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TECHNICAL PAPER
CAA-TP-89-2

AD-A208 604

ANALYTIC REPRESENTATION OF THE TARGET ACQUISITION PROCESS IN NUFAM III

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	UNCLASSIFIED/UNLIMITED

4. PERFORMING ORGANIZATION REPORT NUMBER(S) CAA-TP-89-2	5. MONITORING ORGANIZATION REPORT NUMBER (S)

6a. NAME OF PERFORMING ORGANIZATION US Army Concepts Analysis Agency	6b. OFFICE SYMBOL (if applicable) CSCA-RQN	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) 8120 Woodmont Avenue Bethesda, MD 20814-2797		7b. ADDRESS (City, State, and ZIP Code)

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (if applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS			
	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO

11. TITLE (Include Security Classification)
Analytic Representation of the Target Acquisition Process in NUFAM III

12. PERSONAL AUTHOR(S)
MAJ Mark A. Youngren, D.Sc.

13a. TYPE OF REPORT Final	13b. TIME COVERED FROM Nov 88 TO Mar 89	14. DATE OF REPORT (Year, Month, Day) 89/03	15. PAGE COUNT 54

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Target Acquisition, Nuclear Weapons, Theater Combat; Simulation Models, Renewal Process.
17	08		
15	06	04	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
The paper describes a methodology for determining the probability that a combat unit is acquired, available for fire by a nuclear weapon, is fired on, and is hit by a nuclear weapon. This methodology has been incorporated as part of the Nuclear Fire Planning and Assessment Model III (NUFAM III) used at the Concepts Analysis Agency (CAA). The target acquisition data required are the Probability of Operational Target Acquisition (POTA) values generated by the CAA Target Acquisition Study III (TAS III). The basic approach used alternating renewal processes to represent the acquisition status and movement state of each unit in NUFAM III. Keywords Tactical ... ; Model nuclear targets.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL MAJ Mark A. Youngren	22b. TELEPHONE (Include Area Code) (202) 295-1625	22c. OFFICE SYMBOL CSCA-RQN

DD Form 1473, JUN 86   Previous editions are obsolete.   SECURITY CLASSIFICATION OF THIS PAGE

ANALYTIC REPRESENTATION OF THE TARGET ACQUISITION PROCESS IN NUFAM III

MARCH 1989

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By		
Distribution /		
Availability Codes		
Dist	Avail and / or Special	
A-1		

INSPECTED

This document was prepared as part of an internal CAA project.

	ANALYTIC REPRESENTATION OF THE TARGET ACQUISITION PROCESS IN NUFAM III	STUDY SUMMARY CAA-TP-89-2

THE REASON FOR PERFORMING THIS WORK was to improve the accuracy and efficiency of the target acquisition representation in the Nuclear Fire Planning and Assessment Model, Version III (NUFAM III), and develop a methodology for estimating the sensitivity of the model output to inputs related to acquisition and movement.

THE PRINCIPAL FINDINGS of the work reported in this paper follow.

(1) The target acquisition and unit movement processes may be represented using alternating renewal processes.

(2) The current data for target acquisition used within NUFAM III, the probability of target acquisition (POTA) values for a 2-hour glimpse period derived from the Target Acquisition Study, Version III (TAS III), require the number of periods until acquisition to be distributed as geometric.

(3) The process of tracking nuclear targets after acquisition may be represented based on POTA values if certain assumptions are accepted.

THE MAIN ASSUMPTIONS follow.

(1) The POTA values derived from TAS III may be used to represent the probability of acquisition during successive, independent 2-hour glimpse periods.

(2) The uncertainty about the probability of continuing to detect during a 2-hour glimpse period, given that the target has been previously detected (i.e., the probability of tracking) can be described using a uniform (POTA, 1) distribution.

(3) Units are acquired and move independently of each other.

THE PRINCIPAL LIMITATION of this paper is the reliance on the POTA values, which do not provide an accurate representation of the time-dependent probability that a unit is on the acquisition list at any given period of time. Nevertheless, the POTAs represent the best available data until new acquisition and tracking distributions are developed from the Target Acquisition Study IV (TAS IV) study.

THE SCOPE OF THE TECHNICAL PAPER is limited to representing the target acquisition process in the March 1989 version of NUFAM III using the current input data values and formats.

THE TECHNICAL OBJECTIVES were to compute the probabilities that a unit in NUFAM III:
(1) is acquired and remains on the acquisition list,
(2) is available for fire,
(3) has moved at the time of detonation,
(4) can be fired at by a nuclear weapon, and
(5) is hit by a nuclear weapon.

THE BASIC APPROACH used alternating renewal processes to represent the acquisition status and movement state of each unit within NUFAM III.

THE TECHNICAL PAPER SPONSOR was the US Army Concepts Analysis Agency.

THE TECHNICAL PAPER was written by MAJ Mark A. Youngren, Requirements Directorate, US Army Concepts Analysis Agency.

COMMENTS AND QUESTIONS may be addressed to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-RQN, 8120 Woodmont Avenue, Bethesda, MD 20814-2797.

Tear-out copies of this synopsis are available at back cover.

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CHAPTER 1

BACKGROUND

1-1. INTRODUCTION. The US Army Concepts Analysis Agency's (CAA) Nuclear Fire Planning and Assessment Model III (NUFAM III) explicitly simulates the acquisition of mobile nuclear targets in order to generate an acquired unit set for targeting at the time of fire planning. The version as of March 1989 relies on input values (defined below) such as the POTA, retention time, travel time, and stay time. This paper addresses an analytic representation of the target acquisition model *using the data and assumptions contained in the current version of NUFAM III.* As a result, it does not incorporate more advanced methods for modeling the stay, move, acquisition, and retention times. It is intended to be used in support of NUFAM III -- based analyses (such as the NUX-97 study) and in support of the nuclear effects module (NEMESIS) of the Integrated Warfare Force Evaluation Model (IWFORCEM), until improved data and methodology are available from the TAS IV (Target Acquisition Study IV) and TAME (Target Acquisition Methodology Enhancement) Study (Bauman [1989]). A basic knowledge of the terms associated with the NUFAM III model and input data base is assumed. A complete explanation may be found in the NUFAM III User's Manual (Schuetze and Albrecht [1986]).

1-2. THE TARGET ACQUISITION PROCESS

a. The process of *target acquisition* is defined as the process that searches for, detects, and identifies potential target units for placement on an acquisition list. A *unit* is a military organization composed of personnel and equipment (*target elements*) that is identified in NUFAM III as a potential target. All units are categorized into *type units* according to their characteristics (types of equipment, movement rates, etc.). There are usually many units of each type to be found within the NUFAM III data base. A unit is *acquired* when it is placed on the list; acquisition is lost when it is removed from the list. A unit may be *dropped* (removed) from the list either due to a negative sensor report (i.e., we no longer detect its presence when looking) or it may be dropped after some period of time when the acquisition information cannot be updated. The *time to acquisition*, T_A, is the time it takes to acquire a unit once any previous acquisition has been dropped. The *retention time*, T_R, is the length of time the unit is retained on the acquisition list.

b. *Targets* are units that are planned for engagement by tactical nuclear weapons. *Relocatable targets* are units which have the capability to move during the scenario of interest (although they may or may not retain mission capability during movement). As a consequence of this movement capability, units do not remain acquired indefinitely (unless they can be tracked indefinitely once acquired); at some time, they move and the acquisition is lost. Even if a monitoring or tracking capability exists, there is a probability that such tracking will be lost over time. NUFAM III has recently been modified to include fixed targets as well as relocatable targets, but the fixed targets have predefined locations and are considered to be "acquired" at the beginning of the simulation and retained indefinitely -- thus the various measures discussed in this paper do not apply.

c. Units that are engaged using conventional weapons are generally fired upon soon after acquisition. Nuclear targets, however, differ from conventional as they are planned for specific purposes dictated by the overall theater situation. As a result, nuclear targets are not normally engaged as they are acquired; rather, nuclear fires are directed at targets that are acquired and perceived to still be in place at the time that the weapons are approved for fire.

d. Figure 1-1 illustrates a representative acquisition sequence for a relocatable unit. Any given unit will be placed in only one of two states: acquired or not acquired. Once the unit has been dropped from the list, it is immediately subject to being reacquired. A target acquisition process is therefore a temporal series of such acquisition states. Since nuclear weapons use will generally occur only after the conventional battle has been underway for some time, we are interested in the target acquisition probabilities at some point in time well after target acquisition has begun. For relocatable targets, the target acquisition process can be approximated as an alternating renewal process of indefinite length. Both the time that the target is dropped from the list and the time that the target is acquired are renewal points of this alternating renewal process.

Acquired T_R T_R

Not Acquired T_A T_A T_A

Time   ──────────────────────────────────────▶
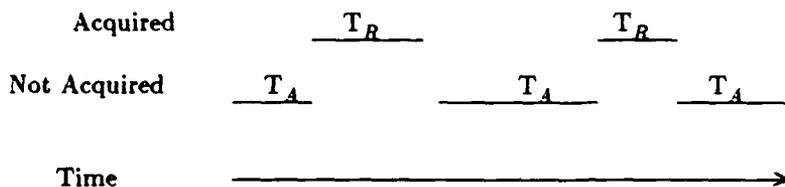
Figure 1-1. Possible Target Acquisition Sequence

1-3. THE FIRE PLANNING AND EXECUTION SEQUENCE IN NUFAM III. NUFAM III represents tactical nuclear warfare as a series of engagements. For each engagement, each side has a fire planning cycle of fixed duration during which nuclear fires are planned against available units on the acquisition list. The fire planning cycle terminates at the beginning of some specified time interval within which weapons may be fired, which is referred to as the *fire execution cycle*. The time that the fire planning cycle ends is denoted as t_p; the NUFAM III model assumes that any unit on the acquisition list at time t_p is available for fire planning. A unit that may be planned as a target will have had a recent acquisition time T_{acq} prior to t_p. The nuclear round will detonate at some time T_d during the fire execution period (Figure 1-2).

Fire Planning Cycle │ Fire Execution Cycle   ──▶ Time

T_{acq} t_p T_d
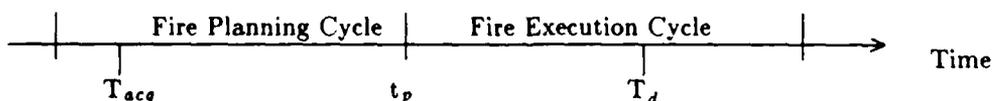
Figure 1-2. Acquisition, Planning and Detonation Events

1-4. SOME CONCEPTS BEHIND ALTERNATING RENEWAL PROCESSES

a. Representing the target acquisition process as an alternating renewal process allows us to use the extensive theory developed around such processes. If we enter such a process for a unit at a point T in time chosen independently of the target acquisition sequence of that unit, the point T is more likely to lie in a long interval than a short interval. This fact needs to be kept in mind when determining the distribution of the time remaining from our entry point T until the next transition (acquisition or drop event), or the time since the last transition to the time T. Fortunately, these distributions have been worked out (cf. Ross [1983]). Following the terminology common in reliability, I call the time from the last transition to our random entry point the *age* and the time from our random entry point to the next transition the *residual life*. An example using the acquisition sequence showing the age and residual life of the distribution of the retention time T_R is shown in figure 1-3.
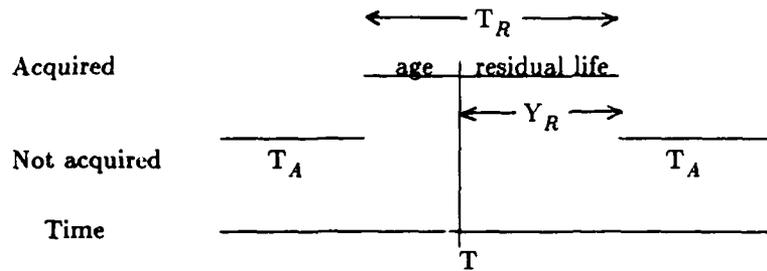
Figure 1-3. Age and Residual Life of the Retention Time Distribution

b. Using the retention time T_R as an example, the random variable for the time remaining from our entry point T until the next transition (drop event), which is the residual life of the retention time, is denoted as Y_R. The general form for a distribution of a residual life is:

$$P(\ Y_R > y\ ) = 1 - \frac{1}{E[T_R]} \int_0^y P[\ T_R > u\ ]\ du, \tag{1}$$

where $E[T_R]$ denotes the mathematical expectation of the random variable T_R (the expectation may be thought of as the "average" of T_R). Since T_R is fixed a. .· constant value R in NUFAM III, $E[T_R] = R$ and

$$\overline{F}_{T_R}(u) = P[\ T_R > u\ ] = 1, \qquad 0 \le u < R, \quad 0 \text{ for } u \ge R. \text{ Thus}$$

$$P(\ Y_R > y\ ) = 1 - \frac{1}{R} \int_0^y 1\ du = 1 - \frac{y}{R}, \qquad 0 \le y < R; \qquad 0 \text{ for } y \ge R. \tag{2}$$

1-5. CURRENT TARGET ACQUISITION MODELING IN NUFAM III

a. The target acquisition process presently modeled in NUFAM III relies on a *probability of operational target acquisition* (POTA), provided by the Target Detection Routine (TADER) model as part of the TAS III study (Penn and Bauman [1987]), as a measure from which the time to acquisition, T_A, can be computed. Given many characteristics of the sensors, target units. and battlefield scenario, the POTA represents "the probability of detecting, recognizing, and locating various types of potential targets at prescribed distances from the forward line of own troops (FLOT) during a random but limited period of time in a day of intense combat" (Bauman and Penn [1987]), where the period of time is currently set at 2 hours. The POTA varies by type unit and zone, where the zone defines a FLOT-to-target range band; Zone 1 is 0-3 km. Zone 2 is 3-12 km, Zone 3 is 12-25 km. Zone 4 is 25-100 km, and Zone 5 is 100-300 km.

b. In order to apply the POTA to a time period larger than 2 hours. NUFAM III assumes that the probability of detecting a unit during any 2-hour period is independent of the probability of detecting that unit during any different 2-hour period. Thus the probability of first detecting a unit during the k+1st 2-hour time period is equal to POTA · (1 − POTA)^k, $k = 0, 1, \ldots$, which is a simple *geometric* distribution. NUFAM III draws against the POTA every 2 hours. which is equivalent to drawing from this geometric distribution. Once NUFAM III has determined the 2-hour period within which an acquisition is to occur, it schedules the actual time of acquisition randomly within the 2-hour interval. Thus, if a random variable X is defined distributed as geometric (POTA), and a random variable U is defined distributed as uniform (0,2), then the random variable for the time to acquisition. T_A, is identically distributed as (thus can be represented as) 2X + U.

c. The retention time, T_R, varies with the capabilities of the sensor systems to revisit or track previously acquired units. Various sensor system scenarios are discussed in Chapter 2. The retention time is currently modeled in NUFAM III as a fixed input value R which represents the expected value of T_R; at some future time, it may be desirable to model the retention time as a random variable for some sensor system scenarios.

CHAPTER 2

ANALYTIC REPRESENTATION OF TARGET ACQUISITION

Section I. INTRODUCTION

2-1. RENEWAL REPRESENTATION OF THE TARGET ACQUISITION PROCESS

a. It is not necessary to explicitly represent the target acquisition process in NUFAM III. In NUFAM III, we are interested in two things related to the acquisition process: the probability that a unit is on the acquired unit list at the time that we conduct our nuclear fire planning, and the probability that the unit will remain on the acquisition list until engagement by a nuclear weapon. Representing the target acquisition process as an alternating renewal process allows us to easily find the probability of acquisition at any random point of time, p_{acq}:

$$p_{acq} = \frac{E[T_R]}{E[T_A] + E[T_R]} \qquad \text{(Youngren [1988b]),} \qquad [3]$$

b. In NUFAM III, $T_A = 2X + U$ and $E[T_R] = R$, where R is a fixed retention time for th. type unit and zone (recall that $X \sim$ geometric (POTA) and $U \sim$ uniform (0,2)). Thus for any given unit,

$$p_{acq} = \frac{R}{2E[X] + E[U] + R} = \frac{R}{2\frac{(1-POTA)}{POTA} + 1 + R}$$

$$= \frac{R}{\frac{2}{POTA} - 1 + R} \cdot \qquad [4]$$

c. Example. Suppose that the POTA IS 0.20 and the retention time R is 2 hours. Then $E[\,T_A\,]$ $= E[\,2X\,] + E[\,U\,] = 2\frac{(1-POTA)}{POTA} + 1 = 2\frac{(0.8)}{0.2} + 1 = 9$ hours and $p_{acq} = \frac{R}{2E[X] + E[U] + R}$ $= \frac{2}{2\cdot4 + 1 + 2} = \frac{2}{11} = 0.18.$

2-2. RELATIONSHIP TO THE STAY-MOVE CYCLE

a. NUFAM III simulates movement by causing the unit to vary between two alternating states: the *move* state (moving), or the *stay* state (stationary). We define the variable $\overline{S}$ to represent the average length of time that a unit is stationary, each time that it is stationary in the scenario, and the variable $\overline{M}$ to indicate the average length of time that it is moving, each time that it is moving in the scenario. These values $\overline{S}$ and $\overline{M}$ are the best estimates of the average stay and move times for each type unit represented in the model scenario. The random variables representing the stay and move times *generated by the model* are denoted as S_M and M_M, respectively. The unit movement is represented as an alternating renewal process just like the target acquisition process (Figure 2-1).

|		
Acquired	T_R	T_R
Not Acquired	T_A	T_A T_A
Stay	S	S
Move	M	M
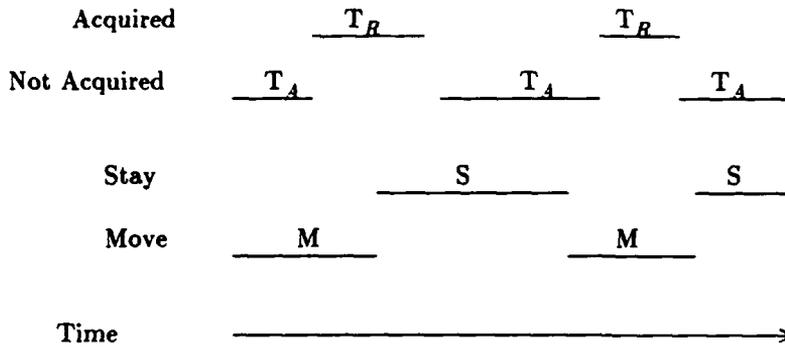
Time ⟶

Figure 2-1. Possible Target Acquisition and Stay/Move Sequences

b. The values input into the NUFAM III Type Unit file are the "average stay time" denoted as $\overline{S}_M$ and the "travel time" denoted as $\overline{M}_M$ for each type unit. In some cases, we need to provide NUFAM III with numbers that do not reflect the scenario in order for the probability that a unit will be hit by a nuclear weapon to be computed correctly within the model. In these cases, we distinguish between the average stay and move time numbers input to NUFAM III, $\overline{S}_M$ and $\overline{M}_M$, with the scenario driven average stay and move times $\overline{S}$ and $\overline{M}$.

c. A target can be acquired and observed while either stationary or moving, so T_A and T_R may be dependent upon the distributions of the scenario stay and move times S and M. However, in NUFAM III we assume that if the *parameters* of the distributions of T_A, T_R, S and M are known, then the *variables* T_A, T_R, S and M are mutually independent. Thus we can examine the unit move/stay state at any arbitrary point in time independently of the target acquisition state. Within the NUFAM III model, the probability that a unit is in the stay state (stationary) at any arbitrary point in time is p_{stay} :

$$p_{stay} = \frac{\overline{S}_M}{\overline{S}_M + \overline{M}_M}, \qquad [5]$$

where $\overline{S}_M$ is the model "average stay time" and $\overline{M}_M$ is the model "travel time" for each type unit. p_{stay} is always computed from the NUFAM III input values $\overline{S}_M$ and $\overline{M}_M$ rather than the scenario average stay and move time values $\overline{S}$ and $\overline{M}$.

d. If we enter the stay/move process at some random time T, at times we will be interested in the amount of time remaining after T until the unit moves. This is, of course, the residual life of the stay time, denoted as Y_S. The distribution of Y_S is given in Appendix C.

e. When the POTAs are calculated in TADER, it relies on input that specifies for each type unit the proportion of time the unit is stationary and moving (broken down into "in the open" and "under cover"). Thus the POTA is already weighted for the relative amount of time any given type unit spends stationary or moving. As a result, we do *not* check in NUFAM III to see whether a unit is stationary or moving at the time that an acquisition event occurs. The proportion of time spent stationary or moving is accounted for when we compute the probability that a unit is available for being fired upon by a nuclear weapon (section IV).

f. **Example.** Suppose that a unit is fairly mobile with a model input average stay time $\overline{S}_M$ of 15 minutes and an average move time $\overline{M}_M$ of 45 minutes. Then $p_{stay} = \frac{15}{15 + 45} = 0.25$.

Section II. MODELING THE RETENTION TIME

2-3. DEFINING THE RETENTION TIME T_R

a. The retention time is the length of time that an acquired unit will remain on the acquisition list. A fixed target remains on the acquisition list indefinitely (thus the methodology discussed in this paper does not apply -- see Youngren [1988a]). A relocatable target unit, on the other hand, may be removed from the acquisition list under the circumstances discussed below. The retention time is currently represented in NUFAM III as a fixed input value R. R (which will vary by type unit and zone) is set equal to $E[T_R]$.

b. Although all relocatable NUFAM III units can move at least once during a simulation, some unit types are engaged while moving in column. We refer to them as "units engaged while moving"; this designation indicates a unit in march column detected moving as part of a longer column of similar units. We refer to all other type units as "engaged while stationary."

c. Given the acquisition systems represented in the TAS III study, we find that some of the short-range sensors which can reach out to the limits of Zone 3 (25 km) can provide a nearly continuous tracking capability - that is, they can continue to observe a unit for some period of time after acquisition. The tracking may be continuous or nearly continuous (observed frequently). Units in Zones 4 and 5, on the other hand, cannot be tracked after acquisition. The distribution used to determine the retention time R will depend upon the tracking capability and whether the unit is engaged while stationary or moving. Three cases are discussed below.

2-4. TARGETS ENGAGED WHILE STATIONARY THAT FACE NO TRACKING CAPABILITY

a. This case models the situation where a unit is acquired while stationary, planned for fire at that location, and there is *no* capability to track the location of a unit after acquisition. All units in Zones 4 and 5 except those which are identified by type as "engaged while moving" fall into this category. The unit may be acquired while stationary or moving; however, the acquisition is useful for fire planning only if it was acquired while it was stationary or when it was moving into an identified stationary location (e.g., assembly area). NUFAM III handles this by checking acquisitions of units of this type to see if it was *stationary at the time of the last observation*, denoted by $P_{last\ stay}$. Only units that were stationary at the time of the last observation (in this case, at the time of acquisition) are planned for fire; thus the retention time is an estimate of the length of time that the target unit will remain at the location at which it was acquired, given that it was acquired while stationary or when moving into an identified stationary location. If it was stationary at the time of acquisition, this will be the residual life of the stay time, Y_S. If the unit was moving into the stationary location when acquired, it will have the entire stay period S ahead of it. Thus

$E[T_R \mid stay] = E[\ Y_S \mid$ stationary at time of acquisition]

$+ E[\ S \mid$ moving into a stationary location at time of acquisition while moving]

$\cdot \ P[$ moving into a stationary location at time of acquisition $\mid$ moving at acquisition]

$= E[Y_S] + E[S] \cdot \delta,$ [6]

where δ represents the probability that a unit acquired while moving is moving into a stationary location. We can derive δ from the scenario stay/move times for the unit by considering how long a period is possible between time of acquisition of a moving unit and the time it stops in order for the stationary location to be known at the time of acquisition. We denote this time by d; thus $\delta = P[\ Y_M \leq d\ ]$, where Y_M is the residual life of the move time. Obviously, d will be small; if desired, d can be approximated as zero.

b. Since d is assumed to be small, we can safely assume that d is less than $\frac{2}{3} \bar{M}$ (if it were not, a different equation for $P[\ Y_M \leq d\ ]$ could be used; the equation used in this section only holds for $d \leq \frac{2}{3} \bar{M}$ (see Appendix C)). Thus, $P[\ Y_M \leq d\ ] = \frac{d}{\bar{M}}$ (Appendix C) and

$$E[T_R \mid stay\ ] = E[Y_S] + E[S] \cdot \delta = \frac{14}{27} \bar{S} + \bar{S} \cdot \frac{d}{\bar{M}} = \bar{S} \cdot \left[\ \frac{14}{27} + \frac{d}{\bar{M}}\ \right]. \tag{7}$$

c. Example. Suppose that $\bar{S}$ = 120 minutes and $\bar{M}$ = 45 minutes. If d = 5 minutes, then using the formulas in Appendix C, $P[\ Y_M \leq d\ ]$ = 5 min / 45 min = 0.11 and $E[\ Y_S\ ]$ = (14/27) · 120 min = 62.2 min. Thus $E[\ T_R \mid stay\ ]$ = 120 min · (0.52 + 0.11) = 75.6 min.

2-5. TARGETS ENGAGED WHILE MOVING, FACING NO TRACKING CAPABILITY

a. This case represents the situation where a unit is acquired as part of a column of units moving along a specified route; we assume no capability to track the location of the unit after acquisition (zones 4 and 5 using TAS III data). In this case, the nuclear weapon is targeted at the location of the unit when acquired -- because the unit is one of many moving along that route, it is anticipated that a similar unit will be within the radius of effects of the weapon when it detonates. The retention time is an estimate of the length of time that other units will be moving through the location at which the unit was acquired. An estimate of this time in hours would be:

$$E[T_R] = \text{Expected length of column remaining (km) / speed of column (km/hr)}$$
$$= (\text{ Column length in km / 2 }) / \text{ speed of column (km/hr).} \tag{8}$$

b. Example. Suppose that a column moving at 20 km/hr is 2 km long. Then $E[\ T_R\ ]$ = [(2 km/2) / 20 km/hr] = 0.05 hour = 3 min.

2-6. TARGETS FACING A NEARLY CONTINUOUS TRACKING CAPABILITY

a. This case models circumstances where there is a capability to continuously (or intermittently with a short duty cycle) monitor or track the location of an acquired unit. All units placed in NUFAM III zones 1, 2, and 3 are assumed to fall into this category. However, this tracking capability cannot last forever -- at some point, the unit will be obscured, a gap in coverage will occur, etc. In this case, the retention time is an estimate of how long the unit can be tracked using the same sensors that initially detected the unit. Estimating the retention time under these circumstances is discussed below.

b. Approximating a continuous or nearly continuous observation with frequent glimpses is reasonable if the time between glimpses is small. All units in NUFAM III are acquired using the 2-hour POTA glimpse cycle. Thus our representation of tracking must use the same 2-hour glimpse cycle. Using a 2-hour glimpse cycle is not a nearly continuous observation model, but it is the only model that can be based on current POTA data. Ongoing data collection efforts at CAA (TAS IV) will permit a better representation of continuous observation in the future.

c. A unit is initially detected as a result of one or more acquisition cycles, during which there is a probability that the unit will be detected by one or more operational sensors. The probability of independently detecting any given unit during each 2-hour cycle, the POTA, is implicitly defined as Pr[detect during this cycle | no detection previous cycle]. When tracking a unit after it has been detected using the same sensors, the probability that at least one of the sensors will continue to detect the unit is Pr[detect during this cycle | detection previous cycle], which would be a type of cued detection. We denote Pr[detect during this cycle | detection previous cycle] as P_D. Clearly,

$P_D \geq$ POTA, so a useful representation of P_D is $P_D = \gamma \cdot$ POTA, where $1 \leq \gamma \leq 1/$POTA. Thus, γ is a factor that indicates how much better the sensors are at detecting a unit previously detected and identified than detecting a new, previously undetected unit.

d. Just as the use of the POTA in NUFAM III leads to a geometric distribution for the number of cycles before a target is acquired, the retention time should have the same distributional form. The probability of tracking a unit for k acquisition cycles is equal to $(\gamma \cdot$ POTA$)^k$, $k = 0, 1, \ldots$, and the probability that a unit is lost (dropped from acquisition list) on the $(k + 1$st) cycle is $(1 - \gamma \cdot$ POTA$)(\gamma \cdot$ POTA$)^k$, which is also geometric. Thus, if X_R is defined as a geometric $(1 - \gamma \cdot$ POTA$)$ random variable, and U_R is defined as a uniform $(0,2)$ random variable, then the retention time T_R can be represented as $2X_R + U_R$. Clearly

$$E[T_R \mid \gamma, \text{POTA}] = 2\, E[X_R] + E[U_R] = \frac{(\gamma \cdot \text{POTA})}{(1 - \gamma \cdot \text{POTA})} + 1 = \frac{1}{1 - \gamma \cdot \text{POTA}} \quad [9]$$

e. It may be possible to estimate P_D (i.e., $\gamma \cdot$ POTA) by running the TADER model using different parametric assumptions. Alternatively, we handle the uncertainty associated with the parameter γ by placing a prior distribution on γ that reflects our uncertainty. Without further information, a flat prior [Uniform (1, 1/POTA)] is reasonable. Our expected value for P_D using this uniform prior halves the probability that a subsequent glimpse will fail to detect the target. The expected value for T_R, $E[T_R]$, is found using standard techniques: $E[T_R] = \int_\gamma E[T_R|\gamma]\, dG(\gamma)$, where $G(\gamma)$ is the cumulative distribution function (cdf) of γ. Unfortunately, T_R does not have a finite mean because γ is allowed to become arbitrarily close to 1/POTA (equivalent to $P_D = 1$, which yields an infinite T_R). However, NUFAM III is intended to be used to model only a brief period within a theater battle (not to exceed 12 hours). With the 24-hour initial acquisition cycle used in NUFAM III runs to achieve an approximate steady-state acquisition process, the simulation will always terminate in less than 36 hours of simulated time. As a result, any time $T_R \geq 36$ hours is equivalent to an arbitrarily long time in a NUFAM III simulation run. Since we are only interested in the expectation $E[T_R]$, we can truncate our distribution for $E[T_R \mid \gamma]$ as follows:

$$E[T_R \mid \gamma, \text{POTA}] = \frac{1}{1 - \gamma \cdot \text{POTA}} \qquad \text{for } \gamma < \frac{1}{\text{POTA}}\left[1 - \frac{1}{36}\right],$$

$$= 36 \qquad \text{for } \gamma \geq \frac{1}{\text{POTA}}\left[1 - \frac{1}{36}\right]. \qquad [10]$$

f. We calculate that $\frac{1}{\text{POTA}}\left[1 - \frac{1}{36}\right] \simeq \frac{0.97}{\text{POTA}}$. Since $\gamma \geq 1$, γ can be $< \frac{0.97}{\text{POTA}}$ only when POTA < 0.97; thus, $E[T_R \mid \gamma] = 36$ when POTA ≥ 0.97. For values of POTA less than 0.97, we must compute the expectation of T_R:

For POTA ≥ 0.97, $E[T_R] = 36$ hrs.

For POTA < 0.97, recalling that $\gamma \sim$ Uniform (1, 1/POTA),

$$E[T_R] = \int_{\gamma=1}^{\frac{0.97}{\text{POTA}}} \frac{1}{1 - \gamma \cdot \text{POTA}} \cdot \frac{d\gamma}{\frac{1}{\text{POTA}} - 1} + \int_{\gamma=\frac{0.97}{\text{POTA}}}^{\frac{1}{\text{POTA}}} 36 \cdot \frac{d\gamma}{\frac{1}{\text{POTA}} - 1}$$

$$= \int_{\gamma=1}^{\frac{0.97}{\text{POTA}}} \frac{1}{1 - \text{POTA}} \cdot \left(\frac{1}{\text{POTA}} - \gamma\right)^{-1} d\gamma \; + 36 \cdot \frac{1 - 0.97}{1 - \text{POTA}} \; .$$

After some calculation, we find that

$$E[T_R] = \frac{POTA - 0.97}{(1 - POTA)^2} \ln\left[\frac{1 - 0.97}{1 - POTA}\right] + \frac{1.08}{1 - POTA} \quad \text{for } POTA < 0.97;$$

$$E[T_R] = 36 \quad \text{for } POTA \geq 0.97. \tag{11}$$

We use this value as the retention time, R, input to NUFAM III for all units in Zones 1, 2, and 3.

g. Example. Suppose that a unit had a POTA of 0.2. Then

$$E[T_R] = \frac{0.2 - 0.97}{(1 - 0.2)^2} \ln\left[\frac{1 - 0.97}{1 - 0.2}\right] + \frac{1.08}{1 - 0.2} = \left[-\frac{0.77}{0.64}\right] \cdot (-3.28) + 1.35 = 5.3 \text{ hrs.}$$

Section III. COMPUTING THE PROBABILITY THAT AN ACQUIRED UNIT WILL REMAIN ON THE ACQUISITION LIST

2-7. DEFINITION

a. The probability that the unit on the acquisition list at the end of the fire planning cycle (time t_p as shown in Figure 1-2) will remain on the acquisition list until some elapsed time s after t_p is simply the survival function of the residual life of the retention time at t_p, Y_R, evaluated at s. Recall from [2] (paragraph 1-4b) that

$$P[Y_R > s] = 1 - \frac{1}{R}\int_0^s 1 \, du = \frac{R - s}{R} \qquad 0 \leq s < R; \qquad 0 \text{ for } s \geq R. \tag{12}$$

Combining this probability with p_{acq} given in [2] (paragraph 2-1b), the probability that a unit is on the acquisition list at time t_p and remains on the list for s additional hours is

$$P[\text{ on acquisition list at } t_p \text{ and still on the list at time } t_p + s] = p_{acq} \cdot P[Y_R > s]$$

$$= \frac{R}{\dfrac{2}{POTA} - 1 + R} \cdot \frac{R - s}{R}$$

$$= \frac{R - s}{\dfrac{2}{POTA} - 1 + R} \quad 0 \leq s < R \text{ ; } 0 \text{ otherwise.} \tag{13}$$

b. In NUFAM III, we are currently using a 1-hour fire execution cycle, with targets planned for fire if they are acquired at any time up to the beginning of the fire execution cycle. Thus, the probability that we acquire a unit that remains available for engagement through the end of a 1-hour fire execution cycle is

$$P_{retain\ 1} \equiv P[\text{ unit is acquired and is retained for one hour }]. \text{ Setting } s = 1 \text{ in [13] yields}$$

$$P_{retain\ 1} = \frac{R - 1}{\dfrac{2}{POTA} - 1 + R}, \qquad (R \geq 1).$$

c. A more sophisticated representation recognizes that a target need only be retained until it is fired upon. The time of detonation is denoted as T_d (Figure 1-2). Thus the remaining (residual) retention time, Y_R, must be greater than $T_d - t_p$ in order for the target to be fired upon.

$$P[\, Y_R > T_d - t_p \mid T_d - t_p \,] \quad = 1 - \frac{T_d - t_p}{R} \qquad 0 \le T_d - t_p < R;$$

$$= 0 \qquad\qquad \text{for } T_d - t_p \ge R. \qquad\qquad [14]$$

Section IV. COMPUTING THE PROBABILITY THAT A UNIT IS AVAILABLE FOR FIRE

2-8. DEFINITION

a. In order for a unit to be available for engagement (subject to fire) by nuclear weapons, it must be acquired and retained as a target at least until the scheduled firing time. The target is evaluated at some known planning time t_p; during the fire planning cycle, the detonation will be scheduled to occur at some time T_d within the fire execution cycle. Thus a target that is on the acquisition list at time t_p must be retained at least until T_d; this occurs when the residual time on the acquisition list, Y_R, is greater than or equal to ($T_d - t_p$). We define the probability P[unit is acquired and retained until detonation] as p_{avail}. Thus

$$p_{avail} = P[\, Y_R > T_d - t_p \mid \text{acquired at } t_p \,] \cdot P[\, \text{acquired at } t_p \,].$$

b. In the absence of further information about the firing times, it is reasonable to assume that any detonation time within the fire execution cycle is equally likely. If the fire execution cycle is 1 hour long, and the cycle begins immediately after the planning cycle ends, then $(T_d - t_p) \sim U[\,0,1\,]$. If a different distribution for $(T_d - t_p)$ can be inferred or if the time at which the fire planning cycle ends is not the time at which the fire execution cycle begins, the probability $Y_R > (T_d - t_p)$ is computed in the same manner as shown, but a different set of limits on the integral and a different formula for $P[\, T_d - t_p = t \,]$ will be required. We calculate p_{avail} using the residual life of the retention time :

$$P[\, Y_R > T_d - t_p \,] = \int_0^D P[\, Y_R > T_d - t_p \,] \, P[\, T_d - t_p = t \,] \, dt = \int_0^D \overline{F}_{Y_R}(\,t\,) \cdot 1 \; dt,$$

$$\text{where } D = \min\{\, R, 1 \,\} .$$

Thus

$$P[\, Y_R > T_d - t_p \,] = \int_0^D \left[\, 1 - \frac{t}{R} \,\right] dt = D - \frac{D^2}{2R} = D\left[\, 1 - \frac{D}{2R} \,\right].$$

Therefore,

$$p_{avail} = \frac{R}{\dfrac{2}{\overline{POTA}} - 1 + R} \cdot D\left[\, 1 - \frac{D}{2R} \,\right]. \qquad\qquad [15]$$

This yields:

$$R \le 1: \qquad p_{avail} = \frac{R^2}{2\left[\, \dfrac{2}{\overline{POTA}} - 1 + R \,\right]},$$

$$R > 1: \qquad p_{avail} = \frac{2R - 1}{2\left[\, \dfrac{2}{\overline{POTA}} - 1 + R \,\right]}. \qquad\qquad [16]$$

c. In some cases, all weapons of a particular type will detonate before the end of the fire execution cycle. An alternative distribution for $(T_d - t_p)$ that requires no additional calculation is $(T_d - t_p) \sim$ uniform $[\, 0 \,, L \,]$, where L is the time to the last detonation after the beginning of the fire execution cycle ($L < 1$ hour). Thus $D = \min\{\, R, L \,\}$ and equation [15] can be used to calculate p_{avail}. This approach may also be used to generalize the results for fire execution cycles of length L, where $L \ne 1$ hour.

d. Example. Suppose that the POTA is 0.2 and the retention time R is 5.3 hours. Using a 1-hour fire execution cycle, D = min{ 1 hour, 5.3 hours } = 1 hour and

$$P_{avail} = \frac{2R - 1}{2\left[\frac{2}{POTA} - 1 + R \right]} = \frac{2(5.3) - 1}{2\left[\frac{2}{0.2} - 1 + 5.3 \right]} = \frac{9.6}{28.6} = 0.34.$$

Section V. COMPUTING THE PROBABILITY THAT A UNIT HAS MOVED AT THE TIME OF DETONATION

2-9. DEFINITION. Because we cannot track acquired units after a certain point (that point depending upon our capability for near continuous tracking), it is possible for a weapon to miss a target. We plan the weapon to detonate at the Desired Ground Zero (DGZ) at which the weapon effects against the target are maximized (subject to some physical constraints). If a target unit has moved since the time the last DGZ adjustment was completed, the weapon will miss the target. Unit movement is represented through the NUFAM III inputs "average stay time" $\bar{S}_M$ and the "travel time" $\bar{M}_M$ discussed previously. The values for $\bar{S}_M$ and $\bar{M}_M$ are determined differently for different tracking capabilities.

2-10. TARGETS ENGAGED WHILE STATIONARY, FACING NO TRACKING CAPABILITY.

a. When we have no capability to track units, we can only successfully engage them if they are stationary at the time of acquisition (which is assumed to hold for units in this category by definition) and they remain in place at the point at which they were acquired. The average stay time $\bar{S}_M$ input to the model equals the scenario average time the unit will remain stationary $(\bar{S})$; $\bar{M}_M$ equals the scenario average time that a unit will be moving from stationary location to stationary location $(\bar{M})$.

b. NUFAM III will check the probability that the unit was stationary at the time it was last observed before launching the weapon. We denote this probability as $p_{last\ stay}$; it is stored in the TUNIT file as PROB.STAY.LOBS. The last observation for units in this category was at the time of acquisition, so referring back to paragraph 2-4a,

PROB.STAY.LOBS = $p_{last\ stay}$ = P[stationary at time of acquisition]
+ $\Big\{$ P[moving into a stationary location at time of acquisition | moving at acquisition]

· P[moving at acquisition] $\Big\}$

$$= \frac{\bar{S}}{\bar{S} + \bar{M}} + \delta \cdot \frac{\bar{M}}{\bar{S} + \bar{M}}, \qquad [17]$$

where $\delta = P[\ Y_M \leq d\ ] = \frac{d}{\bar{M}}$, $d \leq \frac{2}{3}\bar{M}$ (Appendix C).

c. The probability that the unit remains stationary at its identified location until the time of detonation is equal to the probability that the time that the unit remains at the location at which it was acquired is greater than the time from acquisition to detonation. If we denote the time of acquisition as T_{acq} and the time of detonation as T_d, then the probability that the unit remains at its identified location until the time of detonation is P[$Y_S > T_d - T_{acq}$ and stationary at T_{acq}]. The time T_{acq} is not known, but the difference ($t_p - T_{acq}$) is distributed as the age of the distribution of the time of acquisition, denoted as Y_{RA}, independent of the distribution of $T_d - t_p$. The variable $T_d - t_p$ can have any nonnegative distribution; this paper uses either a constant (η) or uniform (0, L) distribution, where L is the length of the fire execution cycle and t_p is the time that

the fire planning cycle ends (and the fire execution cycle begins). The formula for determining $P[\ Y_S > T_d - T_{acq}$ and stationary at $T_{acq}\ ]$, taking into account the distributions of $(\ T_d - t_p\ )$ and $(\ t_p - T_{acq}\ )$, may be found in Appendix C.

   d. To simplify matters, the current studies using NUFAM III determine the probability that the unit is stationary at that identified location at the time of detonation by drawing against p_{stay} ([5], paragraph 2-2c). This is equivalent to ignoring the move/stay state at the time of acquisition T_{acq} or any time between T_{acq} and T_d. Given the definition of these units (units acquired and engaged while stationary), this is not unreasonable. Thus

$$P_{last\ stay} = 1, \quad P_{stay} = \frac{\overline{S}_M}{\overline{S}_M + \overline{M}_M}, \quad \overline{S}_M = \overline{S}, \text{ and } \overline{M}_M = \overline{M}. \tag{18}$$

Future studies may set $p_{last\ stay} = \dfrac{\overline{S}}{\overline{S} + \overline{M}}$ and set $\overline{S}_M$ and $\overline{M}_M$ such that $\dfrac{\overline{S}_M}{\overline{S}_M + \overline{M}_M} =$

$P[\ Y_S > T_d - T_{acq}$ and stationary at $T_{acq}\ ]$.

2-11. TARGETS ENGAGED WHILE MOVING, FACING NO TRACKING CAPABILITY

   a. When we have no capability to track units detected while moving, we cannot successfully engage them--however, if the acquired unit is part of a column moving along a defined path, we may be able to successfully engage a different unit that subsequently moves through the point at which the original unit was acquired. The average stay time $\overline{S}_M$ input to the model represents the average time that a unit is passing through the acquisition point; $\overline{M}_M$ represents the average time that no unit is passing through the acquisition point. If the column is composed of several units of the same type (the basic assumption defining a type unit "engaged while moving"), then we may compute $\overline{S}_M$ and $\overline{M}_M$ as follows:

$$\overline{S}_M = \frac{\text{length of each unit in column (km)}}{\text{speed of column (km/hr)}}$$

$$\overline{M}_M = \frac{\text{length of gaps between units in the column (km)}}{\text{speed of column (km/hr)}}. \tag{19}$$

   b. The probability that the unit was stationary at the time it was last observed immediately before firing the weapon, $p_{last\ stay}$, is not applicable for units in this category. As a result, we set PROB.STAY.LOBS $= p_{last\ stay} = 1$.

   c. Example. Suppose that the length of each unit is 250 meters $= 0.25$ km, and the column is traveling at 20 km/hr. Suppose that the length of the gaps between each unit is 50 meters $= 0.05$ km. Then $\overline{S}_M = 0.25 / 20 = 0.0125$ hr. and $\overline{M}_M = 0.05 / 20 = 0.0025$ hr.

2-12. TARGETS FACING A NEARLY CONTINUOUS TRACKING CAPABILITY (SCHEDULED FIRE)

   a. We have assumed that the target units of this type may be tracked until shortly before the weapon detonates at the time T_d; T_d having been *scheduled* (set during the time of fire planning). The tracking is discontinued shortly before firing due to the necessity of communicating the fire order to the firing unit and adjusting the delivery system for fire. Thus the DGZ of the weapon has been shifted along with the unit until the order to fire is given. From another perspective, the unit (whether stationary or moving) has remained stationary *with respect to the DGZ* during this entire tracking period. As a result, the ratio $\overline{S}_M / [\ \overline{S}_M + \overline{M}_M\ ]$ denotes the probability that the unit remains stationary *with respect to the last computed DGZ*. If it does, then it will be subject to nuclear effects. If not, the target unit must have shifted unexpectedly during the interval between final fire adjustment and firing, causing the weapon to miss.

b. Let η denote the amount of time between the final observation (time of final DGZ adjustment) and the detonation. This delay accounts for the time necessary to transmit the order to fire to the firing unit, adjust the delivery system, etc. If the unit is stationary at the time that the last observation is made, then the DGZ will be the final location. The weapon will miss if the unit moves during the period η (Note: If the unit moves at a time slightly less than η, it may still be within the effects radii for the burst, but this adjustment is dependent on weapon as well as target characteristics; its contribution to the probability is normally small for small yield weapons and will be ignored). Thus the probability that the unit has not moved from the location at which it was last observed, denoted by $p_{no\ move}$, is $P[\ Y_S > \eta$ and the unit stationary at time of last observation], which is:

$$p_{no\ move} = P[\ Y_S > \eta \mid \text{stationary at time of last observation}\ ] \cdot p_{last\ stay}.$$

c. Let $\overline{S}$ and $\overline{M}$ denote the scenario average stay and move times for a particular type of unit under consideration. For $\eta < \frac{2}{3}\overline{S}$ (Appendix C),

$$P[\ Y_S > \eta \mid \text{stationary at time of last observation}\ ] = 1 - \frac{\eta}{\overline{S}}.\ \text{Thus}$$

$$p_{no\ move} = \frac{\overline{S}-\eta}{\overline{S}} \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} = \frac{\overline{S}-\eta}{\overline{S} + \overline{M}}. \tag{20}$$

d. NUFAM III will check the probability that the unit was stationary at a time η before detonation during the fire execution cycle immediately before firing the weapon. We denote this probability as $p_{last\ stay}$ computed as follows:

$$\text{PROB.STAY.LOBS} = P[\ \text{stationary at last observation}\ ] = p_{last\ stay} = \frac{\overline{S}}{\overline{S} + \overline{M}} \tag{21}$$

e. When NUFAM III computes the probability that a detonated round missed the target unit, it checks to see if the unit has moved. Thus to force the current NUFAM III code to record a miss with the appropriate probability, the values $\overline{S}_M$ and $\overline{M}_M$ *entered into the data base* must solve

$$\frac{\overline{S}_M}{\overline{S}_M + \overline{M}_M} = P[\ Y_S > \eta \mid \text{stationary at time of last observation}\ ] = \frac{p_{no\ move}}{p_{last\ stay}} = \frac{\overline{S}-\eta}{\overline{S}}.$$

We suggest setting $\overline{M}_M = \eta$ and $\overline{S}_M = \overline{S} - \eta$.

f. Example. Suppose that $\overline{S} = 15$ min, $\overline{M} = 45$ min, and $\eta = 10$ min, which is $\leq \frac{2}{3}\overline{S}$, thus $P[\ Y_S > \eta\ ] = 1 - \frac{10}{15} = \frac{5}{15}$, and $p_{no\ move} = \frac{15-10}{15+45} = 0.083$. $\overline{S}_M = \overline{S} - \eta = 5$ min and $\overline{M}_M = \eta = 10$ min.

2-13. TARGETS FACING A NEARLY CONTINUOUS TRACKING CAPABILITY (WITHOUT SCHEDULED FIRE)

a. This case represents the same situation as paragraph 2-12 where at least one sensor can observe or "track" the target continuously (or nearly continuously) after acquisition. However, it is reasonable to suppose that if we can track the target unit continuously, we would give the order to fire when the unit stops for the first time within the fire execution period. Until now, we have given the order to fire (determined the time T_d) at time t_p. In this case, we define a (random) time to give the fire order, T_f, during the fire execution period which depends upon the time that the unit stops. It is reasonable to assume that some time will need to be spent getting the order to the

delivery unit, adjusting the aimpoint to the location at which the unit stopped, confirming that the location meets the criteria for employment of the weapon, etc. As in the previous section, this time $(T_d - T_f)$ is denoted as η, and for simplicity, η is assumed constant. If the length of the fire execution cycle is L, then clearly $0 < \eta < L$.

b. If the unit is stationary at time t_p, $T_f = t_p$; otherwise, T_f equals the first time the unit stops between t_p and $t_p + (L - \eta)$ (Figure 2-2). If the move time distribution is not bounded from above at some value less than the fire execution period duration, there is a positive probability that the unit will be moving during the entire fire execution period. Therefore, the stay and move times $\overline{S}_M$ and $\overline{M}_M$ must be adjusted to account for this. Because the stay/move process is assumed independent of the acquisition process, T_f is random with respect to the acquisition process. Therefore, the probability that the unit is on the acquisition list at the time T_f is simply p_{acq}.

Acquired

T_R

Not Acquired

T_A

T_A

Stay

S

S

Move

M

M

L

Fire Planning Cycle | Fire Execution Cycle

η

Time

T_{acq}

t_p

T_f T_d

Time
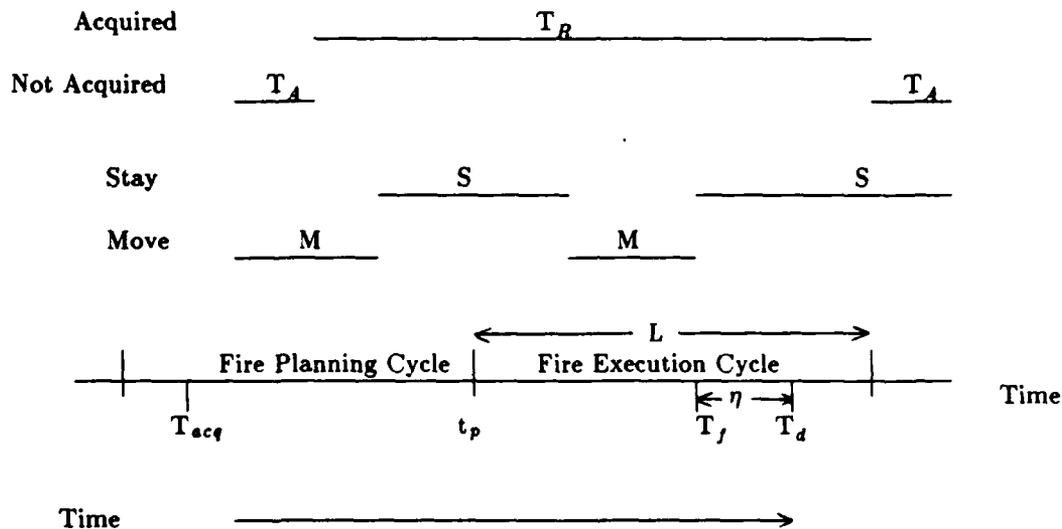
Figure 2-2. Engaging a Target Unit Faced with Nearly Continuous Tracking, without Scheduled Fire

c. The probability that the unit is available for fire, given that it was on the acquisition list at T_f is still the probability that the acquisition is stationary at T_f and retained between T_f and T_d. However, if we issue the fire order at time T_f, we will retain the unit between T_f and T_d by definition. Thus the conditional probability P[unit retained until T_d | unit on the acquisition list at T_f] = 1. Thus $p_{avail} = 1 \cdot p_{acq}$. [22]

d. Let η denote the amount of time between the final observation (time of final DGZ adjustment) and the detonation. Since the unit is stationary at the time that the last observation is made by definition, then the DGZ will be the final location. The weapon will miss if the unit moves during the period η. If $T_f = t_p$, the unit was somewhere in its stay period at time t_p and the probability that it will remain in place until T_d is P[$Y_S > \eta$], where Y_S is the residual life of the stay time. If $t_p < T_f < t_p + (L - \eta)$, then the unit stopped during the fire execution period and the time T_f is equal to the time that the unit's stay period begins. Thus P[stationary at T_d | $t_p < T_f < t_p + (L - \eta)$] = P[S > η].

e. The probability that the unit was stationary at time t_p or stopped during the fire execution cycle, and it has not moved from the location at which it was last observed ($p_{no\ move}$) is:

$$p_{no\ move} = \left\{ P[\ Y_S > \eta\ |\ \text{stationary at } t_p\ ] \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} \right.$$

$$+ P[\ S > \eta\ |\ \text{stopped at } T_f;\ \text{moving at } t_p\ ] \cdot P[\ Y_m < L-\eta\ |\ \text{moving at } t_p\ ] \cdot \left. \frac{\overline{M}}{\overline{S} + \overline{M}} \right\}$$

$$= P[\ Y_S > \eta\ ] \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\ S > \eta\ ] \cdot P[\ Y_m < L-\eta\ ] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}}, \qquad [23]$$

f. $p_{last\ stay}$, the probability that the unit was stationary η time units before detonation during the fire execution cycle immediately before firing the weapon, is:

$$\text{PROB.STAY.LOBS} = p_{last\ stay} = \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\ Y_m < L-\eta\ ] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}}. \qquad [24]$$

g. When NUFAM III computes the probability that a detonated round missed the target unit, it checks to see if the unit has moved. Thus to force the current NUFAM III code to record a miss with the appropriate probability, the values $\overline{S}_M$ and $\overline{M}_M$ *entered into the data base* must solve

$$\frac{\overline{S}_M}{\overline{S}_M + \overline{M}_M} = \frac{p_{no\ move}}{p_{last\ stay}}$$

$$= \frac{P[\ Y_S > \eta\ ] \cdot \dfrac{\overline{S}}{\overline{S} + \overline{M}} + P[\ S > \eta\ ] \cdot P[\ Y_m < L-\eta\ ] \cdot \dfrac{\overline{M}}{\overline{S} + \overline{M}}}{\dfrac{\overline{S}}{\overline{S} + \overline{M}} + P[\ Y_m < L-\eta\ ] \cdot \dfrac{\overline{M}}{\overline{S} + \overline{M}}}$$

We suggest setting $\overline{S}_M = P[\ Y_S > \eta\ ] \cdot \dfrac{\overline{S}}{\overline{S} + \overline{M}} + P[\ S > \eta\ ] \cdot P[\ Y_m < L-\eta\ ] \cdot \dfrac{\overline{M}}{\overline{S} + \overline{M}} =$

$p_{no\ move}$ and $\overline{M}_M = P[\ Y_S \le \eta\ ] \cdot \dfrac{\overline{S}}{\overline{S} + \overline{M}} + P[\ S \le \eta\ ] \cdot P[\ Y_m < L-\eta\ ] \cdot \dfrac{\overline{M}}{\overline{S} + \overline{M}} =$

$p_{last\ stay} - p_{no\ move}$.

h. Example. Suppose $\eta = 10$ min, $L = 1$ hour, $\overline{S} = 15$ min, and $\overline{M} = 45$ min. Then $(L-\eta) = 50$ min. From Appendix C,

$$P[\ Y_m < L-\eta\ ] = 1 - P[\ Y_m > L-\eta\ ]$$

$$= \frac{1}{\overline{M}} \left[2(L-\eta) - \frac{3(L-\eta)^2}{4\ \overline{M}} \right] - \frac{1}{3}, \quad 30\ \text{min} = \frac{2}{3}\ \overline{M} \le (L-\eta) \le \frac{4}{3}\ \overline{M} = 60\ \text{min}.$$

$$= 0.962. \text{ Also } \frac{\overline{S}}{\overline{S} + \overline{M}} = 0.25 \text{ and } \frac{\overline{M}}{\overline{S} + \overline{M}} = 1 - 0.25 = 0.75$$

Thus $p_{last\ stay} = \dfrac{\overline{S}}{\overline{S} + \overline{M}} + P[\ Y_m < L-\eta\ ] \cdot \dfrac{\overline{M}}{\overline{S} + \overline{M}} = 0.25 + 0.962 \cdot 0.75 = 0.972.$

$$p_{no\ move} = P[\ Y_S > \eta\ ] \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\ S > \eta\ ] \cdot P[\ Y_m < L-\eta\ ] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}}$$

$$= \frac{5}{15} \cdot 0.25 + 1 \cdot 0.962 \cdot 0.75 = 0.805,$$

$$\overline{S}_M = p_{no\ move} = 0.805, \quad \overline{M}_M = p_{last\ stay} - p_{no\ move} = 0.972 - 0.805 = 0.167.$$

Section VI. COMPUTING THE PROBABILITY THAT A UNIT
IS FIRED AT BY A NUCLEAR WEAPON

2-14. DEFINITION

a. The probability that a unit can be fired at by a nuclear weapon, denoted as p_{fired}, is the joint probability that a unit is on the acquisition list, is not dropped from the acquisition list prior to detonation, and is stationary at the time of last observation. This probability calculation implicitly assumes that a weapon of the appropriate yield is available and will be fired at the target if it meets these criteria. Recalling that $p_{avail} = P[$ unit is acquired and retained until detonation$]$,

$$p_{fired} = p_{avail} \cdot p_{last\ stay}$$

$$p_{fired} = p_{acq} \cdot \int_0^D P[\ T_R > t_d - t_p \mid T_d - t_p = t\ ]\ P[\ T_d - t_p = t\ ]\ dt \cdot p_{last\ stay}$$

Units in Zones 4 and 5, regardless of whether they are "engaged while stationary" or "engaged while moving", are assumed to be stationary or moving by definition at the time of acquisition, and they are not tracked during the interval between acquisition and firing. Thus the probability that they where stationary with regard to the location at which they were acquired, $p_{last\ stay}$, can be ignored, so it is set equal to 1. Units in Zones 1, 2, and 3 are tracked during the interval between acquisition and firing, thus the probability $p_{last\ stay}$ is equal to p_{stay} (scheduled fire) or is equal to $p_{stay} + P[\ Y_m < L - \eta\ ] \cdot p_{move}$ when the fire order is not planned.

b. For units in Zones 1, 2, and 3, scheduled fire, from [15] and [21] we get

$$p_{fired} = \frac{R}{\frac{2}{POTA} - 1 + R} \cdot D\left[\ 1 - \frac{D}{2R}\ \right] \cdot \frac{\overline{S}}{\overline{S} + \overline{M}}\ , \tag{25}$$

for units in Zones 1, 2, and 3, without scheduled fire, from [15] and [24] we get

$$p_{fired} = \frac{R}{\frac{2}{POTA} - 1 + R} \cdot D\left[\ 1 - \frac{D}{2R}\ \right] \cdot \left[\ \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\ Y_m < L - \eta\ ] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}}\ \right], \tag{26}$$

and for all units in Zones 4 and 5, from [15] and [18] we get

$$p_{fired} = \frac{R}{\frac{2}{POTA} - 1 + R} \cdot D\left[\ 1 - \frac{D}{2R}\ \right] \cdot 1\ , \tag{27}$$

where $D = \min\{\ R, L\ \}$, L is the length of the fire execution time (hours), R is the retention time in hours, η is the length of time between the time the fire order is given (time the unit was last observed when tracked) and the time of detonation, and $\overline{S}$ and $\overline{M}$ are the scenario average stay and move times respectively.

c. Example. Suppose $\eta = 10$ min, $L = 1$ hour, $\overline{S} = 15$ min. and $\overline{M} = 45$ min as given in the previous example. Also suppose that the POTA is 0.2 and $R = 5.3$ hours. Then for units in Zones 1, 2, and 3 with scheduled fire,

$$p_{fired} = \frac{5.3}{\frac{2}{0.2} - 1 + 5.3} \cdot 1\left[\ 1 - \frac{1}{2 \cdot 5.3}\ \right] \cdot \frac{15}{15 + 45} = 0.371 \cdot 0.906 \cdot 0.25 = 0.084.$$

For units in Zones 1, 2, and 3 without scheduled fire,

$$P_{fired} = \frac{5.3}{\frac{2}{0.2} - 1 + 5.3} \cdot [1 - \frac{1}{2 \cdot 5.3}] \cdot [\frac{15}{15 + 45} + 0.962 \cdot \frac{45}{15 + 45}] = 0.371 \cdot 0.906 \cdot 0.972 = 0.319.$$

For units in Zones 4 and 5 using the same $\overline{S}$ and $\overline{M}$ values as used in Zones 1, 2, and 3,

$$P_{fired} = \frac{5.3}{\frac{2}{0.2} - 1 + 5.3} \cdot 1 [1 - \frac{1}{2 \cdot 5.3}] \cdot 1 = 0.371 \cdot 0.906 = 0.336.$$

Section VII. COMPUTING THE PROBABILITY THAT A UNIT IS HIT BY A NUCLEAR WEAPON

2-15. DEFINITION

a. The probability that a unit is hit, denoted as p_{hit}, is the joint probability that a unit is on the acquisition list, is not dropped from the acquisition list prior to detonation, is stationary at the time of last observation, and does not move from the last computed DGZ. This calculation implicitly assumes that a weapon is available for firing at the unit, the weapon is fired, and both the weapon system and warhead function correctly. Thus

$$p_{hit} = p_{avail} \cdot p_{no\ move},$$

$$p_{hit} = p_{acq} \cdot \int_0^D P[\ T_R > t_d - t_p \mid T_d - t_p = t\ ]\ P[\ T_d - t_p = t\ ]\ dt \cdot p_{last\ stay} \cdot \frac{p_{no\ move}}{p_{last\ stay}},$$

$$p_{hit} = p_{fired} \cdot \frac{\overline{S}_M}{\overline{S}_M + \overline{M}_M}, \qquad [28]$$

where p_{fired} is computed as given in the previous section and $\overline{S}_M$ and $\overline{M}_M$ are the values entered into the NUFAM III data base for the unit stay time and move time respectively.

b. Example. Suppose the probabilities that a unit will be fired on are as given in the previous example, with $\overline{S}_M$ and $\overline{M}_M$ as calculated in the examples at the end of each paragraph in Section V. For the purpose of comparison, the same $\overline{M}$ and $\overline{S}$ values are used for all of the examples below. In actual scenarios, the $\overline{M}$ and $\overline{S}$ values would vary considerably by zone, type unit, and unit activity.

(1) For units in Zones 1, 2, and 3 with scheduled fire, $p_{fired} = 0.084$, $\overline{M}_M = \eta = 10$ min, $\overline{S}_M = \overline{S} - \eta = 5$ min, and $p_{hit} = 0.084 \cdot (5/15) = 0.028$.

(2) For units in Zones 1, 2, and 3 without scheduled fire, $p_{fired} = 0.319$, $\overline{S}_M = p_{no\ move} = 0.805$, $\overline{M}_M = p_{last\ stay} - p_{no\ move} = 0.167$, and $p_{hit} = 0.319 \cdot (0.805/0.972) = 0.264$.

(3) For units in Zones 4 and 5 engaged while stationary, $p_{fired} = 0.336$, $\overline{M}_M = \overline{M} = 45$ min, $\overline{S}_M = \overline{S} = 15$ min, and $p_{hit} = 0.336 \cdot (15/60) = 0.084$.

(4) For units in Zones 4 and 5 engaged while moving, $p_{fired} = 0.336$, $\overline{S}_M = 0.0125$ hr, $\overline{M}_M = 0.0025$ hr, and $p_{hit} = 0.336 \cdot (0.0125/0.0150) = 0.280$.

CHAPTER 3

SUMMARY

SUMMARY.

a. The principal findings of the work reported in this paper are:

(1) The target acquisition and unit movement processes may be represented using alternating renewal processes.

(2) The current data for target acquisition used within NUFAM III, the probability of target acquisition (POTA) values for a 2-hour glimpse period derived from the Target Acquisition Study, Version III (TAS III), require the number of periods until acquisition to be distributed as geometric.

(3) The process of tracking nuclear targets after acquisition may be represented based on POTA values if certain assumptions are accepted.

b. The efficiency of NUFAM III can be increased by eliminating the tedious, time consuming simulation of target acquisition cycles by drawing against probabilities of acquisition, engagement, hit, etc. This representation allows us to easily compute offline the distribution of the number of units of a given type that will be available for nuclear engagement for any pair of POTA and retention time. We can use this to determine the effect of variations in the POTA and the retention time on the acquisition, engagement, and defeat of various units without running NUFAM III.

c. The accuracy of the NUFAM III output can be enhanced when the input values $\overline{S}$, $\overline{M}$ and R (the stay time, move time and retention time respectively) are computed for each type unit using the equations documented in this paper. This permits the NUFAM III model to distinguish between different types of target units and different sensor capabilities, yielding a correct representation of the probability that a unit can be engaged with a nuclear weapon.

d. This technique may also be applied to support the nuclear effects module for the IWFORCEM model, NEMESIS, until more accurate data and methodology are available. We can compute the acquired target sets that are eligible for engagement by computing p_{avail} as stated above. The probability that acquired units are hit may be calculated using p_{hit}.

APPENDIX A

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APPENDIX B

GUIDANCE TO NUFAM III USERS

Section I. PARAMETERS

B-1. INTRODUCTION

In this appendix, we discuss how to determine the values for the average stay time, the travel time, and the retention time for each type unit input to NUFAM III.

B-2. PHYSICAL PARAMETERS PROVIDED BY DOCTRINE OR THE SCENARIO

$\overline{S}$ = the scenario average time the type unit will remain stationary.

$\overline{M}$ = the scenario average time that a unit will be moving from stationary point to the next stationary point.

δ = the probability that a unit detected while moving is moving into a stationary location.

d = how soon a unit acquired must arrive at its stationary location in order for the stationary location to be known, if the unit is detected while moving (i.e., the maximum interval of time between detection and arrival for the arrival location to be identified).

η = the amount of time between the final observation (time of final fire adjustment) and the detonation when tracking a target unit. This is primarily a function of the command and control time associated with firing a nuclear round from the designated firing unit.

γ = a multiplicative factor that indicates how much better the sensors are, relative to the POTA, at continuing to track a unit previously detected and identified than detecting a new, previously undetected unit.

L = the length of the fire execution cycle.

B-3. NUFAM III INPUT VALUES

$\overline{S}_M$ = AVERAGE.STAY.TIME (by type unit and zone)

$\overline{M}_M$ = TRAVEL.TIME (by type unit)

R = RETENTION.TIME (by type unit)

NOTE: Parameters in paragraph B-2 are stated in terms of hours, which lead to the calculation of $\overline{S}_M$, $\overline{M}_M$, and R in hours. Before entry into the NUFAM III input data base, $\overline{S}_M$, $\overline{M}_M$, and R must be converted to minutes (by multiplying by 60). Also note that NUFAM III will compute S and M for each unit in minutes, using $\overline{S}_M$ and $\overline{M}_M$. The actual values of $\overline{S}_M$ and $\overline{M}_M$ used are not critical; what matters is the ratio $\overline{S}_M$ / ($\overline{S}_M$ + $\overline{M}_M$). Therefore, in order to avoid roundoff problems, the computed values $\overline{S}_M$ and $\overline{M}_M$ should be multiplied by some constant such that $\overline{S}_M$ + $\overline{M}_M$ is approximately equal to 120 min.

Section II. MAJOR ASSUMPTIONS

B-4. ASSUMPTIONS MADE WITHIN THE NUFAM III MODEL

a. POTA values from TAS III represent the independent probability of acquisition during successive, 2-hour glimpse periods.

b. Units are acquired and move independently of each other.

c. The end of the fire planning time, t_p, is set independently of the acquisition and move/stay processes. The fire execution cycle begins at t_p and has a length of L (L = 1 hour in current studies using NUFAM III).

d. The number of 2-hour periods required to detect a unit, is distributed as Geometric(POTA).

e. For a detected unit, the specific time of acquisition in the (2-hour) period detected, is distributed as Uniform (0,2).

f. Retention time, R, is set to a constant value equal to $E(T_R)$ = average retention time.

g. All sensor suites observing target units in zones 1-3 have nearly continuous tracking capability.

h. All sensor suites observing targets in zones 4 and 5 are incapable of tracking after acquisition.

i. Within the model, the duration of stay time for a unit is distributed as Uniform($\frac{2}{3}\bar{S}_M$, $\frac{4}{3}\bar{S}_M$), where $\bar{S}_M$ = average stay time input to NUFAM. The scenario stay time S is assumed to be distributed as Uniform($\frac{2}{3}\bar{S}$, $\frac{4}{3}\bar{S}$) where $\bar{S}$ = average stay time for the type unit in the scenario.

j. Within the model, the duration of move time for a unit is distributed as Uniform($\frac{2}{3}\bar{M}_M$, $\frac{4}{3}\bar{M}_M$), where $\bar{M}_M$ = average move time input to NUFAM. The scenario move time M is assumed to be distributed as Uniform($\frac{2}{3}\bar{M}$, $\frac{4}{3}\bar{M}$) where $\bar{S}$ = average stay time for the type unit in the scenario.

B-5. ASSUMPTIONS MADE WITHIN THIS PAPER

a. For a unit detection, in a 2-hour period, which is cued by a detection in the preceding 2-hour period, Pr[detect during this cycle | detection previous cycle] = $\gamma \cdot$ POTA, $1 \leq \gamma \leq 1/$POTA. We assume that γ has a Uniform (1, 1/POTA) distribution.

b. The number of (whole) 2-hour periods after initial acquisition, during which a cued acquisition is retained, is distributed as Geometric (1 - $\gamma \cdot$ POTA)

c. The average retention time, $E[T_R]$, has a maximum value of 36 hours in NUFAM because any T_R exceeding 36 hours is equivalent to an arbitrarily long time in a NUFAM simulation.

B-6. ASSUMPTIONS MADE IN NUFAM III RELATING TO THE ENGAGEMENT OF A UNIT BY NUCLEAR WEAPONS

a. In order for a unit to be available for engagement (subject to fire) by nuclear weapons, it must be on the acquisition list at the end of the fire planning time (time t_p) and it must retained as a target at least until the detonation time T_d. We define the probability P[unit is acquired and retained until detonation] as p_{avail}. Thus

$$p_{avail} = P[\, Y_R > T_d - t_p \mid \text{acquired at } t_p \,] \cdot P[\, \text{acquired at } t_p \,].$$

$$p_{avail} = \int_0^D P[\, T_R > t_d - t_p \mid T_d - t_p = t \,] \, P[\, T_d - t_p = t \,] \, dt \cdot p_{acq}.$$

b. The probability that a unit can be fired at by a nuclear weapon, denoted as p_{fired}, is the joint probability that ⴰ unit is on the acquisition list, is not dropped from the acquisition list prior to detonation, and is stationary at the time of last observation. This probability calculation implicitly assumes that a weapon of the appropriate yield is available and will be fired at the target if it meets these criteria.

$$p_{fired} = p_{acq} \cdot \int_0^D P[\, T_R > t_d - t_p \mid T_d - t_p = t \,] \, P[\, T_d - t_p = t \,] \, dt \cdot p_{last\ stay}$$

where $p_{last\ stay} = P[\, \text{unit was stationary at the time it was last observed} \,]$.

c. The probability that a unit is hit, denoted as p_{hit}, is the joint probability that a unit is on the acquisition list, is not dropped from the acquisition list prior to detonation, is stationary at the time of last observation, and does not move from the last computed DGZ. This calculation implicitly assumes that a weapon is available for firing at the unit, the weapon is fired, and both the weapon system and warhead function correctly. Thus

$$p_{hit} = p_{avail} \cdot p_{no\ move},$$

$$p_{hit} = p_{acq} \cdot \int_0^D P[\, T_R > t_d - t_p \mid T_d - t_p = t \,] \, P[\, T_d - t_p = t \,] \, dt \cdot p_{last\ stay} \cdot \frac{p_{no\ move}}{p_{last\ stay}},$$

$$p_{hit} = p_{fired} \cdot \frac{\overline{S}_M}{\overline{S}_M + \overline{M}_M}, \qquad\qquad [29]$$

where p_{fired} is computed as given in the previous section and $\overline{S}_M$ and $\overline{M}_M$ are the values entered into the NUFAM III data base for the unit stay time and move time respectively.

Section III. TARGETS ENGAGED WHILE STATIONARY
THAT FACE NO TRACKING CAPABILITY

B-7. SCENARIO. The type unit is any type other than those identified as an "engaged while moving" type. We assume that the target is sufficiently deep relative to the FLOT such that there is no capability to continuously (or intermittently with a short duty cycle) monitor or track the target. The target is acquired while stationary or is acquired in movement toward an identified stationary location (e.g., assembly area), where d represents the time remaining from the time of acquisition until it stops at this stationary location. We only successfully engage the unit if it remains in place at the point at which they were acquired. We assume a small d such that $d \leq \frac{2}{3} \bar{M}$.

B-8. ZONES. NUFAM III zones 4 and 5.

B-9. STAY TIME AND MOVE TIME.

$\bar{S}_M = \bar{S}$, the scenario average time the type unit will remain stationary.

$\bar{M}_M = \bar{M}$, the scenario average time that a unit will be moving from stationary location to stationary location.

B-10. PROBABILITY THAT A UNIT WAS STATIONARY AT THE TIME OF LAST OBSERVATION. This probability is equal to the probability that the unit was stationary at the time of acquisition or that it was moving but within d minutes of stopping at a definable stationary location. Thus

$$\text{PROB.STAY.LOBS} = \frac{\bar{S}}{\bar{S} + \bar{M}} + \delta \cdot \frac{\bar{M}}{\bar{S} + \bar{M}} = \frac{\bar{S}}{\bar{S} + \bar{M}} \cdot \left[1 - \frac{d}{\bar{M}} \right] + \frac{d}{\bar{M}} \cdot$$

B-11. RETENTION TIME. The retention time is an estimate of the length of time that the target unit will remain at the location at which it was acquired.

$$R = \frac{14}{27} \bar{S} + \bar{S} \cdot \frac{d}{\bar{M}} = \bar{S} \cdot \left[\frac{14}{27} + \frac{d}{\bar{M}} \right] \cdot$$

B-12. ALTERNATE REPRESENTATION. Another representation does not use the probability that the unit was stationary at the time of last observation (the same as setting PROB.STAY.LOBS equal to 1). It is not quite as accurate as the preceding, but does not require the code associated with a check of PROB.STAY.LOBS. As a result, we need to calculate the *unconditional* retention time expectation $E[T_R]$ rather than the conditional expectation $E[T_R \mid \text{stay}]$. The formula for $E[T_R]$ is:

$E[T_R] = E[Y_S \mid \text{stationary at time of acquisition}] \cdot P[\text{stationary}]$

$+ E[S \mid \text{arriving at a stationary location at time of acquisition while moving}]$

$\cdot P[\text{arriving at a stationary location at time of acquisition} \mid \text{moving}] \cdot P[\text{moving}]$

Thus $R = E[Y_S] \cdot \dfrac{\bar{S}}{\bar{S} + \bar{M}} + E[S] \cdot \delta \cdot \dfrac{\bar{M}}{\bar{S} + \bar{M}} = \dfrac{\bar{S}}{\bar{S} + \bar{M}} \cdot \left[\frac{14}{27} \bar{S} + d \right] \cdot$

This formula should only be used if PROB.STAY.LOBS = 1.

Section IV. TARGETS ENGAGED WHILE MOVING
THAT FACE NO TRACKING CAPABILITY

B-13. SCENARIO. The type unit is identified as a "engaged while moving" type. We assume no capability to continuously monitor or track the location of a target unit acquired as part of a column of units moving along a specified route. The nuclear weapon is targeted at the location of the unit when acquired -- because the unit is one of many moving along that route. it is anticipated that a similar unit will be within the radius of effects of the weapon when it detonates.

B-14. ZONES. NUFAM III zones 4 and 5.

B-15. STAY TIME AND MOVE TIME.

$$\overline{S}_M = \frac{\text{length of each unit in column (km)}}{\text{speed of column (km/hr)}}$$

$$\overline{M}_M = \frac{\text{length of gaps between units in the column (km)}}{\text{speed of column (km/hr)}}.$$

NOTE: Since $\overline{M}_M$ must be the same for all zones (only one entry is allowed for each type unit), $\overline{S}_M$ for each type unit will normally be the same for all zones as well.

B-16. PROBABILITY THAT A UNIT WAS STATIONARY AT THE TIME OF LAST OBSERVATION. This probability is not applicable in this situation; we ensure that NUFAM III does not consider it by setting the input value to 1.

PROB.STAY.LOBS = 1

B-17. RETENTION TIME. The retention time is an estimate of the length of time that other target units will be moving through the location at which the unit was acquired.

$$R = \frac{(\text{ Column length in km }) / 2}{\text{speed of column (km/hr)}}$$

Section V. TARGETS FACING A NEARLY CONTINUOUS TRACKING CAPABILITY
(WITH SCHEDULED FIRE)

B-18. SCENARIO. The unit is located relatively close to the FLOT. We assume that we can continuously (or intermittently with a short duty cycle) monitor or track the location of an acquired target. The desired ground zero (DGZ) of the weapon is shifted along with the unit until the final adjustment is made. Let η denote the amount of time between the final observation (time of final fire adjustment) and the detonation. We assume a small $\eta < \min\{ \frac{2}{3} \overline{S}, \frac{2}{3} \overline{M} \}$, where $\overline{S}$ and $\overline{M}$ are defined below. From another perspective, the unit (whether stationary or moving) remains stationary *with respect to the DGZ* during this entire tracking period. The time of detonation, T_d, is scheduled (set) during the time of fire planning.

B-19. ZONES. All units in NUFAM III zones 1, 2, and 3 engaged with scheduled fire.

B-20. STAY TIME AND MOVE TIME.

Let $\overline{S} = $ the scenario average time the type unit will remain stationary.

Let $\overline{M} = $ the scenario average time that the type unit will be moving from stationary location to stationary location.

Let $\overline{S}_M$ and $\overline{M}_M$ denote the stay and move times actually used as NUFAM III input.

η denotes the amount of time between the final observation (time of final fire adjustment) and the detonation.

$$\overline{M}_M = \eta$$

$$\overline{S}_M = \overline{S} - \eta$$

B-21. PROBABILITY THAT A UNIT WAS STATIONARY AT THE TIME OF LAST OBSERVATION.

$$\text{PROB.STAY.LOBS} = P_{last\ stay} = \frac{\overline{S}}{\overline{S} + \overline{M}} .$$

B-22. RETENTION TIME. The retention time is an estimate of how long the target can be tracked using the same sensors that initially detected the unit. Let γ be a factor that indicates how much better the sensors are at detecting a target previously detected and identified than detecting a new, previously undetected target unit, where $1 \leq \gamma \leq 1/\text{POTA}$, and all values of γ between 1 and $1/\text{POTA}$ are considered equally likely.

POTA < 0.97:

$$R = \frac{\text{POTA} - 0.97}{(1 - \text{POTA})^2} \ln\left[\frac{1 - 0.97}{1 - \text{POTA}} \right] + \frac{1.08}{1 - \text{POTA}} \quad \text{(in hrs)}.$$

POTA ≥ 0.97: $R = 36$ hrs.

Section VI. TARGETS FACING A NEARLY CONTINUOUS TRACKING CAPABILITY
(WITHOUT SCHEDULED FIRE)

B-23. SCENARIO. The unit is located relatively close to the FLOT. We assume that we can continuously (or intermittently with a short duty cycle) monitor or track the location of an acquired target. The desired ground zero (DGZ) of the weapon is shifted along with the unit until the beginning of the fire execution cycle, if the unit is stationary at the time that the fire execution cycle begins, or until the first time that the unit stops during the fire execution cycle, if it was moving at the time of the beginning of the fire execution cycle and it does stop during the fire execution cycle. At that time, the time of the detonation, T_d, is scheduled. Let η denote the amount of time between the final observation (time of final fire adjustment) and the time of detonation. From another perspective, the unit (whether stationary or moving) remains stationary *with respect to the DGZ* during this entire tracking period.

B-24. ZONES. All units in NUFAM III zones 1, 2, and 3 engaged without scheduled fire.

B-25. STAY TIME AND MOVE TIME.

Let $\overline{S}$ = the scenario average time the type unit will remain stationary.

Let $\overline{M}$ = the scenario average time that a unit will be moving from stationary location to stationary location.

Let $\overline{S}_M$ and $\overline{M}_M$ denote the stay and move times actually used as NUFAM III input.

η denotes the amount of time between the final observation (time of final fire adjustment) and the detonation.

L denotes the length of the fire execution cycle.

$$\overline{S}_M = P[\, Y_S > \eta\,] \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\, S > \eta\,] \cdot P[\, Y_m < L-\eta\,] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}},$$

$$\overline{M}_M = (\, 1-P[\, Y_S > \eta\,]\,) \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} + (\, 1-P[\, S > \eta\,]\,) \cdot P[\, Y_m < L-\eta\,] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}},$$

where

$$P[\, Y_S > \eta\,] = 1 - \frac{y}{\overline{S}}, \qquad\qquad 0 \le y < \tfrac{2}{3}\overline{S},$$

$$= \tfrac{4}{3} - \frac{1}{\overline{S}}\left[\, 2y - \frac{3\,y^2}{4\,\overline{S}}\,\right], \qquad \tfrac{2}{3}\overline{S} \le y \le \tfrac{4}{3}\overline{S}.$$

$$P[\, S > \eta\,] \;\; = 1, \qquad\qquad 0 \le u < \tfrac{2}{3}\overline{S},$$

$$= 2 - \frac{3u}{2\overline{S}}, \qquad\qquad \tfrac{2}{3}\overline{S} \le u \le \tfrac{4}{3}\overline{S}.$$

$$P[\, Y_M < L-\eta\,] = \frac{L-\eta}{\overline{M}}, \qquad\qquad 0 \le L-\eta < \tfrac{2}{3}\overline{M},$$

$$= \frac{1}{\overline{M}}\left[\, 2\,(L-\eta) - \frac{3\,(L-\eta)^2}{4\,\overline{M}}\,\right] - \tfrac{1}{3}, \qquad \tfrac{2}{3}\overline{M} \le L-\eta \le \tfrac{4}{3}\overline{M}.$$

B-26. PROBABILITY THAT A UNIT WAS STATIONARY AT THE TIME OF LAST OBSERVATION.

$$\text{PROB.STAY.LOBS} = \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\, Y_m < L - \eta \,] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}}$$

B-27. RETENTION TIME. The retention time is an estimate of how long the target can be tracked using the same sensors that initially detected the unit. Let γ be a factor that indicates how much better the sensors are at detecting a target previously detected and identified than detecting a new, previously undetected target unit, where $1 \le \gamma \le 1/\text{POTA}$, and all values of γ between 1 and $1/\text{POTA}$ are considered equally likely.

POTA $\le$ 0.97:

$$R = \frac{\text{POTA} - 0.97}{(1 - \text{POTA})^2} \ln\left[\frac{1 - 0.97}{1 - \text{POTA}}\right] + \frac{1.08}{1 - \text{POTA}} \quad \text{(in hrs)}.$$

POTA $>$ 0.97: $R = 36$ hrs.

NOTE: When NUFAM III determines if a target unit will be retained at least until the time of detonation, the P[unit retained until time T_d | unit on acquisition list at time T_f] should equal 1 for units in this category. The current code does not distinguish between this type of unit and the scheduled fire type of unit, so the probability P[unit retained until time T_d | unit on acquisition list at time T_f] = P[$Y_R > \eta$ | unit on acquisition list at time T_f]. However, the error introduced by this calculation should be very small since R tends to be much greater than η.

B-28. ALTERNATE REPRESENTATION. Another representation does not use the probability that the unit was stationary at the time of last observation (the same as setting PROB.STAY.LOBS equal to 1). It is not quite as accurate as the preceding, but does not require the code associated with a check of PROB.STAY.LOBS. As a result, $\overline{S}_M$ is calculated in the same way as given above but $\overline{M}_M = 1 - \overline{S}_M$:

$$\overline{S}_M = P[\, Y_S > \eta \,] \cdot \frac{\overline{S}}{\overline{S} + \overline{M}} + P[\, S > \eta \,] \cdot P[\, Y_m < L - \eta \,] \cdot \frac{\overline{M}}{\overline{S} + \overline{M}},$$

$$\overline{M}_M = 1 - \overline{S}_M.$$

APPENDIX C

PROBABILITY DISTRIBUTIONS

C-1. GEOMETRIC DISTRIBUTION

Suppose that $X \sim$ Geometric(p). Then

$$P[\, X = x \,] = p \, (\, 1-p \,)^x, \qquad x = 0, 1, \ldots$$

The expectation $E[\, X \,] = \dfrac{1-p}{p}$.

C-2. STAY TIME DISTRIBUTION

Uniform ($\frac{2}{3}\overline{S}$, $\frac{4}{3}\overline{S}$), where $\overline{S}$ is the Average Stay Time.

$$P[\, S > u \,] = \overline{F}_S (u) = 1, \qquad\qquad\qquad 0 \leq u < \tfrac{2}{3}\overline{S},$$

$$= \frac{\tfrac{4}{3}\overline{S} - u}{\tfrac{4}{3}\overline{S} - \tfrac{2}{3}\overline{S}} = 2 - \frac{3u}{2\overline{S}}, \qquad \tfrac{2}{3}\overline{S} \leq u \leq \tfrac{4}{3}\overline{S}.$$

C-3. RESIDUAL STAY TIME DISTRIBUTION

Let Y_S denote the residual life of the stay time. Then

$$P[\, Y_S > y \,] = 1 - \frac{1}{E[S]} \int_0^y \overline{F}_S(u)\, du = 1 - \frac{1}{E[S]} \int_0^y 2 - \frac{3u}{2\overline{S}}\, du,$$

$$= 1 - \frac{y}{\overline{S}}, \qquad\qquad\qquad 0 \leq y < \tfrac{2}{3}\overline{S},$$

$$= \frac{4}{3} - \frac{1}{\overline{S}}\left[\, 2y - \frac{3\,y^2}{4\,\overline{S}} \,\right], \qquad \tfrac{2}{3}\overline{S} \leq y \leq \tfrac{4}{3}\overline{S}.$$

C-4. EXPECTATION OF THE RESIDUAL STAY TIME

$$E[Y_S] = \int_0^\infty y \cdot f_{Y_S}(y)\, dy = \int_0^\infty y \cdot \frac{\overline{F}_S(y)}{E[S]}\, dy,$$

$$= \frac{1}{E[S]} \int_0^{\frac{2}{3}\overline{S}} y \cdot 1 \, du + \frac{1}{E[S]} \int_{\frac{2}{3}\overline{S}}^{\frac{4}{3}\overline{S}} 2y - \frac{3y^2}{2\overline{S}}\, dy = \frac{14}{27}\overline{S}.$$

C-5. MOVE TIME DISTRIBUTION

Uniform ($\frac{2}{3}\bar{M}$, $\frac{4}{3}\bar{M}$), where $\bar{M}$ is the Average Stay Time.

$$\bar{F}_M(u) = \frac{\frac{4}{3}\bar{M} - u}{\frac{4}{3}\bar{M} - \frac{2}{3}\bar{M}} = 2 - \frac{3u}{2M}, \quad \frac{2}{3}\bar{M} \le u \le \frac{4}{3}\bar{M}.$$

C-6. RESIDUAL MOVE TIME DISTRIBUTION AND EXPECTATION

Let Y_M denote the residual life of the move time. Since S and M have the same distribution, so do Y_S and Y_M; simply replace $\bar{S}$ with $\bar{M}$ in the formula in paragraph C-3.

C-7. COMPUTING $P[\,Y_S > T_d - T_{acq}\,]$

The time T_{acq} is not known, but the difference ($t_p - T_{acq}$) is distributed as the age of the distribution of the time of acquisition, Y_{RA} (the age and the residual life are identically distributed), independent of the distribution of $T_d - t_p$. $T_d - t_p$ can have any nonnegative distribution; in this case, a uniform(0, L) distribution is assumed, where L is the length of the fire execution cycle. Thus

$$P[\,Y_S > T_d - T_{acq}\,] = \left\{ \int_{y=0}^{\infty} \int_{t=0}^{D^*} P[Y_S > (T_d - t_p) + (t_p - T_{acq}) \mid T_d - t_p = t;\ (t_p - T_{acq}) = y] \right.$$
$$\left. \cdot\ P[T_d - t_p = t]\ P[t_p - T_{acq} = y]\ dt\ dy \right\},$$

where $D^* = \min\{\, L, \frac{4}{3}\bar{S}\, \}$

$$P[\,Y_S > t + y\,] = 1 - \frac{t+y}{\bar{S}}, \qquad\qquad 0 \le t+y < \frac{2}{3}\bar{S},$$

$$= \frac{4}{3} - \frac{1}{\bar{S}}\left[\, 2(t+y) - \frac{3(t+y)^2}{4\bar{S}}\,\right], \quad \frac{2}{3}\bar{S} \le t+y \le \frac{4}{3}\bar{S}.$$

$$P[T_d - t_p = t] \quad = \frac{1}{L}, \qquad\qquad 0 \le t \le L.$$

$$P[t_p - T_{acq} = y] \quad = \frac{P[\,T_R > y\,]}{E[\,T_R\,]} = \frac{1}{R}, \ 0 \le y \le R.$$

NOTE: Since $(T_d - t_p)$ and $(t_p - T_{acq})$ are both distributed as uniform random variables, it is easier computationally to determine the distribution of $W \equiv (T_d - t_p) + (t_p - T_{acq})$ and calculate

$$P[\,Y_S > T_d - T_{acq}\,] = \int_W P[Y_S > W \mid W = w]\ P[\,W = w\,]\ dw .$$

APPENDIX D

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