Various extensions of the transmission line model are introduced to find the resistance for current flow in MOSFET source/drain regions. The geometry is taken to be a rectangular box with a rectangular contact on the upper surface. Explicit formulae are derived by assuming that the current flow is restricted to various geometrical planes. Comparison of basic results with simulation and experimental data is good. Comparison with simulation results for misalignment is less good.
THREE-DIMENSIONAL MODELLING FOR CONTACT RESISTANCE OF CURRENT FLOW INTO A SOURCE/DRAIN REGION

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Abstract

Various extensions of the transmission line model are introduced to find the resistance for current flow in MOSFET source/drain regions. The geometry is taken to be a rectangular box with a rectangular contact on the upper surface. Explicit formula are derived by assuming that the current flow is restricted to various geometrical planes. Comparison of basic results with simulation and experimental data is good. Comparison with simulation results for misalignment is less good.

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I. Introduction

Various models have been suggested in attempts to find resistance formulae for current flow in source/drain regions of metal-oxide-semiconductor field-effect transistors (MOSFET's). This parasitic resistance becomes relatively more important under device miniaturization, and its dependence on geometrical and physical parameters is sought. Measurement of resistance across single contacts ([1],[2]) indicate large variability neighbor to neighbor. This portends the use of statistics in estimating the yield of a device, [3], and it has implications for the quality control of the fabrication process. Some of the variability is random, due, say, to the imperfect removal of photo-resist, but some has global features. Chief of the latter derive from misalignment: contact regions have been etched in non-symmetrical locations. In order to estimate the effects of contact size and location some two and three dimensional models are constructed here, resulting in resistance formulae of varying complexity. Results for typical parameter ranges are compared with simulation and with test data.

The simplest model, the transmission line model (TLM), [4], [5], is one-dimensional as it neglects both width and depth variations. Improvements on the TLM have been provided in [6], where depth dependence was included, and in [7]-[9], where modelling of contact width and side tabs was presented.
The physical set-up is shown in Figure 1. The source/drain region is taken to be a rectangular block of width \( w \), depth \( t \). The contact is rectangular of length \( b \), width \( w_c \), and has flange distances \( w_1, w_2, c \) from the source/drain boundary, as shown.

Current, uniformly distributed across the end surface \( S \), flows along the block and exits across the contact. The electrostatic potential, \( \phi \), satisfies a zero normal derivative boundary condition on bounding surfaces except on \( S \), and except on the contact where an ohmic interfacial resistance holds. This gives rise to the boundary condition

\[
\phi_y - \tau \phi = 0 \quad \text{on the contact}
\]

where \( \tau = 1/(\sigma \rho_c) \).

Here \( \sigma \) is the material conductivity and \( \rho_c \) the contact resistivity.

The models developed in this paper are extensions of the TLM, and of the work done in [6]. Model 0 assumes that current enters the contact only from a region directly upstream of the contact, and uses the TLM formula. Model I adds to this the contribution of the flanges. Model II is fully three-dimensional, extending the analysis in [6]. The key assumption is to prescribe that current lines remain on planes parallel to the edges of the box. Current flow between two adjacent planes is then two-dimensional, and the full result is achieved by integration in the third direction. Models III and IV are variants of II taking into account different current distributions.
II. Models for contact resistance

**Model 0** For a contact of width \( w_c \) the series or front resistance, \( R_F \), is given by the TLM

\[
R_s / R_F = (w_c / L) \tanh(b / L), \quad \text{where} \quad L = (t \sigma c)^{1/2}
\]

and \( R_s = \rho_c / L^2 \) is the sheet resistance of the material. Substitution of \( \sinh \) for \( \tanh \) in (2.1) yields the end resistance, \( R_{END} \). Model 0 involves neither the flange widths \( w_1, w_2 \) nor the depth \( t \) (except as in \( L \)). The models introduced below account for these thickness and flange effects. As a result, Model 0 cannot be compared properly in the graphs presented. It is quoted only to introduce the formula (2.1).

**Model 1** The flanges are assumed to generate additional (parallel) resistance, computed by viewing the flange regions as having width \( t \) and depths \( w_1, w_2 \). However, poor results from this at small values of \( w_c \) (where \( R_F \sim w_c^{-2} \) is expected) made us reflect on the current flow interactions. The current lines squeezing in from the flange regions are competing with the center current lines which have depth \( t \). Details of the current flowing in from the sides will not be felt by the contact until that current is at a similar distance from the side edge of the contact. Consequently (and after some experimentation) the flange regions were taken to have depths \( t_1, t_2 \) where

\[
t_1 = \min(w_1, t), \quad t_2 = \min(w_2, t).
\]
The front resistance is then given by

\[ R_s/R_F = (w_c/L) \tanh(b/L) + (w_1/L_1) \tanh(w_c/L_1) + (w_2/L_2) \tanh(w_c/L_2) \]

(2.3) where \( L_i = (t_i\rho_c)^{1/2} \), \( i = 1, 2 \).

**Model II** This is an attempt to construct a three-dimensional approximation to current flow in the complicated geometry of Figure 1. The main assumption is that current lines remain on planes parallel to the x-axis (the planes are not parallel to each other). The planes, and the reduction of the three-dimensional problem to a sequence of two-dimensional ones, are found by the following two prescriptions.

**Prescription (1):** Consider the end surface \( S \) and a line through \( z = r \). The line divides \( S \) into two areas. See Figure 2. Consider the plane defined by extending this line parallel to the x-axis. The assumption that current lines do not cross such planes implies that current entering area \( A \) exits on the portion of the contact on \( w_1 < z < r \). The geometrical relation

\[ (r - w_1)/w_c = A/\text{area of } S = A/tw \]

(2.4) is taken to define the plane through \( z = r \). It follows that there is a point \( r_0 \) where the area \( A \) becomes rectangular, of area \( A = r_0 t \). Hence (2.4) gives

\[ r_0 = w_0/(w - w_c). \]

(2.5)
It is now permissible to concentrate only on the portion of the source/drain region defined by $0 < r < r_0$, as the remaining portion will have the same characteristics under this model, and its resistance can be found by appropriate substitutions. For the rectangle defined by $0 < r < r_0$ the plane passing through the corner $y = t, z = 0$, occurs at $r = r^*$, say, where

$$\text{(2.6)} \quad r^* = \frac{2w w_1}{(2w - w_c)}.$$

(See Figure 3). Planes through $z = r$ for $r^* < r < r_0$ intersect the bottom of the region, giving an intercept $h(r)$ and a plane depth $e_1(r)$ where

$$\text{(2.7)} \quad h = \frac{r(2w - w_c) - 2ww_1}{w_c},$$

$$\text{(2.8)} \quad e_1^2 = t^2 + 4[ww_1 - r(w - w_c)]^2/w_c^2,$$

whilst planes through $z = r$ for $w_1 < r < r^*$ intersect the side of the region with intercept $p(r)$ and plane depth $e_2(r)$ given by

$$\text{(2.9)} \quad p = \frac{2tw(r - w_1)}{(rw_c)},$$

$$\text{(2.10)} \quad e_2^2 = r^2 + 4t^2w^2[1 - w_1/r]^2/w_c^2.$$

Prescription (2): Consider an area element formed between the planes $z = r$ and $z = r + dr$.

These are shown in Figure 4 for the cases $r^* < r < r_0$ and $w_1 < r < r^*$. 

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The area elements are replaced by the sector area element shown in Figure 4(c), of equal area. Note that, from (2.4),

\[(2.11) \quad dA = \left(\frac{wt}{w_c}\right)dr.\]

The current flow in an area element is now assumed to take place in the sector element of Figure 4(c), and the equation to be solved is Laplace’s equation in cylindrical coordinates, assuming angular independence. The analysis for this is given in the Appendix. The resistance for the element \(dA\) given in (A.37) is integrated over \(S\) to give \(R_F\) as

\[(2.12) \quad \frac{R_s}{R_F} = \frac{1}{L} \cdot \sqrt{\frac{w}{w_c}} \left\{ \int_{r_0}^{r} \sqrt{\frac{t}{\varepsilon_1}} \tanh\left(\sqrt{\frac{t}{\varepsilon_1}} \cdot \frac{w_c}{w} \cdot \frac{b}{L}\right) dr \right. \\
+ \int_{\omega_1}^{r} \sqrt{2 \frac{r - w_1}{r} \cdot \frac{t}{\varepsilon_2} \cdot \frac{w}{w_c}} \tanh\left(\sqrt{2 \frac{r - w_1}{r} \cdot \frac{t}{\varepsilon_2} \cdot \frac{b}{L}}\right) dr \right\}\]

where \(\varepsilon_1, \varepsilon_2\) are given in (2.8). Note that the result (2.12) is for resistance in the region \(0 < r < r_0\). If the contact is symmetrically placed, \(r_0 = w/2\), and \(R_s/R_F\) for the whole device will be twice that of (2.12); if not, the expression (2.12) has to include a similar expression obtained by replacing \(w_1\) by \(w_2\).

**Model III** Prescription (1) of Model II implies that an element \(dr\) of the contact width is proportional to \(dA\), an element of \(S\), see (2.11). Since current is uniformly distributed across \(S\) this means that equal elements of the contact receive equal
amounts of current; that is, the current density is uniform across the contact width. However a non-uniform distribution is expected as current entering from the flanges will be confined to a layer close to the sides. An estimate of the non-uniformity is provided by the TLM, that is by Model I. The current density for the region defined the contact, i.e., \( w_1 < r < r_0 \), is taken as uniform. The current coming from the flange, \( 0 < r < w_1 \), is estimated by the TLM, as proportional to

\[
\frac{w_1}{L_1} \cosh \left( \frac{r - r_0}{L_1} \right) \left\{ \sinh \left( \frac{r_0 - w_1}{L_1} \right) \right\}^{-1} \quad \text{for} \quad w_1 < r < r_0
\]

Combining these yields the replacement

\[
dA = t \left[ 1 + \frac{w_1}{L_1} \cosh \left( \frac{r - r_0}{L_1} \right) \left\{ \sinh \left( \frac{r_0 - w_1}{L_1} \right) \right\}^{-1} \right] dr
\]

for (2.11), and an appropriate change in the formula for \( R_F \) in (2.12). Also the calculation for \( r^* \) is changed to the solution to

\[
r^* = 2w_1K(r^*)
\]

where

\[
K(r) = \sinh \left( \frac{r_0 - r}{L_1} \right) \left\{ \sinh \left( \frac{r_0 - w_1}{L_1} \right) \right\}^{-1}
\]

Similar changes are made for the current contribution from the flange \( w_2 \).

**Model IV** The introduction in Model III of a relation between \( dA \) and \( dr \) which is non-uniform also gives \( A \) as a function of \( r \) (by integrating (2.14) ) different from
(2.4). There follows new formulae for $\varepsilon_1$, $\varepsilon_2$ viz.

\[ \varepsilon_1^2 = t^2 + 4w_1^2K^2(r) , \quad \varepsilon_2^2 = r^2 + 4t^2\left(1 - \frac{w_1}{r}K(r)\right)^2 \]

which are incorporated in Model IV.

III. Results Various results derived from the formula (2.12) and the corresponding ones from Models III and IV are shown in Figures 7 to 10. In the TLM the thickness $t$ is parametrically involved only in the definition of $L$ and this carries over to most simulations as the latter are derived from the numerical solution of two-dimensional partial differential equations (in $z = 0$). Loh et al, [9], have carried out some three-dimensional simulations and show differences between 1-D, 2-D, and 3-D results for $R_k$, the Kelvin resistance, which is not appropriate for the model constructed here. There are more simulations for the 2-D case and we use these for comparison. However we note that

(1) we are comparing different models, and

(2) as is evident from formula (2.12), and as noted in [9], Appendix, the thickness $t$ enters the 3-D models as a parameter to be chosen in relation to the other length scales $w_1, w_2, L$. As a consequence we can vary $t$ to see its effect on the results.
Figures 7 (a) – (d) show log($R_F/R_i$) versus log($w_c/w_1$) for $L/w_1 = .358$ for Models I-IV for $t/w_1 = .1, .5, 1, 10$ respectively. Simulation data from [9], Figure 12, is also shown. It is noted that $t/w_1 = .5, 1, 10$ give good comparisons, indicating some insensitivity to $t$ over this range. As above we note that we are comparing different models. Figures 8 (a) – (d) repeat the same results for the case $L/w_1 = .367$.

Figure 9 shows log($R_E/R_i$) versus log($w_c/w_1$) plus simulations from [9], Figure 11, for $t/w_1 = 1$ and for $L/w_1 = .567$ (upper curve), $L/w_1 = .358$ (lower curve). These show reasonable comparisons.

Results relevant to misalignment estimates are shown in Figure 10, where also are drawn the simulation data of [10].

IV. Comments

There are small differences between the results presented here and the simulations in Figures 7,8,9 and substantial ones in those of Figure 10. Apart from the fact that $R_{END}$ is more volatile than $R_F$, there are significant differences in the approaches, which bear comment.
The 2-D simulations are based on solving

\[ L^2 \nabla^2 \phi = \phi \text{ on the contact,} \]
\[ \nabla^2 \phi = 0 \text{ off it} \]

by numerical methods over the plane \( z = 0 \), imposing a constant voltage upstream at a station such as \( S \). This turns out to be equivalent to the TLM. A derivation of (4.1) based on averages across a thin layer is given in [9]. An alternative is as follows. Note that the boundary-value problem posed by (1.1) is equivalent to minimizing the integral

\[ J = \int_{\text{volume}} |\nabla \phi|^2 dv + \tau \int_{\text{contact}} \phi^2 dS. \]  

Now assume at small \( t \) that the volume integral may be approximated by \( t \) times the surface integral and assume that the \( \tau \) term applies only on the contact. The zero variation of \( J \) yields (4.1). We remark in passing that numerical simulations based on (4.2) may be more efficient than 3-D finite difference schemes.

The work in [6] improved the TLM to account for depth effect (to first order). It showed

(a) that (4.1) must be supplemented by a transition layer of width \( L \) on either side of the contact boundary. A more complicated boundary value problem holds in this layer.

(b) that the effect of including depth is substantial on \( R_{E,N,D} \), less so on \( R_F \).
The formulae presented in Section II do not include the depth effects calculated in [6]. The latter could be included: easily in Model I; approximately, and at the expense only of more complicated formula, in Models II-IV. However the models presented do take into account the fact that current lines have a three-dimensional trajectory, and it is argued that they may be more realistic than simulations based on (4.1) which is a two-dimensional approximation.

The divergences between the two approaches as seen in Figure 10 suggest that the modeling is inappropriate for misalignment effects in one or both the approaches. The modelling here assumes current flow along planes parallel to the x-axis. As a consequence the formulae derived involve the contact length $b$ as an argument. In practice, current lines, on approach to a square contact, may react to the proximity of contact side edges as well as the rear edge, thereby reducing $b$ in the argument. This effect shows more dramatically in $R_{ENV}$ which evaluates sinh of arguments than in $R_F$ which evaluates tanh.
APPENDIX

We wish to solve Laplace's equation in cylindrical coordinates, with no angular variation. That is

\[(A.1) \quad \frac{\partial^2 \phi}{\partial r^2} + \left(\frac{1}{r}\right) \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0\]

for the boundary value problem shown in Figure 5. The sector occupies the region \(r_1 < r < r_2\) and \(-\infty < x < \infty\). (The source/drain boundary at \(x = -(b+c)\) is removed in this analysis as its effect on the results is assumed to be weak. An approximation which corrects for this omission can be introduced without difficulty). The sector depth

\[(A.2) \quad \epsilon = r_2 - r_1\]

is assumed to be small, and an asymptotic result based on \(\epsilon \ll 1\) similar to the analysis for the plane case presented in [6] is obtained. That analysis was predicated on the existence of boundary layer solutions in regions close to the ends of the contact, i.e. on planes \(x = 0\) and \(x = -b\). Consequently there are five regions where separate solutions are found (see Figure 6), and these solutions are joined in the conventional approach of the method of matched asymptotic expansions ([11] or [12]). Solutions for the five regions are as follows:

**Region I:** This region contains no current lines and the potential is constant. Hence

\[(A.3) \quad \phi = \alpha + \epsilon r \beta + \cdots,\]

where \(\alpha, \beta\) are constants to be found subsequently.
Region II: In terms of scaled variables for the region near the contact end

\[(A.4) \quad m = (x + b)/\varepsilon, \quad n = (r - r_1)/\varepsilon,\]

an asymptotic expansion for the solution is

\[(A.5) \quad \phi = \alpha + \varepsilon^r [\beta + \alpha \phi_1(m, n)] + \cdots\]

The perturbation potential, \(\phi_1\), satisfies boundary conditions

\[(A.6) \quad \partial \phi_1 / \partial n = 0 \quad \left\{ \begin{array}{l} \text{on } n = 0, m < 0 \\ \text{on } n = 1, \text{all } m \end{array} \right. \]

\[\text{and } \partial \phi_1 / \partial n = 1 \quad \text{on } n = 0, m > 0.\]

In the plane case, [6], an explicit integral expression was found for \(\phi_1\); here a series expansion is obtained.

In \(m < 0\)

\[(A.7) \quad \phi_1 = \sum_N \varepsilon^{Nm} F_N(s) \quad \text{where } s = n + r_1/\varepsilon,\]

\[(A.8) \quad \text{and where } F_N'' + (1/s) F_N' + N^2 F_N = 0.\]

Hence

\[(A.9) \quad F_N = A_N J_0(Ns) + B_N Y_0(Ns).\]

The boundary conditions that \(F_N = 0\) on \(n = 0\) and \(n = 1\) determine the eigenvalues \(N\) and the ratios \(A_N : B_N\). The purposes for which we require the solution do not require us
to complete this part of the calculation.

In $m > 0$

The condition that the boundary condition (A.6) yields the correct current exiting the contact implies that $\phi_1$ is $O(m^2)$ as $m \to \infty$, and that

\begin{equation}
(A.10) \quad \phi_1 = k_1 [m^2 - s^2/2] + k_2 \ln s + k_3 + \sum_N e^{-Nm} G_N(s),
\end{equation}

\begin{equation}
(A.11) \quad \text{where} \quad k_1 = e r_1 / (r_2^2 - r_1^2), \quad k_2 = r_1 r_2^2 / (r_2^2 - r_1^2).
\end{equation}

The solution for the eigenfunctions $G_N$ is as in (A.9) with constants again determined (in ratio) by the boundary conditions. The continuity of $\phi_1$ and $\phi_{1m}$ across $m = 0$ is required to supply sufficient conditions to fix the constants in (A.9) and similar ones in (A.10).

There remains the constant $k_3$ in (A.10) to be determined. In the plane case, [6], this constant was determined as a consequence of the availability of an exact result. However it was noticed there that the constant could also be determined from the physical principal that the average potential increase in the $x$-direction is a consequence of current flow across the contact. This latter condition is invoked here and gives

\begin{equation}
(A.12) \quad k_3 = (r_1 / 4 e \Delta^2) \left\{ (1 + r_1 / e)^4 - (r_1 / e)^4 \right\}
\end{equation}

\begin{equation}
-2(1 + r_1 / e)^2 \left[ 2(1 + r_1 / e)^2 \ln(1 + r_1 / e) - 2(r_1 / e)^2 \ln(r_1 / e) - \Delta \right]\}
\end{equation}

\begin{equation}
(A.13) \quad \text{where} \quad \Delta = (1 + r_1 / e)^2 - (r_1 / e)^2
\end{equation}
Note that the series terms in (A.10) are exponentially decreasing as $x$ increases into the contact region, and the non-series terms carry the dominant information concerning the current flow to be matched with the solution in Region III.

**Region III:** The scaling relevant here is

\[(A.14)\]
\[\xi = (r/e)^{1/2} x, \quad \eta = (r - r_1)/\epsilon.\]

The analysis is a little more sophisticated than that for Region II, so we present more of the details. The differential equation (A.1) becomes

\[(A.15)\]
\[\phi_{\eta\eta} + \phi_\eta/(\eta + r/\epsilon) = -\tau e\phi_{\xi\xi},\]

and the boundary condition (2.1) becomes

\[(A.16)\]
\[\phi_\eta = \tau e\phi.\]

The asymptotic series is taken as

\[(A.17)\]
\[\phi = \phi_0 + \epsilon r\phi_1 + \epsilon^2 r^2\phi_2 + \cdots\]

so that the first two terms satisfy

\[(A.18)\]
\[\phi_{\eta\eta} + a\phi_\eta/(\eta + r_1/\epsilon) = 0\]

with boundary conditions

\[(A.19)\]
\[\phi_\eta = 0 \quad \text{at } \eta = 0 \text{ and } \eta = 1,\]
(A.20) \[ \phi_{1\eta\eta} + \phi_{1\eta}/(\eta + r_1/e) = -\phi_0 \xi, \]

with boundary conditions

(A.21) \[ \phi_{1\eta} = \phi_0 \text{ at } \eta = 0, \text{ and } \phi_{1\eta} = 0 \text{ at } \eta = 1. \]

The solution of (A.17) and (A.18) is

(A.22) \[ \phi_0 = M(\xi) \]

and that of (A.19)

(A.23) \[ \phi_1 = M''(\eta + r_1/e)^2/4 + N(\xi)ln(\eta + r_1/e) + P(\xi), \]

with boundary conditions (A.21) giving

(A.24) \[ N\epsilon/r_1 - M''r_1/2\epsilon = M \]

and \[ N/(1 + r_1/e) - M''(1 + r_1/e)/2 = 0 \]

Hence, eliminating \( N \),

(A.25) \[ M''\Delta = M2r_1/\epsilon \]

The solution of (A.25) which matches with the leading term from (A.10) for Region II gives

(A.26) \[ \phi_0 = \alpha \cosh[\mu(\xi + \Gamma)] \]
(A.27) \[ \Gamma = (\tau/e)^{1/2}b, \quad \mu = (2r_1/e\Delta)^{1/2}. \]

The solution for \( \phi_1 \) is found in similar fashion:

(A.28) \[ \phi_1 = (1/2)\alpha \mu^2 \{(1 + r_1/e)^2 ln(\eta + r_1/e) - (\eta + r_1/e)^2/2\} \cosh \mu(\xi + \Gamma) \]
\[ + k_4 \alpha (\xi + \Gamma) \sinh \mu(\xi + \Gamma) + (\beta + \alpha k_3) \cosh \mu(\xi + \Gamma), \]

where

\[ \Delta k_4 = \mu^2[(1 + r_1/e)^4 - (r_1/e)^4]/16 - \mu (r_1/e)^3/4 \]

(A.29) \[ -\mu^2(1 + r_1/e)^2[(1 + r_1/e)^2 ln(1 + r_1/e) - r_1/e)^2 ln(r_1/e) - \Delta/2]4 \]
\[ + (1/\mu\Delta)(r_1/e)^2(1 + r_1/e)^2 ln(r_1/e) \]

Region IV: The scaling used near the contact leading edge is

(A.30) \[ p = x/e, \quad q = (r - r_1)/e. \]

Matching with the solution from Region III indicates that a series in powers of \((e)^{1/2}\) is required, and the solution is developed as

(A.31) \[ \phi = \alpha \cosh \mu \Gamma + (\tau e)^{1/2} \alpha \mu p \sinh \mu \Gamma + \tau e \phi_2(p, q) + \cdots \]

Here \( \phi_2 \) is harmonic and satisfies the boundary conditions

(A.32) \[ \partial \phi_2/\partial q = 0 \quad \text{on } q = 0, \ p > 0 \]
\[ \partial \phi_2/\partial q = 0 \quad \text{on } q = 1, \ \text{all } p, \]

and \[ \partial \phi_2/\partial q = \alpha \cosh \mu \Gamma \text{ on } q = 0, \ p < 0. \]
The solution for $\phi_2$ is

\begin{equation}
(\text{A.33}) \quad \phi_2 = \alpha k_4 \Gamma \sinh \mu \Gamma + \{\beta + \alpha k_3 + \alpha \mu^2[p^2/2 - (q + r_1/e)^2/4]

+ (1/2)(1 + r_1/e)^2 \ln(q + r_1/e)] \} \cosh \mu \Gamma + \alpha \sum_{N} e^{\nu \Gamma} H_N(q + r_1/e) \quad \text{for } p < 0,
\end{equation}

and

\begin{equation}
\phi_2 = (\beta + \alpha k_3) \cosh \mu \gamma + k_4 \alpha \Gamma \sinh \mu \Gamma + \alpha \sum_{N} e^{-\nu \Gamma} I_N(q + r_1/e) \quad \text{for } p > 0.
\end{equation}

The terms $H_N, I_N$ in the infinite series above are linear combinations of Bessel functions and are evaluated in a similar fashion to those in the Region II solutions. However, their exact nature is not required.

**Region V:** The asymptotic behaviour of the Region IV solution as $p \to \infty$ provides the solution here. This is found to be

\begin{equation}
(\text{A.34}) \quad \phi = (\tau/e)^{1/2} x \alpha \tau \sinh \mu \Gamma + \alpha \cosh \mu \Gamma + \tau e[(\beta + \alpha k_3) \cosh \mu \Gamma + (1/2)k_4 \alpha \Gamma \sinh \mu \Gamma] + \cdots
\end{equation}

The first term in (A.34) represents the uniformly distributed current.

This completes the analysis for generating and matching the solutions in Regions I-V. The constants $\alpha, \beta$ assumed for the solution in Region I, equation (A.3), may now be specified. If the total current in Region V is $I$, and the cross-sectional area is $S$, the current density is $I/S$. Equating this with $\sigma \partial \phi/\partial x$ from (A.34) yields

\begin{equation}
(\text{A.35}) \quad \alpha = (I/S)\{\sigma(\tau/e)^{1/2} \mu \sinh \mu \Gamma\}^{-1}.
\end{equation}
(A.36) \[ \beta = -\frac{k_1 \Delta \alpha}{4r_1} (1 + \mu \Gamma \tanh \mu \Gamma) \]

The resistance of an element \(dS\) may now be calculated. The end resistance is related to the potential at \(x = -b\), and the front resistance to that at \(x = 0\). Hence

\[
R_{END} = \frac{(\alpha + \epsilon \tau \beta + \cdots)}{(\frac{1}{2}dS)}
\]

\[
= \left\{ \sigma (\tau / \epsilon)^{1/2} \mu \sinh \mu \Gamma dS \right\}^{-1}
\]

to first order. Likewise \(R_F\) has a similar form, replacing \(\sinh\) by \(\tanh\).
References:


FIGURE 1 Physical set-up.

FIGURE 2 End surface $S$.

FIGURE 3 End surface for $0 < z < r_0$.

(a) Area element for $r<r_0$
(b) Area element for $w_1<r<r^*$
(c) Replacement element

FIGURE 4

FIGURE 5 Boundary value problem.

FIGURE 6 Regions for separate asymptotic expansions.
FIGURES 7 (a) – (d) Graphs of log($R_F/R_s$) versus log($w_c/w_1$) for Models I, II, III, IV, for $t/w_1 = .1, .5, 1, 10$, respectively, plus simulation data from [9]. The graphs may be interpreted as keeping $t$ and $w_1$ fixed and varying $w_c$.

FIGURES 8(a) – (d) Same as Figure 7 with $L/w_1 = .567$.

FIGURE 9 Graph of log($R_{END}/R_s$) versus log($w_c/w_1$) for $t/w_1 = 1$ and for $L/w_1 = .567$ (upper curve), $L/w_1 = .358$ (lower curve).

FIGURE 10 Graph of $R_E$ versus $w_1$ to show the effect of misalignment. The results are for a square contact with $w_c = 5 \mu m$ and $R_s = 20 \Omega/\square$ and for $t = .5, 1 \mu m$. The simulation data is taken from [10], Figure 4.
Model I
Model II
Model III
Model IV
Simulation Data

\[ \frac{R_f}{R_s} \]

[FIGURE 7a] L/W1 = 0.358  t/W1 = 0.1

\[ \text{Log}(T_c/T_0) \]
Figure 7b

$\log\left(\frac{R_f}{R_s}\right)$ vs $\log\left(\frac{W_c}{W_1}\right)$

- Model I
- Model II
- Model III
- Model IV

Simulation Data

$L/W_1 = 0.358$  \hspace{1cm}  $t/W_1 = 0.5$
Figure 7c

Log($R_f/R_s$) vs Log($\gamma_c/\gamma_1$)

--- Model I
--- Model II
--- Model III
--- Model IV

Simulation Data

$L/W_1 = 0.358$  \hspace{1cm}  $t/W_1 = 1.0$
FIGURE 7d

Log(\(R_f/ks\))

Log(\(\tau_c/\tau_1\))

--- Model I
--- Model II
--- Model III
--- Model IV

\(L/W_1=0.358\) \hspace{1cm} \(t/W_1=10.0\)

Simulation Data
FIGURE 8a
FIGURE 8b

$\log(Tc/Y_1)$

Model I
Model II
Model III
Model IV
Simulation Data

$L/W_1=0.567$ $t/W_1=0.5$
\[ \log\left(\frac{W_f}{W_s}\right) \]

Model I
Model II
Model III
Model IV

Simulation Data

\[ L/W_1 = 0.567 \quad t/W_1 = 1.0 \]

FIGURE 8c
Model I
Model II
Model III
Model IV

Simulation Data

L/W1 = 0.567  \quad t/W1 = 10.0

FIGURE 8d
\[ \text{Log}(\frac{\text{Re}}{\text{Rs}}) \]

\[ \text{Log}(\frac{T_c}{T_1}) \]

\[ t/W_1 = 1.0 \]

\[ L/W_1 = 0.567, 0.358 \]

\[ \text{FIGURE 9} \]
Model II with $t=1.0$

Model II with $t=0.5$

Simulation Data from [10]

FIGURE 10