work has continued during this period on the following projects: (1) The role of regularity in multigrid methods -- computational experiments and analytical studies on the V-cycle in an L-shaped domain; (2) Preconditioning and boundary values -- study of preconditioning of elliptic operators; and (3) Preconditioning, boundary values and mixed mode -- extension of (2) above to hyperbolic operators.
Annual Scientific Report
on
Air Force Office of Scientific Research

Contract No. AFOSR-86-0163
June 15, 1987 - June 14, 1988

Principal Investigator:
Seymour V. Parter
Computer Sciences Department
University of Wisconsin
Madison, WI 53706
I. Objectives of the Program

A major thrust of this project is the study of effective means of solving large systems of linear equations with a particular emphasis on those problems which arise from the discretization of elliptic and parabolic partial differential equations. The primary topics are Multigrid and Preconditioning. The study of preconditioning leads to related studies of Condition Numbers and their behavior as $h$, the discretization parameter, tends to zero.

II. Personnel

A. Senior Investigator: Seymour V. Parter

B. Research Assistant: Naomi H. Decker

Ms. Decker's research was the basis for her Ph.D. thesis. The Ph.D. degree was awarded in August 1987.

III. General Activities

A. Part of the summer (1987) was spent in rewriting, revising and polishing several reports. To be specific we revised:

1. Estimates for Multigrid Methods Based on Red-Black Gauss-Seidel Smoothing (by S. V. Parter). This work was accepted for publication in Numerische Mathematik in early 1988.


3. The Fourier Analysis of a Multigrid Preconditioner (by N. H. Decker). This work has appeared in: Proceedings Third Copper Mountain Multigrid Conference, April 1987.

B. Visit to Courant Institute

Professor Parter spent the fall semester at the Courant Institute, New York University. This visit was motivated (in part) by the desire for close relationships with researchers at this Institute. Of particular interest is the work on Domain Decomposition (Olof Widlund) and Conjugate Gradient Methods (Anne Greenbaum).

IV. Publications: The following reports and/or published papers appeared during this period.


V. Lectures

Professor Parter has given several lectures on this research.


VI. Ongoing Work

Work has been continuing on the following projects:

A. *The Role of Regularity in Multigrid Methods* (N. H. Decker, Jan Mandel and S. V. Parter). The ongoing work on the topic consists of computational experiments and analytical studies. In particular, we have done extensive calculations on the San Diego Cray studying the V-cycle in an L-shaped domain. The theory developed earlier yields bounds on the rates of convergence which approach "one" as the number of grids increases. These calculations show that our bounds are qualitatively correct. Moreover, certain other estimates not emphasized in our earlier published reports seem to be sharp. At the same time we are working on extending our theoretical approach to describe the rates of convergence for the F-cycle in problems without $H_2$ regularity.

B. *Preconditioning and Boundary Values* (T. A. Manteuffel and S. V. Parter). This project is concerned with the following question: Let $\{A_n\}$ be a family of discretizations of an elliptic operator $A$, and let $\{B_n\}$ be a family of discretization of an elliptic operator $B$. Find conditions to guarantee estimates of the form

\[
\| B_n^{-1} A_n \|_{L_2} \leq k, \\
\| B_n^{-1} A_n \|_{H_2} \leq k, \\
\| A_n^{-1} B_n \|_{L_2} \leq k
\]
C. Preconditioning, Boundary Values and Mixed Mode (David Gottlieb and S. V. Parter). This research project is concerned with the extension of the ideas developed and being developed in (B) above to the cases where (i) $A$ is a first order hyperbolic operation and $B$ is an elliptic operator and (ii) $A_n$ is a spectral discretization of $A$ while $B_n$ is a finite-difference discretization of $B$. 