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This final report covers research in the following three areas:

1. Nonstrictly hyperbolic conservation laws: change of type of equations modelling three phase flow in porous media, solution of Riemann problems.

2. Plastic flow in two dimensions: linear stability of homogeneous deformations, justification of the quasudynamic approximation.

3. Glimm's method for the vibrating string' discovery of exact solutions related to a periodic motion.

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Final Report:

NONLINEAR SYSTEMS OF CONSERVATION LAWS

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This final report covers research in three areas:

1. Nonstrictly hyperbolic conservation laws.

2. Plastic flow in two and three space dimensions.

3. Computations for the elastic string equations.

1. Nonstrictly hyperbolic conservation laws. The classification [A] of 2x2 systems of nonstrictly hyperbolic conservation laws with quadratic nonlinearities identifies four different types of equations. The Riemann problem was solved in detail in [B] for three of the four types. The fourth type of equation, Case I, is the most significant for applications to models of multiphase flow in oil reservoirs, as discussed in [A]. This case involves undercompressive shocks, which are physical shock waves closely associated with systems that change type.

Using ideas from dynamical systems to understand the role of undercompressive shocks, the Riemann problem was solved for Case I equations in [1]. This paper exploits the special properties of quadratic nonlinearities. In particular, the solution of the Riemann problem in [1] is not structurally stable to perturbation of the equations by higher order nonlinearities. This problem has been addressed using a combination of equilibrium bifurcation theory and the theory of heteroclinic orbits of vector fields. Preliminary results of this approach were written up in a conference proceedings [2]. A paper giving the full analysis of perturbations of Case I quadratic nonlinearities is in preparation [3].

A detailed study of change of type for model equations of three phase flow in porous media is given in [4]. These equations typically have small elliptic regions, which are somewhat accentuated by the inclusion of gravity effects. Special classes of equation appear to lose strict hyperbolicity, but not change type. We also examined degenerate umbilic points that appear in the corners of the physical domain, corresponding to single phase flow, and found numerically that change of type can occur near these corners in the presence of gravity.

2. Plastic flow. This research is part of a project directed by David Schaeffer. Schaeffer and Bruce Pitman have made progress over the last few years on understanding the loss of well posedness in the equations of motion of granular materials. My research has focused on stability questions. The starting point was the equations of two dimensional motion of a rigid-plastic material, with an associative flow rule and volumetric strain hardening. In [5], we establish the correctness of the quasidynamic approximation that is being implemented for simulation, as part of the project. The central result is that the equations are linearly stable if and only if the quasidynamic approximation is linearly stable. We also establish a criterion for stability that is useful for computations, and is the basis for the stability results and their interpretation in [6]. A third paper in preparation studies stability in three dimensions under the same constitutive relations, where preliminary results suggest the quasidynamic approximation is less trustworthy.

3. Computations for the elastic string equations. Glimm's method was implemented for the full initial boundary value problem describing the motion of an elastic string stretched between two fixed points. The details of the algorithm are described in a techical report [7], which was subsequently rewritten as a journal paper [8]. The numerical results are strongly indicative of a periodic smooth solution, and this is reinforced by the discovery of two exact solutions that describe the motion of sections of the string. The analytic solutions can be combined to form a periodic function that resembles the numerical solution. The string equations serve as a start-up system for computations for large systems of conservation laws, but beyond this, the results may be a first indication that shock formation is not automatic for large nonlinear systems with periodic initial data.

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2. (with S. Schecter) Riemann problems involving undercompressive shocks. Proceedings, Univ. of Nice, Jan. 1988.

3. (with S. Schecter) The use of Melnikov's method to characterize shock waves with viscous profiles. In preparation.

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