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NUMERICAL STUDY OF ORBITAL

TRAJECTORIES ABOUT PHOBOS

THESIS

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AFIT/GSO/AA/88D-16

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Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In partial fulfillment of the

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Master of Science in Space Operations

Robert B. Teets

Captain, USAF

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Preface

Interest in a spacecraft's orbital trajectories about the Martian moon, Phobos, has arisen with renewed U.S. and Soviet emphasis on exploration of the Martian system. The Report of the National Commission on Space, appointed by President Reagan and chaired by Thomas Paine, recommended a thorough, efficient and systematic progression towards Mars (11:193) to give NASA focus and revive a sagging U.S. space program. The report recommends unmanned probes, penetrators, and sample return missions to the Moon, to Mars and its moons, to some promising asteroids, and to the outer planets and their moons followed by automated mining and materials processing plants, and eventually by manned explorations and human outposts. The Planetary Society is pushing for the U.S. to make an official declaration to strive towards human exploration of Mars (12:3). The Soviets have already stated they are pursuing the possibility of landing a man on Mars (11:161; 6:14-15). Soviet records for long duration space flight set in their earth orbiting space station provide groundwork for a long duration manned flight to Mars. The Soviets currently have two spacecraft on their way to the Martian system and have another Mars mission scheduled for 1994 (6:15; 2:16; 4:392). The next approved U.S. mission to Mars, the Mars Observer, is scheduled to launch in 1992 (4:392).

The two Soviet spacecraft, Phobos I and Phobos II, were launched July 7 and July 12 of 1988 and should be arriving in the Martian system near the end of January 1989 (4:392). The 480 million dollar (10:9b) mission includes a lander carried on each spacecraft and Phobos II carries a surface hopper intended to land and probe the surface of Phobos (4:392-393: 5:183). With some success, Phobos could "become the fourth extraterrestrial body on which spacecraft from earth have landed."(4:392) On September 2, a Soviet ground controller sent Phobos I an incorrect command causing loss of attitude control of the spacecraft and its solar panels (5:183). With the solar panels improperly aligned, there was not enough energy to sustain the transmitter. Soviet officials are hoping the solar panels will get aligned with the sun and restore power. Early Soviet missions to Mars, all before 1974, did not meet with great success (4:392). They either crashed, missed their target or stopped transmitting data early.

The last earth vehicles to go to Mars were the successful U.S. Viking orbiters and landers which reached the planet in the summer of 1976 (4:392). Viking flybys of Phobos provided the estimate of its gravitational parameter used in this study (13:35).

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Abstract

Orbital trajectories obtained from numerical integration of over two thousand sets of initial conditions for equations of motion of a spacecraft in the Mars Phobos system are examined in a Phobos centered rotating coordinate frame. The equations of motion, simplified with the choice of the system of units, are integrated using a Hamming fourth order predictor corrector algorithm. The trajectories were examined by plotting the position of the spacecraft and by listing of the state vector values at each crossing of the X, Y, and Z axes.

As initial velocity and altitude are varied, trajectories in the orbital plane of Phobos about Mars follow an orderly pattern. A range of initial velocities resulted in orbital trajectories about Phobos at a given altitude. Near the center of this range of initial velocities, termed the orbital window, is a unique initial velocity that resulted in a closed periodic orbit (at a given altitude) in the Phobos centered rotating coordinate frame. Initial velocities greater than or less than the velocity needed for a closed periodic orbit result in trajectories that move away from the periodic orbit in a predictable manner and eventually leave the orbit window.

Trajectories outside the orbit window either collide with Phobos or leave the vicinity of Phobos.

Trajectories out of the orbital plane of Phobos about Mars are more complex, three dimensional (helical shaped) paths that do not remain in an inclined plane, but do exhibit some order. Again there is a range of initial velocities that fall within an orbital window. No orbital trajectories were found that remained in a plane which contained Phobos and was inclined to the orbital plane of Phobos about Mars. No closed periodic orbits were found outside the orbital plane of Phobos about Mars.

NUMERICAL STUDY OF ORBITAL TRAJECTORIES ABOUT PHOBOS

I. Introduction

Background

Phobos is the larger of two small moons orbiting Mars. Phobos has been described as a potato shaped rock (4:392) and has been modeled mathematically as a triaxial ellipsoid by Werner (13:1). Phobos is a gravity gradient stabilized satellite with its long axis, length of 27 ± 1 Km (13:35), maintained in a Mars pointing orientation by the gravity gradient torques. Phobos's length along the axis in its direction of motion is 21.6 ± 1.4 Km and its length along the axis out of its orbit plane is 18.8 ± 1.4 Km (13:35). The gravitational parameters for Phobos and Mars are 6.6e-4 and 42826.32 Km^3/sec^2 respectively (13:35). Because the mass of Phobos is about 65 million times less than the mass of Mars, Mars is the dominating force on a spacecraft orbiting in the Mars Phobos system. Phobos is in a circular orbit 9378 Km from the center of Mars and completes an orbit in 7.65 hours (13:35).

Werner used Hamiltonian mechanics to develop restricted three body equations of motion for a spacecraft in orbit in the Mars Phobos system (13:1-12). He then searched for and found closed periodic orbits about Phobos in the plane of Phobos's orbit about Mars. He showed these were unstable orbits and suggested the existence of bifurcation regions caused by nearby inclined orbits but found no inclined periodic orbits that did not pass through Phobos (13:19-20). A bifurcation is "where a sudden change in behavior occurs as a parameter passes through a critical value" (8:317). It was conjectured that the equations of motion, being nonlinear, could give rise to a variety of interesting orbital trajectories as some initial conditions or control parameters are varied.

Werner's numerical exploration of his equations of motion was limited to finding closed periodic orbits in the plane of Phobos's orbit about Mars and discussing their stability using Floquet theory. Additional work was necessary to classify a broader spectrum of the possible orbital trajectories about Phobos both in and out of the plane of its orbit about Mars.

Problem Statement

The intent of this research effort is to begin with Werner's equations of motion, develop and apply computer software to explore, examine, and classify the possible orbital trajectories near Phobos both in and out of its orbital plane and to look for possible bifurcations.

Approach

The general approach was to numerically integrate the equations of motion beginning with a large number of different sets of initial conditions chosen in a way that would yield a set of orbital trajectories adequately representing all the possible trajectories. Orbital trajectories in the plane of Phobos's orbit about Mars were obtained by integrating over 1500 different sets of initial conditions. Orbital trajectories out of the plane of Phobos's orbit about Mars were obtained by integrating over 700 different sets of initial conditions.

The trajectories obtained by these numerical integrations were examined using plots and listings extracted from the state vector time history. Plots for a few hundred of the trajectories were produced from samples of the state vector. The state vector was sampled at approximately 44 second intervals to keep the size of the plot files small while using enough data for the plots to appear continuous. Listings produced for all of the trajectories displayed the state vector at each crossing of the X, Y, or Z axes, the period about Phobos in the XY plane, and messages to inform when a closed orbit was completed, when a collision occurred, or when the distance from Phobos exceeded the Mars-Phobos distance.

The set of possible trajectories were separated into those that remained in Phobos's orbital plane about Mars, and those that did not. These were further divided into trajectories that circled Phobos at least once (orbital) and those that did not.

Closed periodic orbits about Phobos were special cases of the orbital trajectories that were investigated. A closed periodic orbit was one that later returned to the set of initial conditions for position and velocity. A closed orbit was detected by the software if the state vector returned to the initial conditions to within a set of tolerances after circling Phobos at least once. This set of tolerances was defined as the set containing the largest change in each state variable encountered during any integration step since the beginning of the integration for a particular trajectory.

XY Planar Trajectories

The initial values of the Y coordinate (altitude) and the X velocity are varied to generate families of orbital trajectories in the XY plane (the plane of Phobos's orbit about Mars). All the other initial conditions are taken to be zero. With these initial conditions, the total spacecraft velocity is simply the initial value of the X velocity. Therefore, the initial value of the X velocity for a given initial +Y coordinate is chosen as the control parameter to generate orbital trajectories about Phobos in the XY plane. Zero values for the initial Z coordinate and the initial Z velocity restrict the motion of the satellite to the XY plane since each term in the equations for Z and $\dot{Z}$ contains a Z or a $\dot{Z}$. The choice of zero initial values for the X coordinate and the Y velocity makes the initial velocity of the satellite simply the chosen control parameter, the X velocity, which is then tangent to the trajectory. Choosing the initial conditions this way enabled thinking of the selection of an orbital trajectory as a choice of the kinetic energy (determined by the choice of the X velocity) and the potential energy (determined by the choice of the Y coordinate). Then, the problem of selecting a particular type of orbit about Phobos could be related to the simpler two-body problem of orbit selection. Given a particular altitude in the two-body problem, a certain velocity generally defines a closed orbit. However, in the restricted three-body problem, given a Y coordinate (altitude), selection of the X velocity defines an orbital trajectory which in general, is not a closed path.

Three Dimensional Trajectories

To generate trajectories out of the XY plane, the initial values of the Z coordinate and the X velocity were varied for a given initial Y coordinate value. Again, since the initial values of the Y velocity and the Z velocity were zero, the total spacecraft initial velocity was simply the initial X velocity which again could be treated as the control parameter. In an attempt to find indications of a bifurcation, the initial Y coordinate of 20 Km was chosen because it fell within a region described as à possible bifurcation region by Werner (13:19).

Overview

The introduction presented in this chapter gives some background information leading to the problem statement, then gives the general approach taken to solve the problem.

Chapter II begins the problem development with Werner's restricted three body equations of motion for a spacecraft in the Mars Phobos system. The Phobos centered rotating coordinate frame of reference used for these equations is described. A system of units for mass, length, and time is chosen which simplify the equations of motion.

Chapter III describes the method of solution. To solve the problem of describing the possible orbital trajectories about Phobos, software was needed to enable examination of a large number of numerical solutions to equations of motion. The software developed to integrate the equations of motion from many sets of initial conditions for the spacecraft's position and velocity is provided in the appendix.

The results are presented in Chapter IV using graphs that divide the solution space into several categories representing different outcomes for the trajectories obtained from the integrations. A number of plots are presented showing typical trajectories that fall into these categories. The results are discussed in three sections that separate the trajectories into those that result in closed periodic orbits in the XY plane, XY planar trajectories in general, and three dimensional trajectories.

Chapter V presents conclusions about practical implications when orbiting a spacecraft about Phobos which are drawn from the results of Chapter IV. They include discussion of the amount of precision in control of the state vector needed and the amount and frequency of velocity adjustments needed to stay in the orbit window.

II. Problem Development

The motion of a spacecraft of negligible mass orbiting in the Mars Phobos system, a restricted three body problem, was described mathematically by Werner (13:11-12). He modeled the attraction due to the irregular shape of Phobos using the potential energy field of a homogeneous triaxial ellipsoid rather than a sphere which is usually treated as a point source. Using Hamiltonian mechanics, he derived the following equations of motion for the system.

$$\dot{X} = P_x + \Omega(Y - D) \tag{1}$$

$$\dot{Y} = P_y - \Omega X \tag{2}$$

$$\dot{Z} = P_z \tag{3}$$

$$\begin{aligned}\dot{P}_x = \Omega P_y - GM_o(X/D^3 - 3XY/D^4 + 3R^2X/2D^5) - GM_1X/R^3 \\ + 3GIX/4R^5 - 3GX/4R^7[(5X^2 - 2R^2)(I-2I_{xx}) \\ + 5Y^2(I - I_{yy}) + 5Z^2(I - 2I_{zz})]\end{aligned} \tag{4}$$

$$\begin{aligned}\dot{P}_y = - \Omega P_x + GM_o[1/D^2 - 4Y/D^3 - 3(YR^2 - DR^2 - \\ 2DY^2)/2D^5] - GM_1Y/R^3 + 3GIY/4R^5 - \\ 3GY[(5Y^2 - 2R^2)(I - 2I_{yy}) + 5X^2(I - 2I_{xx}) + \\ 5Z^2(I - 2I_{zz})]/4R^7\end{aligned} \tag{5}$$

$$\dot{P}_z = -GM_o(Z/D^3 - 3YZ/D^4 + 3ZR^2/2D^5) - GM_1Z/R^3 + 3GZI/4R^5 - 3GZ[(5Z^2 - 2R^2)(I - 2I_{zz}) + 5X^2(I - 2I_{xx}) + 5Y^2(I - 2I_{yy})]/4R^7 \quad (6)$$

Where X, Y, Z are the coordinates of the spacecraft position (altitude) and $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are the components of the spacecraft velocity expressed in the coordinate frame described below. P_x, P_y, and P_z are the conjugate momenta (9:172-173).

The other terms used in eqs (1)-(6) are

Ω = angular velocity of XYZ coordinate frame

$R^2 = X^2 + Y^2 + Z^2$

M_o = mass of Mars

M_1 = mass of Phobos

G = universal gravitational constant

D = Mars to Phobos distance

$I = I_{xx} + I_{yy} + I_{zz}$ = sum of the principal moments of inertia of Phobos

The principal moments of inertia of a homogeneous ellipsoid are (7:501)

$$I_{xx} = M_1(b^2 + c^2)/5 \quad (7)$$

$$I_{yy} = M_1(a^2 + c^2)/5 \quad (8)$$

$$I_{zz} = M_1(a^2 + b^2)/5 \quad (9)$$

Where

a = half length of Phobos along X axis

b = half length of Phobos along Y axis

c = half length of Phobos along Z axis

The coordinate system for eqs (1)-(6) is a rotating cartesian system with the positive Y axis always pointed towards Mars and the positive X axis pointed in the direction of Phobos motion in its orbit about Mars. Figure 1 shows Mars and Phobos in the XY plane. The size of Phobos is scaled up 10 times to make it just visible. The Z axis points out of the plane of the orbit of Phobos (out of the page) forming a right-handed coordinate system.

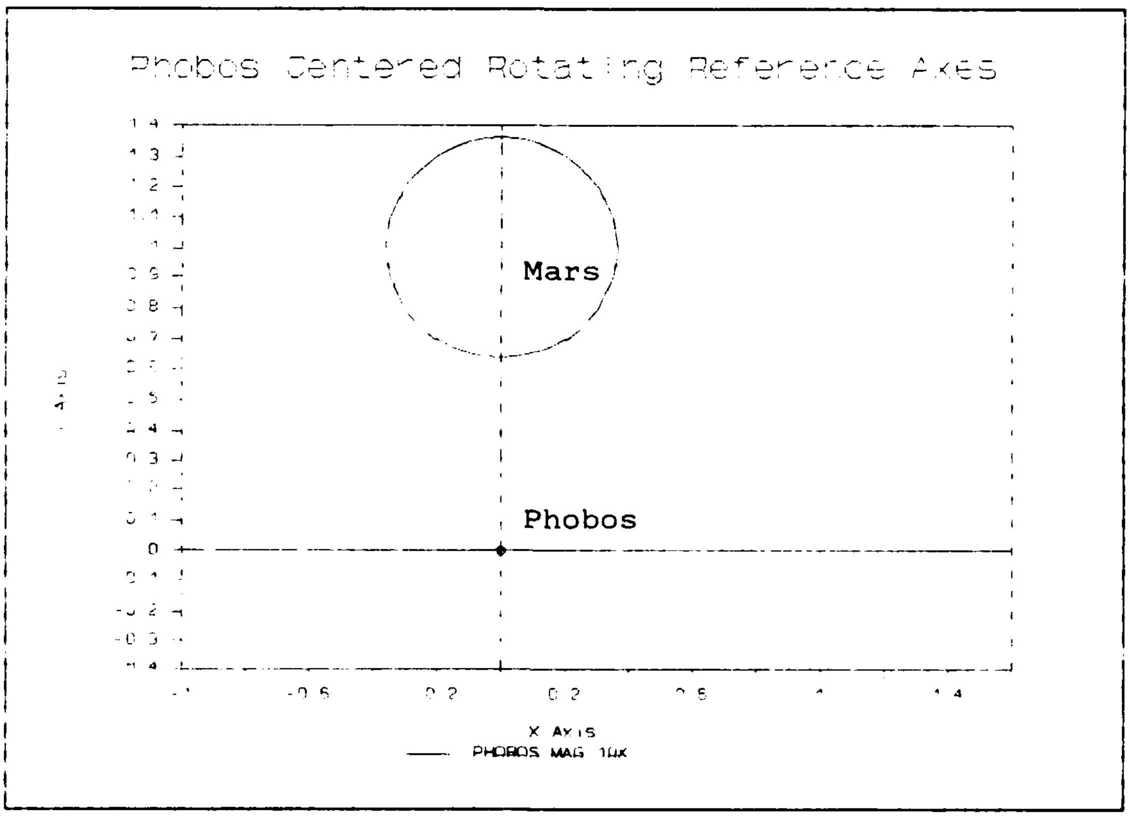

Figure 1. Phobos Centered Rotating Coordinate System

The choice of the system of units generally used with restricted three body problems allows some simplification of the equations of motion. The mass unit is defined by letting the sum of the masses of the two primary bodies be equal to 1 mass unit (M.U.). The unit of length, 1 distance unit (D.U.), is the distance between the two primaries (9378 Km). The time unit (T.U.) is chosen so that the gravitational constant G is equal to 1. This happens when the period of the two primaries in their orbit about each other (7.65 hours) is equal to 2π T.U. One T.U. is then equal to 4383 seconds (about 1.2 hours). The angular velocity, Ω, of the coordinate frame is then 1 radian/T.U. With these units, and letting

$$I_1 = -I_{xx} + I_{yy} + I_{zz}$$

$$I_2 = I_{xx} - I_{yy} + I_{zz}$$

$$I_3 = I_{xx} + I_{yy} - I_{zz}$$

where the principal moments of inertia are calculated using eqs (7)-(9) with the just defined units for the mass and length terms, Eqs (1) - (6) simplify to

$$\dot{X} = P_x + Y - 1 \tag{10}$$

$$\dot{Y} = P_y - X \tag{11}$$

$$\dot{Z} = P_z \tag{12}$$

$$\dot{P}_x = P_y - M_o(X - 3XY + 3R^2X/2) - M_1X/R^3 + 3IX/4R^5 - 3X[(5X^2 - 2R^2)I_1 + 5Y^2I_2 + 5Z^2I_3]/4R^7 \tag{13}$$

$$\dot{P}_y = - P_x + M_0[1 - 4Y - 3(YR^2 - R^2 - 2Y^2)/2] - M_1Y/R^3$$
$$+ 3IY/4R^5 - 3Y[(5Y^2 - 2R^2)I_2 + 5X^2I_1 + 5Z^2I3]/4R^7 \quad (14)$$

$$\dot{P}_z = - M_0(Z - 3YZ + 3ZR^2/2) - M_1Z/R^3 + 3ZI/4R^5 -$$
$$3Z[(5Z^2 - 2R^2)I_3 + 5X^2I_1 + 5Y^2I_2]/4R^7 \quad (15)$$

III. Method of Solution

Eqs (10)-(15) are the coupled nonlinear differential equations to integrate to get the path of a spacecraft in the Mars Phobos system. The integration is performed by the FORTRAN program listed in the Appendix. Eqs (10)-(15) appear in the subroutine "RHS.for", called by subroutine "Haming.for", a fourth order predictor corrector integration algorithm. The main program, "Phobos.for", reads a set of initial conditions and the length of time to integrate from an input file, "IC.dat". The program integrates the equations of motion until the trajectory terminates in a collision with Phobos or Mars, exceeds 1 D.U. from Phobos, or the specified length of time is reached. Then a new set of initial conditions are read from the input file and the program integrates again and again until the end of file is read from "IC.dat".

The program types selected information to the screen (which can be saved in a log file) and writes the state vector (position and velocity) and the elapsed time to data files if plots are desired. The information typed to the screen includes a notice when a coordinate is passing through a value of zero, the rate of change of that coordinate, and the value of the X coordinate (or the Y coordinate if X is passing through zero). Also typed to the screen are the time to complete each orbit, the orbit count, and a message to indicate the reason for early termination of the integration if needed.

The two options to write the state vector and elapsed time to data files allows the data to be plotted using any plotting software that plots columns of data against another column (like a spreadsheet). The first option, to write the state vector and the elapsed time to an output file named "state.dat;n" each .01 T.U. (43.83 sec), enabled the plotting of the orbital trajectories. The second option, to write the same information to a file named "section.dat;n" each time the trajectory crosses the Y = 0 plane, is intended to produce plane section plots.

The integration step size was chosen large enough so that the computer run time for a group of trajectories was reasonably short, yet small enough so that a smaller step size wouldn't significantly change the output state vector or the accumulated period of an orbital trajectory. A significant change in the state vector was one that was larger than the largest change encountered during any integration step for the orbit. A significant change in the orbital period was one that was larger than the step size chosen. The choice of 0.0001 T.U. (.4383 seconds) for the step size accomplished this goal. A 10 T.U. trajectory would run in about a minute. Using a 0.00001 T.U. step size didn't change the results but took much more computer time.

A closed orbit is detected by the software when the state vector returns to the initial conditions within an amount equal to the largest step size of the state vector during the integration. Because that step size depends on the integration time step, a small time step is desirable. The time reported for the orbital period also depends on the step size. The chosen step size, less than 0.5 seconds, makes the reporting of the period to the nearest second simpler and more convenient than a larger step size which would have required some interpolation between steps to report to that accuracy.

The trajectories are not allowed to pass through the surface of Phobos or Mars. The position of the spacecraft is checked each step of the integration to ensure it falls outside the equation of an ellipsoid centered on the origin with Phobos's dimensions. The integration is stopped if the position of the spacecraft falls inside the ellipsoid for three reasons. First, the equations of motion are no longer valid inside the surface of Phobos because the potential energy expression used to develop the equations is different inside the ellipsoid (3:98-107). Second, numerical difficulties can arise if the position of the spacecraft is near the origin (R approaching zero) because the equations are full of terms divided by R. Third, the spacecraft crashes into Phobos and the flight is over. The position of the spacecraft is also checked to ensure it remains outside a sphere with a Mars radius centered on Y equals 1 D.U.

because that also represents an undesirable trajectory that crashes into Mars.

No trajectories were encountered that ended in collision with Mars because all the trajectories considered were near Phobos and therefore had plenty of orbital velocity about Mars.

IV. Results

Closed Periodic Orbits in the XY Plane

Werner found closed periodic orbits are possible in the XY plane from near the surface of Phobos to beyond 5000 Km (13:23). With proper selection of the initial X velocity (the chosen control parameter), a closed orbit in the XY plane can be obtained for any practical altitude. All these closed planar orbits proceed in a clockwise direction as seen looking down the +Z axis. Figure 2 shows some closed periodic orbits about Phobos with Y axis crossings at 20, 40, 60, 80, 100, 125 Km altitude.

Figure 3 shows that for periodic orbits the value of the X velocity is generally positive (spacecraft velocity greater than Phobos velocity) for Y greater than zero (closer to Mars) and is always negative (spacecraft velocity less than Phobos velocity) for Y less than zero (farther from Mars). This is the expected result for a spacecraft in a purely two-body orbit about Mars. The velocity of the orbit decreases with increasing distance from Mars as kinetic energy is traded for potential energy. The values of the X velocity for orbits with higher altitudes about Phobos decrease some because they are in lower energy elliptical orbits about Mars than the circular orbit of Phobos about Mars. The result is the bowed look of the plot in figure 3. Orbits in the opposite direction

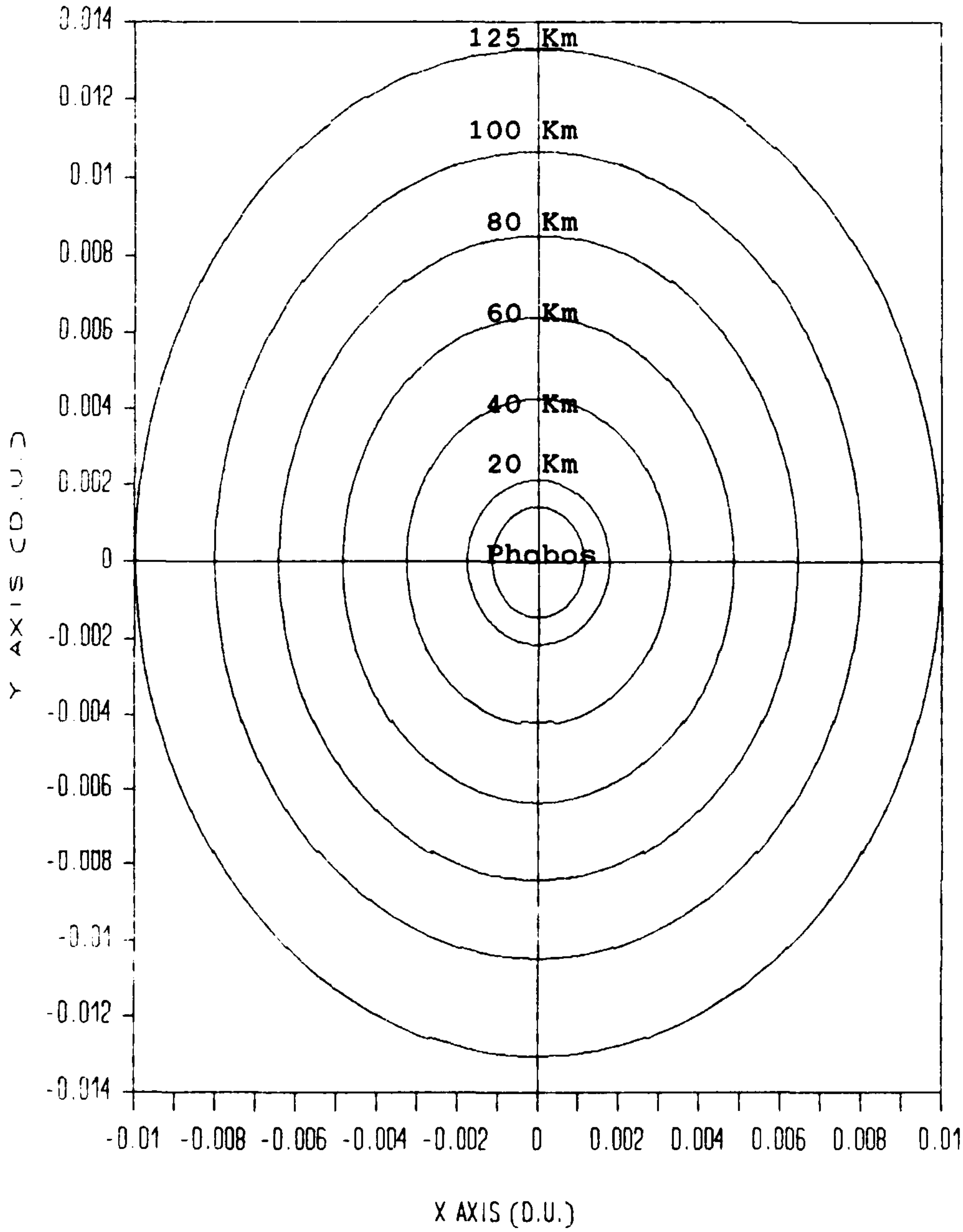

Figure 2. Closed Periodic Orbits About Phobos in XY Plane

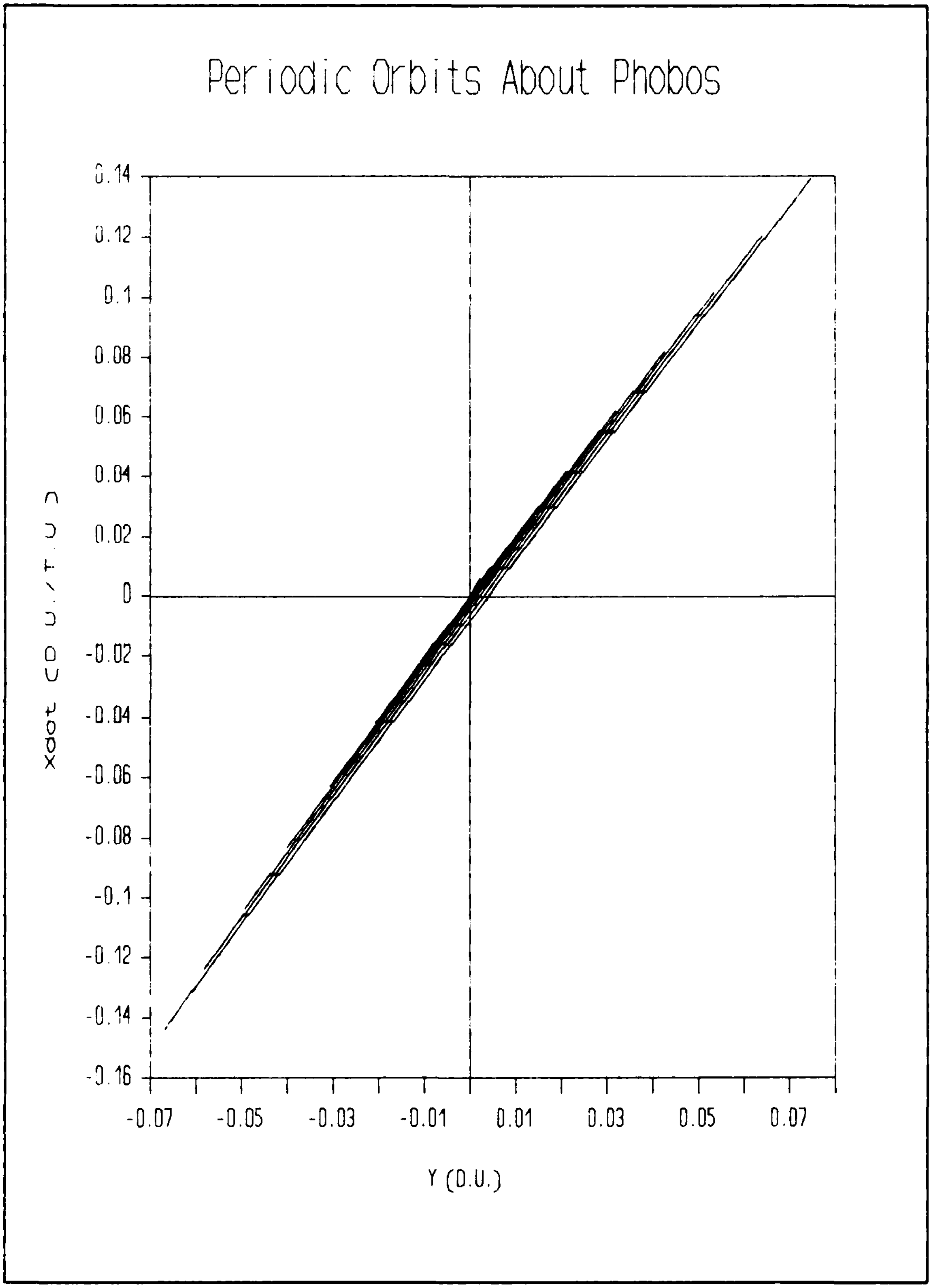

Figure 3. X Velocity for Periodic Orbits in the XY Plane

(counterclockwise) are not found and are unlikely because Mars gravity and the resulting orbital motion about Mars dominates even when fairly close to Phobos. Figure 4 shows for periodic orbits, the Y velocity must be greater than zero (moving closer to Mars) when X is negative (spacecraft behind Phobos in Mars orbit) and the Y velocity must be less than zero (moving away from Mars) when X is positive (spacecraft ahead of Phobos in Mars orbit).

Using figures 3 and 4, given any set of X and Y coordinates, the components of the spacecraft's velocity needed in the X and Y directions to achieve a closed periodic orbit can be approximately determined. The choice of the particular X velocity along the width of the bow in figure 3 for a given Y value depends on the altitude of the orbit about Phobos.

With the initial Y velocity set to zero, the initial X velocity needed to produce a closed periodic orbit at a desired Y altitude is plotted in figures 5-7. Figure 5, which shows the initial values of the X velocity for orbits near Phobos, reveals a noticeable bend in the curve around a Y altitude of about 20 Km. The slow bend in the line of periodic orbits around the 20 Km Altitude may be related to the appearance of the additional non-zero pair of Poincaré exponents found by Werner at altitudes near 20 Km (13:23). Figure 6 extends the data out to 900 Km altitude and appears almost linear. Figure 7 extends the data further to 5500 Km and clearly shows the nonlinearity of the data.

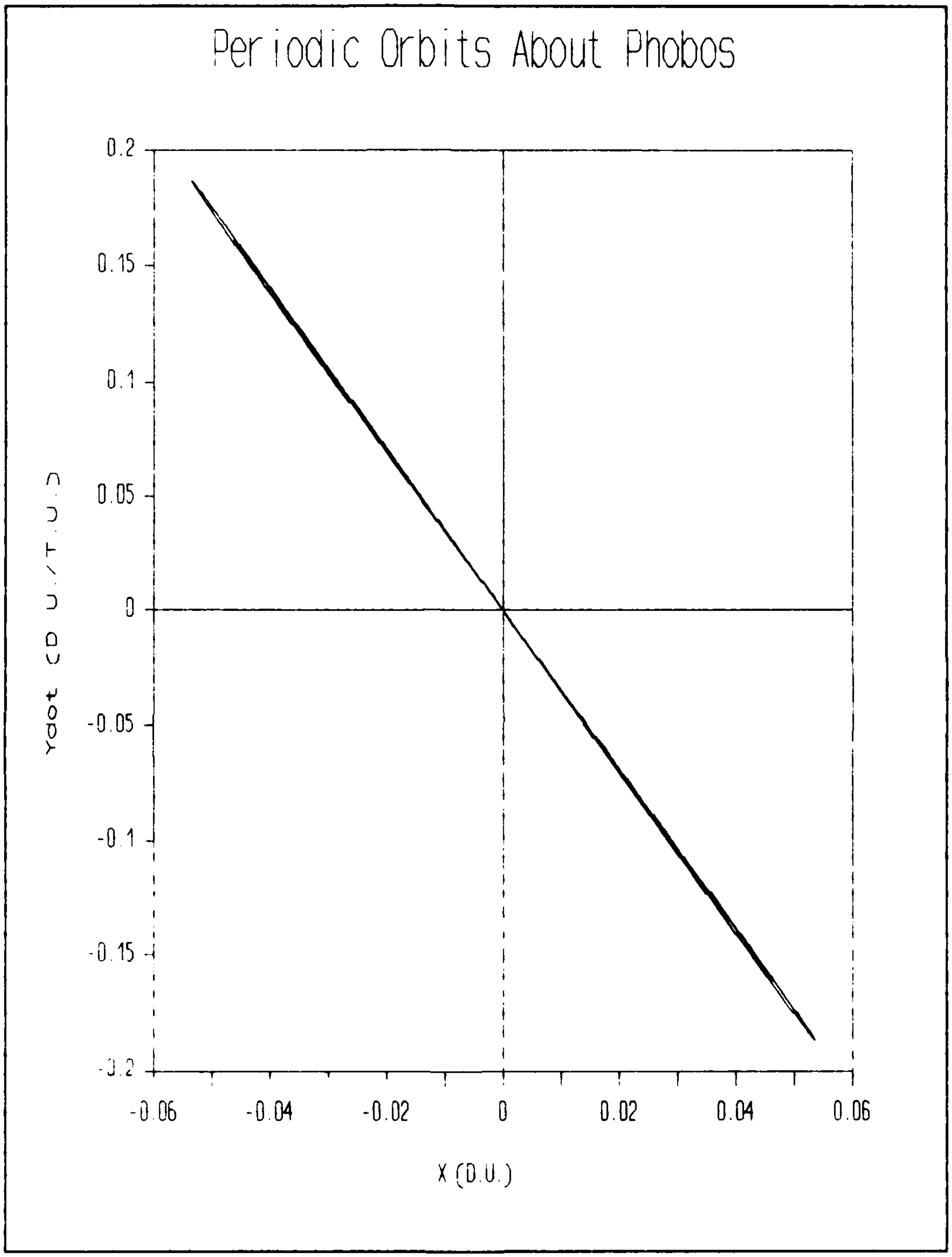

Figure 4. Y Velocity for Periodic Orbits in the XY Plane

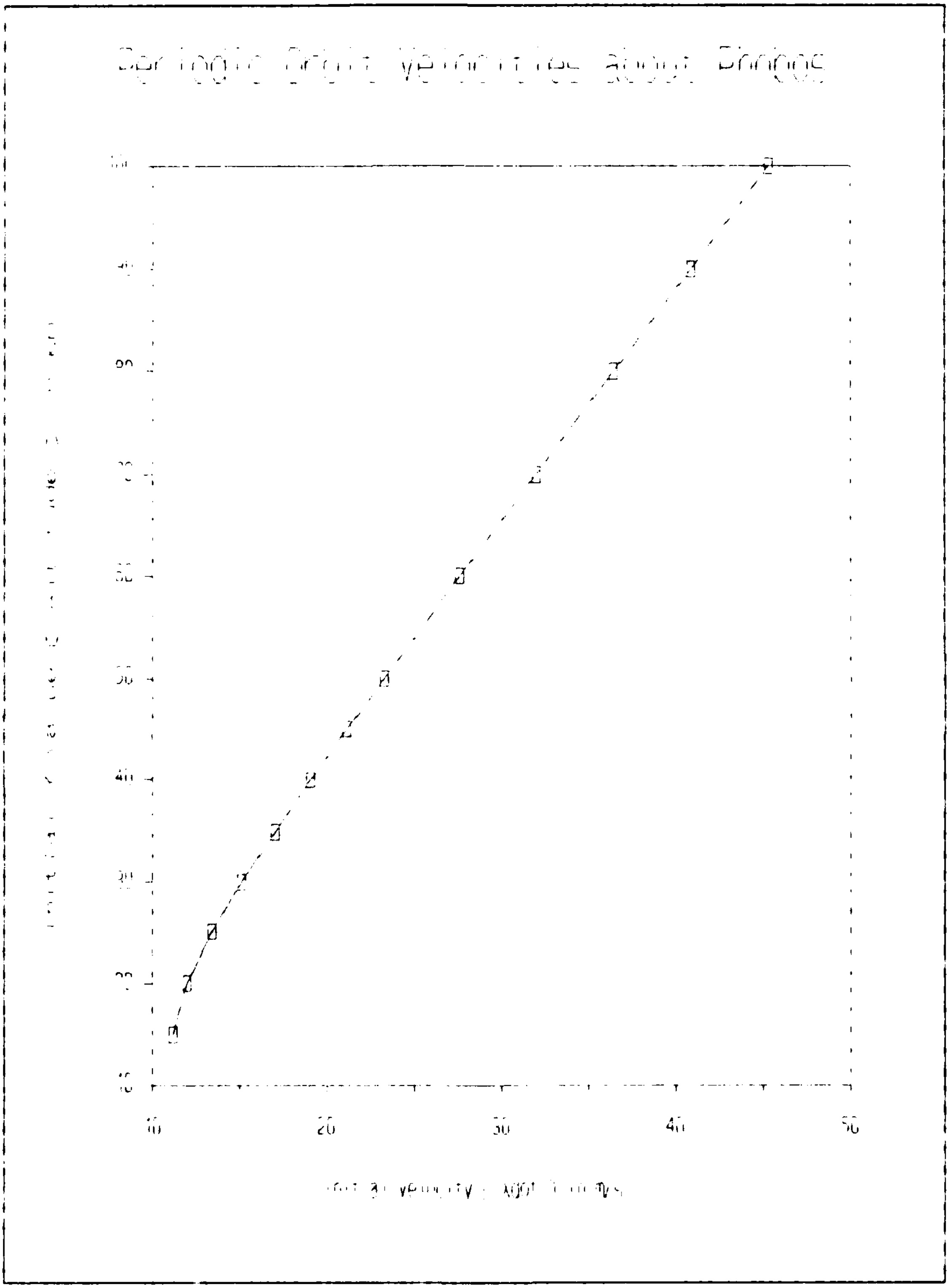

Figure 5. Initial Velocities (Xdot) for Periodic Orbits near Phobos in the XY Plane

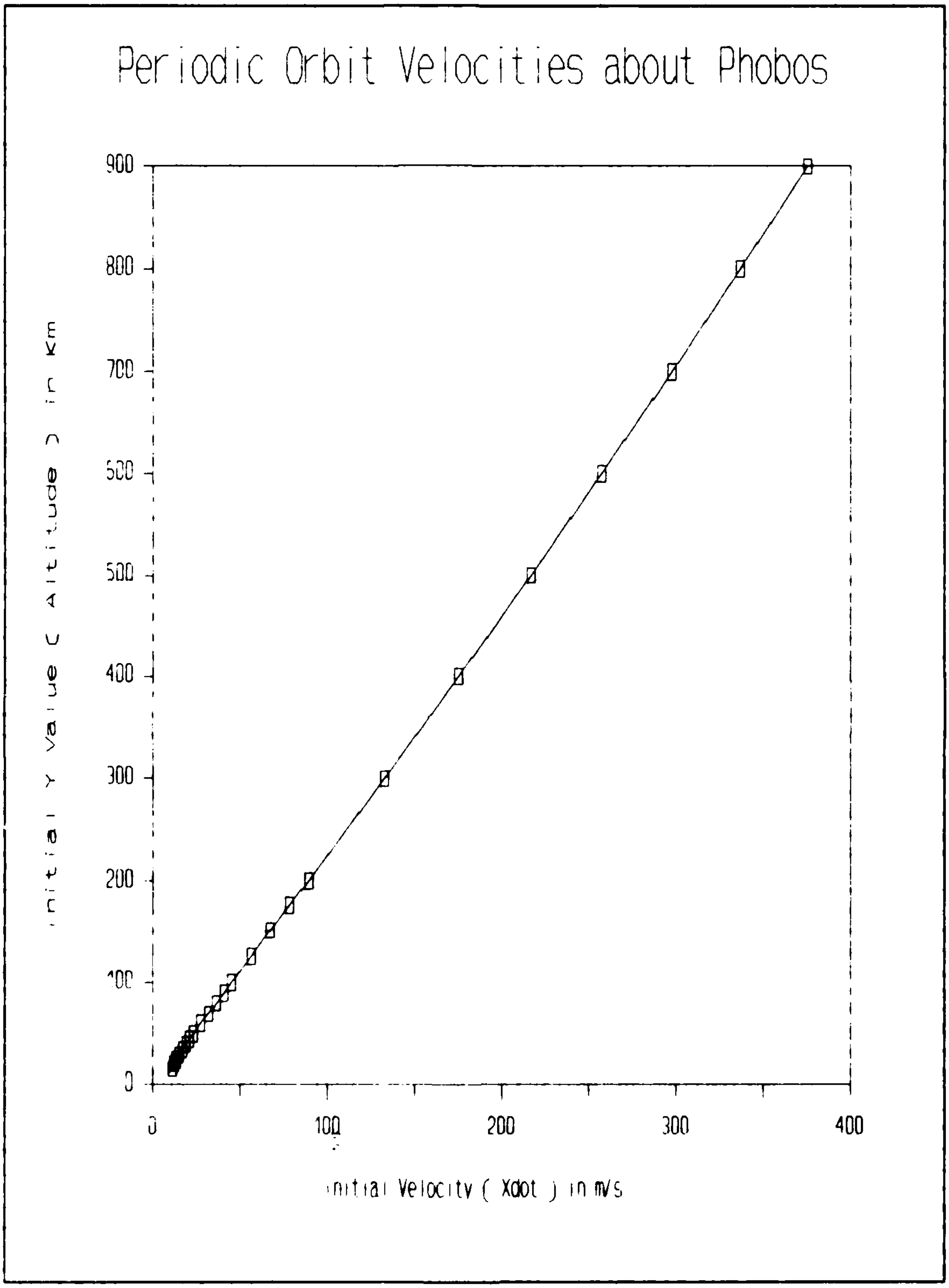

Figure 6. **Initial Velocities (Xdot) for Periodic Orbits out to 900 Km in the XY Plane**

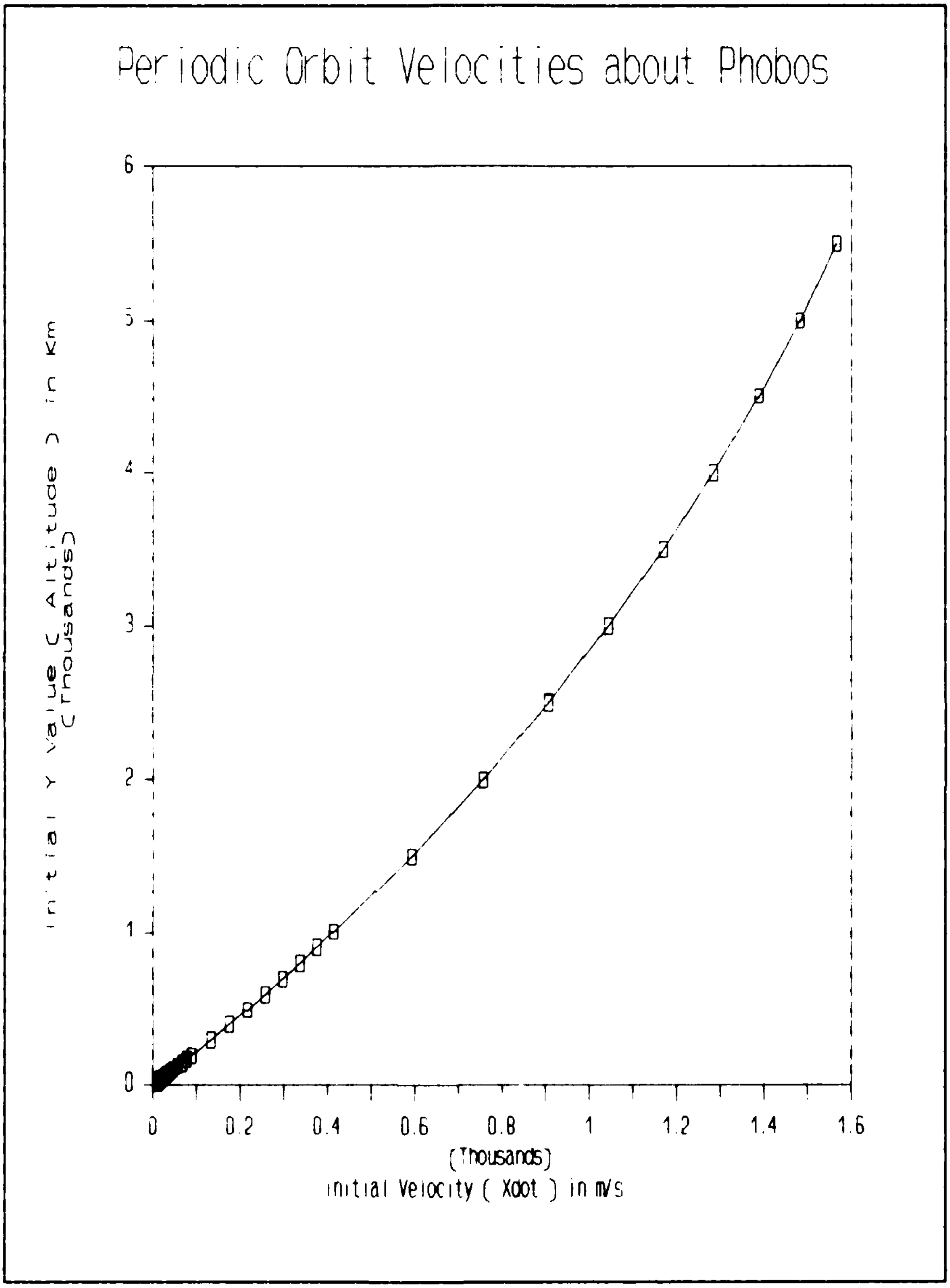

Figure 7. Initial Velocities (Xdot) for Periodic Orbits out to 5500 Km in the XY Plane

The initial X velocity, $\dot{X}_p$, needed to generate a periodic orbit at a given Y altitude near Phobos varies nearly linearly and can be approximated by a linear expression given by

$$\dot{X}_p(m/s) = .44\ Y(Km) + 1.2$$

After a second guess for the correct initial X velocity for a periodic orbit, a few (one or two) simple linear interpolations quickly converges on the correct value. The interpolation is accomplished by adjusting the initial X velocity to eliminate the difference in the first two X axis crossings for the trajectories obtained from the integration of the previous two guesses.

Although time is not explicit in the equations of motion, the orbital period of a spacecraft in a closed periodic orbit about Phobos is of interest. The orbital period is obtained after the integration of the complete orbit path and is the result of summing the individual integration time steps. The orbital period of orbits about Phobos increases rapidly from 7310 seconds (about 2 hours) for orbits at 15 Km altitude to 10240 seconds (almost 3 hours) at 50 Km altitude. Figure 8 shows a plot of the orbital periods for closed periodic orbits in the XY plane from 15 Km to 200 Km and shows a drastic change in slope around 50 Km. Above 50 Km altitude the periods of the orbits are about 3 hours and increase slowly with increasing altitude. Figure 9 extends the plot of orbital periods out

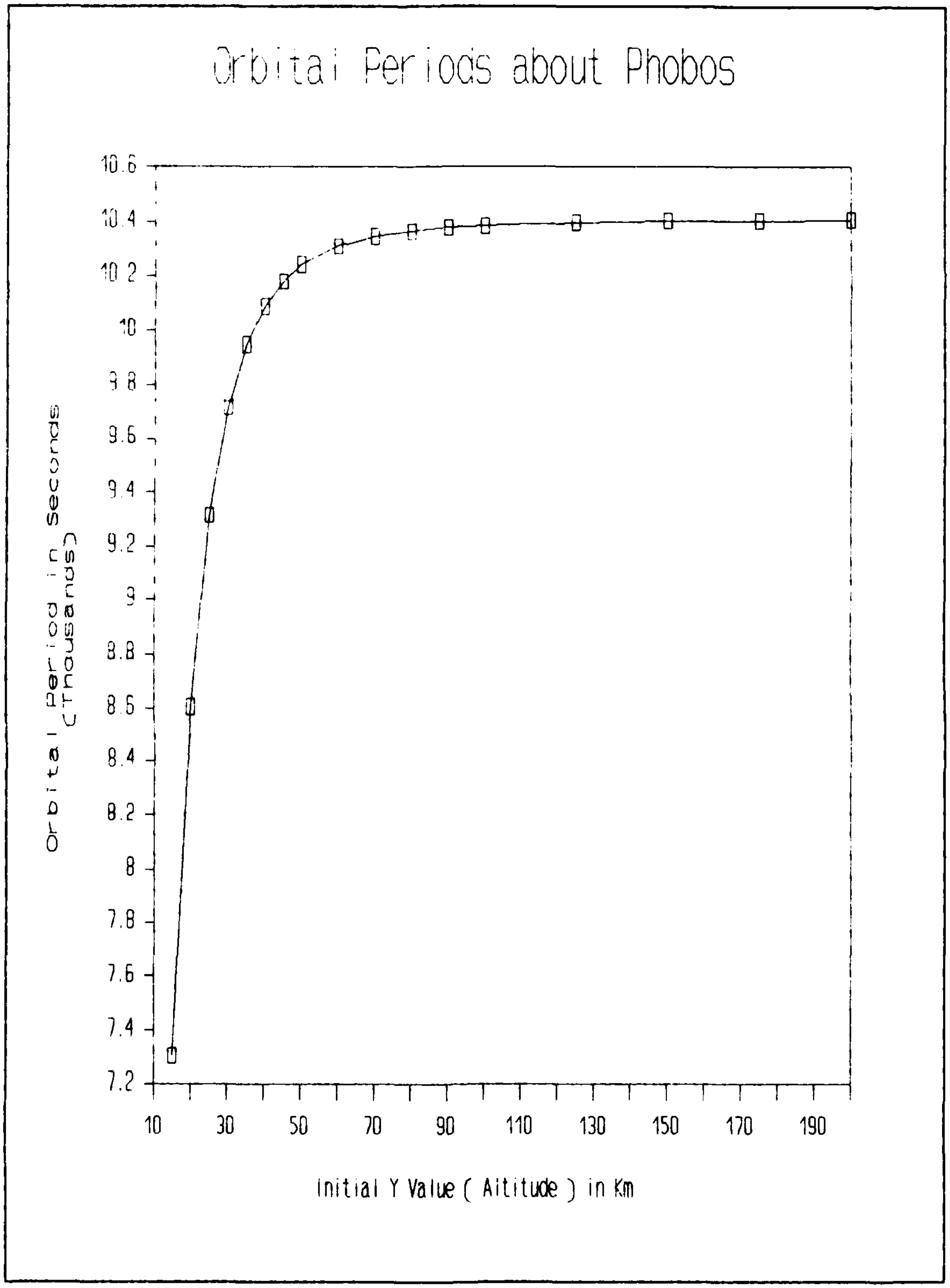

Figure 8. Orbital Periods of Closed Periodic Orbits Near Phobos in the XY Plane

to 5500 Km and shows the slowly increasing trend. The sharp bend (big decrease in slope) around 55 to 100 Km or 10300 second period is where Werner found an additional non-zero pair of Poincaré exponents (13:23).

Werner found the initial Y coordinates (altitudes) needed to produce closed periodic orbits in the XY plane with orbital periods in increments of 100 seconds beginning with 8200 seconds and ending with 11500 seconds (13:23). The large jump in altitude he shows from 105 Km with a 10400 second period to 1284 Km with a 10500 second period shows the wide range of altitudes that have approximately the same orbital period as seen in figures 8 and 9.

Figure 10 shows how the orbital periods of the closed periodic orbits in the XY plane found by this study compares to those found by Werner. Because Werner's data are given by incremental period and the data in this study are given by incremental altitude, Werner's data are adjusted by interpolation to give periods at incremental altitudes. There is some disagreement (13% or less) in the data for orbits close to Phobos. This may be due to differences in the details of the numerical integration such as the time step used or due to numerical round-off differences arising from the different magnitudes of the numbers resulting from different choices for units of measure. The agreement is very good for orbits beyond 50 Km altitude.

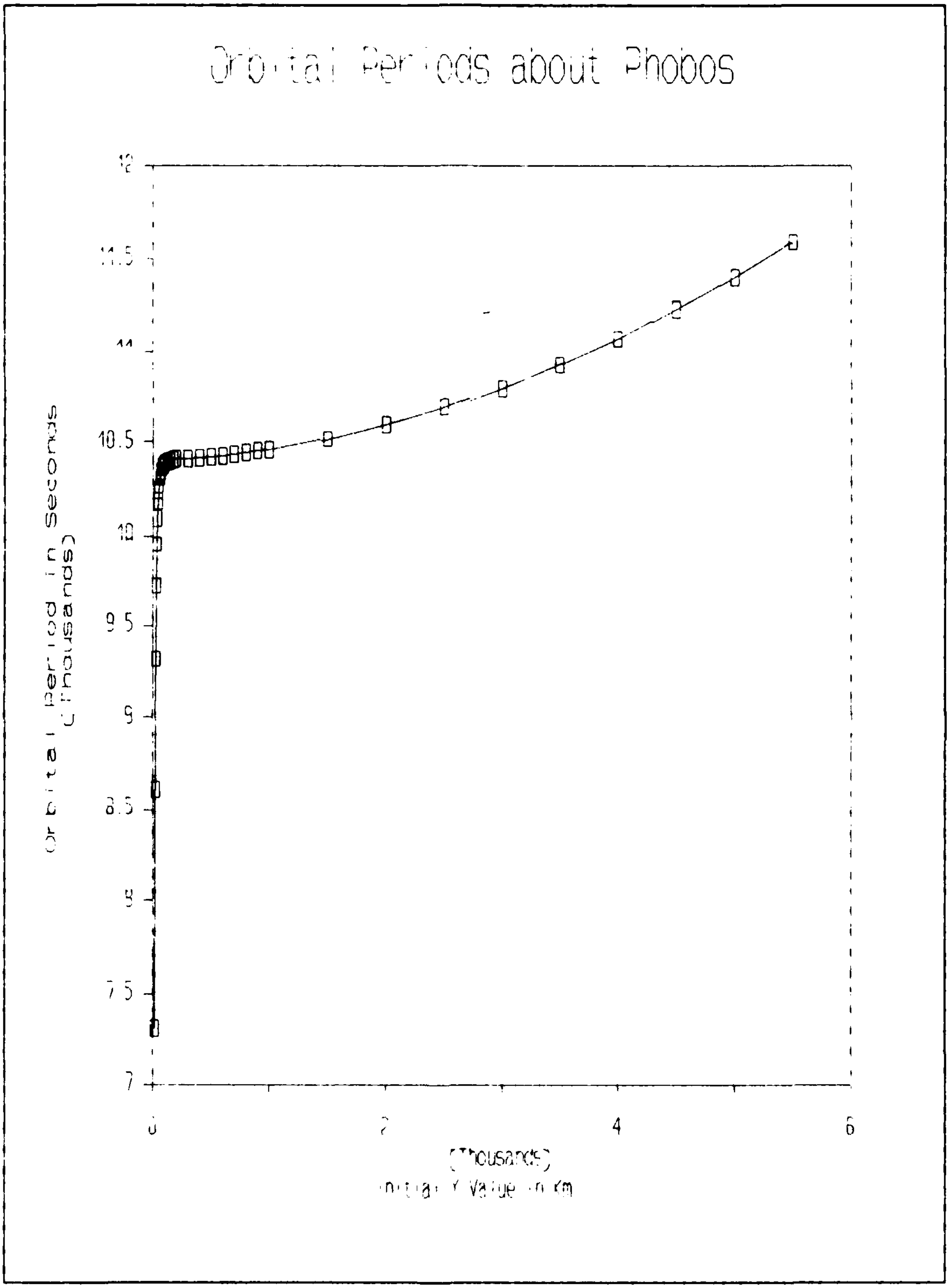

Figure 9. Orbital Periods of Closed Periodic Orbits about Phobos

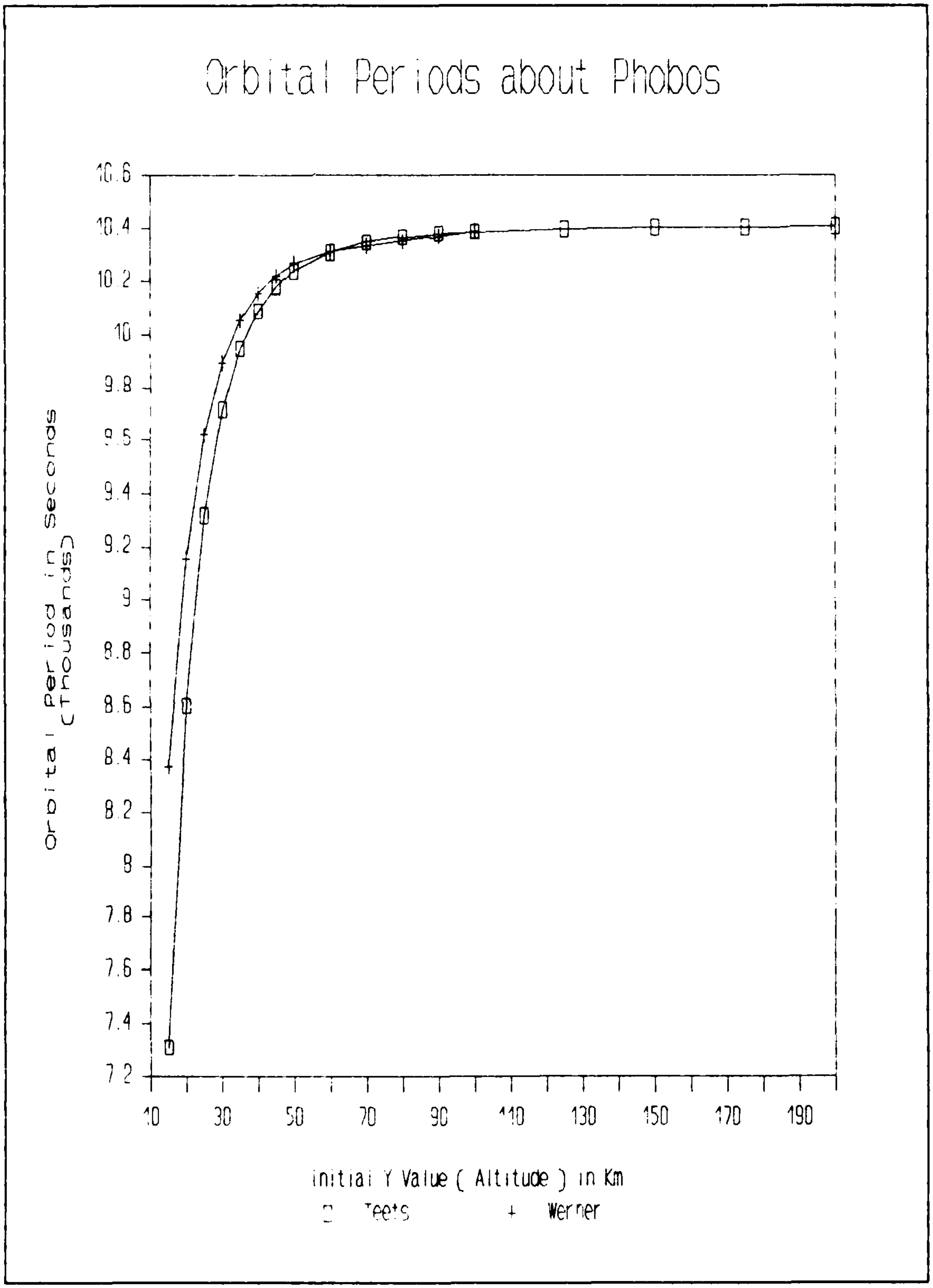

Figure 10. Orbital Periods Compared with Werner's

XY Planar Trajectories

Figure 11 is a map of the type of trajectories obtained by varying the initial X velocity (kinetic energy) for a given Y altitude (potential energy). The orbital trajectories that do not form a closed orbital path about Phobos eventually either collide with Phobos or leave the vicinity of Phobos (escape). In either case, the spacecraft can circle Phobos not at all, once, twice, or more times before collision or escape. The number of orbits about Phobos before collision or escape is related to how close the initial value of the X velocity is to the value needed for the periodic orbit at that altitude.

Figure 11 shows there are three regions of escape velocities. A spacecraft beginning at a Y coordinate with an initial X velocity that falls in any one of these three regions will leave the vicinity of Phobos without circling Phobos.

Figure 11 also shows three regions of velocities that result in collisions with Phobos before completing an orbit about Phobos. One of these regions is much narrower than the other two. It appears to be a sliver split from a larger collision region by an escape region which appears only at altitudes above 40 Km.

The remainder of the area on Figure 11 represents trajectories that circle Phobos at least once and is called the orbit window. The line of velocities for closed

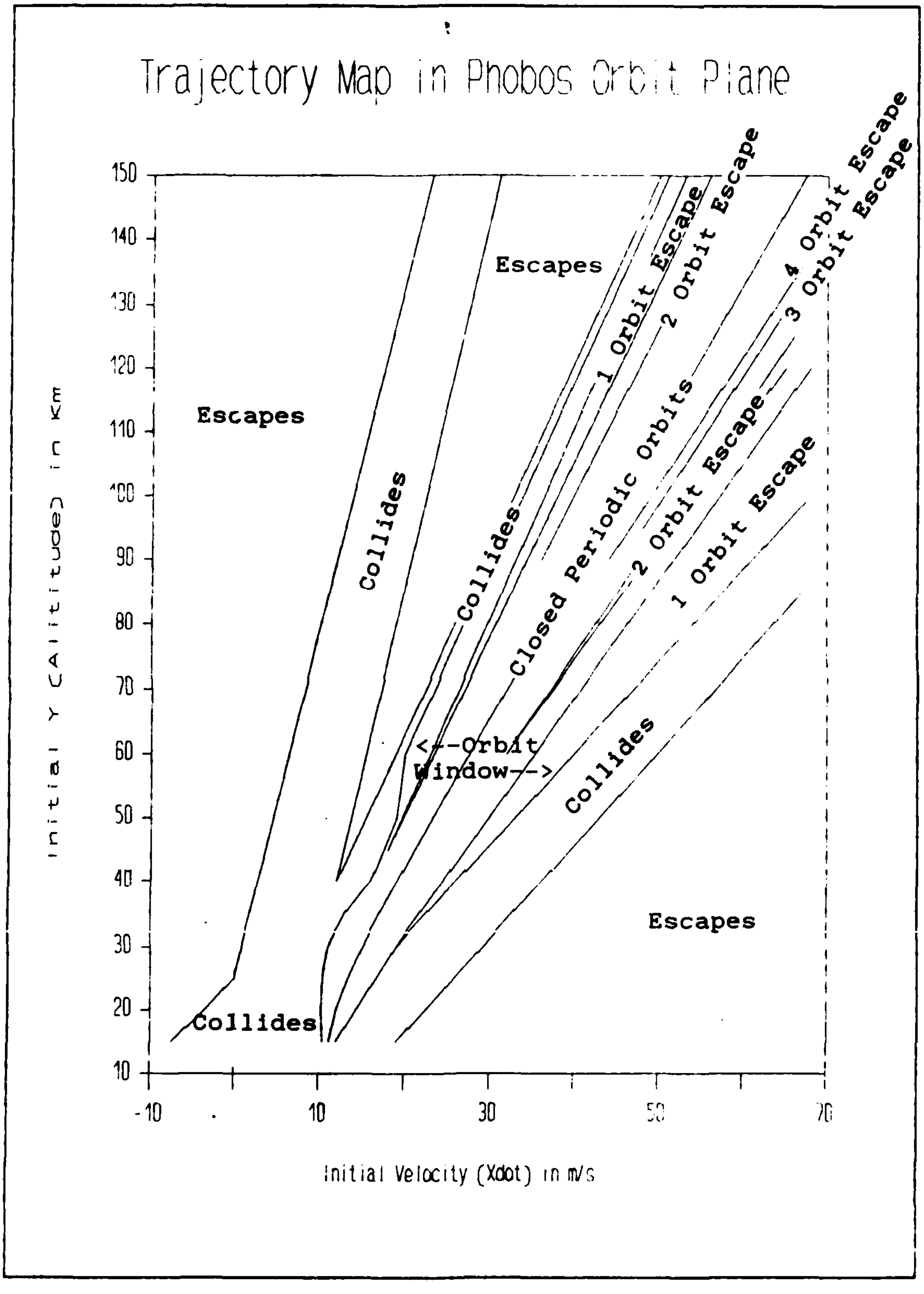

Figure 11. Trajectory Map in Phobos Orbit Plane

periodic orbits of Phobos is at the center of the orbit window. The orbit window is narrow at low Y altitudes but expands as the altitude increases.

Two notable rapid expansions of the orbit window occur that stand out from the general expansion trend. The first is between Y altitudes of 19 and 25 Km. The slope of the left-hand boundary of the orbit window is greater than 90° between 19 and 20 Km. The slope of this same boundary is less than 90° between 23 and 24 Km. Assuming the boundary is continuous, the slope must be 90° (infinite) at some Y altitude between 20 and 23 Km. The second rapid expansion of the orbit window is between 50 and 60 Km. The slope of the left-hand boundary of the orbit window in this range appears to approach infinity also. These two ranges of altitudes correlate with the two regions where Werner found the additional non-zero Poincaré exponents and suggested they indicated the existence of bifurcation regions in the solution space of the equations of motion (13:19-20). They certainly signaled the existence of these two ranges of infinite slopes.

Figure 12 shows orbital trajectories beginning at a Y of 20 Km altitude with initial $\dot{X}$ values of -4, -2, 0, 2, 4, 6, 8, and 10 m/s which are all less than the velocity needed for a periodic orbit. The trajectory with an initial $\dot{X}$ of - 4 m/s escapes and falls in the first escape region shown in Figure 11. The rest of the trajectories in Figure

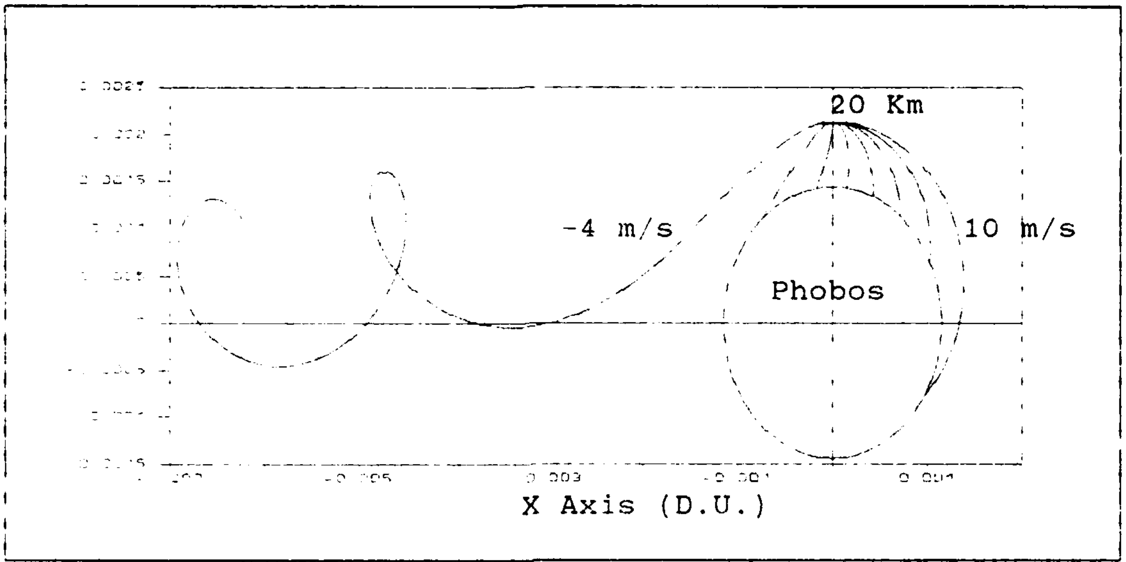

Figure 12. Initial Velocities Less Than Closed Periodic Orbit Velocity

12 all collide with Phobos without circling the moon. These trajectories all fall in the first collision region shown in Figure 11.

Figure 13 shows trajectories beginning at a Y of 20 Km altitude with velocities of 14, 16, 18, 20, and 22 m/s which are all greater than the velocity for a periodic orbit. The trajectory that begins with an initial velocity of 22 m/s just barely misses Phobos then escapes. This trajectory is just over the boundary into the third escape region identified in figure 11. The rest of the trajectories in figure 13 are in the third collision region (counting from left to right) shown in Figure 11.

Values of the initial X velocity greater than $\dot{X}_p$ resulted in orbits whose X axis crossing, X_c, shifted right

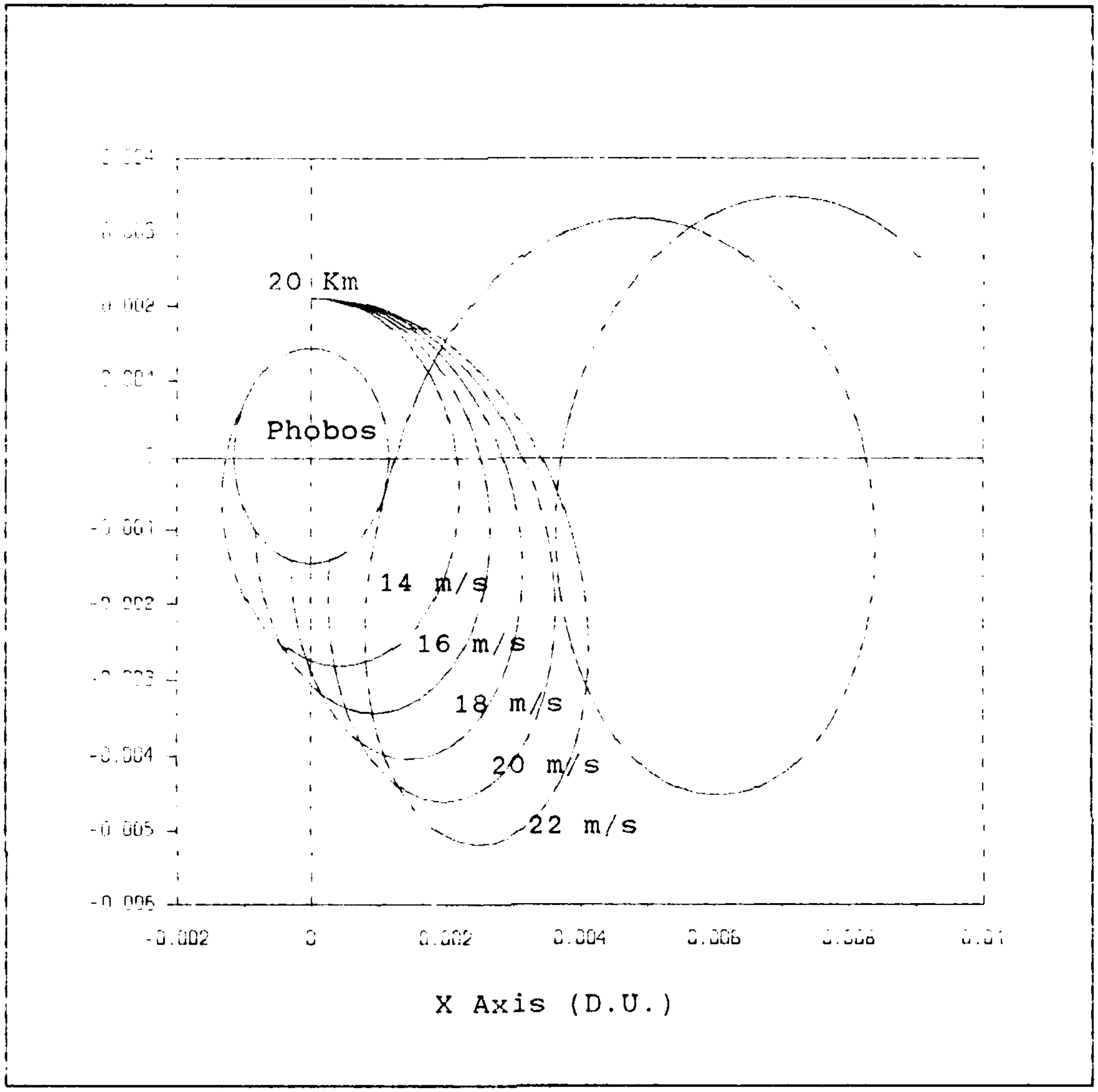

Figure 13. Initial Velocities More Than Closed Periodic Orbit Velocity

(increased) by increasing amounts until the orbit either collided with Phobos or escaped after one or more orbits.

Figure 14 shows an orbital trajectory about Phobos that begins at a Y altitude of 20 Km with an initial X velocity of 12 m/s. This velocity is slightly more than the initial velocity needed for a periodic orbit at that altitude. The trajectory ends in a collision with Phobos after four orbits.

Figure 15 shows an orbital trajectory about Phobos that begins at a Y altitude of 90 Km with an initial X velocity of 44 m/s. This velocity is a little more than the initial X velocity needed for a periodic orbit at that altitude. This time, the spacecraft escapes after four orbits. After the trajectory stepped over Phobos, the increase in the X axis crossings continued but by decreasing amounts until the trajectory appears to orbit a point ahead of Phobos in its orbit about Mars.

Values of the initial X velocity less than $\dot{X}_p$ resulted in orbits whose X axis crossing, X_c, shifted left (decreased) by increasing amounts until the orbit either collided with Phobos or escaped. If the change in X_c was larger than the X diameter of Phobos, the trajectory could step over Phobos and escape after completing one or more orbits of Phobos. Then, the shift left in the x axis crossings continues but by decreasing amounts until the trajectory appears to orbit a point behind Phobos in its orbit about Mars.

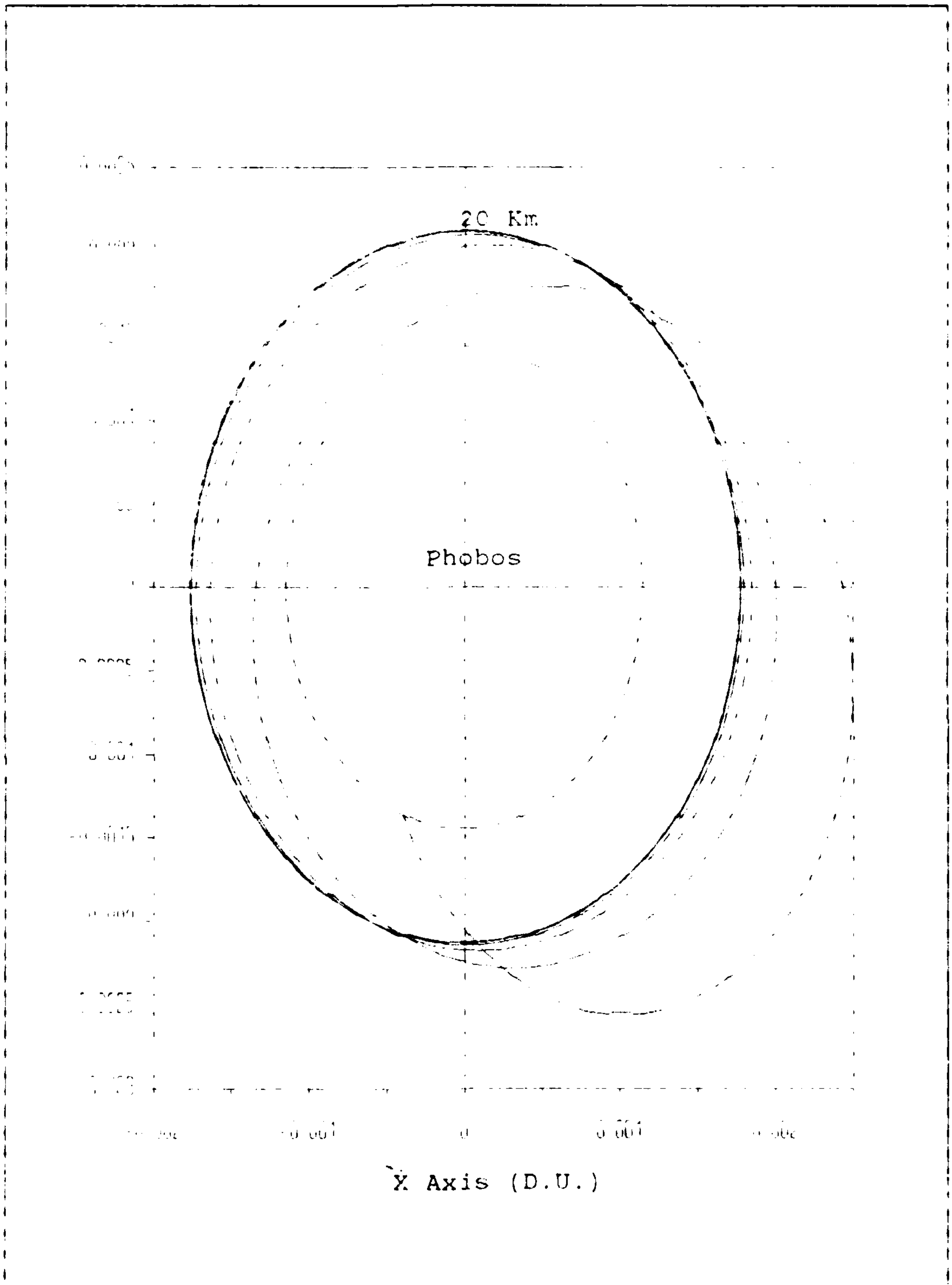

Figure 14. Velocity Slightly More Than Needed for a Periodic Orbit Results in Collision

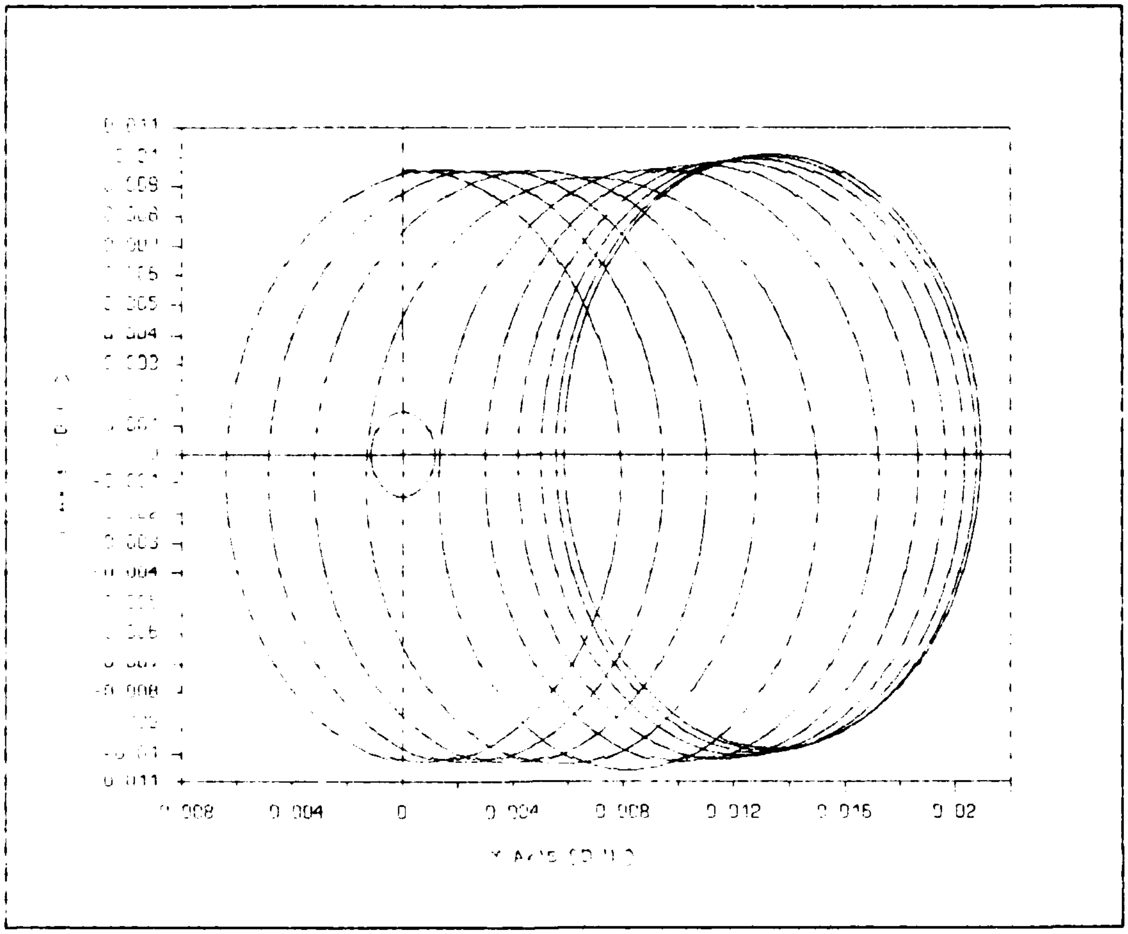

Figure 15. Initial Velocity a Little More Than Needed for a Closed Periodic Orbit Resulting in Escape

Three Dimensional Trajectories

Figure 16 is a map of the type of trajectories obtained by varying the initial X velocity (kinetic energy) and the initial Z altitudes with a fixed initial Y altitude of 20 Km. The line at Z equals zero represent the trajectories in the XY plane at a Y altitude of 20 Km as shown in the XY plane trajectory map in Figure 11.

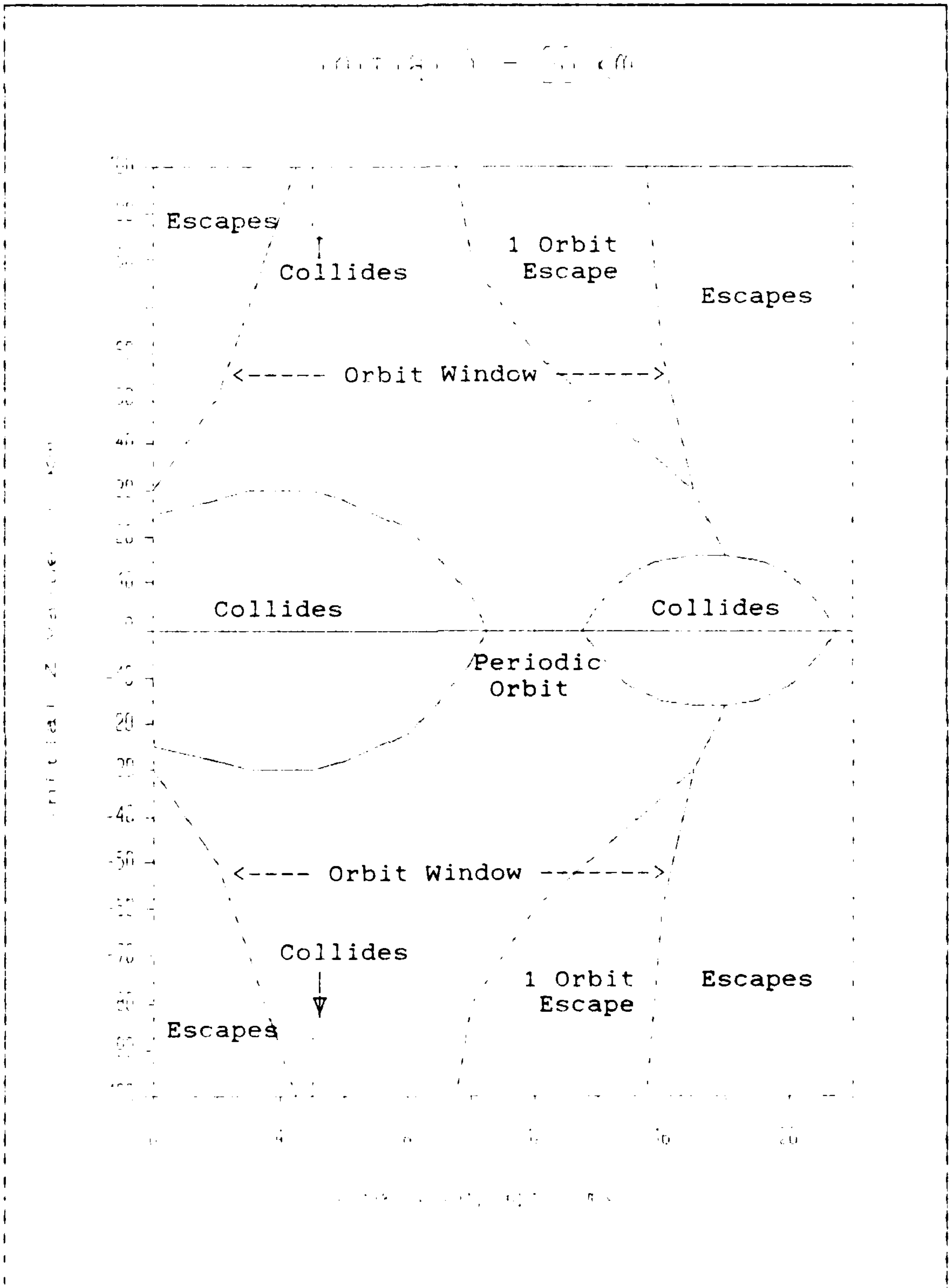

Figure 16. Trajectory Map Out of the Plane of Phobos's Orbit

The trajectory map shown in Figure 16 is symmetric about the line Z equals zero. There are two regions where the trajectories end in collision with Phobos before completing an orbit. These two collision regions give a third dimension to the collision regions shown in Figure 11 and discussed with the XY planar trajectories. The two regions are ellipsoid shapes with the larger ellipsoid corresponding to the smaller X velocities. The other regions, two escape regions and the orbit window, represented at a Y altitude of 20 Km in the XY planar trajectory map of Figure 11, are also represented in Figure 16 and thus given a third dimension.

The closed periodic orbit at the center of the three dimensional orbit window lies on a line in the XY plane trajectory map of Figure 11 but appears to be a singular point in the trajectory map of Figure 16. No closed periodic orbits are found other than the one at Z equals zero.

Figure 16 shows a region of trajectories that orbit once before escaping. These trajectories appear at Z altitudes greater than 30 Km. This region is connected to a similar region that appears in the XY plane trajectory map of Figure 11 at Y altitudes above 30 Km.

Figure 16 also shows a narrow collision region appears at Z altitudes above 85 Km with an initial X velocity of 5 m/s.

Figure 17 shows an orbital trajectory projected on the XY plane that begins with an initial Y of 20 Km, an initial Z altitude of 10 Km, and an initial X velocity of 12 m/s. The trajectory ends in a collision with Phobos after five orbits. Figure 18 shows the Z amplitude versus time for this trajectory increases as does the period of the oscillation.

Figure 19 shows an orbital trajectory projected on the XY plane that begins with an initial Y of 20 Km, an initial Z altitude of 60 Km, and an initial X velocity of 8 m/s. The trajectory ends in a collision with Phobos after seven orbits. Figure 20 shows the Z altitude versus time for this trajectory has a period of about 2π T.U. which is characteristic of its orbit about Mars.

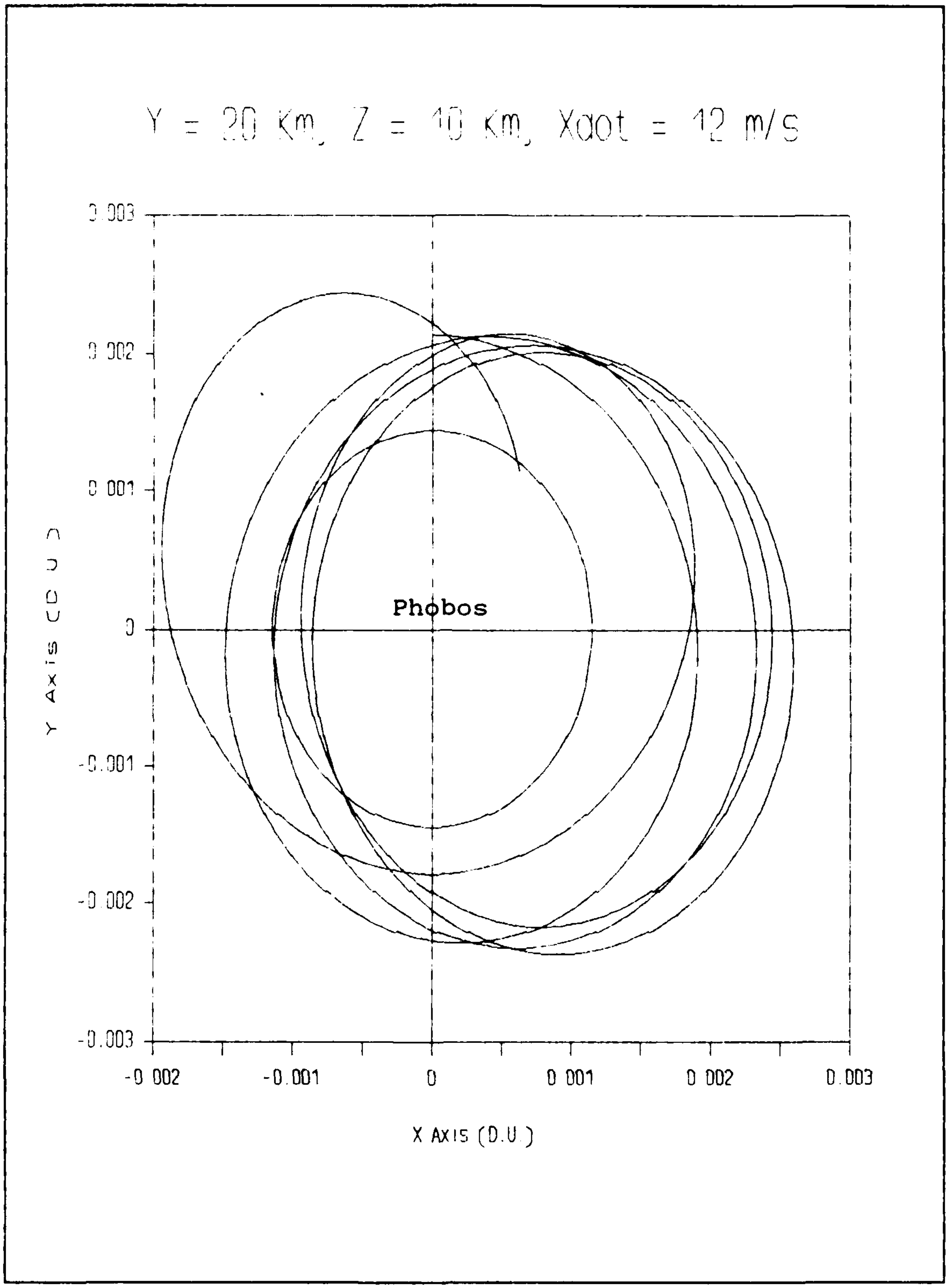

Figure 17. Three Dimensional Orbital Trajectory Projected on the XY Plane

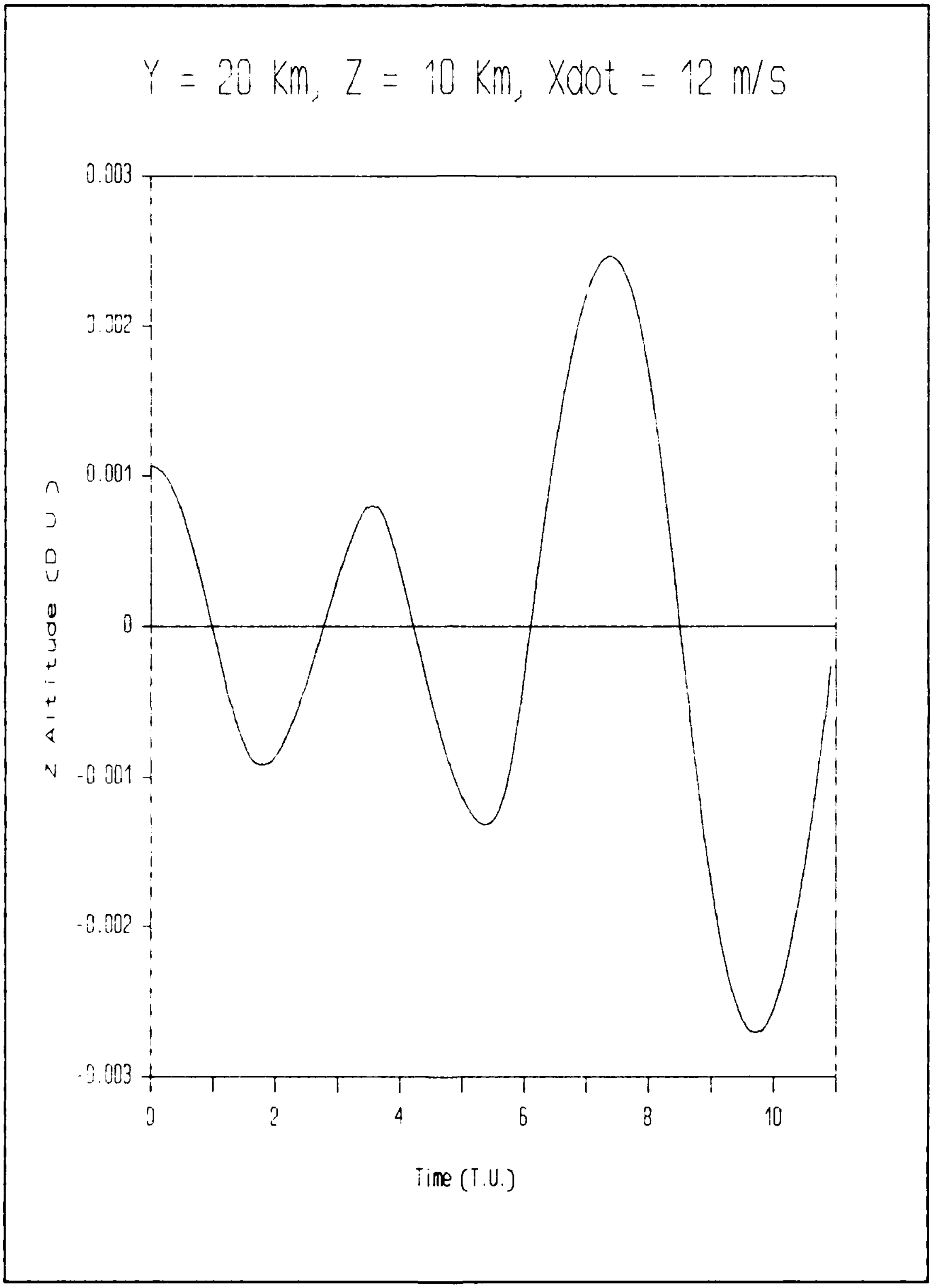

Figure 18. Z Altitude for the Trajectory in Figure 17

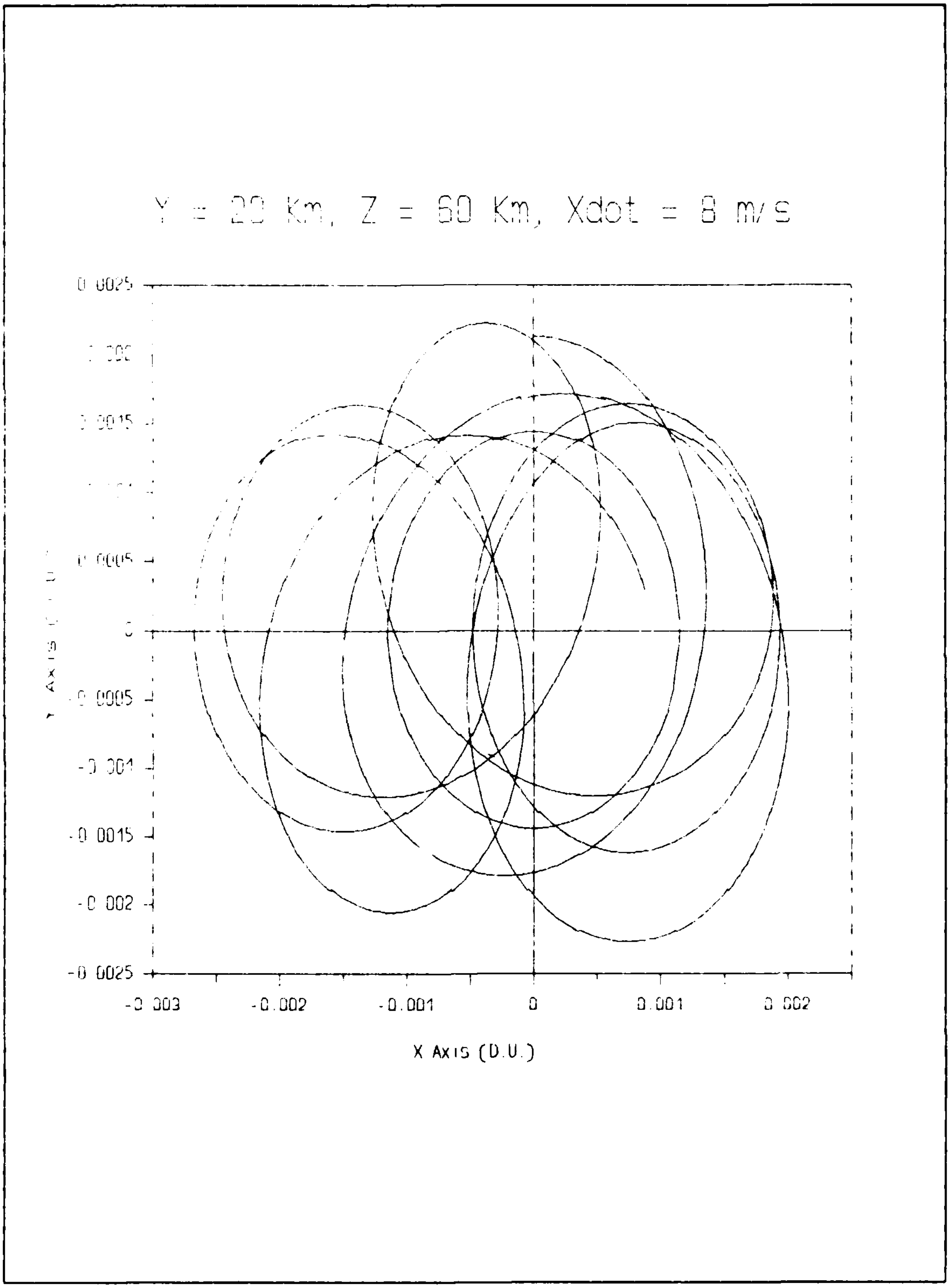

Figure 19. Three Dimensional Trajectory from a Z Altitude of 60 Km Projected on the XY Plane

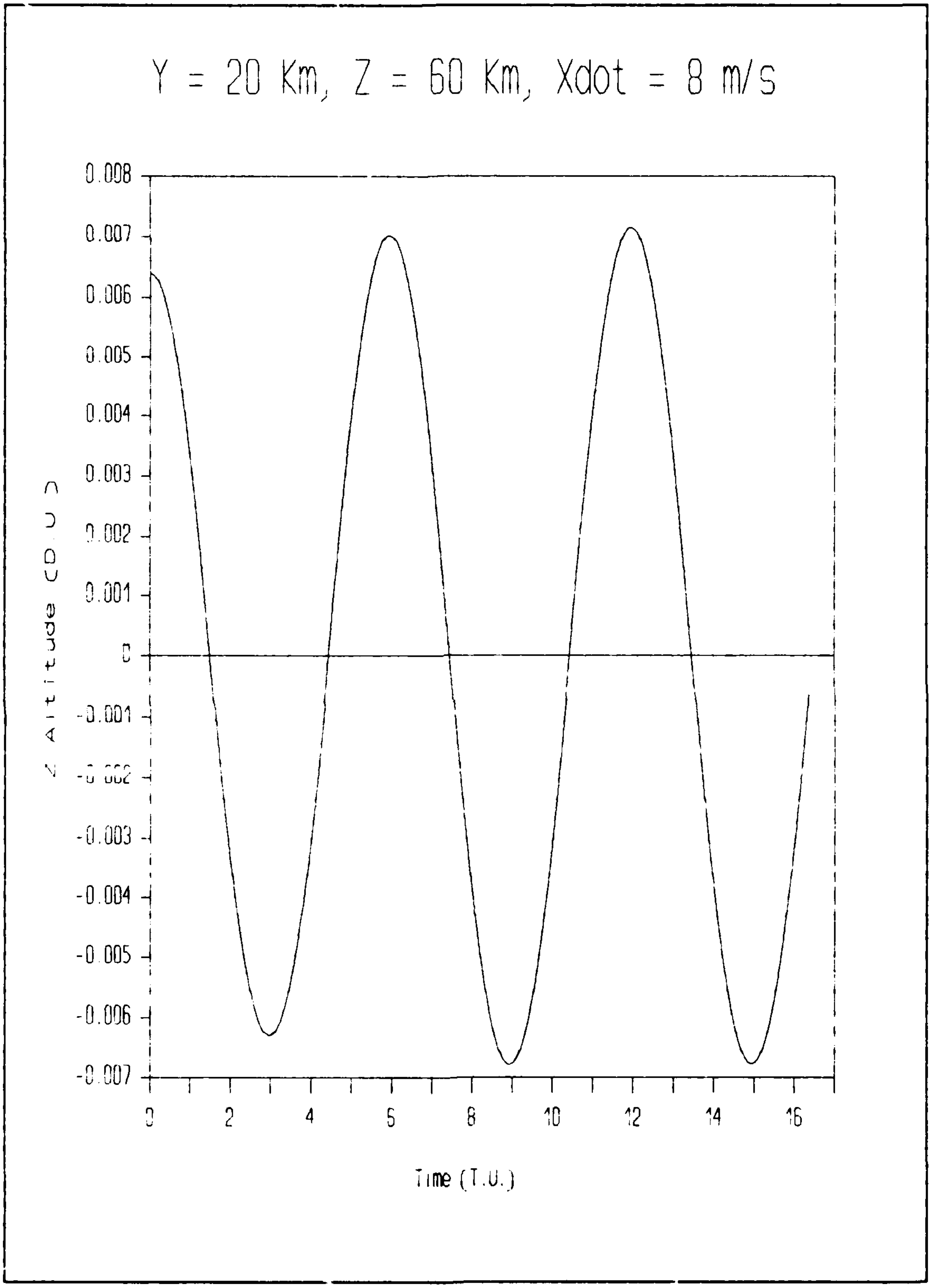

Figure 20. Z Altitude for the Trajectory in Figure 19

Conclusions

Mars gravitational potential is by far the dominant force acting on a spacecraft orbiting in the Mars Phobos system even for orbits close to Phobos. It can be said that orbits about Phobos are merely orbits about Mars that happen to go around Phobos with some perturbation effects due to the gravitational potential of Phobos.

If a spacecraft is to orbit Phobos, it must control its velocity to stay inside the orbit window. The window is small at low altitudes, only a few meters per second wide, but widens as the altitude increases.

Closed periodic orbits are available near the center of the orbit window in the plane of Phobos's orbit about Mars. A spacecraft whose velocity can be controlled to within a few cm/s can achieve these closed periodic orbits. Technically, these orbits are unstable because small errors in the state vector cause the spacecraft to drift away from the closed periodic solution. But the drift rate is slow for orbits near the center of the orbit window. Because this drift rate is slow, the closed periodic solution is a solution that can represent a practical stability similar to the practical stability exhibited by a bullet spinning about its least moment of inertia axis. Although the bullet is technically unstable spinning about that axis, the time of flight is small enough compared to the amplitude of the instability that it is insignificant. For a spacecraft in orbit about Phobos, small velocity adjustments (a few meters per second) provided at relatively infrequent intervals, 12 hours or longer depending on altitude, can keep the spacecraft in the orbit window.

No closed periodic orbit solutions were found outside of the plane of Phobos's Orbit about Mars. Nevertheless, a spacecraft can be maintained inside the three dimensional orbit window with relatively small and infrequent velocity adjustments by picking regions away from the edges of the window.

The solution space of trajectories about Phobos is well behaved and predictable. The search for bifurcations and unexplained chaotic behavior failed. The two regions where Werner suggested the appearance of additional Poincaré exponents indicated possible bifurcation regions (13:19-20) were found to correlate with two rapid expansions of the orbit window.

Appendix

```
c   Program Phobos     written by Bob Teets       Summer 1988
c   integrates a version of Werner's equations of motion for
c   a massless satellite in orbit about Phobos. The equations
c   are modified by the introduction of the system of units
c   customary for the three body problem.  The program allows
c   for output of the state vector to data files for plotting
c   or simply screen output which can captured in a log file
c   if desired.
       common /ham/ t,s(6,4),ds(6,4),err(6),n,h
       double precision r0(6),r(6),dr(6),drm(6)
       double precision t,s,ds,err,h,xtu
       logical closed,plot,plane
       character*1 ans
       type 10
10     format(5x,'Do you want orbit plots ? ',$)
       accept 15,ans                  ! If so, the state vector
15     format(a1)                     ! will be written to a
                                      ! file state.dat;n, where
       if((ans.eq.'y').or.(ans.eq.'Y'))  then
                                      ! version, n, corresponds
         plot=.true.                  ! to the record # read
       else                           ! from the file, IC.dat,
         plot=.false.                 ! the initial conditions.
       endif
       type 20
20     format(5x,'Do you want plane section plots ? ',$)
       accept 15,ans
       if((ans.eq.'y').or.(ans.eq.'Y'))  then
                                     ! If so, the state vector
         plane=.true.                ! will be written to file
       else                          ! section.dat;n twice each
         plane=.false.               ! orbit as y passes zero.
       endif
       open(unit=10,file='ic.dat',status='old')
1      read(10,25,end=100) (r0(i),i=1,6),xtu
                                 ! input initial pos, vel, tf
25     format(7f10.5)            ! in Km, m/sec, T.U.
       type *,'inputs ',(r0(i),i=1,6),xtu   ! echo back input
       do i=1,3
         r0(i)=r0(i)/9.378d+03            ! converts Km to D.U.
         r0(i+3)=r0(i+3)/9.378d+06*4.383d+03   ! converts
       enddo                              ! m/sec to D.U./T.U.
       do i=1,6
         s(i,1)=r0(i)            ! puts initial pos and vel in s
         dr(i)=0.                ! initialize delta state vector
         drm(i)=0.               ! initialize max delta state
       enddo
       s(4,1)=r0(4)+1.0d+00-r0(2)         ! changes xdot to Px
       s(5,1)=r0(5)+r0(1)                 ! changes ydot to Py
```

```
      norb=0                               ! orbit counter
      nxt=0                        ! haming initialization flag
      n=6                          ! # of equations of motion
      h=1.0d-04             ! delta t, time step for integration
      t=0.0d+00             ! set initial time = 0
      tlast=0.              ! time at end of last orbit
      tstep=.01             ! time step for print interval
      nsteps=xtu/h          ! lets integration go for x T.U.
      tp=0.                        ! time for next print
      call haming(nxt)             ! initialize haming
      if (nxt .eq. 0) then
         type *, 'haming failed to initialize'
         stop
      endif
      if(plot)  open(unit=11,file='state.dat',status='new')
      if(plane)
     *  open(unit=12,file='section.dat',status='new')
      if(plot)  write(11,30)
     *                 t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
30    format (1x,7e11.4)
      do it=1,nsteps
        do i=1,3
          r(i)=s(i,nxt)            ! save old position vector
          r(i+3)=ds(i,nxt)         ! save old velocity vector
        enddo
        call haming(nxt)           ! integrate for new
                                   ! position and velocity
        if(t.ge.tp)  then          ! print state vector
          tp=tp+tstep              ! set next print time
          if(plot)  write(11,30)
     *                 t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
        endif
        r2=s(1,nxt)*s(1,nxt)+s(2,nxt)*s(2,nxt)
     *      +s(3,nxt)*s(3,nxt)
        if(r2.ge.1)  then
          type *,'orbit',norb,' beyond 1 D.U.'
          if(plot)  write(11,30)
     *                 t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
         if(plot)  close(11)
          if(plane)  close(12)
          go to 1                    ! new initial conditions
        endif
        phobos=s(1,nxt)*s(1,nxt)/1.326e-6
     *         +s(2,nxt)*s(2,nxt)/2.072e-6
     *         +s(3,nxt)*s(3,nxt)/1.005e-6
        if(phobos.le.1.)  then
          type *,'orbit',norb,' ends in collision with
     *Phobos'
          if(plot)  write(11,30)
     *                 t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
          if(plot)  close(11)
          if(plane)  close(12)
          go to 1                    ! new initial conditions
```

```
      endif
      do i=1,3
        dr(i)=abs(s(i,nxt)-r(i))       ! change in position
        dr(i+3)=abs(ds(i,nxt)-r(i+3)) ! change in velocity
        if(dr(i).gt.drm(i))  drm(i)=dr(i)     ! max delta r
        if(dr(i+3).gt.drm(i+3))  drm(i+3)=dr(i+3) ! max
                                                   !delta v
      enddo
      if(((s(1,nxt).lt.0.0d+00).and.(r(1).gt.0.0d+00)).or.
     *   ((s(1,nxt).gt.0.0d+00).and.(r(1).lt.0.0d+00)))
     * then
        type *,'passing x = 0, xdot = ',ds(1,nxt),' y = '
     *   ,s(2,nxt)
        if(ds(1,nxt).gt.0)  then
          norb=norb+1                  ! increment orbit count
          period=(t-tlast)*4383.
          tlast=t
          type *,'period =',period,' seconds for orbit'
     *      ,norb
        endif
        if(plot)  write(11,30)
     *              t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
      endif
      if(((s(2,nxt).lt.0.0d+00).and.(r(2).gt.0.0d+00)).or.
     *   ((s(2,nxt).gt.0.0d+00).and.(r(2).lt.0.0d+00)))
     * then
        type *,'passing y = 0, ydot = ',ds(2,nxt),' x = '
     *     ,s(1,nxt)
        if(plot)  write(11,30)
     *              t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
        if(plane)  write(12,30)
     *              t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
      endif
      if(((s(3,nxt).lt.0.0d+00).and.(r(3).gt.0.0d+00)).or.
     *   ((s(3,nxt).gt.0.0d+00).and.(r(3).lt.0.0d+00)))
     * then
        type *,'passing z = 0, zdot = ',ds(3,nxt),' x = '
     *    ,s(1,nxt)
        if(plot)  write(11,30)
     *              t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
      endif
      if(norb.ge.1)  then          ! check if orbit closed
        closed=.true.
        do i=1,3
          if(abs(r0(i)-s(i,nxt)).gt.drm(i))
     *       closed=.false.
          if(abs(r0(i+3)-ds(i,nxt)).gt.drm(i+3))
     *       closed=.false.
        enddo
        if(closed) then
          type *,'orbit closed, # orbits = ',norb
          if(plot)  write(11,30)
     *              t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
```

```
            if(plot)  close(11)
            if(plane)  close(12)
            go to 1                     ! new initial conditions
          endif
        endif
      enddo
      if(plot)  write(11,30)
     *              t,(s(i,nxt),i=1,3),(ds(i,nxt),i=1,3)
      if(plot)  close(11)
      if(plane)  close(12)
      go to 1                           ! new initial conditions
100   stop
      end

      subroutine haming(nxt)
c
c     haming is an ordinary differential eqns integrator
c     it is a fourth order predictor-corrector algorithm
c     which means that it carries along the last four
c     values of the state vector, and extrapolates these
c     values to obtain the next value (the prediction part)
c     and then corrects the extrapolated value to find a
c     new value for the state vector.
c
c     the value nxt in the call specifies which of the 4
c     values of the state vector is the "next" one.
c     nxt is updated by haming automatically, and is zero on
c     the first call
c
c     the user supplies an external routine rhs(nxt) which
c     evaluates the equations of motion
c
      common /ham/ x,s(6,4),ds(6,4),errest(6),n,h
      double precision x,s,ds,errest,h,hh,xo,tol
c
c     all of the good stuff is in this common block.
c     x is the independent variable ( time )
c     s(6,4) is the state vector- 4 copies of it, with nxt
c                 pointing at the next one
c     ds(6,4) are the equations of motion, again four copies
c                 a call to rhs(nxt) updates an entry in ds
c     errest is an estimate of the truncation error -
c     normally not used
c     n is the number of equations being integrated - 6 here
c     h is the time step
c
      tol = 0.0000000001d+0
c     switch on starting algorithm or normal propagation
      if(nxt) 190,10,200
c
c     this is hamings starting algorithm....a predictor -
c     corrector needs 4 values of the state vector, and you
c     only have, one- the initial conditions.
```

```
c     haming uses a Picard iteration (slow and painful) to
c     get the other three.
c     if it fails, nxt will still be zero upon exit,
c     otherwise nxt will be 1, and you are all set to go
c
   10 xo = x
      hh = h/2.0d+00
      call rhs(1)
      do 40 l = 2,4
      x = x + hh
      do 20 i = 1,n
   20 s(i,l) = s(i,l-1) + hh*ds(i,l-1)
      call rhs(l)
      x = x + hh
      do 30 i = 1,n
   30 s(i,l) = s(i,l-1) + h*ds(i,l)
   40 call rhs(l)
      jsw = -10
   50 isw = 1
      do 120 i = 1,n
      hh = s(i,1) + h*( 9.0d+00*ds(i,1) + 19.0d+00*ds(i,2)
     1     - 5.0d+00*ds(i,3) + ds(i,4) ) / 24.0d+00
      if( dabs( hh - s(i,2)) .lt. tol ) go to 70
      isw = 0
   70 s(i,2) = hh
      hh = s(i,1) + h*( ds(i,1) + 4.0d+00*ds(i,2) +
     *     ds(i,3))/3.0d+00
      if( dabs( hh-s(i,3)) .lt. tol ) go to 90
      isw = 0
   90 s(i,3) = hh        hh=s(i,1)+h*(3.0d+00*ds(i,1)
     *   +9.0d+00*ds(i,2)+9.0d+00*ds(i,3)
     1     + 3.0d+00*ds(i,4) ) / 8.0d+00
      if( dabs(hh-s(i,4)) .lt. tol ) go to 110
      isw = 0
  110 s(i,4) = hh
  120 continue
      x = xo
      do 130 l = 2,4
      x = x + h
  130 call rhs(l)
      if(isw) 140,140,150
  140 jsw = jsw + 1
      if(jsw) 50,280,280
  150 x = xo
      isw = 1
      jsw = 1
      do 160 i = 1,n
  160 errest(i) = 0.0
      nxt = 1
      go to 280
  190 jsw = 2
      nxt = iabs(nxt)
c
```

```
c       this is hamings normal propagation loop -
c
  200 x = x + h
      np1 = mod(nxt,4) + 1
      go to (210,230),isw
c     permute the index nxt modulo 4
  210 go to (270,270,270,220),nxt
  220 isw = 2
  230 nm2 = mod(np1,4) + 1
      nm1 = mod(nm2,4) + 1
      npo = mod(nm1,4) + 1
c
c     this is the predictor part
c
      do 240 i = 1,n
      ds(i,nm2) = s(i,np1) + 4.0d+00*h*( 2.0d+00*ds(i,npo) -
     1   ds(i,nm1) + 2.0d+00*ds(i,nm2) ) / 3.0d+00
  240 s(i,np1) = ds(i,nm2) - 0.925619835*errest(i)
c
c     now the corrector - fix up the extrapolated state
c     based on the better value of the equations of motion
c
      call rhs(np1)
      do 250 i = 1,n
      s(i,np1) = ( 9.0d+00*s(i,npo) - s(i,nm2) + 3.0d+00*h*(
     1 ds(i,np1) + 2.0d+00*ds(i,npo) - ds(i,nm1))) / 8.0d+00
      errest(i) = ds(i,nm2) - s(i,np1)
  250 s(i,np1) = s(i,np1) + 0.0743801653 * errest(i)
      go to (260,270),jsw
  260 call rhs(np1)
  270 nxt = np1
  280 return
      end

      subroutine rhs(nxt)
c
c     rhs contains the differential equations of motion.
c     the basic function of rhs is to calculate the
c     equations of motion (ds = f(s,t)) from the given
c     current state (stored in s) and the time t.
c     the state vector,s, is defined as follows:
c     s(1-3,nxt) are the x,y,z components of the position
c     vector,r. s(4-6,nxt) are the generalized momenta, Px,
c     Py, Pz.
c
c     the haming common
c
      common /ham/ t,s(6,4),ds(6,4),err(6),n,h
      double precision t,s,ds,err,h
      double precision
     *                r,r2,r3,r4,r5,r7,x,x2,y,y2,z,z2,Px,Py,Pz
      double precision
            r2x,r2y,r2z,xy,yz,Ixx,Iyy,Izz,I1,I2,I3,It,m0,m1
```

```
c
c     ****************************************************************
c
c          EVALUATE THE EQUATIONS OF MOTION
c
c     ****************************************************************
c
      m1=1.541036375d-08             ! M.U.
      m0=1.0d+00-m1                  ! M.U.
      Ixx=m1*6.153941736d-07         ! M.U.*D.U.^2
      Iyy=m1*4.661904792d-07         ! M.U.*D.U.^2
      Izz=m1*6.797057187d-07         ! M.U.*D.U.^2
      I1=-Ixx+Iyy+Izz
      I2=Ixx-Iyy+Izz
      I3=Ixx+Iyy-Izz
      It=Ixx+Iyy+Izz
      r2=s(1,nxt)*s(1,nxt)+s(2,nxt)*s(2,nxt)
     *   +s(3,nxt)*s(3,nxt)
      r=dsqrt(r2)
      r3=r*r2
      r4=r2*r2
      r5=r3*r2
      r7=r3*r4
      xy=s(1,nxt)*s(2,nxt)
      yz=s(2,nxt)*s(3,nxt)
      r2x=r2*s(1,nxt)
      r2y=r2*s(2,nxt)
      r2z=r2*s(3,nxt)
      x2=s(1,nxt)*s(1,nxt)
      y2=s(2,nxt)*s(2,nxt)
      z2=s(3,nxt)*s(3,nxt)
      ds(1,nxt) = s(4,nxt)+s(2,nxt)-1.0d+00
      ds(2,nxt) = s(5,nxt)-s(1,nxt)
      ds(3,nxt) = s(6,nxt)
      ds(4,nxt) = s(5,nxt)-m0*(s(1,nxt)-3.0d+00*xy
     *   +1.5d+00*r2x)-m1*s(1,nxt)/r3+7.5d-01*It*s(1,nxt)/r5
     *   -7.5d-01*s(1,nxt)/r7*((5.0d+00*x2-2.0d+00*r2)*I1
     *   +5.0d+00*y2*i2+5.0d+00*z2*I3)
      ds(5.nxt) = -s(4,nxt)+m0*(1.0d+00-4.0d+00*s(2,nxt)
     *   -1.5d+00*(r2y-r2-2.0d+00*y2))-m1*s(2,nxt)/r3
     *   +7.5d-01*It*s(2,nxt)/r5-7.5d-01*s(2,nxt)/r7*
     *   ((5.0d+00*y2-2.*r2)*I2+5.0d+00*x2*I1
     *   +5.0d+00*z2*I3)
      ds(6,nxt) = -m0*(s(3,nxt)-3.0d+00*yz+1.5d+00*r2z)-
     *   m1*s(3,nxt)/r3+7.5d-01*It*s(3,nxt)/r5
     *   -7.5d-01*s(3,nxt)/r7*((5.0d+00*z2-2.0d+00*r2)*I3
     *   +5.0d+00*x2*I1+5.0d+00*y2*I2)
      return
      end
```

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Orbital trajectories obtained from numerical integration of over two thousand sets of initial conditions for equations of motion of a spacecraft in the Mars Phobos system are examined in a Phobos centered rotating coordinate frame. As initial velocity and altitude are varied, trajectories in the orbital plane of Phobos about Mars follow an orderly pattern. Trajectories out of that plane are more complex, three dimensional (helical shaped) paths that do not remain in an inclined plane, but do exhibit some order.

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