DECREASING NONCONFORMANCE OF PARTS
IN THE AIR FORCE SUPPLY SYSTEM

THESIS

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Captain, USAF

AFIT/GOR/MA/88D-6

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Wright-Patterson Air Force Base, Ohio

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Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
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Master of Science in Operations Research

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Preface

I believe that the Air Force needs to find a logical way to lower the nonconformance rate of its parts to an appropriate level. I use the phase 'appropriate level' because I do not believe that the Air Force should try to decrease nonconformity to zero; the cost to do this would be unrealistically high. The Air Force needs a framework for balancing the costs associated with nonconforming parts against the costs associated with decreasing that nonconformance. I believe this study has taken a step in the right direction to do just that.

In developing this model and writing this thesis, I have had help from many people. My thesis advisor, Capt Joe Tatman, provided many insights into the problem. His enthusiasm and optimism were much needed and greatly appreciated. My appreciation is also extended to Maj Joe Litko and Maj Ken Bauer for their advice. Their ideas helped to make my work better. I also wish to thank my thesis sponsor, Bruce McKalip, from the headquarters of the Air Force Logistics Command. His assistance in helping me to better understand the problem was invaluable. Finally, I wish to thank my wife, Santa, whose understanding, support, and good humor enabled me to give my best effort on this thesis.

Matthew A. Stone
# Table of Contents

Preface ........................................................................................................ ii
List of Figures .......................................................................................... v
Abstract ..................................................................................................... vi

I. Introduction ............................................................................................ 1
   Background ............................................................................................ 1
   Research Objective ............................................................................... 2
   Benefits ............................................................................................... 3
   Relationship to MIL-STD-105D ......................................................... 3
   Assumptions ....................................................................................... 5
   Scope of this Study ............................................................................. 5
   Decisions ............................................................................................. 6
   Summary of Results ........................................................................... 7

II. Methodology .......................................................................................... 9
   Introduction .......................................................................................... 9
   Defining the Decision Maker ............................................................ 9
   Previous Analysis .............................................................................. 9
   The Problem ....................................................................................... 11
   The Decision ...................................................................................... 12

III. Model Structure .................................................................................. 14
   A Simple Model ................................................................................ 15
   Merits of the Simple Model ........................................................... 20
   Limitations of the Simple Model .................................................... 21
   An Expanded Model ......................................................................... 21
   Merits of the Expanded Model ....................................................... 23
   The Value Function of the Expanded Model .................................. 25
   Assumptions of the Expanded Model ............................................. 26
   Limitations of the Expanded Model ................................................ 26

IV. Applying Response Surface Methodology to Perform
   Sensitivity Analysis on a Decision Analysis Problem ....................... 28
   Inputs to the Model .......................................................................... 29
   Outputs from the Model .................................................................... 30
   Design of the Experiments ................................................................ 31
      First Design .................................................................................. 31
      Results from the First Design ..................................................... 33
      Second Design .............................................................................. 34
      Results from the Second Design ................................................ 35
   Conclusions from the Sensitivity Analysis ...................................... 41
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Influence Diagram of the Simple Model</td>
<td>16</td>
</tr>
<tr>
<td>2. Discretizing a Continuous Probability Distribution</td>
<td>18</td>
</tr>
<tr>
<td>3. Influence Diagram of the More Complex Model</td>
<td>24</td>
</tr>
<tr>
<td>4. Residual Plot for the Second Design</td>
<td>37</td>
</tr>
<tr>
<td>5. Response Surface Used for the Sensitivity Analysis</td>
<td>39</td>
</tr>
<tr>
<td>6. Influence Diagram When the Government Action (GA) Decision Is Always Forced to 0 (Accept)</td>
<td>44</td>
</tr>
<tr>
<td>7. Influence Diagram When the Government Action (GA) Decision Is Always Forced to 1 (Reject)</td>
<td>45</td>
</tr>
<tr>
<td>8. Response Surface When GA Decision Is Forced to 0</td>
<td>48</td>
</tr>
<tr>
<td>9. Response Surface When GA Decision Is Forced to 1</td>
<td>50</td>
</tr>
<tr>
<td>10. Intersection of the Two Response Surfaces</td>
<td>51</td>
</tr>
<tr>
<td>11. Line Where the Two Response Surfaces are Equal</td>
<td>52</td>
</tr>
</tbody>
</table>
Abstract

The purpose of this study was to provide AFLC (Air Force Logistics Command) with a prototype decision model which will help AFLC engineers decide when post-production testing for a particular NSN (National Stock Number) is beneficial to the Air Force, as well as what action the Air Force should take as a result of that testing. The problem is a two sided issue. On one hand, nonconformance costs the Air Force in terms of damaged equipment and potential harm to personnel. On the other hand, the Air Force cannot afford to test everything. The study provides an initial step in developing a decision aid to help AFLC allocate its scarce testing resources.

Sensitivity analysis was performed on this model by using the techniques of Response Surface Methodology. For the data used in this study, the random variable which has the greatest impact on the expected cost to the Air Force is the expected dollar value that a single part can be expected to cause given that the part is in nonconformance with its specifications. This result was confirmed using traditional EVPI (expected value of perfect information) calculations.

The most significant contribution of this study is the development of a graphic decision aid. With this aid, the decision maker can graphically see how the present situation relates to a 'break-even line', or indifference curve. In addition to showing the decision maker which alternative is optimal, this method shows how much the independent variables would have to change for the optimal alternative to change.
I. Introduction

Background

The Air Force Logistics Command spends $13 billion a year procuring products for its supply system (5). The total Air Force budget for FY 88 excluding military pay and retirement is over $62 billion (2:C-4). Therefore, over twenty percent of the Air Force budget is spent every year just to replenish its supply inventory. The Air Force supply system is so large that even a small percentage of nonconformity in the inventory can have a major impact on the Air Force budget. When nonconformance in present, money that could have gone to other Air Force missions must be used to replace parts that do not work or last as long as they should have. In either case, when nonconformance exists, the Air Force is not getting what it's paying for.

The government goes to great lengths to ensure that the parts procured for use in all military equipment are able to perform the mission reliably. Military contract specifications are written, government inspectors are stationed in contractor factories through the DCAS (Defense Contract Administration Service) system, and first article testing is performed, all in an effort to try to prevent poor quality parts and material from entering the supply system of the
military services. However, as one might expect, a certain proportion of these parts and supplies do not meet the military specification for their manufacture and performance. When a part fails to meet its specification, the consequences are often minor, but sometimes, such a failure results in a major weapons system failure. Being the manager of the Air Force supply system, the Air Force Logistics Command (AFLC) is understandably concerned about the problem of nonconforming parts and material. A preliminary report of some nonconformance testing performed at Warner Robins ALC in 1986 and 1987 indicates that a significant number of parts in their inventory do not conform to contract specifications. The report recommends that post-production testing (the testing of items after they are already in the inventory) be performed on all Federal Stock Classes (FSCs) which are important and which have demonstrated high nonconformance in the past.

Obviously, AFLC does not have the resources to test all of its Federal Supply Classes. Furthermore, it may not be economically sound to try to do testing on all FSCs. The purpose of this thesis is to provide a prototype tool which will assist AFLC in deciding how much testing should be done on any given Federal Supply Class (FSC) or National Stock Number (NSN).

Research Objective

The objective of this research is to provide a prototype decision tool to AFLC engineers to help them make the following decisions:

1. When is post-production testing of a particular NSN economically wise?
2. Once AFLC engineers decide which NSNs to test, how many items in each NSN should be tested?

3. Based on the results of the post production testing for a particular weapon system, what actions should AFLC take to insure the operational safety of the weapon system?

Benefits

The benefits to be gained from this sort of post production testing are numerous. The first benefit is short-term in that we can identify and eliminate nonconforming parts from the inventory today. The second benefit is the creation of a database which would provide an indication of how well products from different contractors conform to specifications. The third benefit is more long-term. Just the existence of a testing program in itself and a database of contractor performance could raise the quality of parts that are received from the private industry. The testing results could help to provide an objective way for the Air Force to prevent poor quality manufacturers from winning government contracts. These actions would alert the defense manufacturing community of a new tougher stance by the Air Force and should be expected to increase overall quality in the long run.

Relationship to MIL-STD-105D

Military Standard 105D is a widely accepted tool which contains many detailed tables and charts useful for acceptance testing. It is used when the military is going to test an NSN and either accept or reject a lot of that NSN based on the outcome of that testing. If one
knows the population size and the Acceptable Quality Level (AQL), one can determine from Mil-Std-105D how many units should be tested from that NSN and how many have to be good in order to accept the lot.

The drawback to using Military Standard 105D is that the numbers it provides were derived in isolation from the specific problem at hand. When Mil-Std-105D says that X units need to be sampled in order to be 90 percent sure that we have achieved the minimum accepted quality level, it does not consider the costs involved with testing, the costs associated with rejecting the lot, or the cost of having nonconforming parts in the inventory which may eventually cause significant damage to a weapons system and/or crew. Or, it may be that testing the number of units recommended by Mil-Std-105D may simply be too expensive due to large set-up costs, limited testing resources, a large manpower requirement to perform the testing, or any combination of these factors.

A pure statistician may find these costs to be annoyances which muddy the water. However, a good decision maker will either consciously or subconsciously take all of these costs into account. Therefore, a good model of the decision problem must also try to model all of these costs. The complication is that almost all of these costs are very uncertain and therefore difficult to quantify. No real data exists for these unknown costs, however, there may exist experts who can provide their subjective inputs as to what these costs might be. Decision analysis provides a methodology for quantifying the subjective opinions of experts and including them in the analysis. This thesis uses decision analysis methods to try to do just that.
This thesis is not trying to suggest a way to replace Mil-Std-105D, only a way to supplement it. Whereas Mil-Std-105D sees the accept or reject question from a purely statistical point of view, the model presented in this thesis tries to account for the real world subjectivity that truly affects the decision. The mathematics of Mil-Std-105D are clean and precise yet too simplistic. The model in this thesis may be called fuzzy because it contains much subjective data yet therein lies its strength in that it attacks the decision from a holistic point of view. In addition to Mil-Std-105D, the model in this thesis can help the decision maker make a better decision of how many items to test.

Assumptions

The assumption of this thesis is that AFLC has subjective knowledge about which NSNs or FSCs have a problem with nonconformance. There are several ways in which this subjective information could be quantified. The number of Quality Deficiency Reports (QDRs) or Material Deficiency Reports (MDRs) issued on an item could be a good indication that a problem with nonconformity exists. Another indication of a problem with nonconformance could also be provided by the engineers at the ALC where the item is managed. Once subjective information is gained that a problem with nonconformance exists, the decision tool of this thesis should be applied to determine what actions should be taken.

Scope of this Study

This thesis effort is designed to provide a prototype model of the
decision process just described. Although the model correctly captures
the essence of the problem, the alternatives for the two decisions need
to be expanded and made more realistic. Currently the alternatives for
NTEST (the number of items to test) are limited to three items tested,
and the alternatives for GA (government action) are limited to accept
and reject.

To be truly beneficial to the end user, a user friendly "front
end" must be added to this model, as well as graphical outputs that are
meaningful to the user.

Decisions

There are two main decisions facing the decision maker. The first
decision is how many units to sample and test for conformance to
specifications. In the formulation of the problem that follows, this
first decision is referred to as NTEST. The second decision is what
action the government (specifically the Air Force) should take. This
second decision is made in light of the results of any testing that is
performed. This second decision is referred to as GA. The goal of
using this model is to provide the decision maker with the optimal
alternatives for both of these decisions which minimize the expected
cost to the Air Force.

The alternatives for the second decision, GA, are 0 and 1. If GA
is set to 0, it implies no action taken by the Air Force. If GA is set
to 1, it implies action taken by the Air Force. It may seem that
limiting the alternatives of GA to only two values is not a realistic
representation of all the alternatives that are truly available to the
Air Force in this situation. However, this seeming lack of realism is
compensated for by using a variable to represent the cost of the Air Force's action, COSTGA. The actual value of the cost of the Air Force's action varies depending upon the probability distribution that defines it. This probability distribution is provided by the user. Therefore, instead of having only two alternatives for the Air Force's action, this problem can represent a whole range of actions which are identified by their cost. Each different value for the cost of the Air Force's action (COSTGA) represents a different alternative for the Air Force's action (GA).

Summary of Results

This study provides AFLC with a prototype decision model which can be adapted to any particular NSN which is thought to have a nonconformance problem. This prototype decision model is designed to help AFLC decision makers decide when post-production testing is beneficial to the Air Force, as well as what action the Air Force should take as a result of that testing. The use of this tool will give AFLC insight about how to spend its scarce testing resources. It will also provide a logical way to start gathering an historical database of contractor performance in the production of specific items.

A new type of sensitivity analysis is used on the model in this thesis. The techniques of Response Surface Methodology (RSM) are used to determine which random variable has the greatest impact on the response variable, expected cost to the Air Force. For the data used in the example, the response variable is most sensitive to changes in the variable DAMGBP (Damage Given the Part is Bad). The results of the
sensitivity analysis are also confirmed by traditional EVPI (Expected Value of Perfect Information) calculations.

The most significant contribution of this thesis is the development of the methodology used in chapter V. Figure 11 is an example of the kind of graphic decision aid which can be produced with this methodology. A graph, such as the one in Figure 11, has two major features which make it extremely useful to the decision maker.

The first feature is the fact that two of the independent variables are varied over a wide range. This feature allows the decision maker to use a wide range of values for some unknown random variable instead of having to restrict the analysis to a more precise estimate of the unknown random variable.

The second feature is that the decision maker can graphically see how the present situation relates to the so-called 'break-even line'. Instead of just telling the decision maker which alternative is optimal, this method shows the decision maker how much the independent variables would have to change for the optimal alternative to change.
II. Methodology

Introduction

During the course of this study, much time and effort was spent just trying to define the problem. There are many competing political interests as well as many technical aspects to this problem. In order to better understand all the issues surrounding the problem, the author spent much time learning about how the Air Force manages its material inventory. This investigation focused on who is involved in managing any particular item and who the actual end user of a decision support tool would be. The author also examined the preliminary results of the study being conducted by the Inspector General of the Department of the Air Force (DoD/IG).

Defining the Decision Maker

Within AFLC, the activity of managing supply items is performed by many specialized career fields. Item managers, equipment specialists, engineers, primary contracting officers, and administrative contracting officers are all involved at some point in the process. When an item is believed to have problems with nonconformance to specifications however, it is the engineers who are primarily responsible for rectifying the problem. Therefore, the engineers were targeted as the decision makers in this problem and the decision tool was tailored for their use.

Previous Analysis

The next step was to learn more about the DoD/IG's investigation
of nonconforming parts in the Air Force. In 1986, the Inspector General of the Department of Defense (DoD/IG) began a program to determine the extent that products in the DoD were in nonconformance to the specifications for their manufacture and performance. The first stop in their audit was Warner-Robins Air Logistics Center (WR-ALC) in Georgia. Because of a cutback in funding, the DoD/IG was forced to scale back the number of items to be tested as well as the number of Federal Supply Classes from which the items would be sampled. The preliminary findings of DoD/IG's audit report indicates that a very significant percentage of parts in the Warner-Robins AFB inventory are in nonconformance of their specifications. As a result of the cutback and subsequent small sample sizes, AFLC is concerned about the accuracy of the DoD/IG's sampling techniques and the validity of the inferences that will be drawn from the sampling.

If the DoD/IG made correct use of sampling during their audit, they would be able to observe a small portion of some population and draw inferences about the population as a whole. Of course, the validity of their inferences depends greatly on how representative their sample was of the entire population.

In order to learn more about how the DoD/IG collected their sample, this author contacted the project officer of the DoD/IG audit as well as the statistician who designed the sampling strategy used during the audit. Neither one was able to provide any details about the sampling strategy.

For lack of better knowledge, this author assumed that the audit used some type of attributes sampling plan. An attributes sampling
plan is a sampling plan where a particular attribute of an item is tested and the only information recorded is whether or not the item tested conforms to some specification (3:63). A single attribute sampling acceptance plan (SASAP) is designed to provide a consumer with a desired level of protection against accepting a lot in which the proportion of defectives associated with a certain attribute exceeds an acceptable level. Military Standard 105D (Mil-Std-105D) contains extensive tables useful for designing an attributes sampling plan.

The audit is also believed to have used quota sampling as opposed to full probability sampling. Quota sampling is a type of sampling where the number to be sampled from each stratum is decided in advance and the experimenter continues to sample items until the necessary quota is obtained in each stratum (1:105). Quota sampling is preferred by some survey researchers because it is easier and more inexpensive to implement than full probability sampling. Cochran describes quota sampling as "stratified sampling with a more or less nonrandom selection of units within strata" (4:890). According to Benjamin King, the most important shortcoming of quota sampling, and nonprobability sampling in general, is the inability to calculate the mean square estimation error (4:890). The mean square estimation error cannot be determined due to the inherent selection bias of quota sampling. Without knowing the mean square estimation error, there is no way of knowing how accurate the estimate really is.

The Problem

In response to the preliminary findings of the DoD/IG's audit
report, the Air Force Logistics Command (AFLC) could decide to institute a post-production testing program that will test all critical parts to insure that they conform to their specifications. AFLC does not have the testing resources to test every single part that it manages, therefore, AFLC must decide which National Stock Numbers (NSNs) should come under the testing plan if one is implemented. The Air Force cannot afford to test its entire inventory, however, the Air Force also cannot afford to have parts in its inventory that are in significant nonconformance to specifications and could possibly cause major malfunctions. Therefore, the challenge is to find a logical way to balance the risk associated with nonconforming parts against the cost of testing them.

The Decision

The decision analysis methodology is used to structure the problem and to aid AFLC decision makers in understanding the problem. There are two decisions which are modeled. The first decision is how many units to sample and test for conformance to the specifications. The second decision is what action the Air Force should take in light of the testing that is performed. The outcome of the model is the expected total cost to the Air Force. In decision analysis terms, expected total cost to the Air Force is the value node, and the objective is to minimize this cost. The expected total cost to the Air Force is comprised of three costs considered in this model: the testing costs; the costs associated with nonconforming parts; and, the costs for the Air Force to take some type of action to correct the problem.

12
The author assigned the ranges and the probability distributions associated with each unknown cost. The emphasis was on developing a model that is structurally correct. The true ranges and probability distributions of each cost need to be obtained to reflect the true knowledge from the decision makers who will actually use this model when it is expanded from a pilot model into a full-scale model.

The initial model was analyzed using a decision analysis software program called Performa. Deterministic sensitivity analysis was performed. The model was then expanded and analyzed using another decision analysis program called InDia. A different type of sensitivity analysis using response surface methodology was then conducted on the expanded model. The results of the sensitivity analysis provide a measure of the importance of quantifying the unknown probability distributions. More time and effort should be spent on getting an accurate probability distribution on those variables to which the response is most sensitive. Finally, regression techniques are used to show how the optimal alternative for the GA decision (whether or not the Air Force should take action) changes as the unknown independent variables change. By doing so, it allows the decision maker to graphically see how changes in the independent variables can affect the decision of whether or not the Air Force should take action.
III. Model Structure

The model of this decision process is represented with an influence diagram. Influence diagrams provide a convenient way of representing a decision analysis problem more compactly than using a conventional decision tree. An influence diagram is a way of representing a decision process with nodes and arcs. Different nodes represent different types of variables which can be decisions, random variables, deterministic functions or value nodes. The arcs between the nodes represent dependencies of one node upon another. Once the influence diagram is built, the influence diagram can be reduced and solved to provide the optimal decision alternative and the expected value of that optimal decision alternative. Several other analyses are available to the decision analyst to gain a better understanding of the problem.

The three main costs that are considered in both models are: the testing costs; the costs associated with nonconforming parts; and the cost for the Air Force to take some type of action to correct the problem.

The testing costs are structured to account for fixed testing costs and variable testing costs. Fixed testing costs include special testing equipment and tools and any other start-up costs. Variable testing costs include the manpower to perform the testing and the cost of the actual unit that is sampled if the testing is destructive.

The costs of nonconformance are incurred when parts are in nonconformance to their specifications yet they remain in the inventory.
and have the potential to cause damage. The costs due to nonconformance include the cost of the defective unit, the cost to buy another unit to replace it, and the cost of higher assemblies that may be damaged due to failure of the nonconforming part. The decision maker must base his decision on the relationship between all of these costs. The decision maker must weigh the costs of an extensive testing program against the risks and costs associated with high nonconformance of parts in the inventory.

The following constants are used throughout each run of the model. The user of the model can change any constant before each run, however, its value will not vary during the run.

1. Fixed testing costs
2. Variable testing costs
3. The total number of units in the population
4. The maximum number of units that can be sampled from the population

A Simple Model

The first attempt at formulating a model of this decision process focuses on what appears to be the main decision which is to determine the number of units that should be sampled for a particular NSN that is suspected to have a nonconformance problem. This decision is labeled as NTEST in the influence diagram which is shown as Figure 1. The goal of using this simple model is to provide the decision maker with the optimal alternative for NTEST which minimizes the expected cost to the Air Force. The alternatives for NTEST can theoretically be anything from zero up to N, the total number of units in the NSN. If NTEST is...
Figure 1. Influence Diagram of the Simple Model
zero, it implies that no testing for conformance to specifications is performed. If NTEST is N, it implies 100 per cent testing for conformance to specifications. For the sake of simplicity, NTEST is artificially limited to range from 0 to 3 in the simple example.

TPG (True Percent Good) is the true percentage of units in the inventory of this type that meet the desired specification. For example, if the mean of the values for TPG is 0.8, then we can expect that 8 out of 10 units conform to specifications. It is important to note that in this model TPG is not just one number; it is a continuous random variable. The same is true for all the single circle nodes (also called chance nodes) in the influence diagram. Before the problem can be solved, however, all of the continuous random variables must be discretized. The author assumed that all of the random variables were approximately normally distributed. See Figure 2 for an example of how the variable TPG is discretized. Notice in Figure 2 that TPG is discretized to three possible values where 0.73625 is the mean of the lower quartile, 0.80 is the mean of the two middle quartiles, and 0.86375 is the mean of the upper quartile.

The nodes X1, X2, X3 represent the first, second, and third units that are sampled from the NSN. For this simple problem, a maximum of three units can be tested. The outcome for X1, X2, and X3 is one if the item conforms to specification, and zero if the item does not conform. The probability of whether X1, X2, or X3 is a one is dependent on the mean value used for TPG (true percentage of units in the inventory that conform). For example, if a mean value of 0.8 is used for TPG, then there is an 80 percent chance that X1 will be one.
Figure 2. Discretizing a Continuous Probability Distribution
NUMGOOD is a deterministic function that merely sums the results of testing sample units X1, X2, and X3. It could happen that no units are sampled, and in that case, NUMGOOD would be zero.

In Figure 1, there is only one decision node present, NTEST. NTEST is a decision node for how many units should be tested. The input for NTEST is all the possible alternatives for the number of units to be tested. The optimal number of units to test is obtained by solving the influence diagram. In this small example, the decision alternatives for NTEST range from zero to three units that can be tested.

FLAG is only used to signal to the value function whether at least one unit has been tested.

The value node in this influence diagram is the cost to the Air Force. The function which represents the Air Force's cost is:

\[
\text{FLAG} \times [-1000.0 - 240.0 \times \text{NTEST} - \exp(\text{ABS}((\text{TPG} \times \text{NTEST}) - \text{NUMGOOD})) + 1.0]
\]

where FLAG is a binary variable indicating whether any testing was done, NTEST is the number of units that are sampled, TPG is the true percentage of units that conform, and, NUMGOOD is the number of units that conform among those sampled.

In this function, FLAG is 1.0 if NTEST is greater than or equal to one. The fixed testing costs include special testing equipment and any other costs to start a new testing program of this sort. In this small example, the fixed testing costs are set at $1000.00. The variable testing costs include the manpower to perform the testing and the cost of the unit itself if the testing is destructive. The variable testing
costs are set at $240.00 per unit tested. The remainder of the value function is based on the idea that there is a cost associated with not knowing the true value of TPG. In other words, a cost is incurred which is proportional to the error in the estimate of TPG. The Air Force places a high value on knowing the true value of TPG (the true proportion of units that meet the specification). The value function above incurs a smaller cost (greater value) when TPG*NTEST is closer to NUMGOOD. Conversely, the Air Force has a lower value for when TPG and NUMGOOD are farther apart. The reasoning behind this value function is that the Air Force can make better decisions about the management of this particular NSN if the Air Force knows the true percentage of units that conform to the specifications. A better estimate of this true percentage is obtained when more testing is done, therefore, in this small example, the optimal alternative for the number of units to test is always the highest number of units, three in this case. If the alternatives for NTEST were allowed to be sufficiently larger, then there may come a point when the additional testing costs do not warrant the gain in information that results from that testing. At that point, the optimal alternative for NTEST would not be the highest alternative.

Merits of the Simple Model

The advantage of the simple model in Figure 1 is that it is a representation of the decision process which focuses on the main decision which is the number to sample (NTEST). The advantage of this representation is that it focuses on the decision maker's real interest, the number of items that should be tested. The simpler model is also easier to understand. However, by using this simpler model,
realism is sacrificed.

Limitations of the Simple Model

The disadvantage of the simple model in Figure 1 is that the value function is extremely arbitrary. The value function assumes that the Air Force always has more value when the estimate for the percentage that conform (NUMGOOD/NTEST) is closer to the true percentage that conform (TPG). This may in fact be the case, but it is doubtful that the value of that difference is truly represented by an exponential function. In any case, the model is vague about the value of having TPG close to (NUMGOOD/NTEST).

Another disadvantage of the simple model in Figure 1 is its nearsightedness. It does not consider what decisions will be made later as a result of the testing that is done now. As a result, it is impossible to calculate an accurate value of information on the random variables because changes in the random variables do not impact decisions that will be made in the future.

To develop a better value function, it's necessary to know what decisions will be based on the results of the testing. To properly put a value on that testing, it's imperative to know how the results of the testing affect future decisions.

An Expanded Model

In addition to the decision of how many units to test (NTEST), there are at least two more decisions that are relevant to this nonconformance problem: an operational decision; and a financial decision.
The operational decision focuses on what actions the Air Force should take to improve the present quality of the spare parts that are supporting the flying mission. The operational decision is mainly dependent on the results of any testing that is done. The operational decision encompasses the risks associated with using the parts against the cost of taking actions to improve the situation. We can try to estimate the risks associated with using the present inventory of parts and find the point where the risk is balanced by the cost of taking action to correct the problem. The point where these two costs are balanced can be graphed for a differing number of sample sizes and the resultant graph will be split into two regions. In one region, the optimal decision would be to accept the parts and follow the status quo. In the other region, the optimal decision would be to reject the parts and take some type of action to correct the problem such as procuring new parts (preferably from a higher quality source). By presenting the problem in the appropriate way, the decision maker's task is reduced to making a judgment about whether the true value of $P$ is greater than or less than some value. This should be an easier task than trying to determine the prior probability distribution on $P$. In any case, the additional information should be helpful. See chapter V for a further explanation of this use of the model.

The financial decision focuses on what actions the Air Force should take concerning the contract and the manufacturer. The financial decision depends on many other outside factors in addition to the testing results and the operational decision. When making the financial decision, the Air Force must consider the past performance of
the manufacturer, the size of the contract, the seriousness of the nonconformance, the willingness of the manufacturer to correct the defect, and the probability that any legal action taken will be favorable to the Air Force.

Merits of the Expanded Model

The expanded model in Figure 3 is only concerned with the operational decision. It attempts to correct some of the deficiencies of the simple model in Figure 1. The main difference between the two models is the inclusion of a second decision node, Government Action (GA), which represents the operational decision that must be made. By adding the GA node, the model in Figure 3 is a better representation of the decision process because it models how the results of the testing affect the future decision, GA. The alternatives for the GA decision node are 0 and 1 where 0 represents no action taken by the government and 1 represents action by the government. This action taken by the government is not specifically defined. The action could be freezing the entire stock of the item until 100% inspection is made, purging the stock and procuring new stock from a different manufacturer, or any other action the government believes appropriate.

Another addition to the influence diagram in Figure 3 is the addition of the chance node DAMGBP. DAMGBP (Damage Given it's a Bad Part) represents the expected dollar value of the damage that a single nonconforming unit will cause.

The deterministic node DDDDBP represents the expected dollar value of the total damage that will result due to all the nonconforming parts.
Figure 3. Influence Diagram of the More Complex Model
in the inventory. Costs included in DDDBP are the cost of lost use of
the part and also the cost of damage done to higher assemblies. The
function for DDDBP is:

\[ \text{DDDBP} = \text{DAMGBP} \times (1 - \text{TPG}) \times N \]

where DAMGBP is the expected dollar value of the damage that a single
nonconforming unit will cause, (1-TPG) is the percentage of parts that
do not conform to specifications, and N is the total number of units in
the NSN.

COSTGA (COST of the Government's Action) is a chance node that
contains the probability distribution on how much it will cost the
government to take some form of action.

The Value Function of the Expanded Model

The value node, Expected Cost to the Air Force, evaluates all of
the costs that are incurred for each possible alternative and outcome
in the influence diagram. The costs that are considered are the fixed
testing costs, the variable testing costs, the costs associated with a
part that is nonconforming yet remains in the inventory, and the cost
for the government to take some type of action to correct the problem.

The value function for the value node in Figure 3 is:

\[ \text{[FLAG} \times (-1000.0 - 240.0 \times \text{NTEST})] - [\text{GA} \times \text{COSTGA}] - [(1 - \text{GA}) \times \text{DDDBP}] \]

The first half of this value function adds in the fixed and variable
testing costs if any testing is done. The second half of the function
adds in the cost of government action if GA is 1, or it adds in the
costs due to nonconforming parts if GA is 0. The costs due to
nonconforming parts include the cost of the defective units, the cost to buy other units to replace them, and the cost of higher assemblies that may be damaged.

**Assumptions of the Expanded Model**

This model is not intended to be used on every NSN that the Air Force manages. Because of the time involved to input the required information, it would be unrealistic to go through this procedure for every NSN. This model should only be used for those NSNs which are suspected to have a nonconformance problem.

**Limitations of the Expanded Model**

The model in Figure 3, while better than the model in Figure 1, still makes some large generalizations. Specifically, many unknowns are aggregated into one probability distribution. The chance nodes DAMGBP and COSTGA make use of this aggregation to simplify the problem.

The two chance nodes DAMGBP and COSTGA each contain many parts. The chance node DAMGBP (dollar value of the damage that a single nonconforming part will cause) has two main parts: the cost of the part itself; and the cost of damage done to higher assemblies. The cost of damage done to higher assemblies is also made up of several sub-distributions which are based on where the part is used and when the part fails. The actual damage depends on many possible scenarios that could occur such as:

a. the part is used on a piece of ground support equipment and its failure does not cause significant damage to higher assemblies.
b. the part is used on an aircraft but its failure does not threaten the airworthiness of the aircraft.

c. the part is used on an aircraft and its failure does threaten the airworthiness of the aircraft.

d. the part is used on an aircraft and is found and replaced during inspection.

e. the part is nonconforming but its nonconformance does not cause a failure.

The chance node COSTGA (the cost of the government action) also contains many sub-distributions. Each of these sub-distributions depends on the specific action that the government takes. When GA is 0, it represents the government not taking any action and then COSTGA is nothing. When the decision node GA is 1, COSTGA is a distribution of the cost to the government associated with taking any one of several actions or combinations of actions such as:

a. tighter inspections of the part before it is accepted.

b. 100% inspections of the part before it is accepted.

c. freezing the entire stock until 100% inspection can be performed on all parts.

d. negotiating with the manufacturer for reworking the items.

e. terminating the contract for the benefit of the Air Force and seeking a new manufacturer.

In both cases, the chance nodes DAMGBP and COSTGA attempt to place a probability distribution which approximates the combination of all the sub-distributions. While this may make the problem more fuzzy, it also scopes the problem to a manageable size.
A new type of sensitivity analysis is used in this study. Stochastic sensitivity analysis is performed on the model using Response Surface Methodology (RSM). When used in its descriptive sense, the purpose of RSM is to find a parsimonious polynomial representation of a system. RSM can be used to find a polynomial which approximates the response of the actual system under study. The goal of applying RSM is to find a simpler representation of a system which will enable the user to obtain answers to 'what if' questions without having to rerun a large model or computer simulation. RSM uses factorial experimental designs and the method of least squares to find an empirical response surface which is close enough to the real model's response to satisfy the decision maker. Many times, a system of a higher order can be approximated with a lower order polynomial if the region of interest is small.

In common sensitivity analysis, all the variables are held at their nominal values and the variable of interest is varied to see how it affects the response. This is artificial because in the real world, the variables are not likely to vary in this way. Common sensitivity analysis provides a way to calculate the main effect that each input variable has on the response variable. If the experimenter wishes to know the interaction effects between the variables, the experimenter must vary the inputs in an ad hoc way in order to observe what effect this has on the response variable.
RSM provides for a better sensitivity analysis because RSM explores the interaction effects between the variables in a more structured, consistent, and reliable way. For instance, in a full factorial design, all the possible combinations of the high, medium, and low values of the input variables are considered. This is useful, because in the real world, the variables are not going to change one at a time, but rather, all the variables will be changing constantly. That is why it is so important to know not only how varying one variable affects the response, but also how varying two or more of the variables affects the response. Using RSM for sensitivity analysis is more realistic because it can provide the estimates for the interaction effects between the variables.

This study uses Response Surface Methodology (RSM) to find which input variables have the greatest impact on the response variable, minimum expected cost to the Air Force. When used in this way, RSM is a useful tool to perform sensitivity analysis on a decision analysis model. It provides a way to rank order the importance of quantifying all the unknown probability distributions. Those distributions which have a greater regression coefficient also have a greater effect on the response, and therefore more time and money should be spent to better quantify them. The actual influence diagram used for this sensitivity analysis is shown in Figure 3 and is described in chapter III.

**Inputs to the Model**

There are three inputs to this model: the true percentage of items in the National Stock Number (NSN) that conform to the
specifications (TPG); the dollar value of the damage that is expected to occur due to a part that does not conform to the specifications (DAMGBP); and, the dollar value of the expected cost to the government if the government takes action (COSTGA). These inputs are not single numbers, but rather, they are continuous probability distributions. Unfortunately, due to the limitations of the software package used to solve this influence diagram, any continuous probability distribution must first be discretized before the influence diagram can be solved. For this project, each probability distribution is discretized by finding the mean of the lower 0.25 fractile of the cumulative distribution function (CDF), the mean of the upper 0.25 fractile, and the mean of the middle 0.50 fractile.

Outputs from the Model

There are really three 'outputs' that are obtained by solving this particular influence diagram: the optimal alternative for the decision of how many items in the NSN should be tested, the optimal alternative for the decision of what action the Air Force should take in light of the testing that was done, and the minimum expected value which results from following the first two optimal alternatives. For the purposes of this sensitivity analysis, the only output of concern is the minimum expected cost to the Air Force. Therefore, the response variable we are trying to describe is the minimum expected cost to the Air Force. Our goal is to model the response variable the best we can by varying the three input distributions mentioned above.
Design of the Experiments

First Design. For the first experimental design attempt, a three-level, 3 full factorial design is used. Since there are three factors, this requires 27 runs.

Only one run is made at each design point. The influence diagram is a deterministic system. As such, multiple replications were not made at the same design point because they would yield the same output value.

The three factors used in this design are defined as the respective means of the probability distributions on TPG, DAMGBP, and COSTGA. The high and low settings for each of the means of the three distributions are:

<table>
<thead>
<tr>
<th>Factor</th>
<th>High (+1)</th>
<th>Medium (0)</th>
<th>Low (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPG</td>
<td>0.90</td>
<td>0.85</td>
<td>0.80</td>
</tr>
<tr>
<td>DAMGBP</td>
<td>$150,000</td>
<td>$100,000</td>
<td>$50,000</td>
</tr>
<tr>
<td>COSTGA</td>
<td>$1,000,000</td>
<td>$900,000</td>
<td>$800,000</td>
</tr>
</tbody>
</table>

The standard deviations of each distribution are held constant throughout the analysis.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPG</td>
<td>0.05</td>
</tr>
<tr>
<td>DAMGBP</td>
<td>$25,000</td>
</tr>
<tr>
<td>COSTGA</td>
<td>$45,000</td>
</tr>
</tbody>
</table>

The structure of the design appears on the next page:
where \( X_1 \) represents TFG, \( X_2 \) represents DAMGBP, and \( X_3 \) represents COSTGA. The fifth through the eleventh columns of the above matrix are obtained by multiplying the appropriate combination of columns two through four. For example, the \( i \)th element of the \( X_1X_2 \) column is obtained by multiplying the \( i \)th element of the \( X_1 \) column by the \( i \)th element of the \( X_2 \) column.

To get the actual output value for each design point, the three input probability distributions must be changed to correspond to the high, medium, and low settings given above. Once the mean of each distribution is changed, each distribution is then discretized as described earlier.
Results from the First Design. Trying to fit a full second order model against the new vector of outputs resulted in a perfect fit. Whereas a perfect fit sounds good, it is not good in this case because there is no way of getting estimates for the standard error of the coefficients. In this context, the standard error of the coefficients is used as a surrogate for the explained sum of squares. Therefore, without estimates for the standard error of the coefficients, there is no way of knowing how well the regression equation fits the data. In addition, the plot of the residuals is also not indicative of a good fit.

These problems were not expected because there are 27 runs and the regression is trying to estimate 11 parameters. There should have been 16 degrees of freedom left over for error. The reason for a perfect fit is due to having reduced the range covered by the COSTGA factor. It's range was reduced so much that it had no effect on the response variable. Therefore, instead of having 27 runs, there are really only 9 unique runs which are repeated 3 times. As a result, the regression is estimating 11 parameters with only 9 unique runs and a perfect fit will always happen when the number of runs is less than the number of parameters being estimated.

To overcome these problems, two more changes were made. First, the actual output values are transformed by the natural logarithm transformation. Second, instead of trying to estimate 11 parameters, we will only try to estimate those parameters that appeared to be significant on the first regression attempt.
The equation for the model after all these changes is:

\[ Y = 11.918 - 0.347 \, TPG + 0.549 \, DAMGBP - 0.059 \, TPG^2 - 0.144 \, DAMGBP^2 \]

The ANOVA table for this regression equation is:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>Partial ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAMGBP</td>
<td>1</td>
<td>5.43127032</td>
<td>5.43127032</td>
<td>0.702</td>
</tr>
<tr>
<td>TPG</td>
<td>1</td>
<td>2.16203856</td>
<td>2.16203856</td>
<td>0.279</td>
</tr>
<tr>
<td>DAMGBP*DAMGBP</td>
<td>1</td>
<td>0.12414146</td>
<td>0.12414146</td>
<td>0.016</td>
</tr>
<tr>
<td>TPG*TPG</td>
<td>1</td>
<td>0.02080927</td>
<td>0.02080927</td>
<td>0.003</td>
</tr>
<tr>
<td>TOTAL</td>
<td>26</td>
<td>7.73825961</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ R^2 = 1.0000 \quad \text{Adjusted } R^2 = 1.0000 \]

The column labeled 'Partial R' is the sum of squares of each term divided by the total sums of squares. Partial \( R^2 \) is a good indicator of the contribution that each term is making toward describing the response variable.

**Second Design.** The regression equation resulting from the first design shows that the response variable, minimum expected cost to the Air Force, is insensitive to changes in COSTGA. The reason for this is due to the small range over which COSTGA was allowed to vary. In the second design attempt, the range covered by COSTGA is increased greatly. The new high, middle, and low settings of the variables used in the second design are listed below:

<table>
<thead>
<tr>
<th></th>
<th>HIGH (+1)</th>
<th>MEDIUM (0)</th>
<th>LOW (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPG</td>
<td>0.90</td>
<td>0.85</td>
<td>0.80</td>
</tr>
<tr>
<td>DAMGBP</td>
<td>$150,000</td>
<td>$100,000</td>
<td>$50,000</td>
</tr>
<tr>
<td>COSTGA</td>
<td>$500,000</td>
<td>$290,000</td>
<td>$80,000</td>
</tr>
</tbody>
</table>
Again, the standard deviations of the distributions are held constant at their previous values.

<table>
<thead>
<tr>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPG</td>
</tr>
<tr>
<td>DAMGBP</td>
</tr>
<tr>
<td>COSTGA</td>
</tr>
</tbody>
</table>

See appendix A for the variable settings and the output values for each of the runs.

Results from the Second Design. The regression equation resulting from the second experimental design is:

\[
Y = -151,667 + 34,444 \, \text{TPG} - 51,389 \, \text{DAMGBP} - 36.944 \, \text{COSTGA} \\
+ 13,333 \, \text{TPG} \cdot \text{DAMGBP} + 22,500 \, \text{TPG} \cdot \text{COSTGA} - 34,583 \, \text{DAMGBP} \cdot \text{COSTGA} \\
+ 16,250 \, \text{TPG} \cdot \text{DAMGBP} \cdot \text{COSTGA} + 35,833 \, \text{COSTGA} \cdot \text{COSTGA} \tag{1}
\]

The regression equation just given is for the coded parameters.

The decoded regression equation is:

\[
Y = -151,667 + 34,444((\text{TPG}-0.85)/0.05) \\
- 51,389((\text{DAMGBP}-100,000)/50,000) \\
- 36.944((\text{COSTGA}-290,000)/210,000) \\
+ 13,333((\text{TPG}-0.85)/0.05)\cdot((\text{DAMGBP}-100,000)/50,000) \\
+ 22,500((\text{TPG}-0.85)/0.05)\cdot((\text{COSTGA}-290,000)/210,000) \\
- 34,583((\text{DAMGBP}-100,000)/50,000)\cdot((\text{COSTGA}-290,000)/210,000) \\
+ 16,250((\text{TPG}-0.85)/0.05)\cdot((\text{DAMGBP}-100,000)/50,000)\cdot((\text{COSTGA}-290,000)/210,000) \\
+ 35,833((\text{COSTGA}-290,000)/210,000)\cdot((\text{COSTGA}-290,000)/210,000) \tag{2}
\]
The ANOVA table for this regression equation is:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>Partial R</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAMGBP</td>
<td>1</td>
<td>4.7534E10</td>
<td>4.7534E10</td>
<td>0.358</td>
</tr>
<tr>
<td>COSTGA</td>
<td>1</td>
<td>2.4568E10</td>
<td>2.4568E10</td>
<td>0.185</td>
</tr>
<tr>
<td>TPG</td>
<td>1</td>
<td>2.1355E10</td>
<td>2.1355E10</td>
<td>0.161</td>
</tr>
<tr>
<td>DAMGBP*COSTGA</td>
<td>1</td>
<td>1.4352E10</td>
<td>1.4352E10</td>
<td>0.108</td>
</tr>
<tr>
<td>COSTGA*COSTGA</td>
<td>1</td>
<td>7.7042E9</td>
<td>7.7042E9</td>
<td>0.058</td>
</tr>
<tr>
<td>TPG*COSTGA</td>
<td>1</td>
<td>6.0750E9</td>
<td>6.0750E9</td>
<td>0.046</td>
</tr>
<tr>
<td>TPG*DAMGBP</td>
<td>1</td>
<td>2.1333E9</td>
<td>2.1333E9</td>
<td>0.016</td>
</tr>
<tr>
<td>TPG<em>DAMGBP</em>COSTGA</td>
<td>1</td>
<td>2.1125E9</td>
<td>2.1125E9</td>
<td>0.016</td>
</tr>
<tr>
<td>UNEXPLAINED SS</td>
<td>18</td>
<td>6.8146E9</td>
<td>3.7859E8</td>
<td>0.051</td>
</tr>
<tr>
<td>TOTAL</td>
<td>26</td>
<td>1.3265E11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R² = 0.9486  Adjusted R² = 0.9172

The regression equation appears to be a good fit of the 27 data points. The R² value shows that almost 95 per cent of the variability of the 27 data points is accounted for by the regression equation. The other sign of a good fit is the plot of the residuals which is shown in Figure 4. The residuals appear to be randomly scattered with no obvious trends.

These two measures of goodness of fit show that the regression equation (2) is a good model of what is taking place inside the influence diagram. In turn, the influence diagram is a model of what is taking place inside the head of the decision maker who is deciding whether or not an NSN should be tested and if so, how many units should be tested. Indirectly then, equation (2) approximates the real world decision being modeled in this study. Therefore, by examining equation (2), some insights into the real decision problem can be made.

The ANOVA table above also provides information about the model. The ANOVA table shows how much each term in equation (2) contributes
toward modeling the response. The variable DAMGBP has the most impact on the response followed by COSTGA, TPG, the interaction between DAMGBP and COSTGA, and so on down the ANOVA table. This rank ordering of significance is directly related to the sensitivity analysis of the influence diagram. The response, minimum expected cost to the Air Force, is more sensitive to changes in those variables at the top of the ANOVA table. For the ranges of the variables used in this sensitivity analysis, the response is most sensitive to DAMGBP.

The degree of sensitivity of the response to changes in the variables can also be seen from plots of the regression equation. Figure 5 is just one of several ways to plot equation (2). Since there are four variables (3 input and 1 output), one of the variables must be fixed in order to obtain a three-dimensional visual representation. In Figure 5, the mean of the probability distribution on the variable COSTGA is held constant at its lowest value, $80,000. DAMGBP and TPG are on each horizontal axis, while minimum expected cost to the Air Force is on the vertical axis. Notice that the surface is plotted only over the ranges of DAMGBP and TPG which were actually used in the experimental design. This is because the regression equation is valid only for that area of applicability. The area of applicability is that area where TPG is between 0.80 and 0.90; DAMGBP is between $50,000 and $150,000; and, COSTGA is between $80,000 and $500,000. Similarly, these results may also depend on some values that were fixed throughout the experiment like the population size, the fixed and variable testing costs, and the maximum number of units that can be tested (three in this small example).
Expected Cost to the Air Force

COSTGA set to $80,000

Figure 5. Response Surface Used for the Sensitivity Analysis
By examining the plot in Figure 5, it is apparent that the slope of the surface is much greater along the DAMGBP axis than it is along the TPG axis. The differences in the slope of the surface is due to the differences in the regression coefficients of each variable. By actually plotting the equation, the decision maker can visually see how changes in one variable will have a larger impact on the response than changes in another variable.

In terms of the decision that AFLC must make regarding whether or not to take action, we can see that uncertainty about the true value of DAMGBP is more significant than the uncertainty about the true value of TPG.

To confirm these results of the sensitivity analysis, expected value of perfect information (EVPI) calculations were performed on the model. The EVPI results are shown below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>EVPI Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAMGBP</td>
<td>$10,937.50</td>
</tr>
<tr>
<td>COSTGA</td>
<td>$9,743.75</td>
</tr>
<tr>
<td>TPG</td>
<td>$3,000.00</td>
</tr>
</tbody>
</table>

The results for EVPI on these three random variables represent the very most that the decision maker would be willing to pay to remove all uncertainty about the value of these random variables. Any testing or data collecting that could be done to gain information on these random variables is not going to be 100 per cent accurate, therefore, these EVPI results represent an upper bound that should be paid to gain information on the variables.
The results of the EVPI calculations support the results of the RSM sensitivity analysis. The response variable is most sensitive to changes in the DAMGBP variable, less sensitive to changes in the COSTGA variable, and least sensitive to changes in the TPG variable. In terms of the decision, the most that AFLC should pay to know with complete accuracy the damage that a single nonconforming part causes given that the part is in nonconformance is $10,937.50.

Conclusions from the Sensitivity Analysis

The most important conclusion made as a result of applying RSM to this particular influence diagram, for the data used in this example, is that the input distribution on DAMGBP has the largest impact on the response variable, minimum expected cost to the Air Force. This can be seen by the large regression coefficient for DAMGBP. This can also be seen from the graph of the response surface in Figure 5. The slope along the DAMGBP axis is by far greater than the slope along the TPG axis. This implies that the response is much more sensitive to changes in DAMGBP than for changes in TPG. For the analyst of this problem, it means that great care should be taken to obtain an accurate and thorough probability distribution on DAMGBP. A sloppy distribution on any of the variables could bias the results, however, special care should be taken when quantifying DAMGBP because the response is especially sensitive to changes in DAMGBP.

Response surface methodology is a useful tool to perform sensitivity analysis on an influence diagram. It provides a way to rank order the importance of quantifying the input probability distributions. Those distributions which have a greater regression
coefficient (and hence greater slope) are going to have a greater
effect on the response, and therefore more time and money should be
spent to better quantify them.

Moreover, all the ranges over which all the factors are varied
needs to be reexamined to insure that they are realistic. It may be
that the ranges do not need to be increased, but that a whole other
study needs to be done over another range of applicability. The larger
the range of applicability, the sloppier the fit of the response
surface will be. That's why it's important to narrow the required
range of applicability. It may be that several studies over smaller
regions of applicability is more appropriate than one study over a
region of applicability that is too large.
V. Results

In chapter IV, regression was used in a descriptive sense to show the sensitivity of the response to the different independent variables. In this chapter, similar regression techniques are used to graphically show how the optimal alternative changes as the independent variables change. Specifically, this chapter shows how the optimal choice for the GA (government action) decision changes as the TPG (true percentage of parts that conform) and DAMGBP (damage that a single nonconforming part will cause) variables change.

To perform this analysis, the form of the influence diagram must be changed so that the GA node is removed. In the first step, the influence diagram is changed so that the government is always forced to accept whatever degree of nonconformance is present and the costs associated with doing so. This is equivalent to always forcing GA to be 0 (accept). See Figure 6 for the influence diagram when GA is forced to 0.

In the second step, the influence diagram is changed so that the government is always forced to reject whatever degree of nonconformance is present and pay the costs of corrective action (whatever that may be). This is equivalent to always forcing GA to be 1 (reject). See Figure 7 for the influence diagram when GA is forced to 1.

Next, a three level, 3 full factorial experiment is performed on both of the modified models. The probability distribution on the cost of corrective action (COSTGA) is not changed so that it maintains a mean of $80,000 throughout both experiments. A regression is performed
Figure 7. Influence Diagram When the Government Action (GA) Decision Is Always Forced to 1 (Reject)
on the data from the experiments of both of the modified models.

Now each of the regression equations from each of these modified models defines a surface in three dimensions. The variables TPG and DAMGBP make up the two horizontal axes and the response, expected cost to the Air Force, is the vertical axis. If the surfaces defined by these two regression equations intersect, their line of intersection can be found and projected onto the plane defined by the TPG and DAMGBP axes. The usefulness of such a plot is best illustrated by an example.

Example

Forcing GA to be 0 (accept). Figure 6 shows the influence diagram in which the government action (GA) decision is forced to be 0 (accept). A full factorial experiment was performed using this influence diagram. The probability distribution for the COSTGA variable was not changed to a high or medium setting, but rather, the COSTGA distribution kept it's mean constant at $80,000 throughout the experiment. Therefore, there are two independent variables and it takes 9 runs to do a three level 3 full factorial experiment. The high, medium, and low settings for each of the means of the TPG and DAMGBP distributions are:

<table>
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<th></th>
<th>HIGH (+1)</th>
<th>MEDIUM (0)</th>
<th>LOW (-1)</th>
</tr>
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<td>$100,000</td>
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The standard deviations of each distribution are held constant throughout the experiment.

46
See appendix B for the variable settings and the output values for each of the nine runs. A regression is performed against the data from appendix B and the following regression equation gives a perfect fit against the data.

\[ Y = -150,000 + 50,000*TPG - 75,000*DAMGBP + 25,000*TPG*DAMGBP \]  

Note that the above equation is for the coded values of TPG and DAMGBP. The uncoded equation is:

\[ Y = -150,000 + 50,000*\left(\frac{TPG - 0.85}{0.05}\right) - 75,000*\left(\frac{DAMGBP - 100,000}{50,000}\right) + 25,000*\left(\frac{TPG - 0.85}{0.05}\right)*\left(\frac{DAMGBP - 100,000}{50,000}\right) \]

The surface defined by equation (4) is shown as Figure 8. Notice that the ranges of the two horizontal axes, TPG and DAMGBP, do not extend past the high and low settings used in the experiment.

**Forcing GA to be 1 (reject).** Figure 7 shows the influence diagram in which the government action (GA) decision is forced to be 1 (reject). It is unnecessary to perform a full factorial experiment using this influence diagram because when GA is forced to 1 (reject), the expected cost to the Air Force will always be the mean of the COSTGA distribution. In this case, the mean of the distribution of the cost to the government variable (COSTGA) is $80,000. Therefore, the
Figure 8. Response Surface When GA Is Forcetd to 0

Expected Cost to the Air Force

COSTGA set to $80,000
The equation which defines the output for this influence diagram is:

\[ Y = -80,000 \]  \hspace{1cm} (5)

The surface defined by equation (5) is shown as Figure 9.

For this particular example, the surfaces defined by equations (4) and (5) intersect; see Figure 10. In this instance, one alternative does not dominate the other. However, in some instances, one alternative will completely dominate the other alternative in the area of applicability. In that case, there would be no line of intersection.

The equation of the line where the two surfaces intersect can be found by simultaneously solving equations (4) and (5). Solving for DAMGBP gives:

\[
\text{DAMGBP} = \frac{[50,000 \times (70,000 - 50,000((\text{TGP} - 0.85)/0.05))] + 100,000}{(-75,000 + 25,000((\text{TGP} - 0.85)/0.05))} \]  \hspace{1cm} (6)

The line defined by equation (6) is plotted as Figure 11. In Figure 11, the line where the two surfaces intersect is projected down onto the TPG-DAMGBP axis.

The plot has a single line which divides the space into two regions. In one region, the 'accept' alternative is optimal, and in the other region, the 'reject' alternative is optimal. The line can be thought of as a sort of 'break-even line'. On this line, the expected value of the 'accept' alternative is the same as the expected value of the 'reject' alternative.
Figure 10. Intersection of the Two Response Surfaces
Figure 11. Line Where the Two Response Surfaces are Equal
When a plot such as the one in Figure 11 is constructed, it should help the decision maker determine which decision (accept or reject) is best. Such a plot can help the decision maker visualize where present circumstances put him in relation to the 'break-even line'. By entering the plot for a certain value of TPG and DAMGBP, the decision maker can see in which region he falls and how close he is to the 'break-even line'. Hopefully, the decision maker will have some preconceived notion as to what the true values of TPG and DAMGBP are. If so, the decision maker can tell if he is close to the line or not. If his preconceived notions of TPG and DAMGBP put him close to the line, then more time should be spent in getting better estimates for TPG and DAMGBP if it has not already been done.

For example, if the decision maker believes that about 85% of his parts conform to specifications (TPG), and that, on average, a single nonconforming part will cause about $55,000 worth of damage (DAMGBP), then he knows that he is very close to the 'break-even line'. He should then do one of two things. One, he could get better estimates for TPG and DAMGBP if he is unsure about them because any error in their estimates could change the optimal alternative for GA. Or, two, now that he knows that he is close to the 'break-even line', he knows that the expected value of his decision will be nearly the same no matter which alternative he chooses for GA. In light of this, the decision maker may want to base his decision on the other intangible political aspects of the problem which are not included in this model of the technical problem.
On the other hand, if the decision maker believes that significantly less than 85% of his parts conform to specifications, and that, on average, a single nonconforming part will cause significantly more damage than $55,000, then he knows he is not close to the 'break-even line'. The decision maker can then use Figure 11 to see that according to this technical model of the problem, his optimal alternative is to reject the batch of parts. Choosing the other 'nonoptimal' alternative is likely to greatly increase the expected cost to the Air Force.
VI. Conclusions

This study developed a prototype model of the nonconforming parts decision problem which faces AFLC. The model is in the form of an influence diagram which contains two decisions: the number of items from the NSN that should be sampled and tested for conformance to specifications; and, the action that AFLC should take to minimize the cost to the Air Force.

Lack of Real Data

The model takes a very broad view of the problem. The costs that are used as variables in the model are aggregates of many other costs and unknowns. If real data exists that could be used to quantify any of these unknown variables, then using the real data would be the best method to continue the analysis. However, it is most likely that true data does not exist on any of the variables that are used in this model, and in that case, the only logical way to quantify these unknowns is to use the subjective opinions of experts. The lack of real data on a variable does not justify excluding that variable from the model if it has an important impact on the decision problem. The variable must still be quantified the best way possible, and that is through the use of subjective probability assessments from the decision maker and/or the person who knows the most about the unknown variable.

Uses of the Model

The model developed in this study is only a prototype. However, the type of results obtainable from using a full blown version of this
model could be of great use to certain decision makers within AFLC. The model explained in chapter III is helpful because it accounts for those variables which are hard to quantify yet are so important to the decision that needs to be made. The plot such as the one in Figure 11 is very useful to the decision maker who is contemplating action and needs to make a good decision in a short amount of time. Instead of relying on the accuracy of subjective probability assessments on the key unknown variables, Figure 11 shows the unknown variables over their whole range and how they affect the optimal decision. Instead of having to make extra runs of the model in order to answer 'what if' questions, the decision maker can look at Figure 11 and see what effect changing a variable would have.

It is important to note that the results obtained in chapter V not only give information about the value of the expected cost to the Air Force based on the values of the inputs, but it also shows how the optimal decision alternative changes as the unknown variables change. If the decision maker has some prior knowledge about the unknown variables, even if it is purely subjective, he can graphically see where present circumstances place him with respect to the 'break-even line'.

The results of the sensitivity analysis were also very enlightening. Uncertainty about the true value of DAMGBP has a much larger impact on the cost to the Air Force than uncertainty about TPG. The implication is that it is more important to accurately quantify DAMGBP than TPG. However, it takes manpower and resources to accurately quantify an unknown and subjective variable. Therefore, if
both are competing for the same resources, then the money should be spent according to the benefit to be gained from knowing the true value of each. Having an accurate probability distribution on the true value of DAMGBP is going to decrease the expected cost to the Air Force more than having the same degree of accuracy on the TPG distribution.

Procedure vs. Results

The results of this study presented in chapter V apply only for the area of applicability specified. The area of applicability is the area where TPG is between 0.80 and 0.90, DAMGBP is between $50,000 and $150,000, and COSTGA is between $80,000 and $500,000. The same solution procedure could be repeated for a different area of applicability and the conclusions of such an analysis may be different. The particular area of applicability and the ranges of all the unknown variables used in this study were chosen by the author using his best judgment. The important product of this research is not the actual results so much as the development of the methodology which was used to structure and solve the problem.

Figure 11 obviates the need for detailed probability distributions on the DAMGBP and TPG variables thus eliminating much work. However, a graph like the one in Figure 11 can only remove the need for two probability distributions; the other probability distributions are still necessary to perform the analysis. The decision maker may find that other variables in the model are more uncertain and harder to quantify the DAMGBP and TPG variables. In that case, this same procedure can be used to eliminate the need for probability
distributions on the two most uncertain and hardest to quantify variables.

**Necessary Improvements**

This model relies heavily on the subjective inputs from experts, that is why it is so important to spend the effort to perform thorough probability assessments. It is very important not only to perform accurate probability assessments, but also to interview as many of the 'experts' as possible so that their differences can be compared and resolved. The science of combining expert opinions would be extremely applicable for this task.

The model needs to be made more user friendly so that an expert can provide the information that the model requires. The model requires information about two types of random variables. Two of the random variables which are input to the model can be identified by the user as more uncertain and more important than the other random variables. Call these the 'A' variables. Probability distributions must be built for the rest of random variables which are input to the model. Call these the 'B' variables.

For the 'A' variables, the user should be able to designate the ranges over which these random variables are to vary. Instead of being limited to a probability distribution about these 'A' variables, the user only has to give a high and a low value for each 'A' variable. In the example in this thesis, DAMGBP and TPG are the 'A' variables. The decision tool should then take the high, medium, and low values of the ranges given for the 'A' variables and, using the RSM techniques described in this thesis, generate the required runs from the model,
and build the appropriate response surface.

Probability distributions will have to be built for the other 'B' random variables. Most people have a hard time transforming their true knowledge about an unknown event to numbers which describe a probability distribution. This author believes the most efficient and promising method of getting the opinions of experts would be through the use of a computerized graphical display. It should be similar to probability wheels which are used when a decision analyst conducts a personal interview with an expert. Such a graphical display should be used to enable the decision maker or his designated expert to provide their knowledge about a subjective event in such a way that the expert does not require any knowledge about probability or statistics. Artificial Intelligence should be used in order for the machine to learn and build the appropriate probability distribution based on the inputs it receives from the expert or the decision maker.

Finally, the model needs to be expanded to include realistic alternatives for the government action (GA) decision and for the number to test decision (NTEST).
## Appendix A: Data Used in Chapter IV
### Output from the Runs Used for Sensitivity Analysis

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## Appendix B: Data Used in Chapter V

### Variable Settings and Output Values

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VITA

Captain Matthew A. Stone

He graduated from the United States Air Force Academy in May 1984 with the degree of Bachelor of Science in Operations Research, and a commission in the USAF. Upon graduation, he served as an Airborne Warning and Control System Operations Research Analyst in the Directorate of Operations and Requirements, 28th Air Division, Tinker AFB, Oklahoma, until entering the School of Engineering, Air Force Institute of Technology, in May 1987.
DECREASING NONCONFORMANCE OF PARTS IN THE AIR FORCE SUPPLY SYSTEM (UNCLASS)

Matthew A. Stone, B.S., Capt., USAF

MS Thesis

1988 December

72

DECISION MAKING, DECISION THEORY, RESPONSE, LOGISTICS, DECISION ANALYSIS

Thesis Advisor: Joseph A. Tatman, Captain, USAF
Assistant Professor, Dept of Mathematics and Computer Science

Approved for release in accordance with AFR 190-1

12 Jan 1989
Abstract

The purpose of this study was to provide AFLC (Air Force Logistics Command) with a prototype decision model which will help AFLC engineers decide when post-production testing for a particular NSN (National Stock Number) is beneficial to the Air Force, as well as what action the Air Force should take as a result of that testing. The problem is a two sided issue. On one hand, nonconformance costs the Air Force in terms of damaged equipment and potential harm to personnel. On the other hand, the Air Force cannot afford to test everything. The study provides an initial step in developing a decision aid to help AFLC allocate its scarce testing resources.

Sensitivity analysis was performed on this model by using the techniques of Response Surface Methodology. For the data used in this study, the random variable which has the greatest impact on the expected cost to the Air Force is the expected dollar value that a single part can be expected to cause given that the part is in nonconformance with its specifications. This result was confirmed using traditional EVPI (expected value of perfect information) calculations.

The most significant contribution of this study is the development of a graphic decision aid. With this aid, the decision maker can graphically see how the present situation relates to a 'break-even line', or indifference curve. In addition to showing the decision maker which alternative is optimal, this method shows how much the independent variables would have to change for the optimal alternative to change.