30 cm to 5 cm by trimming the width of the common ground-plane conductor of each of the CPW structures and rejoining the two with solder to form a single intervening ground plane. The outer ground conductor width is \( c' - b = 15 \) cm.

As shown in Fig. 4, both coplanar waveguides were terminated in 73 \( \Omega \) which was measured with the aid of a vector impedance bridge. All ground conductors for their termination were connected together. Crosstalk between the coplanar waveguides was then measured by increasing the amplitude of a sinusoidal signal applied to CPW 1 until a predetermined signal amplitude was detected in CPW 2.

A communication receiver with a fixed gain of 95 dB and a passband of 100 Hz was used to detect the crosstalk signal. The experimental results are shown in Fig. 5 as dotted curves for various CPW separations.

\[
\text{Fig. 5. Crosstalk measured between two 59 cm CPWs vs. operating frequency from 5 MHz to 100 MHz. Here } a, b, c, \alpha, \omega \text{ are experimental data points for separation } 2 c = 5 \text{ cm, 10 cm, 20 cm, and 30 cm, respectively. The corresponding theoretical predictions are shown as solid lines. If lateral dimensions are scaled down by a common factor of 100, the crosstalk will be the same for operating frequency 100 times as large (0.5 GHz to 10 GHz).}
\]

4. Discussion and Conclusion

Since the lower plate consists of four disconnected pieces of conductor, the upper plate in Fig. 2 has more metallic area as compared with the lower plate. The relative differences in area are calculated to be 3% for a separation \( 2c = 5 \) cm and 1.6% for \( 2c = 30 \) cm. Thus, the CPW ground conductor in our experiment can be considered as infinitely extended. This is to be expected for two CPWs with large separation, i.e., \( 2c > a \) and \( 2c > b \), because electric fields originated from the signal conductor of CPW 1 find ample ground conductors for their termination.

The substrate thickness is 13 cm which is much larger than the slot width \( b - a = 3 \) mm. Thus the substrate is practically infinite in thickness [4]. With infinite ground conductors and infinite substrate thickness, eq. (7) predicts the characteristic impedance \( Z_0 = 64 \Omega \) for the substrate relative dielectric constant \( \varepsilon_r = 6.14 \). This is in contrast with the measured value 73 \( \Omega \). The difference may be due to the presence of an air gap between the calcite substrate and the manufactured CPWs on the epoxy glass circuit board. Since the air gap and epoxy glass with \( \varepsilon_r = 3.65 \) are so close to both signal and ground conductors, they reduce the CPW capacitance per unit and increase the characteristic impedance (from 64 \( \Omega \) to 73 \( \Omega \)) as can be expected from eq. (7).

The presence of the air gap and the epoxy glass does affect the predicted value for characteristic impedance. However, it does not change our prediction of the crosstalk based on eqs. (5) and (6). A CPW characteristic impedance is determined mainly by the local electric field distribution adjacent to CPW signal conductor, while the crosstalk due to capacitive coupling between two CPWs with \( 2c > b - a \) is determined by the electric field distribution far away from the signal conductor. This latter electric field will pass through the thick (13 cm) substrate. Consequently, the presence of a thin air gap has little effect on crosstalk and the thick calcite substrate is considered as the only substrate in the prediction of crosstalk in eq. (6).

The coupling capacitance \( \Delta cL \) is represented as a lumped circuit element in Fig. 3 and in eq. (6) for crosstalk prediction. It is a valid representation at low frequency, when the CPW length \( (l = 59 \) cm) is much...
less than the operating signal wavelength. We have experimentally found that \( l = 59 \text{ cm} \) was about a quarter wavelength for a 69 MHz signal. Ideally, a distributed capacitance-coupling is needed for operating frequency larger than 69/2 MHz. As shown in Fig. 4, the crosstalk measurements were performed with a transmitter and a receiver at one end of the two CPWs, and two 73 Ω resistors terminated the other end. The traveling wave on CPW 1 from the transmitter will induce traveling waves in both forward and backward directions over the CPW 2 [5]. Only the backward traveling wave on CPW 2 was detected and the forward traveling wave is terminated. The backward traveling waves induced along the whole length (~ 59 cm) of CPW 2 will have different propagating phases at the receiver. Phasor summation of these traveling waves with various phases will result in less crosstalk than eq (6) at high frequency. This in turn causes the measured crosstalk vs. frequency to be flatter than expected, as can be seen from Fig. 5.

The numerical analysis of crosstalk is made for the two CPWs with a slot width \((b - a)\) equal to half the signal conductor width \(a\) (i.e., \((b - a) = a\)). If the slot width \((b - a)\) is less than \(a\), then electric fields will be more tightly confined. Consequently, the crosstalk will be less.

The static capacitive coupling model for crosstalk between two CPWs is a good model at low frequency and reasonably close separation (see Fig. 5). As the frequency increases and CPW separation becomes large, we may have to consider the effect of magnetic coupling, which is neglected in this simplified model. The magnetic field coupling has been known to be long ranged [6] and may be responsible for the departure between analytical and experimental results for \(2c = 20 \text{ cm}\) and 30 cm in Fig. 5.

The coupling capacitance in eq. (5) and the CPW characteristic impedance in eq. (7) do not depend on the absolute CPW dimensions, of \(a, b, c\), and \(c'\). They depend only on the relative dimensions of \(a, c\), and \(c'\) with respect to \(b\). If all dimensions of the CPW in Fig. 4 are scaled down by a factor of 100, then the coupling capacitance per unit length \(Ac\) and the CPW characteristic impedance \(Z_0\) in eq. (7) remain the same. However, the total coupling capacitance \(Acf\) will be 100 times smaller and, thus the operating frequency \(f\) has to increase 100 times (in eq. (6)) to maintain the same crosstalk. Consequently, Fig. 5 is still valid for 100 times down scaled version (with \(l = 5.9 \text{ mm}\)) of the CPW in Fig. 4, provided the horizontal axis is increased by a factor of 100 with the unit of 0.1 GHz rather than MHz.

In conclusion, we have calculated the capacitive crosstalk between two coplanar waveguides by mapping of two adjacent CPWs into a parallel geometry using elliptical integral transformations. Reasonably good agreement between the calculated crosstalk and the measured crosstalk in the range of \(-40 \text{ dB} \) to \(-100 \text{ dB}\) was achieved on two up-scaled CPWs of various separations. The difference between theory and experiment is attributed to long ranged magnetic coupling not included in our model and difficulty in accurately measuring the crosstalk as low as \(-100 \text{ dB}\). Under the process of scaling, both characteristic impedance \(Z_0\) and coupling capacitance per unit length \(Ac\) are unchanged. However, the total coupling capacitance \(Acf\) and the crosstalk at a given frequency are directly scaled with the CPW length.

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References