SUBSET COMPLEMENT ADDITION UPPER BOUNDS - AN IMPROVED INCLUSION/EXCLUSION METHOD

BY

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1. Introduction

In a number of statistical problems, it is desired to know the probability of a union of n events : $P\{\bigcup_{j=1}^{n} A_j\}$ where the A_j are undesirable events such as rejection of the jth null hypothesis when it is true, or the jth confidence interval not covering the true parameter. For many of these situations, it is impossible to calculate $P\{\bigcup_{j=1}^{n} A_j\}$ exactly due to the numerical inability of integrating over n events or incomplete knowledge of the entire union/intersection structure among these events. When this does occur, an attempt is made to be conservative and obtain an upper bound for $P\{\bigcup_{j=1}^{n} A_j\}$.

It may be feasible to integrate over k or less events or otherwise determine $P\{A_{j_1} \cup A_{j_2} \cup \cdots \cup A_{j_{k_o}}\}$ or $P\{A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_{k_o}}\}$ for $k_o \leq k$. This information may then be used in some fashion to derive the upper bound for $P\{\bigcup_{j=1}^{n} A_j\}$. Many approaches for doing this have previously appeared in the literature. The earliest was the inclusion-exclusion formulas of Boole(1854) and Bonferroni(1936) stating :

$$\begin{aligned} P\{\bigcup_{j=1}^{n} A_j\} \leq \sum_{j=1}^{n} P\{A_j\} &- \sum_{j_1<j_2} P\{A_{j_1} \cap A_{j_2}\} + \\ &\sum_{j_1<j_2<j_3} P\{A_{j_1} \cap A_{j_2} \cap A_{j_3}\} \\ &- + \cdots + \sum_{j_1<j_2<\cdots<j_k} P\{A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_k}\} \end{aligned} \tag{1}$$

where k is an odd positive interger. The most familiar of these, of course, is the Standard Bonferroni Inequality where $k = 1$.

$$P\{\bigcup_{j=1}^{n} A_j\} \leq \sum_{j=1}^{n} P\{A_j\}$$

One problem with this approach is that the number of terms one must calculate to implement this formula is $\sum_{j=1}^{k} \binom{n}{j}$ which becomes excessive as k becomes large. For instance, when $n = 10$, if $k = 1$ then 10 terms must be calculated, if $k = 5$ then 637 terms must be calculated.

Another problem is that the upper bound given by the inclusion-exclusion formula does not necessarily become lower as k becomes larger. For example, consider 10 events with the probability of any single event occuring equal to 0.08, the probability of any two events both occuring equal to 0.04 and the probability of any three events all occuring equal to 0.02; then the inclusion-exclusion upper bound with $k = 1$ for the $P\{\bigcup_{j=1}^{n} A_j\}$ is 0.80, while the inclusion-exclusion $k = 3$ upper bound for this probability is 1.40. Not only is 1.40 $>$ 0.80, but 1.40 $>$ 1 - an upper bound for any probability.

A different approach was developed by Kounias and Marin(1976) and modified by Tydeman and Mitchell(1981). It formulates the upper bound of $P\{\bigcup_{j=1}^{n} A_j\}$ as a linear program with 2^n nonnegative variables and $\sum_{j=0}^{k} \binom{n}{j}$ equality constraints. Using this formulation will produce the lowest possible linear upper bound for a given set of probability information. However, even for moderate values of n and k, this linear program will be too complicated to be conveniently evaluated. In fact, no attempt has been made to use this method with k larger than 2. This approach also requires knowledge of $\sum_{j=1}^{k} \binom{n}{j}$ probabilities which, as stated before, may be too many terms to calculate.

It is, therefore, of interest to find methods incorporating knowledge of k event intersection/union probabilities to produce easily calculatable upper bounds for n event union probabilities which are lower than those upper bounds currently used. One such formula has been developed for $k = 2$ by Hunter(1976). It gives:

$$P\{\bigcup_{j=1}^{n} A_j\} \leq \sum_{j=1}^{n} P\{A_j\} - \sum_{e_{ij} \in T} P\{A_i \cap A_j\}$$

where n is finite, T is any spanning tree with vertices $A_1, A_2, \cdots, A_n$; and A_i is connected to A_j in T by edge e_{ij}. Several articles, including those by Stoline(1983) and Bauer and Hackel(1985), have been written evaluating and implementing this method. Hoppe(1985) and Tomescu(1986) expanded on this procedure to develop lower bounds for probabilities of unions. Tomescu(1986) also developed related inequalities which utilize probabilties involving $k > 2$ events to give upper bounds. These bounds, however, become very complicated as k becomes large, and like the inclusion-exclusion inequalities, do not necessarily decrease with k.

In section 2, a general method is developed which expands Hunter's idea to $k > 2$ and n possibly infinite. For a fixed value of n, the number of probabilities needed to apply this new method is a decreasing function of k. It is also shown that when using this method, k can be increased resulting in at worst no improvement in the upper bound. In section 3, the new algorithm is applied to simultaneous confidence intervals and multiple hypothesis testing involving multivariate normal (and t) distributions.

2. The New Upper Bounds

Consider the following representation of the probability for a union of n events (with n possibly ∞).

$$P\{\bigcup_{j=1}^{n} A_j\} = P\{\bigcup_{j=1}^{k} A_j\} + \sum_{j=k+1}^{n} P\{A_j \cap (A_{j-1} \cup A_{j-2} \cup \cdots \cup A_1)^c\} \tag{2}$$

Define the set S_j to contain $k-1$ elements from $1, 2, \cdots, j-1$ for $j \geq k+1$. Without loss of generality, let these elements be $i_1 < i_2 < \cdots < i_{k-1}$. It is true that:

$$(A_{j-1} \cup A_{j-2} \cup \cdots \cup A_1) \supset (A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_{k-1}})$$

which implies that:

$$(A_{j-1} \cup A_{j-2} \cup \cdots \cup A_1)^c \subset (A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_{k-1}})^c$$

and thus that:

$$P\{A_j \cap (A_{j-1} \cup A_{j-2} \cup \cdots \cup A_1)^c\} \leq P\{A_j \bigcap_{\substack{i_1 < i_2 < \cdots < i_{k-1} \\ i_1, i_2, \cdots, i_{k-1} \in S_j}} (A_{i_1} \cup A_{i_2} \cup \cdots A_{i_{k-1}})^c\}$$

from this it follows:

Theorem 1. Subset Complement Addition Upper Bound (SCAUB)

$$P\{\bigcup_{j=1}^{n} A_j\} = P\{\bigcup_{j=1}^{k} A_j\} + \sum_{j=k+1}^{n} \Big[P\{A_j \cap (A_{j-1} \cup A_{j-2} \cup \cdots \cup A_1)^c\}\Big]$$

$$(3) \qquad \leq P\{\bigcup_{j=1}^{k} A_j\} + \sum_{j=k+1}^{n} \Big[P\{A_j \bigcap_{\substack{i_1<i_2<\cdots<i_{k-1} \\ i_1,i_2,\cdots,i_{k-1}\in S_j}} (A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_{k-1}})^c\}\Big]$$

where k is any positive integer smaller than n and S_j is a set with $k-1$ elements from $(1, 2, \cdots, j-1)$ for $j \geq k+1$.

This bound is called a subset complement addition upper bound (SCAUB) since it is created by adding probabilities of intersections of new events with complements of unions of subsets of events that have already been incorporated into the bound. The SCAUB can be shown to be a distribution free analog of Glaz and Johnson's(1984) product type bounds for Multivariate Totally Positive Order Two (MTP_2) distributions. See Glaz(1987) and Hoover(1988). To obtain the upper bound of Theorem 1 requires only the calculation of $n-k+1$ probabilities; each probability involving k events. For $n = 10$ and $k = 5$, this is 6 terms as compared with 637 terms needed to use the inclusion-exclusion upper bound with $n = 10$ and $k = 5$.

When k is 1, the upper bound of Theorem 1 is $P\{A_1\} + \sum_{j=2}^{n} P\{A_j \cap (\phi)^c\} = \sum_{j=1}^{n} P\{A_j\}$ which is the Standard Bonferroni Upper Bound. When k is 2, the upper bound of Theorem 1 becomes:

$$P\{A_1 \cup A_2\} + \sum_{j=3}^{n} P\{A_j \cap (A_i)^c\} \text{ where } i \in S_j \text{ and } j \geq 3$$

$$= P\{A_1\} + P\{A_2\} - P\{A_1 \cap A_2\} + \sum_{j=3}^{n} \Big[P\{A_j\} - P\{A_j \cap A_i\}\Big] \text{ where } i \in S_j \text{ and } j \geq 3$$

$$= \sum_{j=1}^{n} P\{A_j\} - \sum_{j=2}^{n} P\{A_j \cap A_i\} \text{ where } i \in S_j \text{ for } j \geq 3, \text{ and } 1 \in S_2$$

$$= \sum_{j=1}^{n} P\{A_j\} - \sum_{e_{ij}\in T} P\{A_j \cap A_i\} \text{ where } i < j \text{ and } e_{ij} \in T \text{ iff } i \in S_j$$

which is Hunter's upper bound.

An upper bound (B_k^{**}) which gives the same value as the specific SCAUB inequality with S_j containing $j-1, j-2, ..., j-k+1$ for $j > k$ was mentioned by Worsley(1985). The form of this upper bound is only given for $k = 3$, but can, with some work, be extended to all k. (Note that there are typographical errors in the above article which make the result difficult to understand.) Besides being more restrictive than the SCAUB order k inequality, B_k^{**} also requires the calculation of $2^k - 1 + (n-k)2^{k-1}$ terms to obtain a k order upper bound on the probability of the union of n events. When $n = 10$ and $k = 5$, this is 111 terms compared with only 6 terms needed for the bound of Theorem 1. Finally, B_k^{**} uses probabilities of intersections of various events which, in the normal simultaneous confidence interval problem, are numerically much more difficult to calculate than are the probabilities used by the SCAUB (unions of events and intersections of single events with complements of unions).

The value of the upper bound from Theorem 1 with $k \geq 2$ will depend on the ordering of events and choice of elements for the S_j. When k is two, it is always possible to determine an ordering and choice of elements for the S_j that gives the lowest possible Theorem 1 upper bound by using the Minimal Spanning Tree Theorem of Kruskal(1956) with probabilities of intersections as edge weights (see Hunter(1976)). Unfortunately, no method which will always do this for $k > 2$ has been discovered. If the events $A_1, A_2, \cdots, A_n$ are exchangeable (exchangeable means that $P\{A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_m}\} \equiv C_m$ regardless of choice of events), then $P\{A_j \cap (A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_m})^c\}$ will be constant regardless of events; therefore the ordering of events and choice of elements for S_j will not matter. Also, when the events have a natural ordering $1, 2, \cdots, n$ with the overlap between a fixed event and a preceeding event being a monotonically decreasing function of the number of events in the sequence seperating them, then using the natural ordering with $S_j = \{j-1, j-2, \cdots, j-k+1\}$ should give a good upper bound. This type of situation will occur in Markovian processes and time series.

One nice property of the SCAUB which inclusion-exclusion bounds do not have is that k can be made larger with the upper bound at worst becoming no lower and, in many cases, becoming much lower. In other words, as probabilities involving more events are incorporated into deriving the bound, the bound becomes better.

Theorem 2. Monotonicity of the SCAUB

Let φ_1 be a Theorem 1 upper bound derived using a particular value of k : $k < n$, ordering of events and choice of sets $S_{k+1}, S_{k+2}, \cdots, S_n$. It is possible to produce a Theorem 1 upper bound φ_2 using the value $k+1$, the same ordering of events and the sets $S^*_{k+2}, S^*_{k+3}, \cdots, S^*_n$ (the sets now contain k instead of $k-1$ elements) with $S_j \subset S^*_j$: $k+2 \leq j \leq n$ such that:

$$P \bigcup_{j=1}^{n} \{A_j\} \leq \varphi_2 \leq \varphi_1 \tag{4}$$

Proof

The first inequality in (4) follows from the SCAUB. To obtain the second inequality in (4), first note that the following identity holds for the first term in φ_2:

$$P\{\bigcup_{j=1}^{k+1} A_i\} = P\{\bigcup_{j=1}^{k}\} + P\{A_{k+1} \cap \left[\bigcup_{j=1}^{k} A_i\right]^c\}.$$

Next, define the set S^*_{k+1} to be $(1, 2, \cdots, k)$ and j^* to be the unique element such that $\{S_j \cup j^*\} = S^*_j$ for $j = k+1, k+2, \cdots, n$. Now look at the difference:

$$\begin{aligned}
\varphi_1 - \varphi_2 &= \sum_{j=k+1}^{n} P\{A_j \cap \left[\bigcup_{\text{all } i \in S_j} A_i\right]^c\} - \sum_{j=k+1}^{n} P\{A_j \cap \left[\bigcup_{\text{all } i \in S^*_j} A_i\right]^c\} \\
&= \sum_{j=k+1}^{n} P\{A_j \cap A^*_j \cap \left[\bigcup_{\text{all } i \in S_j} A_i\right]^c\} \\
&\geq O.
\end{aligned}$$

3. Application to Multivariate Normal Probabilities Within Rectangles

The SCAUB may be used to produce upper bounds for the probability that the maximum absolute value from a vector of standardized normal (or t) variables is larger than a given value c when the dependence structure (correlation matrix) of the variables is known. Such bounds are of interest in simultaneous hypothesis testing and simultaneous confidence intervals involving multivariate normal (or t) data. In this case, $(X_1, X_2, \cdots, X_n)$ is a multivariate normal vector with mean zero and some known covariance. Event A_j is that variable X_j does not fall in the interval $(-c \cdot \sigma_{x_j}, c \cdot \sigma_{x_j})$.

Examples of such bounds for $c = 1.96$ and 2.50 ; and $n = 5$, 8 and 10 are given in Table 1. We allow k to be 2, 3 and 4 since there are programs by IMSL(1982) and Schervish(1984) which can integrate multivariate normal probability over rectangles with up to four dimensions. Finally, for simplicity, σ_{x_j} is assumed to be 1 for all j, $Corr(X_j, X_i)$ is assumed to be ρ and $Corr(X_{i_1}, X_{i_2})$ is assumed to be ρ for all $i, i_1, i_2 \in S_j$. We allow ρ to be .3, .5, .7, .9 and .99. As a comparison to these upper bounds, the Standard Bonferroni Upper Bound and the Dunnett(1955) Exact Value under the assumption that $Corr(X_i, X_j) \equiv \rho$ for all i,j are given. The numerical values in Table 1 were obtained using the IMSL(1982) procedures DCADRE and MDNOR to integrate (with an accuracy of $\pm$ 0.000001) the Dunnet(1955) Exact Value Formula for equicorrelated multivariate normal distributions .

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Table 1. Upper Bounds for $P\{\bigcup_{i=1}^{n} |X_i| \geq c\}$
where $(X_1, X_2, \cdots, X_n) \sim N(O, \sum)$

ρ	Standard Bonferroni or SCAUB k=1	SCAUB k=2	SCAUB k=3	SCAUB k=4	Exact Value Equicorrelation
$c = 1.96$, $n = 5$					
0.3	0.24997	0.23042	0.21846	0.21175	0.20891
0.5	0.24997	0.21297	0.19482	0.18621	0.18285
0.7	0.24997	0.18379	0.16072	0.15150	0.14839
0.9	0.24997	0.13141	0.11023	0.10354	0.10157
0.99	0.24997	0.07631	0.06739	0.06510	0.06449
$c = 1.96$, $n = 8$					
0.3	0.39996	0.36576	0.34315	0.32472	0.29971
0.5	0.39996	0.33520	0.29890	0.27738	0.25013
0.7	0.39996	0.28414	0.23799	0.21495	0.19087
0.9	0.39996	0.19246	0.15011	0.13339	0.11904
0.99	0.39996	0.09605	0.07820	0.07248	0.06825
$c = 1.96$, $n = 10$					
0.3	0.49996	0.45598	0.42628	0.40003	0.35122
0.5	0.49996	0.41669	0.39829	0.33816	0.28693
0.7	0.49996	0.35104	0.28951	0.25725	0.21306
0.9	0.49996	0.23317	0.17669	0.15329	0.12761
0.99	0.49996	0.10920	0.08541	0.07740	0.06998

Table 1. (continued)

ρ	Standard Bonferroni or SCAUB k=1	SCAUB k=2	SCAUB k=3	SCAUB k=4	Exact Value Equicorrelation
$c = 2.50$, $n = 5$					
0.3	0.06210	0.05999	0.05857	0.05773	0.05773
0.5	0.06210	0.05674	0.05375	0.05218	0.05154
0.7	0.06210	0.05021	0.04534	0.04341	0.04224
0.9	0.06210	0.03635	0.03083	0.02897	0.02840
0.99	0.06210	0.02030	0.01771	0.01703	0.01684
$c = 2.50$, $n = 8$					
0.3	0.09936	0.09567	0.09284	0.09073	0.08712
0.5	0.09336	0.08998	0.08400	0.08008	0.07456
0.7	0.09336	0.07855	0.06882	0.06640	0.05735
0.9	0.09336	0.05430	0.04326	0.03860	0.03439
0.99	0.09936	0.02640	0.02102	0.01934	0.01803
$c = 2.50$, $n = 10$					
0.3	0.12420	0.11945	0.11568	0.11273	0.10553
0.5	0.12420	0.11214	0.10416	0.09868	0.08805
0.7	0.12420	0.09745	0.08447	0.07838	0.06557
0.9	0.12420	0.06626	0.05155	0.04502	0.03742
0.99	0.12420	0.03014	0.02323	0.02088	0.01858

The calculations for the entry in Table 1 with $n = 8$, $k = 3$, $c = 1.96$ and $\rho = 0.9$ are now shown in detail. For the above case, $P\{A_1 \cup A_2 \cup A_3\}$ taken to six digits is 0.083644, while the $P\{A_j \cap (A_{i_1} \cup A_{i_2})^c\}$ taken to six digits is 0.013293 for all j larger than 3 and $i_1, i_2 \in S_j$. So the SCAUB upper bound is:

$$P\{\bigcup_{j=1}^{8}\} \leq P\{\bigcup_{j=1}^{3}\} + \sum_{j=4}^{8} P\{A_j \cap (A_{i_1} \cup A_{i_2})^c\}\ i_1, i_2 \in S_j$$

$$= 0.083644 + 5(0.013293)$$

$$= 0.15011 (\text{rounded to five digits})$$

The bounds in Table 1 do become significantly better as k becomes larger. The improvement is quite dramatic for the higher correlations of 0.9 and 0.99. The biggest improvements occur between $k = 1$ and $k = 2$. The improvements become monotonically smaller as k increases, which is to be expected.

If variables are equicorrelated, then Dunnett's(1955) method produces the exact value for the probability of a union. This exact value under the assumption of equicorrelation is given *is given in column 6* of Table 1 and can be compared to the numbers in columns 2,3,4 and 5 of the same row which are Theorem 1 upper bounds to the exact value. Under equicorrelation, the upper bounds are close to the exact values for n and/or ρ small. It seems reasonable to assume that for a given set of variables, even without equcorrelation, the smaller the number of variables and the smaller the absolute values of the correlation coefficients, the closer the SCAUB inequalities will be to the exact values.

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20. ABSTRACT

This paper presents the "Subset Complement Addition Upper Bound" (SCAUB) procedure which produces upper bounds for probabilities of unions of n events given that probabilities of unions and/or intersections of subsets including up to k events are known. The SCAUB method is an extension of Hunter's (1976) improved Bonferroni bounds. The SCAUB inequality is much simpler to calculate than are other distribution free upper bounds proposed in the past. It is also a distribution free analog of Glaz and Johnson's (1984) product type bounds. We prove that for any fixed n events, the SCAUB inequality monotonically decreases with k. SCAUB upper bounds are applied to the multivariate normal (or t) simultaneous inference interval problem.