PROCEDURE FOR CALCULATING THE INITIAL CONDITIONS OF THE TRACK IN THE TRAXION-M1 COMPUTER PROGRAM

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MECHANICS AND STRUCTURES BRANCH

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(SEE REVERSE SIDE)
ABSTRACT

The normal procedure for the solution of any dynamic system is to derive the equations of motion and solve them, for a given set of initial conditions, by a selected numerical method when closed form solution is not possible. The initial conditions specified usually represent the initial position and the initial velocity of the system. In most problems, the initial conditions are quite simple and can be easily determined. Therefore, the major effort consists of obtaining the desired dynamic solution. Unlike the ordinary dynamic problems, the initial conditions for the dynamic analysis of a track vehicle are not readily obtainable due to the complexity of the track configuration. The determination of the initial positions/location of the track requires a considerable amount of effort. Consequently, it becomes an important part of the dynamic analysis of the track.

The purpose of this report is to present detailed discussions of the method of determination of the initial conditions for the dynamic analysis of tracked vehicles.
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INTRODUCTION

A variety of military vehicles are equipped with tracks which enable the vehicles to be maneuvered in the open field under different terrain conditions. The components of the track must be designed not only to withstand the severe loads from various maneuvers, but also to meet other important requirements which affect the overall performance of the vehicles. These include the useful life of the track, the interior and exterior noise level, and the vibration of the vehicle.

In the past, BBN Systems and Technologies Corporation (BBN) has been funded by several Army installations (TACOM, HEL, MTL) and FMC Corporation for the development of an analytical capability for predicting the dynamic loads in a closed loop, finite pitch track and its motion. The final results are contained in three separate documents.\(^1\text{-}^3\) The results of these projects have been implemented into two different computer programs: TRAXION-M113, which was developed first, is for the dynamic analysis of single-pin track; and TRAXION-M1, developed later, can analyze the dynamic behavior of a double-pin track (Figure 1). Great similarities exist between the two programs and they are now applicable only to the case when the vehicles ride on hard and flat surfaces. At the present time, BBN is under a new MTL contract\(^*\) to modify and extend the current dynamic analysis of single-pin track and the associated TRAXION-M113 computer program to include soft suspension response, noise prediction, and general ground profile contour.

![Figure 1. Types of tracks.](image)

The normal procedure for the solution of any dynamic system is to derive the equations of motion and solve them, for a given set of initial conditions, by a selected numerical method when closed form solution is not possible. The initial conditions specified usually represent the initial position and the initial velocity of the system. In most problems, the initial conditions are quite simple and can be easily determined. Therefore, the major effort consists of obtaining the desired dynamic solution. Unlike the ordinary dynamic problems, the

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initial conditions for the dynamic analysis of a track vehicle are not readily obtainable due to the complexity of the track configuration. The determination of the initial position/location of the track requires a considerable amount of effort. Consequently, it becomes an important part of the dynamic analysis of the track.

Another peculiar feature pertaining to the dynamic analysis of the track is the intermittent nature of the external forces acting on the track shoes as a result of the interaction between the shoes and wheels,* shoes and ground, and connectors and sprocket. These forces must be applied to and removed from the right shoes/connectors at the proper time. Consequently, careful analysis and intricate programming are required to meet such a criterion.

Detailed derivations of the equations of motion and the methods of calculation of the external forces acting upon the shoes/connectors are given in Ref. 1 through 3 for either single-pin track or double-pin track. However, the treatment of the determination of the initial conditions and the procedure of correct application of the external forces on the shoes/connectors are not covered in detail. Instead, only brief discussions are given and more detailed descriptions are contained in certain subroutines of the computer programs.

The purpose of this report is to present detailed discussions of the method of determination of the initial conditions for the dynamic analysis of tracked vehicles. It is hoped that it will serve as a supplement to Ref. 1 through 3 and, furthermore, will give the reader a better understanding of the dynamic analyses of tracks and the associated computer programs.

Finally, it should be noted that in order to maintain some degree of self-content, a small amount of duplication is unavoidable in the discussion of certain topics in this report. Furthermore, this report is concerned only with the double-pin track and the associated computer program TRAXION-M1. The general method for the determination of the initial conditions for the double-pin track systems, nevertheless, is equally applicable to the single-pin track systems.

**DETERMINATION OF INITIAL CONDITIONS**

The method of finite difference is selected to solve the set of equations of motion together with their appropriate initial conditions. The state of the track system at time \( t_{n+2} = t_n + 2\Delta t_0 \) is determined from the known states at two previous time points, \( t_{n+1} = t_n + \Delta t_0 \) and \( t_n \). In order to start the computational scheme, it is clear that the states of the track at two specific time points, \( t_2 = t_1 + \Delta t_0 \) and \( t_1 \) must be known. In other words, the information of the initial conditions is required as the initial input. For the present problem, the position of each shoe/connector at static equilibrium (\( t_1 \)-State) and its velocity at a successive time point (\( t_2 \)-State) are used as the initial conditions.

**Procedure for Determination of \( t_1 \)-State**

The determination of the \( t_1 \)-State of the track system is, in essence, to obtain the position of each shoe/connector at the static equilibrium (i.e., when the vehicle is at rest). Only a general procedure for accomplishing this is outlined in Ref. 3 and the details are not discussed. The procedure is comprised of the following four major steps:

*The word “wheels” is used to include the sprocket, the idler, the roadwheels, and the supporting wheels.
1. Starting with the set of a given position of the sprocket, idler, the rear roadwheel, and the front roadwheel (Figure 2), a continuous elastic band/track loop (without finite pitch) wrapped around these four wheels is constructed. The band is subjected to a tension which is consistent with the designed track tension and bushing stiffness.

![Figure 2. Seven roadwheels track configuration.](image)

2. The length of the band is then computed and compared with the total length of the track under the same designed tension. If the absolute value of the difference of the two lengths exceeds a predetermined tolerance, adjustment is made by moving the idler horizontally to a new position and a new bend length is calculated. This process is repeated until the difference of the two lengths satisfies the tolerance.

3. The track is constructed by placing the links (shoes and connectors) along the final configuration of the elastic band or track loop. The construction process is initiated by placing the first shoe under the rear roadwheel and continuing the placing of the links in the counterclockwise direction. During the process of placement, the rubber-surface wheels are compressed and their radii are reduced by an amount consistent with the track tension, the weights of the track, and vehicle.

4. At the end of the placement of all the shoes/connectors, if the track length is shorter than required to form a closed loop around the wheels, then the idler is shifted to the left; if it is longer than required, the idler is shifted to the right (Figure 3). This step is repeated until the track closes into itself within an acceptable tolerance.

![Figure 3. Adjustment of track length by shifting idler position.](image)
Determination of $t_1$-State

As mentioned earlier in this report, the details for the determination of the $t_1$-State or the position of each shoe/connector at static equilibrium have not been documented explicitly, however, they are contained in the computer program TRAXION-M1. These details will be presented in the following discussions to facilitate understanding of the theoretical foundation of the computer program as well as of the analysis itself. This goal is achieved by providing the missing details in the aforementioned four steps in the procedure for the determination of the $t_1$-State.

Construction of the Track Loop

The determination of the $t_1$-State begins with the construction of a continuous loop or band (without finite pitch) which is wrapped around the sprocket, idler, and all of the roadwheels. However, only the sprocket, the idler, the rear roadwheel, and the front roadwheel are needed for the construction of the track loop. The construction is carried out in three distinct steps: (1) the determination of the tangents between the neighboring wheels, (2) computation of the lengths of the arcs between the tangent points, and (3) determination of the length of the track loop under designed track tension (Figure 4).

Figure 4. Tangents and arcs between tangent points for construction of the track loop.

Determination of Tangents

The process of the construction of the track loop begins with the determination of the tangents between the sprocket, the idler, and the two roadwheels. The criteria employed in the computer program for selecting the correct tangents are: (1) that all of the wheels must occur on the left side of the tangent if one travels along the tangent in the counterclockwise direction, and (2) the tangent does not intersect other wheels. The criteria can be best understood with the aid of Figure 5 which provides clear illustrations of both the correct and incorrect tangents. The first set of pictures (a) shows the correct and incorrect tangents between two neighboring wheels. It can be seen that when one travels along the correct tangent in the counterclockwise direction, the wheels lie on its left side. The same also holds true when other wheels are present, as shown in the second set of pictures (b). The last picture (c) depicts the situation where a tangent becomes an incorrect one when it intersects
another wheel. Following are derivations of the necessary mathematical expressions that can be used to determine whether or not a particular tangent satisfies the criterion. For easy cross reference, the same names or symbols of the quantities employed in the computer program will be used in the following mathematical equations whenever possible. All bold-faced quantities denote two-dimensional vectors, which are represented by complex numbers for achieving simple mathematical operations and easy programming. Details of the representation of vector operations by complex-number operations can be found in Ref. 3.

\[ \text{(a)} \quad \begin{array}{c} \text{Correct} \\
\end{array} \]

\[ \text{(b)} \quad \begin{array}{c} \text{Incorrect} \\
\end{array} \]

\[ \text{(c)} \quad \begin{array}{c} \text{Incorrect} \\
\end{array} \]

T = Trailing Wheel; L = Leading Wheel; O = Other Wheel

Figure 5. Determination of correct or incorrect tangents.

All geometric quantities to be calculated will be expressed in terms of the position vectors of the wheels, radii of the wheels, and the lengths of the shoe/connector. Prior to the derivation, the following quantities are defined (see Figure 6a):

CT = position vector of the center of trailing wheel
CL = position vector of the center of leading wheel
RT = radius of trailing wheel
RL = radius of leading wheel
PT = position vector of the tangent point of trailing wheel
PL = position vector of the tangent point on leading wheel
UNIT = unit vector
TH = angle between the line connecting centers of trailing and leading wheels and line RT (Figure 6a) or line RL (Figure 6b).
Figure 6. Determination of tangent between leading and trailing wheels.
From Figure 6a, one gets the following relationships when \( RT > RL \):

\[
TH = \cos^{-1} \left( \frac{|RL - RT|}{|CL - CT|} \right) \\
UNIT = \frac{(CL - CT)}{|CL - CT|} \\
PL = CL + RL \cdot UNIT \cdot e^{iTH} \\
PT = CL + RT \cdot UNIT \cdot e^{iTH}
\]

where the dot represents simple multiplication.

When \( RL > RT \) (Figure 6b), the value of \( TH \) in the exponent in Equations 3 and 4 is replaced by \((\pi - TH)\).

The tangent between any two wheels (from the trailing wheel to the leading wheel) is obtained by the subtraction of vector \( PT \) from vector \( PL \) (i.e., \( PL - PT \)). The computations of \( PT \) and \( PL \) are carried out within the subroutine TNCIR.

The above relations can also be used to determine the tangent from a point \( Q_1 \) to a circle of radius \( R_2 \) centered at \( Q_2 \). This is accomplished by observing that the point \( Q_1 \) is equivalent to a circle of radius \( R_1 = 0 \) centered at \( Q_1 \).

The immediate concern following the determination of a particular tangent is to verify whether or not it satisfies the criterion previously mentioned. To do so, the following relationships are generated (see Figure 7):

\[
V_1 = PT - C \tag{5}
\]

\[
V_2 = \frac{(PL - PT)}{|PL - PT|} \tag{6}
\]

\[
TR = C + [V_1 \cdot V_2 \cdot \text{Real}(V_1^* \cdot V_2)] \tag{7}
\]

where \( V_2 \) is a unit vector and \( V_1^* \) is the conjugate of \( V_1 \). (In general, the conjugate of any complex number \( Q \) is donated by \( Q^* \).) The above calculations are contained in the subroutine CILID. Let

\[
DIST = C - TR \tag{8}
\]

or

\[
DIST = V_1 + V_2 \cdot \text{Real}(V_1^* \cdot V_2) \tag{9}
\]

The vector \( DIST \) is now employed to verify the right tangent by performing the following tests:

1. **Tangent Intersecting or Not Intersecting the Third Wheel**

   If \((|DIST| < RE)\) (refer to the third wheel represented by the dotted circle in Figure 7a), then the tangent intersects the third wheel and it is the wrong one. In this case, the second wheel is not the "true leading" wheel, therefore, try another "third" wheel. If
(|DIST| > RE) (refer to the third wheel represented by the solid circle in Figure 7b), then the tangent is the right one, provided it satisfies the same test for all remaining wheels. RE is the compressed radius of the wheel as defined in the subroutine RDPRS.

Figure 7: Test of correct tangent.
(2) Wheels on the Correct Side of Tangent

As mentioned earlier, the wheels must occur on the left side of the correct tangent when one travels along the tangent in the counterclockwise direction. The following relationships can be used as a check (see Figure 7):

Set \[ A = PL - PT \] \hspace{1cm} (10)

\[ STH = \text{Imag}(A^* \cdot \text{DIST}) \] \hspace{1cm} (11)

If \( STH > 0 \), the third wheel is on the left side of the tangent and if \( STH < 0 \), the third wheel is on the right side of tangent.

It should be noted here that the same procedure and computations are repeated until the correct tangents to all of the four wheels, as shown in Figure 4, are found. The computations to verify the right tangents are done inside the subroutine TRCSTR.

Computation of Lengths Between Tangent Points

As shown in Figures 4 and 8, the entire track loop is made up by the four tangents and the four arcs between the tangent points. In Figure 8, \( \text{POCN}(j) \) (\( j = 1-8 \)) are the position vectors of the tangent points on the four wheels. Let \( \text{ARC}(i) \) and \( \text{TH}(i) \) (\( i = 1-4 \)) denote, respectively, the length of the \( i \)th arc and the angle between two tangent points; the value of \( \text{ARC}(i) \) is given by the following:

\[ \text{ARC}(i) = \text{RE}(i) \cdot \text{TH}(i). \]
TH(i) can be computed from the orientation angle PHCN(i) of the two neighboring tangents. Specifically, PHCN(i), i = 1, 2, 3, 4 are defined as the angles between the bottom, right, top, and left tangents, respectively, and the positive X-axis. The relationships between TH(i) and PHCN(i) are given below (see Figure 8):

\[
\begin{align*}
\text{TH}(1) &= \text{PHCN}(2) - \text{PHCN}(1) \\
\text{TH}(2) &= \text{PHCN}(3) - \text{PHCN}(2) \\
\text{TH}(3) &= \text{PHCN}(4) - \text{PHCN}(3) \\
\text{TH}(4) &= 2\pi + \text{PHCN}(1) - \text{PHCN}(4).
\end{align*}
\] (12)

Computation of Initial Length of the Track Loop

Let DL(i) denote the length of the ith tangent and its value can be computed by the following:

\[
\text{DL}(i) = |\text{POCN}(2i) - \text{POCN}(2i-1)|. 
\] (13)

Here, the same symbols, POCN(i) (i=1-8), as those employed in TRAXION-MI, are used to denote the position vectors of the tangent points on the wheels. The length of the track loop (STRL) is then the summation of all the tangents and arcs:

\[
\text{STRL} = \sum_{i=1}^{4} [\text{DL}(i) + \text{ARC}(i)]. 
\] (14)

The computation of the length of the track loop using the above equations is carried out in the subroutine STRNGL. However, prior to this, the values of the orientations of the tangents PHCN(i) are determined in the subroutine TRCSTR after the tangents are obtained.

Determination of Final Track Loop

A certain amount of tension (TNS) is applied to the track when it is installed on the vehicle. Therefore, the length of the track shoe (L) is increased by a small amount which is a function of the bushing stiffness (KB). If the total number of shoes is NS, then the final length, DE, of the track under tension is

\[
\text{DE} = NS \cdot (\text{LE} + \text{CLE}) 
\] (15)

where

\[
\begin{align*}
\text{LE} &= L + 2\text{TNS}/\text{KB}: \text{(shoe length under tension)} \\
\text{CLE} &= \text{LC}: \text{(connector length)}.
\end{align*}
\]

DE and LE are computed in the subroutine RDPRS. The particular track loop which is made up by the four tangents and the four arcs between the tangent points is the right one, if its length (STRL) is equal to or within an acceptable tolerance of the value of DE. In other words, the following must hold:
\[ \left| \frac{DE-STRL}{DE} \right| = 0, \text{ or } \left| \frac{DE-STRL}{DE} \right| \leq \varepsilon \]  

(16)

where \( \varepsilon \) is a small tolerance.

If \( \left| \frac{(DE-STRL)}{DE} \right| > \varepsilon \), small adjustment to the position of the idler is adopted. When \( DE > STRL \), the idler should be moved to the right. In the case when \( DE < STRL \), move the idler to the left. After moving the idler to a new position, the entire process of the track loop construction is repeated until the condition \( \left| \frac{(DE-STRL)}{DE} \right| < \varepsilon \), is reached. At this time, the tension in the track loop is equal to the designed value. The computations for determining the final track loop are carried out in the subroutine TNSSTR.

**Construction of a Double-Pin Finite Pitch Track**

The configuration of the final track loop provides a good starting point for the process of construction of a finite pitch track of double pins. The construction will proceed in the counterclockwise direction and will be accomplished in the following steps:

**Construction of Bottom Strand of Track**

The bottom strand of the track or the segment of the track between the rear and the front roadwheels will be built first. As the first step, the tangent between the rear and the front roadwheels and its length are found. It should be noted that, although this tangent and its length were found previously, that information was not documented.

The straight portion of the bottom strand is to be built along the same direction as the “bottom.” For convenience, the first shoe is arbitrarily placed under the rear roadwheel. Furthermore, the center of the shoe is placed directly under the center of the rear roadwheel. This shoe-wheel contact point is also the tangent point on the rear roadwheel of the tangent between the rear and the front roadwheels. The number of pairs of shoes and connectors which can be placed between the two tangent points is determined by dividing the length of the tangent by the sum of the shoe length under tension (LE) and the connector length (CLE). At the same time, the last free link (either being a shoe or a connector) behind the tangent or contact point on the front roadwheel is also determined. The above calculations are carried out in the subroutine ENGWH2.

The positions of all the links (shoes and connectors) in the bottom strand behind the tangent point on the front roadwheel are determined in the subroutine STRAIT with the exception of the position of the first shoe which is defined in the subroutine FITTRK. The word position implies the location of the center of the link and its orientation. Since the links are assumed to be rigid, the information for the position of each link at a given time is sufficient to define the configuration of the track.

**Wrapping Links Around Front Roadwheel**

At the end of the construction of the bottom strand, the position of the last free link (either a shoe or a connector) is known. The head of the last free link is just behind the tangent point on the front roadwheel. The position of the next link to be connected to the last free link is determined in the subroutine WRPWH2. This then commences wrapping the
track around the front roadwheel. The term wrapping is used here to mean the positioning of the shoes contacting the front roadwheel as we move toward the idler.

During the process of wrapping, the angle ANGT of the tangent from the head (PIN) of each new link to the (trailing) front roadwheel and the angle ANGL from the same point to the (leading) third wheel or the idler are computed (Figure 9a). These angles are obtained after the two corresponding point-to-circle tangent line segments PCL-PIN and PCT-PIN (Figure 9a) are determined through the method discussed earlier in this section (Equations 1-4). Wrapping is completed (or the “link has dawned” as it has been referred to in the subroutine DAWN) when ANGT is greater than ANGL (Figure 9b). Otherwise, the process of wrapping is to be continued. The subroutine POCIT is utilized to determine the shoe-wheel contact point and subroutine CIRIN to locate the positions of the connectors between the neighboring shoes during the wrapping process. The angles ANGT and ANGL are computed in the subroutine DAWN.

Construction of Right Strand of Track

Following the completion of wrapping the links around the front roadwheel, the construction of the right strand of the track is initiated. A procedure similar to that employed in the construction of the bottom strand of the track is used here. It is carried out in the subroutine ENGWH3. First of all, the tangent from the head of the last link, which is wrapped around the front roadwheel, to the third wheel (i.e., the idler in the present case) is determined. The number of pairs of shoes and connectors which can be placed along the tangent is determined in the second step. The direction of the right strand is determined in a slightly different manner. It depends on whether the last link is a shoe or a connector and what its points of contact are with the third wheel as determined by various tests in the subroutine ENGWH3. The positions of all the links in the right strand are determined in the subroutine STRAIT. It should be pointed out that none of the links on the right strand are in contact with any wheel.

Wrapping Links Around Third Wheel

After the completion of constructing the right strand, the next step is to wrap the links around the third wheel (idler). The procedure of wrapping is identical to the one used to wrap links around the front roadwheel. The entire wrapping process is carried out in the subroutine WRPWH3.

Construction of Top Strand of Track

The construction of the top strand of the track and the determination of the positions of the free links in the top strand follow the same procedure employed in the construction of the right strand of the track. The details are contained in the subroutine ENGSPR.

Wrapping Links Around Sprocket

The sprocket is directly connected to the engine of the vehicle. Its function is to drive the track, and therefore, the vehicle by engaging its teeth with the connectors (Figure 10a). The number of connectors engaged by the sprocket teeth at a given instance is about four to five depending on the vehicle type. It should be pointed out that, unlike all other wheels which interact only with the shoes, the sprocket interacts only with the connectors.
Accordingly, the track wrapping which is controlled by the soft shoe-wheel contact for all other wheels is governed by the hard (metal-to-metal) connector-tooth contact for the sprocket.
Since only the connectors contact the sprocket, the wrapping of the links around the sprocket is carried out in a different manner. First of all, the shoe-spread angle (AS) and the connector-spread angle (AC) as shown in Figure 10b are computed. Depending on the nature of the first link engaged with the sprocket, the position of the next link (shoe or connector) is determined by rotating the vector [LT-CC(4)] by an appropriate angle (AS or AC) in the counterclockwise direction. The position of the head of the new link is given by LH and the orientation of the new link is given by the angle AUNIT as shown in Figure 10b.

Figure 10. Wrapping links around sprocket.
To test when wrapping should cease, the angle of the tangent from the head of the new link to the first wheel is computed, i.e., AP4 (Figure 10b). If AP4 is greater than AUNIT, continue the wrapping process, otherwise, terminate it at the last link.

The details of wrapping the links around the sprocket are given in the subroutine WRPSPR. The specific connectors which are in contact with the sprocket at a given instant of time are also identified. This information is needed in the computation of the forces and moments acting on the sprocket and the track at a later stage of the analysis.

Construction of Left Strand of Track

The left strand of the track consists of those free links which lie between the sprocket and the rear roadwheel. The method employed to determine their positions at any instance is identical to that used to obtain all the free links on the right strand. The details are contained in the subroutine ENGWH1.

Wrapping Links Around Rear Roadwheels

After the construction of the left strand of the track, the last step is to determine the additional links needed for the completion of the construction of the entire track. The number of the additional links needed to wrap around the rear roadwheel depends on the nature of the last link on the left strand.

When the last link on the left strand is a connector and its link number is equal to the total number of links, it indicates that all the links have been used in the construction of the track. At this point, and in an ideal case, the head of the connector should coincide with the tail of the first link (which is a shoe) or be within an acceptable distance from it. However, in most cases, the head of the connector either does not reach the tail of the first link or passes it. In the first case, the track is too short while in the second case, the track is too long for the given four-wheel arrangement (see Figure 4). These two situations are corrected through an iterative adjustment of the idler's horizontal position (i.e., by moving it to the right or left, as needed, until an acceptable tolerance is reached). However, it should be pointed out that for every adjustment, the entire process of the track construction is repeated.

When the last link on the left strand is a shoe, the next link should be a connector and its position must then be determined in the following manner. If the link number of this connector is equal to the total number of links, the situation becomes identical to what is discussed in the preceding paragraph. On the other hand, if the link number of the connector is less than the total number of links, two additional links (or a pair of shoe and connector) are required to wrap around the rear roadwheel. The process of wrapping is continued until the link number of the last connector is equal to the total number of links.

Determination of Correct Track Configuration

After completion of wrapping links around the rear roadwheel, a test is performed to determine whether any adjustment of the position of the idler, as described earlier, is necessary in order to obtain the correct track configuration.
The particular test used to determine whether or not the correct track configuration has been constructed makes use of two particular angles. The first angle, \((\text{PHO})\), which was computed earlier in the subroutine \textsc{Fittrk}, is the angle of the vector \((\text{PTO} - \text{CL})\) (Figure 11). \text{PTO} is the position vector of the tail of the first link. In the present case, it is the shoe which has been placed under the first wheel at the beginning of the construction of the bottom strand of the track. \text{CL} is the position vector of the center of the rear roadwheel.

The second angle \((\text{PH1})\) is the angle of the vector \((\text{LH} - \text{CL})\) as shown in Figure 11. Here, \text{LH} is the position vector of the head of the lastly determined connector in the subroutine \textsc{Wrpwh1}. If the difference between the two angles is sufficiently small, i.e., \(|\text{DPH}| < 0.001\) and \(\text{DPH} = \text{PH1} - \text{PH0}\), the wrapping process is terminated and the construction of the entire track is completed. In cases when \(|\text{DPH}| \geq 0.001\), continuous adjustments of the positions of the idler are made and the entire process of the track construction is repeated until the tolerance is met. The implementation of this section is contained in the subroutine \textsc{Wrpwh1}.

![Figure 11. Wrapping links around rear roadwheel.](image)

**Determination of \(t_2\)-State**

In the preceding section, the determination of the \(t_1\)-State of the track is discussed. As mentioned before, the \(t_1\)-State represents the track configuration at static equilibrium. The \(t_2\)-State of the track is referred to the track configuration at a specific time \(t_2\), where \(t_2 = t_1 + \Delta t\) and \(\Delta t\) is a small time increment. The determination of the \(t_2\)-State is described in this section.
The position of each shoe/connector at the $t_2$-State is estimated by using the information at the $t_1$-State and the vehicle speed. The estimate is based upon two assumptions about the movements of the links. The first assumption states that at the particular instant of time $t_1$, the links in the straight segments move along the same direction as that of the straight segment to which they belong. The second assumption stipulates that the links which happen to be on the curves around the wheels move rigidly with the wheels.

To estimate the new positions of the links at time $t_2$, let the wheels rotate in the clockwise direction with an angular speed $\omega$. At time $t_2$, the position of a particular link on the straight position of the track is given by (see Figure 12):

$$R_2 = R_1 - v\Delta t \ e^{i\phi_1}$$

$$\phi_2 = \phi_1 \quad (17)$$

$$\phi_2 = iVI \quad (18)$$

$$\text{Figure 12. Position change of link in straight portion of track.}$$

The position of a particular link on the curve around a wheel is given by (see Figure 13):

$$R_2 = C + (R_1 - C) \ e^{i\phi}$$

$$\phi_2 = \phi_1 + \Delta \phi \quad (19)$$

where

$$\Delta \phi = -\omega \Delta t \quad (20)$$

In the above relationships, $R_1$ and $R_2$ are the position vectors of center of mass of the link, and $\phi_1$ and $\phi_2$ are the orientations of the link at times $t_1$ and $t_2$, respectively. $C$ is the position vector of center of the wheel and $V$ is the vehicle speed.
DETERMINATION OF DYNAMIC RESPONSE OF TRACK

The information for the track configurations at two specific instants of time $t_1$ and $t_2$ (or $t_1$-State and $t_2$-State) discussed in the preceding section, provides the initial conditions/inputs to the computational scheme of finite difference selected for determining the time evolution of the track system. The $t_3$-State is derived from the $t_2$-State and $t_1$-State. The $t_4$-State is derived from the $t_3$-State and $t_2$-State. Finally, every $t_n$-State is calculated from the $t_{n-1}$-State and $t_{n-2}$-State. It should be mentioned that the computational process includes the proper external forces acting on the track system. Detailed discussions are contained in Ref. 3.
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The normal procedure for the solution of any dynamic system is to derive the equations of motion and solve them, for a given set of initial conditions, by a selected numerical method when closed form solution is not possible. The initial conditions specified usually represent the initial position and the initial velocity of the system. In most problems, the initial conditions are quite simple and can be easily determined. Therefore, the major effort consists of obtaining the desired dynamic solution. Unlike the ordinary dynamic problems, the initial conditions for the dynamic analysis of a track vehicle are not readily obtainable due to the complexity of the track configuration. The determination of the initial positions/locations of the track requires a considerable amount of effort. Consequently, it becomes an important part of the dynamic analysis of the track. The purpose of this report is to present detailed discussions of the method of determination of the initial conditions for the dynamic analysis of tracked vehicles.