AFIT/GST/ENS/89M-05

A METHODOLOGY FOR DETERMINING THE SURVIVABILITY OF FIXED-WING AIRCRAFT AGAINST SMALL ARMS

THESIS

John M. Grover
Captain, USAF

AFIT/GST/ENS/89M-05

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Distribution unlimited; aproved for public release.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
AFIT/GST/ENS/89M-05

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
School of Engineering

6b. OFFICE SYMBOL *(If applicable)*
AFIT/ENS

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS *(City, State, and ZIP Code)*
Air Force Institute of Technology (AU)
Wright-Patterson AFB, Ohio 45433-6583

7b. ADDRESS *(City, State, and ZIP Code)*

8a. NAME OF FUNDING / SPONSORING ORGANIZATION

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS *(City, State, and ZIP Code)*

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*
See Box 19

12. PERSONAL AUTHOR(S)
John M. Grover, B.A., Capt, USAF

13a. TYPE OF REPORT
MS Thesis

13b. TIME COVERED
FROM ________ TO ______

14. DATE OF REPORT *(Year, Month, Day)*
1989 March

15. PAGE COUNT
117

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
19	06	
23	06	

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Small Arms, Aircraft Survivability

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Title: A Methodology for Determining the Survivability of Fixed-wing Aircraft against Small Arms

Thesis Chairman: Daniel Reyen, Maj (Ret), AD
Instructor of Operations Research

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Daniel Reyen, Maj (Ret), AD

22b. TELEPHONE *(Include Area Code)*
(513) 255-5758

22c. OFFICE SYMBOL
AFIT/ENS

The purpose of this study was to develop an efficient and effective method of evaluating the survivability of a fixed-wing aircraft against small arms. A computer model was created to predict the probability an aircraft survives given an encounter with a small armsweapon, P(s/e), and the expected number of hits, E(hits).

The model was a one-on-one deterministic duel of a small arms weapon and an aircraft. The aircraft was represented on a straight and level flight approaching the weapon at a given velocity, angle, distance, and altitucde. The small arms weapon is located at a fixed position, firing at a fixed point in front of the aircraft. The weapon orientation varied by normal distributions around the fixed aim point. The bullet trajectories were represented using an iterative technique. Bullet and aircraft intersections were calculated using a sphere to represent the aircraft. All probability distributions were broken into discrete intervals providing the means to maintaining a deterministic model. The results of the model supplied a probability of survival and expected hits for a specific weapon and aircraft with a set velocity, distance from aim point, altitude, and angle of approach.

AFIT/GST/ENS/89M-05

A METHODOLOGY FOR DETERMINING THE SURVIVABILITY OF FIXED-WING AIRCRAFT AGAINST SMALL ARMS

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Masters of Science in Operations Research

John M. Grover, B.A.

Captain, USAF

March 1989

Preface

The purpose of this study was to develop an efficient and effective method of evaluating the survivability of a fixed-wing aircraft against small arms. A computer model was created to predict the probability an aircraft survives given an encounter with a small arms weapon, P(s/e), and the expected number of hits it receives, E(h).

While developing and writing this thesis, I pestered several people. I would like to thank Lt. Col. Ronald Bagley (committee member), Lt. Col. Thomas Schuppe, Maj. Kenneth Bauer, and Maj. William Schneider (ASD/XRM) for their patience and comments. Especially, I would like to express my appreciation to my faculty advisor, Maj. (Ret) Daniel Reyen, for listening every time I said "But Sir, ...", for calling me 'stupid' when I deserved it, and for kicking me in the butt when I needed it. Finally, I would like to thank my wife, Linda, for not divorcing me while at AFIT.

John M. Grover

Table of Contents

List of Figures

List of Tables

AFIT/GST/ENS/89M-05

Abstract

The purpose of this study was to develop an efficient and effective method of evaluating the survivability of a fixed-wing aircraft against small arms. A computer model was created to predict the probability an aircraft survives given an encounter with a small arms weapon, P(s/e), and the expected number of hits it receives, E(hits).

The model was a one-on-one deterministic duel of a small arms weapon and an aircraft. The aircraft was represented on a straight and level flight approaching the weapon at a given velocity, angle, distance, and altitude. The small arms weapon was located at a fixed position, firing at a fixed point in front of the aircraft. The weapon orientation varied by normal distributions around the fixed aim point. The bullet trajectories were represented using an interative technique. Bullet and aircraft intersections were calculated using a sphere to represent the aircraft. All probability distributions were broken into discrete intervals providing the means to maintaining a deterministic model. The results of the model supplied a probability of survival and expected hits for a specific weapon and aircraft with a set velocity, distance away, altitude, and angle of approach.

A METHODOLOGY FOR DETERMINING THE SURVIVABILITY OF FIXED-WING AIRCRAFT AGAINST SMALL ARMS

I. Introduction

Small arms have brought down aircraft during every major conflict in which the United States has flown. In the latest conflict, Vietnam, the U.S. lost 410 aircraft to weapons of .51 caliber and below (11:37). Obviously, small arms are a threat to aircraft; but how effective of a threat are they? The answer to this question is currently being sought by the Department of Defense. In March 1987, the Deputy Under Secretary of Defense for Tactical Warfare Programs, Donald N. Fredericksen, requested that the Joint Technical Coordinating Group on Aircraft Survivability (JTCG/AS) include small arms as a threat in their survivability study of new Close Air Support (CAS) aircraft. The JTCG/AS encountered a major pitfall during the preliminary research of the small arms threat: there are no models available which can accurately evaluate the survivability of fixed-winged aircraft against small arms (2).

Problem Statement

The purpose of this study is to develop a methodology which will accurately calculate the probability of survival of a fixed-wing aircraft against small arms.

II. Background

Survivability Models

Aircraft survivability is defined as "the capability of an aircraft to avoid and/or withstand a man-made hostile environment" (1:1). Survivability modelling is the calculation of a measure of this capability. There is a variety of methods used to quantify an aircraft's survivability. The methods range from the compilation of heuristic and historical data to the analysis of complex computer simulations. This study focuses solely on the types of computer models that can be used for the analysis of aircraft survivability and it is further restricted to the aspects of survivability modelling pertainent to small arms.

Classifications of Survivability Models. Computer models of survivability can be described by some general classifications. This section gives brief definitions of these classifications. All of the following definitions are adapted from Hartman's Lecture Notes in High Resolution Combat Modelling (7:1-5,1-6).

Dynamic vs Static. A model is dynamic if it explicitly represents the passage of time. A static model is concerned with only a single instance in time. If a model represents the flight path of an aircraft or bullet over time it is dynamic.

Continuous vs Discrete. A model is continuous if its variables can be updated at any time. A discrete model is restricted to updating at specific instances in time. These instances take place after certain time intervals or they are triggered by events. An example of a discrete model is one that uses a look-up table for variable updates. A continuous model may update the variable by using algorithms.

Deterministic vs Stochastic. A model is deterministic if it contains no random sampling of probability distributions. A stochastic model uses random sampling to decide uncertain occurances. Deterministic models often use an expected value when deciding uncertain occurances.

Elements of Survivability Modelling. There are three key elements of survivability modelling. The first element is the susceptibility assessment, which quantifies an aircraft's inability to avoid being hit by enemy fire. The second is the vulnerability assessment, which quantifies an aircraft's inability to withstand the damage caused by enemy fire. The last element is the survivability assessment, which brings together the results of the susceptability and vulnerability assessments to produce a measure of the aircraft's ability to avoid being destroyed.

Vulnerability Assessment. Survivability models rarely have built-in vulnerability assessments. Most models use data bases containing the results of previous assessments.

Several agencies have the capability to do vulnerability assessments for small arms, a few are: Air Force Aeronautical Systems Division, US Army Ballistic Research Laboratory, and the Naval Weapons Center. Because agencies like these have proven vulnerability assessment methods that can supply results for a survivability model, this study will not examine assessment methodologies, but it will address the form of the assessment results.

The results of most vulnerability assessments are a series of vulnerability measures for multiple combinations of aircraft aspect angle and bullet impact velocity. There are six aspect angles used for a minimum and 26 for a detailed assessment. The aspect angles for a minimum and a maximum assessment are represented in Figures 1 and 2, respectively (1:181,182). Bullet impact velocities are normally given in increments of 500 feet per sec. A sample form used for vulnerability assessment output is given in Figure 3 (1:182).

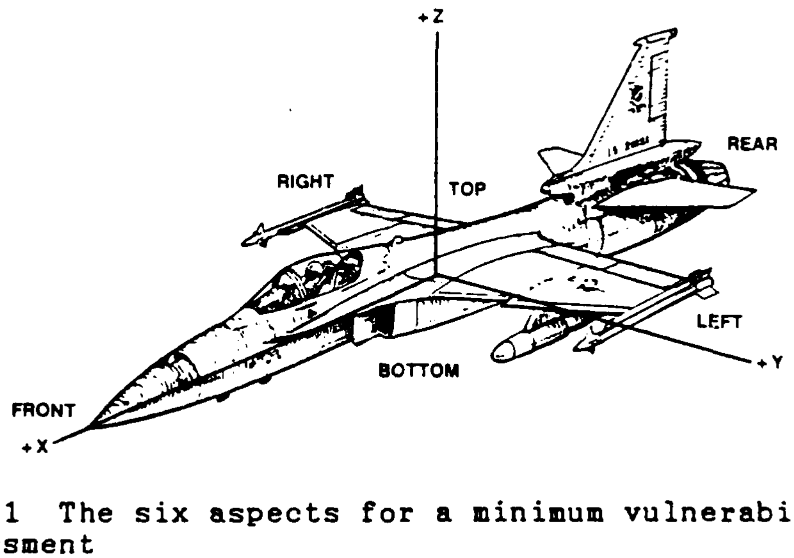


Fig. 1 The six aspects for a minimum vulnerability assessment

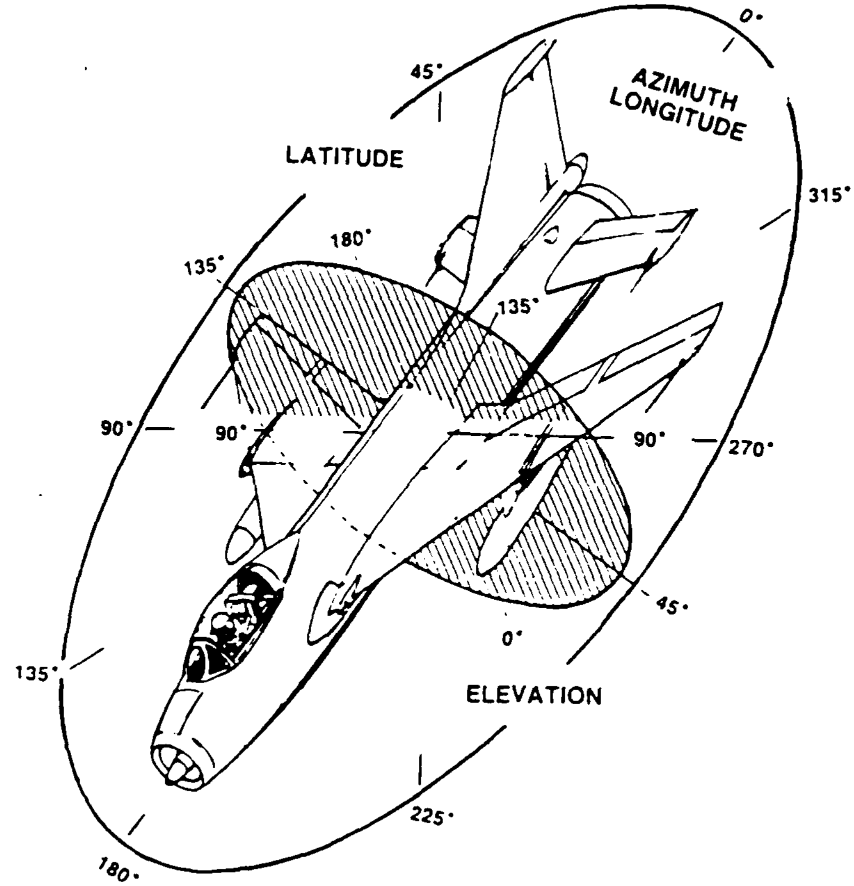


Fig. 2 The 26 aspects for a detailed vulnerability assessment

Assessment date______ Aircraft ______
Performing organization______ Threat ______
Kill category______

Projectile V_S, ft/sec (m/sec)	Total A_V, ft^2 (m^2)					
	Left side	Right side	Top	Bottom	Front	Rear
500 (152.4)						
1,000 (304.8)						
1,500 (457.2)						
2,000 (609.6)						
2,500 (762.0)						
3,000 (914.4)						
3,500 (1066.8)						

Fig. 3 An example of a total aircraft single hit vulnerability summary table

There are two measures of vulnerability useful for small arms. The first is the conditional probability that an aircraft is killed given a random hit on the aircraft. The other is the aircraft's vulnerable area, which is a theoretical area, which, if hit, results in an aircraft kill (1:154). The relationship between these two measures is:

$$A(v) = P(k/h) * A(p)$$

where, for a given aspect angle,

A(v) = aircraft vulnerable area (ft^2)
P(k/h) = probability of kill given a hit
A(p) = aircraft presented area (ft^2)

Survivability models which use A(p) and P(k/h) are usually stochastic. A stochastic model would determine if a hit had occured on A(p) and then compare P(k/h) to a randomly drawn probability to conclude whether the aircraft is killed. A few deterministic models use the P(k/h) as an expected number of aircraft killed, E(k), but this is rare. Most deterministic models determine an aircraft kill solely by a hit on A(v).

Susceptibility Assessment. The susceptibility assessment constitutes the greater part of a survivability model. This assessment must represent all activities that occur from the beginning of an encounter to the bullet impact. The susceptibility assessment can be separated into stages. These stages are aircraft detection, aircraft identification, tracking, weapon firing, bullet flyout, and impact (1:1).

Since small arms do not rely on electronic equipment for any of these stages, the methodologies to represent them are not very complicated. Small arms, as air defense, are most often used for self defense; therefore, detection and identification are assumed to have occured. Tracking is dependent on the method of engagement; therefore, it will be discussed under Engagement Methods. Weapon firing can be represented by two variables: the probability the weapon will jam and the rate of fire. Normally the probability of small arms jamming is assumed to be zero, due to the reliability of the today's weapons and the brievity of encounters. According to Army Field Manual 7-8, The Infantry Platoon and Squad, a small arms' maximum rate of fire should be used against aircraft (6:H-5). Bullet flyout and impact are the most difficult of all the stages to represent. Flyout is the representation of the path of the bullet through space and impact deals with the orientation and force of the bullet at the aircraft. Together flyout and impact are represented by the ballistics of the bullet, which will be discussed under Bullet Ballistics.

One measure of susceptibility useful for small arms is the probability that an aircraft is impacted (hit) by a bullet, P(h) (1:227). Each stage of the susceptibility assessment has certain parameters which will contribute to a hit on the aircraft. These parameters have associated probabilities of occurance. The measure P(h) is the product of all the contributing parameter probabilities. Models that use P(h) directly are usually stochastic. Some models never explicitly calculate P(h). These models determine whether a hit or miss has occured strictly from the geometry of the encounter.

Survivability Assessment. The survivability assessment is the easiest of the three elements of survivability modelling. The most often used measures of survivability are the single shot probability of kill, P(kss) and the probability an aircraft survives given an one-on-one encounter, P(s/e) (1:312). Small arms rely on a volume of fire for effectiveness; therefore, P(kss) is not an appropriate measure. For the simplest case, the following equation gives the relationship between the survivability, susceptibility, and vulnerability measures:

$$P(s/e) = 1 - P(h) * P(k/h)$$

where

P(s/e) = probability of survival given an encounter
P(h) = probability of hit
P(k/h) = probability of kill given a hit

In most stochastic models, the aircraft is either killed or it survives, P(s/e) is not applicable. These models compare P(h) and P(k/h) to randomly drawn probabilities to conclude whether the aircraft is hit and then killed.

Engagement Method

The methods used to engage aircraft using small arms vary from military to military. Even a given military uses different methods depending on the situation. Creating a model which represents all known methods would be very difficult. The alternative is to use the one basic practice which all countries use in at least one of their methods.

Lead point fire is the only practice which is used by all militaries (2). In this practice, the ground combatant picks a point in front of the aircraft and fires his weapon at that point until the aircraft passes it. Methods which use this practice vary by the amount of lead used, for example, the U.S. Army uses a 200 meter lead (5:3-95) and the North Vietnamese used four aircraft lengths.

A country's military picks a lead distance which they believe produces the most hits on an aircraft for the widest variety of situations. To maximize the number of hits on the aircraft the first bullet fired must reach the aim point before the aircraft. If it is assumed that all militaries succeed in maximizing the number hits, the actual lead distance used is not important. Therefore, the basic method this model assumes is the lead point method with the first bullet fired reaching the aim point before the aircraft.

Bullet Ballistics

Ballistics are all of the inflight characteristics of a projectile. These characteristics include location, velocity, orientation, and acceleration. Another term closely associated with ballistics is trajectory. The trajectory of a projectile is the path it follows through space. Therefore, a trajectory is a subset of ballistics that deals with only the location of the projectile. When equations that represent a trajectory over time are known, they can be used to calculate remaining ballistics. The basic mathematical relationships are:

$$s'(t) = v(t)$$

$$s''(t) = a(t)$$

where

$s(t)$ = location vector as a function of time
$v(t)$ = velocity vector as a function of time
$a(t)$ = acceleration vector as a function of time

Because the ballistics of a projectile can be derived from its trajectory, ballistics and trajectory are frequently mistaken for interchangeable terms.

Another word often confused with ballistics, is ballistic. The word ballistic describes a projectile when it is not self-powered. A ballistic missile has a self-powered ascent, but is called ballistic because it free-falls during descent. Most bullets have no internal propulsion mechanism; therefore, their entire flight is ballistic. Any projectile that is self-powered during its entire flight is not ballistic (e.g. air-to-air and ground-to-air missiles). Even though some projectiles are not ballistic, all projectiles have ballistics.

Velocity vs Speed. The words velocity and speed are often used interchangably. Velocity and speed are not identical terms. Velocity is a vector and speed is the magnitude of that vector. The confusion between the terms is not helped by terminology like, "muzzle velocity", which is actually a speed. To alleviate the confusion, "velocity" and "magnitude of velocity" will be synonymous from this point on.

<u>Ballistic Trajectories</u>. The calculation of a realistic bullet trajectory is difficult, but some simplifying assumptions have been made without greatly affecting results. Because small arms bullets usually travel less than a mile above the ground, it is assumed that the earth's gravity and atmospheric density are constants for the entire trajectory. Also, gravity and drag due to air density are assumed to be the only outside forces capable of acting upon the bullet, all other forces, like wind, are too random to be considered. Using only these assumptions, the basic equations of motion are expressed for distance X, and height Z, as follows (4:2-61):

$$x''(t) = - Ro * V(t) * A * CD(V(t)) * x'(t) / (m * 2)$$

$$z''(t) = - Ro * V(t) * A * CD(V(t)) * z'(t) / (m * 2) - g$$

$$V(t) = (x'(t)^2 + z'(t)^2)^{.5}$$

where

$x''(t)$ = acceleration in the x direction at time t
$z''(t)$ = acceleration in the z direction at time t
$x'(t)$ = velocity in the x direction at time t
$z'(t)$ = velocity in the z direction at time t
$V(t)$ = velocity at time t
t = time of flight
g = acceleration of gravity (32.2 ft/sec^2)
m = mass of bullet (slugs)
A = presented area of the bullet (ft^2)
Ro = density of the atmosphere (.002377 slugs/ft^3)
$CD(V(t))$ = coefficient of drag for velocity V(t)

These equations are second order, coupled, nonlinear, differential equations. This interdependence of variables makes the integration of closed form solutions for the trajectory equations very difficult. The most common approach used to avoid this problem is to assume it away by using flat trajectories. The other approach is to numerically estimate the equations using iterative algorithms.

Flat. This trajectory is the most popular type of ballistic trajectory used for anti-aircraft guns. This trajectory assumes the force of gravity over a short distance is negligible and atmospheric drag is a constant (4:2-45). These assumptions allow closed form trajectory equations to be derived. The equations are (4:2-46,2-47):

$$x(t) = x'(0) * t - H * x'(0) * t^2 / 2$$

$$z(t) = z'(0) * t - H * z'(0) * t^2 / 2$$

$$H = Ro * V(0) * A * CD / m / 2$$

where

$x(t)$ = distance down range at time t
$z(t)$ = height at time t
$x'(0)$ = initial velocity in the x direction
$z'(0)$ = initial velocity in the z direction
$V(0)$ = muzzle velocity

Flat trajectories cancel the interdependence of variables and allow for a continuous, dynamic solution, but it has one significant drawback. Flat trajectories are accurate for only a relatively short distance during ascent and are not capable of calculating descent ballistics. This prevents the analysis of a major portion of the bullets trajectory.

Iterative. Trajectories calculated by iteration are the most accurate. This type of trajectory calculation demands the aid of a computer. The basic idea is to assume that the coefficient of drag and bullet angle with respect to the ground are constants for a very short period of time. This assumption permits the ballistics to be calculated at the end of the time period and the results to be used as initial conditions for the next time period. Using this method, it is possible to piece together the complete flight of the bullet with great accuracy. The basic iteration equations for the trajectory are (4:2-61,2-68):

$$x(ta+tp) = x(ta) + x'(ta) * tp - K * x'(ta) * V * tp^2$$

$$z(ta+tp) = z(ta) + z'(ta) * tp - K * z'(ta) * V * tp^2 - g / 2 * tp^2$$

$$K = Ro * A * CD(V) / (m * 4)$$

$$V = (x'(ta)^2 + z'(ta)^2)^{.5}$$

where

$x(ta)$ = position in the x direction at time ta
$z(ta)$ = position in the z direction at time ta
$x'(ta)$ = velocity in the x direction at time ta
$z'(ta)$ = velocity in the z direction at time ta
ta = previous iteration time
tp = time period between iterations

Current Models in use for Small Arms

There are no currently used aircraft survivability models in use which were written for small arms. The Air Force Armament Test Labortory at Eglin AFB developed a model called P001, which is the most widely used model for the larger caliber anti-aircraft guns. P001 is unsuitable for small arms for the following reasons (8):

1. The weapon systems follow the target and try to adjust aim point. This is not an engagement method used for small arms.

2. The ballistics are calculated using flat trajectories. This restricts the number of engagement ranges which can be analyzed.

3. P(kss) is the measure of survivability. Small arms rely on a volume of fire for effectiveness.

Experimental Designs

The main objective of this thesis is to develop a computerized survivability model which can be used to analyze the effectiveness of small arms against fixed-wing aircraft. An important part of the development process is the validation of the model. Doing an experiment using response surface techniques will provide a means of comparing the effects and interactions of variables in the model with reality. The experiment will also provide an example of a type of analysis the model will support.

There are three experimental designs which may be useful for evaluating the first and second order variable responses. They are the 3^k factorial design, the Box and Behnken design, and the central composite design. A brief overview of these designs will be given in this chapter. Before the individual designs are presented, a few terms must be discussed:

Orthogonality: An orthogonal design minimizes the common variance of the estimators which reduces the bias associated with an individual estimator (9:335).

Uniform Precision: Uniform percision in an experimental design causes the variance of the estimate at the origin to equal the variance of the same estimate at a unit distance from the design center (10). The constancy of the variance reduces errors caused by a factor's magnitude at a given level.

Rotatable: A rotatable design is a design in which the variance of the predicted response at some point is function of the distance from the design center; not a function of direction (10).

Factorial design: A factorial design is a design which calculates a response variable for every possible combination of factor levels. For instance, if a model had five factors with three levels each, a full factoral design would have 3^5 or 243 runs.

Full 3^k Factorial Design. This full factorial design, which uses three factor levels, is the smallest full factorial design which can be used to fit a second order response surface. However, as the number of factors (k) gets large the number of runs become excessive (10).

Box and Behnken Design. This design is an incomplete 3^k design, which is rotatable. But, this design is not completely orthogonal and does not possess uniform precision.

The main asset of the design is the realitively few number of runs needed for a large number of factors. The effects of non-orthogonality and lack of uniform precision is lessened as the number of factors increases (3).

Central Composite Design. This design is an incomplete five level design, which is rotatable and can be orthogonal and posess uniform precision. The greater number of levels used for this design provides very accurate estimates but the number of runs is usually greater than the number needed for a Box and Behnken design. This difference does not become substantial until six factors are used (10). Table I summarizes the number of runs required for each experimental design. In Table I a separate number is given for deterministic and stochastic models. The difference in runs is because deterministic models need only one replication of the center point.

Table I. Number of Runs for Experimental Designs

	Factors				
	3	4	5	6	7
Full 3^k Factorial					
Runs	27	81	243	729	2187
Box and Behnken					
Stochastic Runs	15	27	46	54	62
Deterministic Runs	13	25	41	49	57
Central Composite					
Stochastic Runs	23	36	59	100	175
Deterministic Runs	15	25	43	77	143

Conculsion

The history of small arms effectiveness against aircraft has shown it to be a useful air defense weapon. But, very few people have tried to quantitatively model its effectiveness. The mathematics needed to model the bullet ballistics are well known and documented, but the linking of a small arms specific scenario to this mathematics is in the early stages of development and not well defined. The remainder of this study will be devoted to the development of this link.

III. Methodology

The main purpose of this study is to develop a methodology for determining the survivability of fixed-wing aircraft against small arms. The methodology presented in this chapter was designed for use in a computer survivability model. The main portion of this chapter is dedicated to the mathematics of the methodology used in the model. It does not discuss the data base management or the user interface, these topics are discussed in Appendix A. The model code is contained in Appendix G.

Scenario Limitations

There is an infinite number of situations and geometries in which small arms could be used to engage fixed-wing aircraft. To reduce the number of possibilities scenarios, limitations must be applied to the survivability model.

Physical Environment The environment is the most restricted element in the scenario. Different combinations of environmental factors such as weather, air density, terrain, and foliage are too numerous to represent; therefore, only one very simple environment will be used. The setting is a clear windless day on a flat sea-level plain.

Participants. The entities represented in the model include a fixed-wing aircraft and a single ground combatant using a small arms weapon. The participants have no interactions other than the small arms fire. As an example, this study will use a generic 7.62mm round and fixed-wing aircraft. All aircraft and weapon data used in the model are in Appendix B and Appendix C, respectively.

Type of Engagement. The engagement method that this model uses is the lead point method discussed in Chapter 2. Also, the conservation of ammunition is not considered.

Aircraft Flight Path. Manuevering against small arms fire has little benefit since the pilot doesn't know which is the shortest path away from the bullets. Therefore, the flight path of the aircraft is restricted to a straight line at a constant altitude. The aircraft also maintains a constant velocity. The model represents any aircraft angle of approach. The angle of approach is measured from the X-axis as represented in Figure 5.

Aircraft Representation. The aircraft is represented according to the number of aspect angles supplied. In Chapter 2, the six and 26 aspect representation were discussed. Each aspect angle had a presented area, A(p) or a vulnerable area A(v) associated with it. As a first cut approximation, this study uses a sphere with the same total surface area, A(t) as the actual aircraft. This assumes all A(p) are circular and equal for all aspects. The presented aircraft sphere has a radius equal to the square root of A(t) divided by, 4 times pi.

Coordinate System

The ground combatant is the origin of the system. The postive X-axis is the line formed on the ground between the combatant and the aim point. The positive Z-axis is the altitude above the ground. The Y-axis is the off range distance from the aim point. This coordinate system is represented in Figure 4.

Aiming

The aim point is expressed in terms of the ground distance (GRD), off range distance (OFR), and altitude (ALT) from the combatant. This aim point is used to calculate the mean aiming azimuth and elevation angle. The mean azimuth angle always lies on the X-axis, therefore it is always zero. This also causes the OFR to always be zero. The mean elevation angle is:

$$E = \arctan(alt / grd)$$

where

E = mean elevation angle
grd = x coordinate of the aim point
alt = z coordinate of the aim point

Since there is an aiming error caused by the combatant's limitations and weapon inconsistencies between shots, the actual firing elevations and azimuths are described by normal distributions about the angle mean. The use of normal distributions implies that there is an infinite number of initial elevations and azimuths angles available for

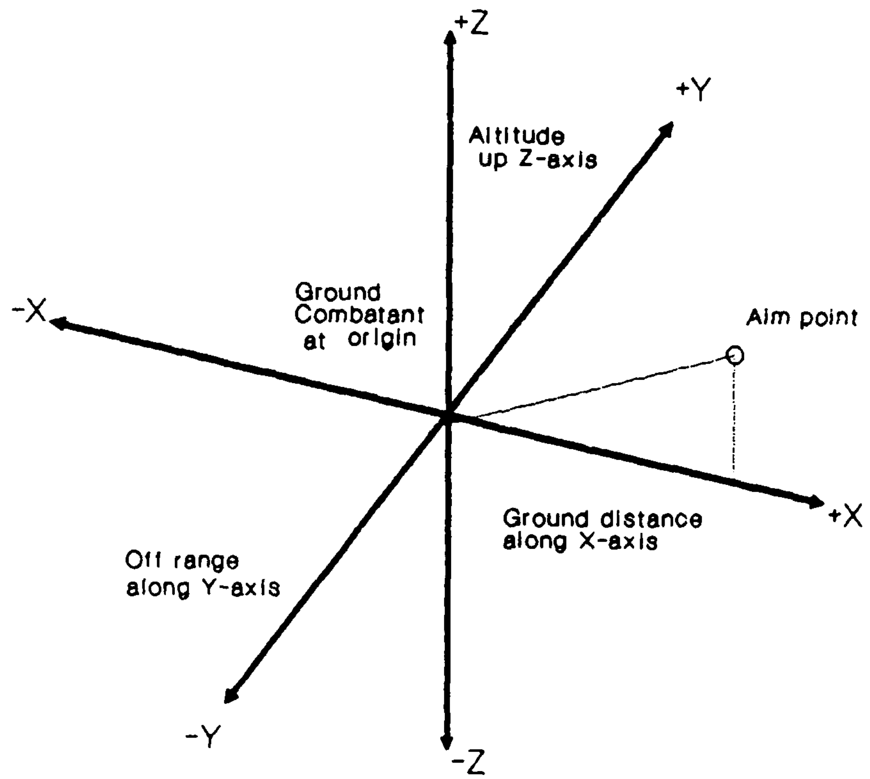


Fig. 4 Coordinate system

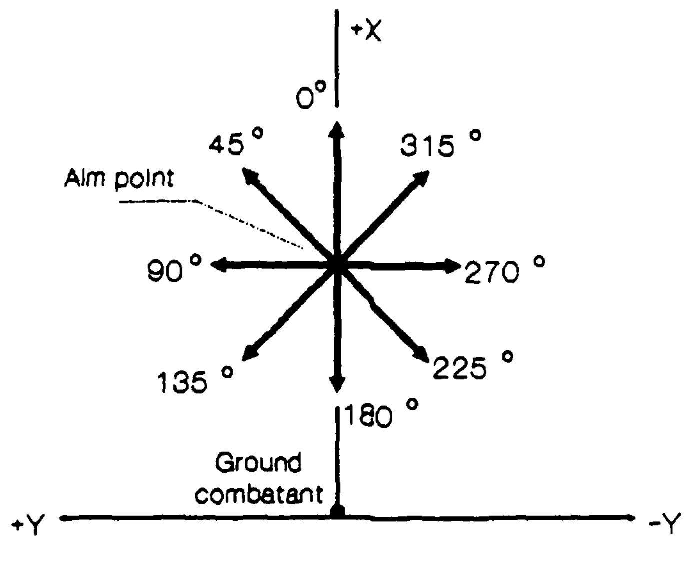


Fig. 5 Eight aircraft approach angles

trajectory calculations and each angle has a zero probability of occuring. These characteristics of a continuous probability distribution makes the direct use of them in a deterministic model difficult. To remedy this problem, only fifteen angles on each side of the mean are considered. The angles are placed every 0.2 standard deviations within three standard deviations of the mean. By separating the normal distributions into descrete intervals, each of the initial angles will also have an associated probability of occurance. The probability of occurance can then be used as the percent of bullets fired at a given angle. In this manner an inherently continuous and stocastic phenomenon can be converted to a descrete and deterministic one, without a great loss of detail.

Trajectories

The positions and component velocities of the bullets' flight paths are first calculated for elevation angles over the mean azimuth. The trajectories are plotted every 0.2 standard deviations within three standard deviations of the mean elevation angle. The trajectory calculation uses the iterative method as described in Chapter 2. The data needed to support the trajectory calculations are in the weapon data base (Appendix C). Once all the trajectories are plotted, in the X,Z-plane, for the mean azimuth, the trajectories for the other azimuths can be plotted by rotating the mean azimuth trajectory plots as follows:

$$(X,Y,Z) = (\cos(A) * Xm,\ \sin(A) * Xm,\ Zm)$$

where

A = azimuth angle
(X,Y,Z) = point on the trajectory for the given azimuth
$(Xm,0,Zm)$ = point on the trajectory for the mean azimuth

Expected Hits

The expected number of hits on the aircraft sphere by a single trajectory, E(hits), is the measure of susceptibility used by this model. It is the most difficult calculation in the model.

The first step is to find the points where the given mean azimuth trajectory cuts the horizontal planes tangent to the presented sphere. There are two types of intersections possible. Type I are those which pass through both tangent planes, as shown in Figure 6. Type II are the trajectories which pass through only the lower tangent plane, as shown in Figure 7. The trajectory is then approximated by two straight lines between these two points. Each line defined by two points is then rotated for every azimuth and used separately in the remaining calculations for expected hits. These points of intersection, after rotation, will be refered to as (X1,Y1,Z1) and (X2,Y2,Z2) for the remainder of this section.

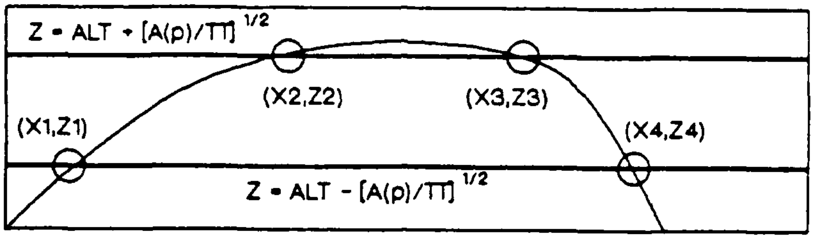


Fig. 6 Type I trajectory Intersection

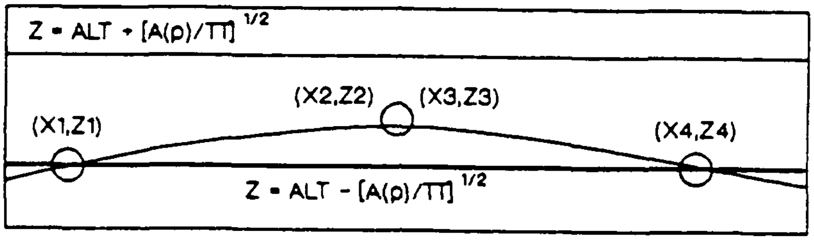


Fig. 7 Type II trajectory intersection

The second step is to transform the coordinate system to where all interactions between the aircraft sphere and the line segment representing the bullet trajectory can be represented in 2-dimensions.

The first transformation is a simple rotation of the aircraft angle of approach to zero degrees. This transformation is accomplished by the following:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos(Aa) & \sin(Aa) & 0 \\ -\sin(Aa) & \cos(Aa) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where

Aa = aircraft angle of approach

The transformed intersection points and aim point are: (X1′,Y1′,Z1′), (X2′,Y2′,Z2′), and (GRD′,OFR′,ALT′).

The second transformation is another axis rotation. The purpose of this transformation is to rotate line between the transformed intersection points to a vertical position in the Y′,Z′-plane. This transformation is accomplished as follows:

$$Th = \arctan[\ (Y2'-Y1') \ /\ (Z2'-Z1')\ ]$$

$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(Th) & -\sin(Th) \\ 0 & \sin(Th) & \cos(Th) \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

An example of the results of the transformations can be seen in Figures 8 and 9, where Figure 8 is the base coordinate system and Figure 9 is the final transformed coordinate system.

Using the projection on the Y′′,Z′′-plane, the length of intersection made by the bullet path and the sphere is:

$$L = 2 * [\ A(t)\ /4\ /pi - (\ Y'' - OFR''\ )\hat{}2\ ]\hat{}.5$$

$$Y'' = Y1'' = Y2''$$

where

L = length of intersection of sphere and trajectory in the Y′′,Z′′-plane
A(t) = total surface area of the aircraft

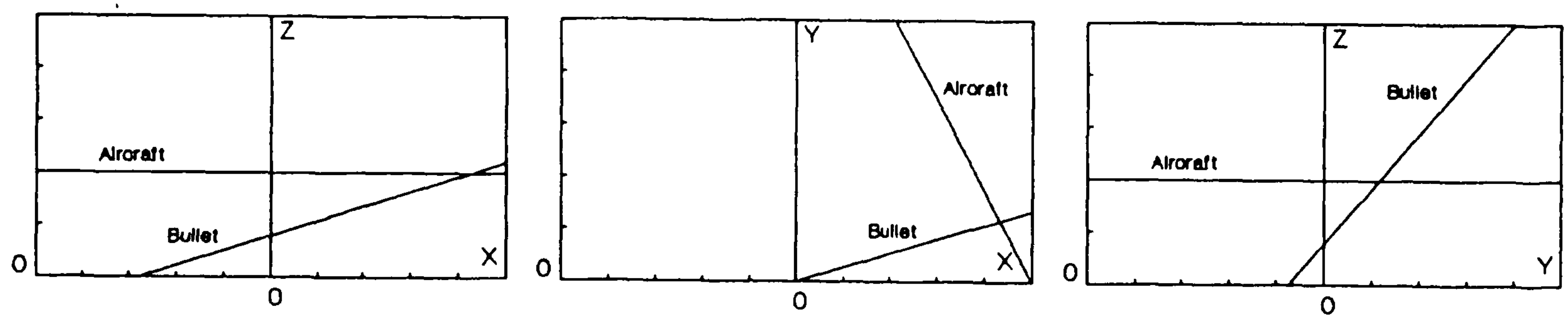


Fig. 8 Three views of the base coordinate system

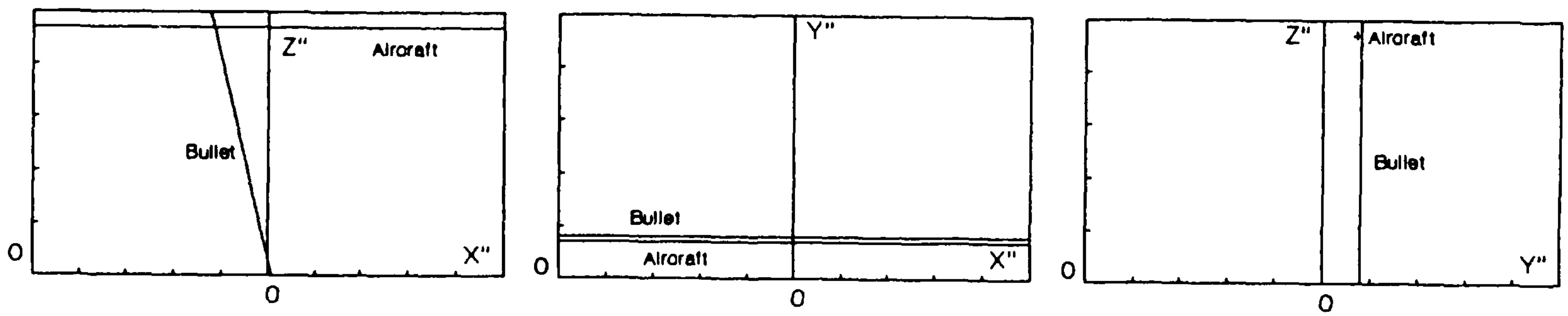


Fig. 9 Transformed coordinate system

If Y´´- OFR´´ is greater than the radius of the sphere, L is imaginary; therfore, the sphere and trajectory do not intersect. The real length of intersection represents the diameter of the circle the bullets´ trajectory cuts out of the sphere when projected in the X´´,Z´´-plane. The length of intersection and the circle cut are represented in Figures 10 and 11, resectively.

Step three is to find the points where the sliced circle first and last intersects the trajectory segment in the X´´,Z´´-plane. These two points are the two most important points needed for the expected hits calculations. The Z´´ coordinates of these points are called I1 and I2, the X´´ coordinates are not necessary for any calculations. The Z´´

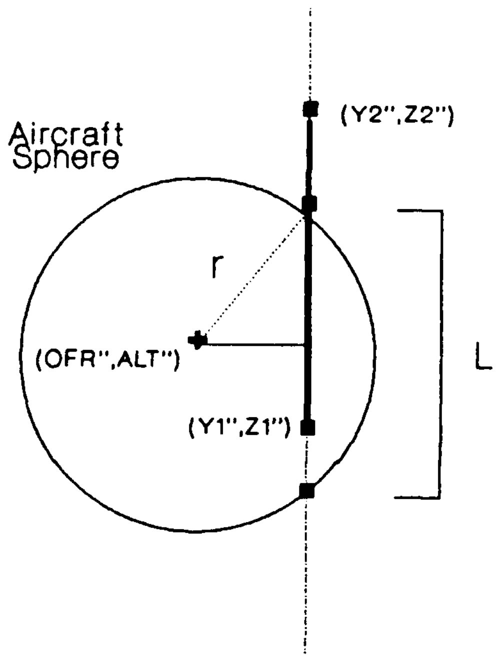


Fig. 10 Y",Z"-projection of intersection

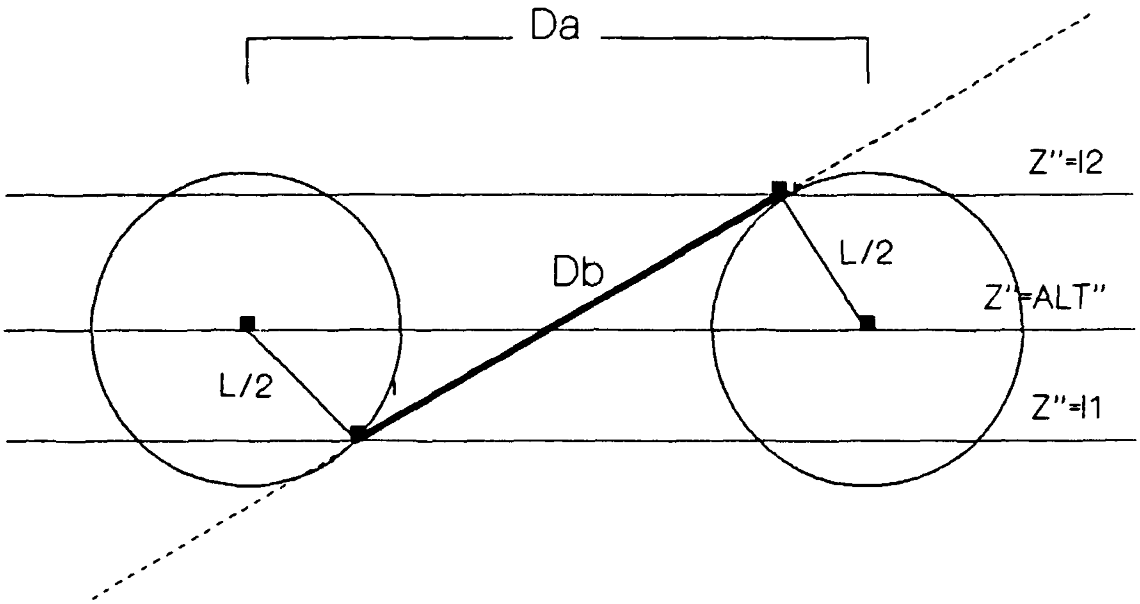


Fig. 11 X'',Z''-projection of circle sliced from sphere

coordinates of the end points of the trajectory segment, Z1'' and Z2'', are prime candidates, but so are the Z'' coordinates of the points where the trajectory segment is tangent to the sliced circle, T1 and T2, which are calculated by:

$$T1 = ALT'' - L/2 * \cos(Ai)$$

$$T2 = ALT'' + L/2 * \cos(Ai)$$

$$Ai = \arctan[(Z2''-Z1'') / (X2''-X1'')]$$

where

Ai = angle of trajectory as it passes through the aircraft sphere

It is also possible that some combination of these points is correct. The simplest way to choose the correct two points is to discard the highest and the lowest Z´´ coordinates of the four. The remaining two are then set to I1 and I2.

The next step is to calculate the length of time the bullets´ path intersects the circle cut from the sphere. There are three mutually exclusive cases where the time of intersection is calculated:

Case 1: The X´´ component of the bullets´ velocity is positive (i.e. the aircraft and bullets close on each other).

Case 2: The X´´ component of the bullets velocity is negative and the component of the aircraft´s velocity parallel to the bullets´ velocity is greater than the bullets´ velocity (i.e. the aircraft catches and hits the bullets).

Case 3: The X´´ componet of the bullets´ velocity is negative and the component of the aircraft´s velocity parallel to the bullets´ velocity is less than the bullets´ velocity (i.e. the bullets catch and hit the aircraft).

For the three cases, the time of intersection is calculated as:

Case 1: ti = ta + tb

Case 2: ti = ta - tb

Case 3: ti = tb - ta

ta = Da / Va

tb = Db / Vb

$$Da = [\ (L/2)^2 - (I1-ALT'')^2\ ]^{.5} + [\ (L/2)^2 - (I2-ALT'')^2\ ]^{.5} + Db * \cos(Ai)$$

$$Db = (\ I2 - I1\ )\ /\ \sin(Ai)$$

$$Ai = \arctan[\ (Z2''-Z1'')\ /\ (X2''-X1'')\ ]$$

where

- Da = distance covered by the aircraft during the intersection
- Db = distance along the trajectory path which intersects the aircraft sphere
- Va = velocity of aircraft
- Vb = average velocity of bullets on segment
- L = diameter of the circle cut from the aircraft sphere
- I1 = altitude of first intersection
- I2 = altitude of last intersection
- Ai = angle of trajectory as it passes through the aircraft sphere

Figure 11 illustrates most of the variables use in the previous equations.

The final calculation for the expected number of hits is:

$$E(hits) = ti * ROF * P(ele) * P(azi)$$

where

- ROF = weapon rate of fire
- P(ele) = percent of bullets fired at the initial elevation required for this trajectory
- P(azi) = percent of bullets fired at the initial azimuth required for this trajectory

The above sets of equations assume that the first bullet fired reaches the farthest point of intersection before the aircraft.

<u>Vulnerability Measure</u>

As stated earlier in this chapter, the aircraft is represented by a sphere with the same total surface area A(t), as the aircraft, but this is not the measure of vulnerablity. The measure of vulnerability is the probability of kill given a hit on the aircraft sphere, P(k/h). Unlike the A(t), P(k/h) is not constant, it varies with the impact velocity of the bullet on the aircraft. The same three cases used for the time of intersection are also used for the impact velocity. The impact velocity is:

Case 1: $Vi = Va * \cos(Ai) + Vb$

Case 2: $Vi = Va * \cos(Ai) - Vb$

Case 3: $Vi = Vb - Va * \cos(Ai)$

where

Ai = angle of trajectory as it passes through the aircraft sphere

The impact velocity is used to interpolate the P(k/h) from the vulnerability data base, Appendix B.

Probability of Survival

The probability of survival is simple to calculate once E(hits) is calcualted and P(k/h) is interpolated for each trajectory segment the aircraft passes through. Using these two numbers the probability of survival for a single trajectory segment is:

$$P(sts) = EXP[\ -P(k/h) * E(hits)\ ]$$

where

P(sts) = probability of survival for a single trajectory segment intersection
P(k/h) = probability of kill given a hit for the trajectory segment

E(hits) = expected number of hits from the trajectory segment

The total probability of survival for the entire engagement is:

$$P(s/e) = \prod_{\substack{\text{for all} \\ \text{trajectory} \\ \text{segments}}} P(sts)$$

Experimental Design for Methodology Verification and Validation

The purpose of this experimental design is to identify the important factors among those selected for the experiment and their relationships to the response variables. This information will then be compared to known information about the same factors.

Response Variables and Factors. This experiment is restricted to a single weapon and a single aircraft. There are two response variables and four factors which are used in the experiment:

Response Variables

1. Probability of Survival, P(s/e)
2. Total Expected number of Hits, E(h)

Factors

1. Aircraft altitude, Alt
2. Ground distance to aim point, Grd
3. Aircraft velocity, Vel
4. Aircraft angle of approach, App

Type of Design. As shown in Chapter 2, the number of deterministic runs for four factors for a full second order factorial, central composite, and Box and Behnken are 81, 25, and 25, respectively. Based solely on the number of runs the full factoral can be eliminated. The Box and Behnken design and the central composite design require the same number of runs, but the central composite design is orthogonal, rotatable, and requires 5 factor levels, which results in more accurate estimations of the factor correlations. Therefore, the central composite design will be used.

Experiment. The experiment is performed using the following steps:

1. Calculate corner points responses using a full 2^4 factorial design.

2. Calculate center points responses.

3. Check for curvature to see if a second order model is needed.

4. If second order necessary, calculate axial points responses.

5. Use stepwise regression to identify important factors and interactions.

The factor levels used are in Table II and the complete listing of runs and levels are given in Table III.

Table II. Factor levels

Factor	Levels				
	-2	-1	0	1	2
Alt	100	200	300	400	500
Grd	100	300	500	700	900
Vel	400	500	600	700	800
Aap	0	45	90	135	180

Table III. Runs with level settings

	Alt	Grd	Vel	Aap
Corner points	1	1	1	1
	1	1	1	-1
	1	1	-1	1
	1	-1	1	1
	-1	1	1	1
	1	1	-1	-1
	1	-1	-1	1
	-1	-1	1	1
	1	-1	1	-1
	-1	1	-1	1
	-1	1	1	-1
	1	-1	-1	-1
	-1	-1	-1	1
	-1	1	-1	-1
	-1	-1	1	-1
	-1	-1	-1	-1
Center points	0	0	0	0
Axial points	2	0	0	0
	-2	0	0	0
	0	2	0	0
	0	-2	0	0
	0	0	2	0
	0	0	-2	0
	0	0	0	2
	0	0	0	-2

IV. Results

Central Composite Design Results

The design was completed using the characteristics of a 7.62mm small arms round and a generic aircraft vulnerability database. The data inputs are in Appendices B and C. The results of the runs described in Chapter 3 are in Appendix D.

Using the corner points and the center points, linear models were fit to the response variables, probability of survival P(s) and expected number of hits E(hits). Plotting the residuals provided a visual check for curvature. Figures 12 and 13 show the residual plots for P(s) and E(hits), respectively. The complete linear regression results with scaled plots for P(s) and E(h) are in Appendix F. The residual plot for each regression model showed a good random distribution for the corner point residuals, but, the center point residual was far from the others. Regardless of scale, center point residual offsets of this relative magnitude, indicated that second order models were necessary for both response variables.

With the axial points added, the second order models were fit using stepwise regression. The results are summarized in Tables IV and V. The Tables use the following abbreviations: aircraft velocity (Vel), aircraft angle of approach (Aap), aim point ground distance (Grd), and aim point altitude (Alt). Complete results are in Appendix F.

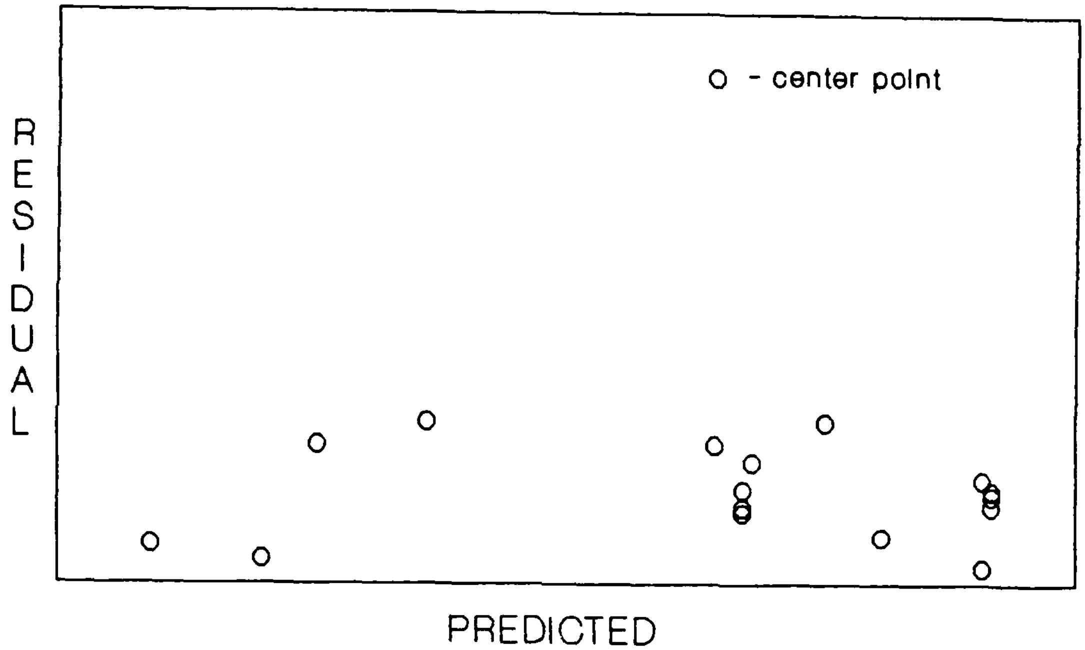


Fig. 12 P(s) linear model residuals

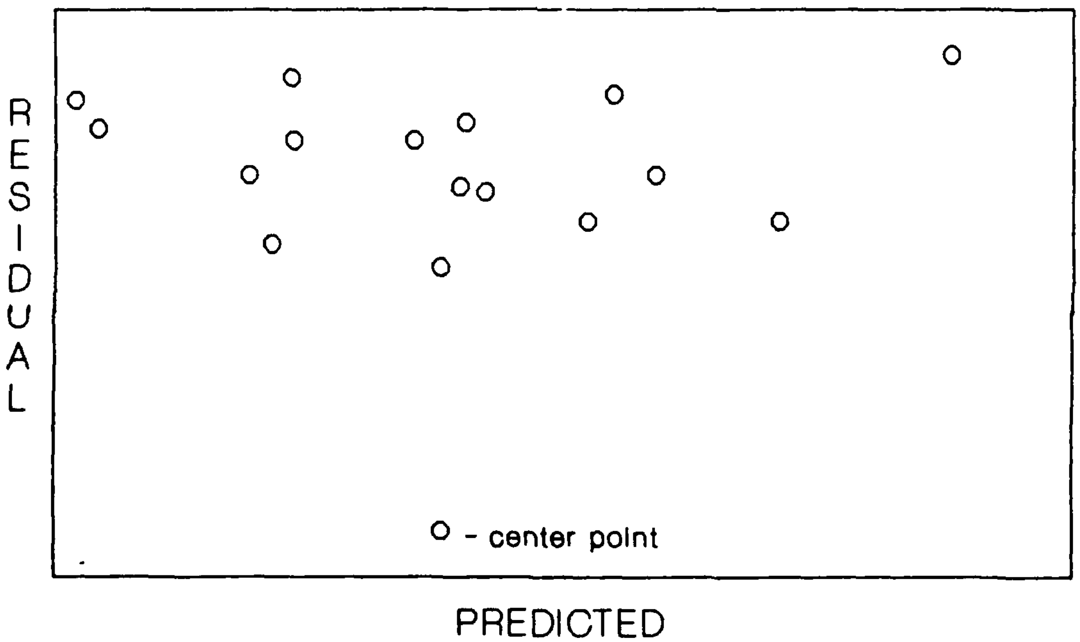


Fig. 13 E(hits) linear model residuals

Table IV. Stepwise Regression Results for P(s)

Variables	B value	Partial R^2
INTERCEPT	0.99987979	
Aap	-0.00021998	.4801
Grd	0.00017788	.3139
Grd * Aap	0.00008512	.0479
Aap * Aap	-0.00008793	.0438
Grd * Grd	-0.00008081	.0620
Grd * Aap * Alt	-0.00004564	.0138
Aap * Alt	0.00004249	.0119
Vel	0.00002615	.0068
Grd * Vel	-0.00003016	.0060
		R Square = .9862

Table V. Stepwise Regression Results for E(hits)

Variables	B value	Partial R^2
INTERCEPT	0.01105365	
Aap * Aap	0.00630314	.3992
Grd	-0.00453835	.2279
Aap	0.00363550	.1482
Vel	-0.00274286	.0832
Alt	-0.00163205	.0295
Grd * Grd	0.00146557	.0227
Grd * Alt	0.00150094	.0166
Aap * Alt	-0.00132092	.0129
		R Square = .9383

Response Surfaces for Verification and Validation

The focus of this thesis was on the development of a computer model and not on the evaluation of a particular problem with a computer model. Therefore, most of this section was devoted to the verification and validation, V & V, of the model. Verification is to ensure all calculations in the model perform as intended. Validation is to substantiate that the model results are realistic. The line between verification and validation is often blurred. Techniques used for one can sometimes provide insight into the other.

The generation of response surfaces for P(s) and E(hits) was useful for the V & V of the model. The response surfaces generated had high R square values; therefore, the results of the response surfaces were considered a reasonable reflection of the computer model. Comparing the response surfaces to reality gave insights into the validity of the model.

Extremes. The first and most obivious advantage of the response surfaces was the extreme values used for their generation. These extremes flushed out many errors in the model coding and verified that the model at least functioned for the entire range of values for which it was intended.

Coefficient of Correlation (R^2). The next use of the response surfaces was to validate that the variables which have known effects, should have R^2 values which match their importance. By far, the variable which should affect P(s) and E(hits) the most is the aircraft angle of approach, Aap. This factor is the major driver of bullet impact angles and hence the velocity of impact. Aap also greatly influences the length of time the aircraft is exposed to a given trajectory. Time exposed dictates the number of hits. The R^2 values concured with these facts. Aap and its interactions made up 60.6 % and 59.5 % of the total R^2 for P(s) and E(hits), respectively. These percentages are calculated by summing the partial R^2 values for Aap and its interactions, then dividing by the total R^2.

The second most important factor should be ground distance, Grd. This factor influences the velocity and angle of impact of the bullet. The greater the distance the slower the velocity and the greater the change in angle. Both of these effects, inturn, influence P(s) and E(hits). The percents of the R^2 values for Grd and its interactions were 44.8 % and 28.5 %, for P(s) and E(hits), respectively.

The effects of altitude, Alt, should have been simular to Grd. But, its R^2 contributions were only 6.3 % and 2.6 %. At first this seemed to signal an error in the methodology or code, but no code errors were found and the model produced expected results for all other factors. The results were finally traced to the factor settings. Alt had settings only half as wide as Grd; therefore, at corner points and axil points Grd dominated the distance and angle equations.

The range of aircraft velocities used should have little affect on either response variable. The weapon used for the experiment had a high initial velocity, 2400 ft/sec. This suggests that the range of aicraft velocity used, 400 to 800 ft/sec, would have little affect on the impact velocity of the bullets and therefore little effect on the P(s). All the variables and interactions which contain Vel, amounted to only 1.3 % and 8.9 % of the total R^2 value for P(s) and E(hits).

Another known correlation is between the two response variables. Since, the number of hits on an aircraft is directly related to it probability of survival, the factors which most affect E(hits) should be the factors which most affect P(s). Reviewing the percentages stated above proved that this coorelation was true for the model.

Sign of Factor Coefficients. The last use of the response surfaces was to ensure that the sign of the factor coefficients related properly to reality. In other words, if the sign is negative, the response variable should decrease as the factor increases. In reality, P(s) should increase as Grd, Alt, and Vel increase and P(s) should decrease as Aap increases. E(hits) should react exactly opposite of P(s); because, as the number of hits increases the probability of survival decreases. All the coefficient signs, as seen in Tables IV and V, corresponded to reality.

Face Validity

The model was demonstrated and the methodology was reviewed by the Advanced Systems Analysis Directorate, Aeronautical Systems Division (ASD/XRM), Wright-Patterson AFB and the model was briefed to the Assistant Deputy for Development Planning (ASD/XR), Col. Samuel Boykin. The model methodology was considered "excellent", by Col. Boykin and ASD/XRM has adopted the model for use in future studys. A letter of appreciation from Col. Boykin to AFIT/CC, for the work done in this thesis, is given in Appendix H.

V. Conclusions and Recommendations

Strengths and Limitations

This model has three major strengths: (1) It represents a small arms specific method of engagement. (2) Because small arms rely on volume fire, the model calculates multiple hits on the aircraft from a single shooter. (3) The model acurately represents complete bullet trajectories not flat line estimates.

The model also has three limitations: (1) The aircraft is represented by a sphere, detailed aspects are not used. (2) The scenario is a 'worse case' for the aircraft. The aircraft is restricted to a constant velocity, altitude, and approach angle (i.e. no evasive maneuvers). Also, no environmental effects are modeled. (3) The model represents only one ground combatant.

Practical Implications

There are two major uses for this model. The first is to provide an aid for survivability studies which analyze competing aircraft designs. The model provides P(s), but it also calculates the expected number of hits, E(hits), for battle damage repair models.

The other use is for wargamming. For high resolution combat, the model could be incorporated directly into the wargame. The model could also be used as a data generator for lower resolution wargames. For example, all memebers of an infantry platoon and their various small arms could be evaluated separately and the results integrated into a larger model.

Recommendations

This study has presented a first step in solving a heretofore unanswered problem; much remains to be done. Future studies should be conducted; specifically, a larger model could be developed which would calculate the survivability of an aircraft against a platoon, company, or larger unit. The core model would be the one presented here. The remainder of the proposed model would consolidate the single shooter results from the core model into results for a combat unit of the desired resolution. Additional projects should include eliminations of the model's other limitations listed above.

Conclusion

The model developed for this thesis provides a link between established bullet ballistics equations and high resolution combat scenarios using small arms as air defense. Being one of the first to provide such a link, it should prove to be a useful tool for the analysis of aircraft survivability against small arms.

Appendix A: Computer Model I/O

This appendix was written for the person tasked to make runs of the computer model. This section covers two major areas. The first provides an discription of model setup and execution procedures. The second gives a brief description of the program routines. The model's code is listed in Appendix G. The comments provided in the code listing will supplement the brief program descriptions given in this appendix.

Required Databases

There are two small data bases which are required for execution. These data bases are the aircraft vulnerability file and the small arms characteristics file. Both databases are very small and could easily be combined into one file; but, to allow the matching of various aircraft and small arms, they are maintained separately.

The entries in the databases are read using a free format; therefore, only the order matters. Appendices B and C show examples of the aircraft and small arms databases, respectively. The comments added to these examples are not required in the actual files.

Runtime Inputs

During model execution several inputs are required. The runtime data can be entered interactively or through a database. Mulitple model runs must be databased because the model does not loop back for multiple interactive inputs. The interactive inputs are entered using forms developed in SIMGRAPHICS. The forms require the use of a mouse and key board inputs. Databased runs are executed by entering the database name at the appropriate prompt. Appendix E contains an example of a runtime input database.

Output

The output file produced by the model contains all runtime inputs and the probability of survival of the aircraft and the expected number of hits on the aircraft. If the interactive inputs were used, the results will printed in the database and interactively.

Program Specifications

Hail Storm is written in PC SIMSCRIPT II.5 version 2.3 and is currently hosted on a Zenith 248 using MS-DOS operating system. The program currently requires a math co-processor, a digital mouse, and an EGA moniter. PC SIMSCRIPT II.5 requires hard disk storage and 640 KB of RAM memory.

The program includes about 1000 lines of code and requires approximately 300 KB of memory to store code, databases, forms, and graphs. Execution time is approximately 3 minutes per run.

Routine Calling Pattern and Descriptions

Below is a list of the routines, with cross indexed calls and brief bullets on routine functions. It should be noted, only routines explicitly written for the assessment portion of the model are listed. Calls to SIMSCRIPT library and graphics routines are not considered.

Routine: PREAMBLE
Called by: "System"
Calls:
Functions: - identitfies and sets mode of global variables

Routine: MAIN
Called by: "System"
Calls: INTERACT, SET.DATA, INIT.RUN, TRAJECT, SURVIVAL.MAIN
Functions: - central processing routine
- loops for multiple runs

Routine: INTERACT
Called by: MAIN
Calls:
Functions: - prompts user for runtime inputs

Routine: SET.DATA

Called by: MAIN

Calls: PERTIZE

Functions: - reads aircraft and small arms databases

- sets probabilities by standard deviations

Routine: INIT.RUN

Called by: MAIN

Calls:

Functions: - initializes expected hits, probability of survival, and coordinate arrays

Routine: TRAJECT

Called by: MAIN

Calls: INIT.CUTS, PLANE.CUTS

Functions: - calculates trajectories for each elevation on mean azimuth

Routine: SURVIVAL.MAIN

Called by: MAIN

Calls: TRANSFORM, PROB.SURVIVE, EXPECT.HITS

Functions: - rotates intersections to azimuths

- calculates diameter of circle cut from aircraft sphere

Routine: PERTIZE

Called by: SET.DATA

Calls:

Functions: - interpolates coefficient of drag and probability of kill values based on percentages of maximum reference values of velocity

Routine: INIT.CUTS

Called by: TRAJECT

Calls:

Functions: - initializes points of intersection for each elevation trajectory

Routine: PLANE.CUTS

Called by: TRAJECT

Calls:

Functions: - calculates coordinates of trajectory intersections with the Z-planes tangent to aircraft sphere

- calculates slope of intersecting trajectory line segments

- calculates average velocity for intersecting trajectory line segments

Routine: TRANSFORM

Called by: SURVIVAL.MAIN

Calls:

Functions: - transforms any point to new coordinate system for use in expected hits calculations

Routine: PROB.SURVIVE

Called by: SURVIVAL.MAIN

Calls:

Functions: - calculates impact velocities

- calculates proability of survival

Routine: EXPECT.HITS

Called by: SURVIVAL.MAIN

Calls:

Functions: - calculates expected number of hits on the aircraft sphere

Appendix B: Sample Aircraft Data

''Aircraft Surface Area (ft^2)
650.0

''Impact Velocities (ft/sec) 1 thru 10

3500	3000	2500	2000	1500	1000	500	0000	0000	0000

''Probability of Kill given a Hit 1 thru 10

.0797	.0635	.0480	.0325	.0164	.0004	.0002	0000	0000	0000

''Impact Velocities (ft/sec) 11 thru 20

0000	0000	0000	0000	0000	0000	0000	0000	0000	0000

''Probability of Kill given a Hit 11 thru 20

0000	0000	0000	0000	0000	0000	0000	0000	0000	0000

Appendix C: Sample Small Arms Data for 7.62mm Round

''Bullet Mass (slugs) Bullet Presented Area (ft^2)
.00055 .0005

''Muzzle Velocity (ft/sec) Elevation Standard Deviation (deg)
2400. 5.

''Azimuth Standard Deviation (deg) Rate of Fire (bullets/sec)
5. 6.

''Bullet Velocity (expressed in sea level Mach) 1 thru 10
4.000 3.500 3.000 2.500 2.000 1.500 1.250 1.125 1.063 1.000

''Coefficient of Drag 1 thru 10
0.207 0.233 0.249 0.286 0.318 0.384 0.442 0.445 0.430 0.318

''Mach 11 thru 20
0.938 0.875 0.750 0.500 0.000 0.000 0.000 0.000 0.000 0.000

''Coefficient of Drag 11 thru 20
0.255 0.239 0.223 0.221 0.220 0.000 0.000 0.000 0.000 0.000

Appendix D: <u>Results of Runs</u>

GRD	ALT	VEL	AAP	P(S)	HITS
700.00	400.00	700.00	45.00	.9999981	.0064318
700.00	400.00	700.00	135.00	.9997284	.0141047
700.00	400.00	500.00	45.00	.9999963	.0105391
700.00	400.00	500.00	135.00	.9997214	.0182120
700.00	200.00	700.00	45.00	.9999987	.0061644
700.00	200.00	700.00	135.00	.9997269	.0174373
700.00	200.00	500.00	45.00	.9999971	.0108847
700.00	200.00	500.00	135.00	.9997495	.0221577
300.00	400.00	700.00	45.00	.9998470	.0130998
300.00	400.00	700.00	135.00	.9994168	.0189204
300.00	400.00	500.00	45.00	.9997195	.0195039
300.00	400.00	500.00	135.00	.9992893	.0253245
300.00	200.00	700.00	45.00	.9999604	.0161519
300.00	200.00	700.00	135.00	.9991810	.0289398
300.00	200.00	500.00	45.00	.9998561	.0251703
300.00	200.00	500.00	135.00	.9990700	.0379582
900.00	300.00	600.00	90.00	.9999948	.0107397
100.00	300.00	600.00	90.00	.9991484	.0256314
500.00	500.00	600.00	90.00	.9998482	.0112881
500.00	100.00	600.00	90.00	.9999819	.0115086
500.00	300.00	800.00	90.00	.9999140	.0086654
500.00	300.00	400.00	90.00	.9998292	.0173308
500.00	300.00	600.00	0.00	.9999905	.0345000
500.00	300.00	600.00	180.00	.9990957	.0405717
500.00	300.00	600.00	90.00	.9998858	.0115539

Appendix E: Sample Multiple Run Input Database

```
''Output file name   Small Arms file name   Aircraft file name
    out.dat              arm.dat                 air.dat

''Number of runs
       3

''Ground Distance    Altitude    Velocity    Approach Angle
      500              200         300            180
      500              100         400             90
      345.8            90.4        540.0          15.125
```

Appendix F: Regression Output

Part 1

Stepwise Regression and Residual Plot

of a Linear Model of P(s)

STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE PS

NOTE: SLENTRY AND SLSTAY HAVE BEEN SET TO .15 FOR THE STEPWISE TECHNIQUE.

STEP 1 VARIABLE APP ENTERED R SQUARE = 0.53645953
C(P) = 8.03590418

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	0.00000076	0.00000076	17.36	0.0008
ERROR	15	0.00000066	0.00000004		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
APP	-0.00021812	0.00005235	0.00000076	17.36	0.0008

BOUNDS ON CONDITION NUMBER: 1, 1

STEP 2 VARIABLE GRD ENTERED R SQUARE = 0.82881005
C(P) = -3.23123851

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	0.00000118	0.00000059	33.89	0.0001
ERROR	14	0.00000024	0.00000002		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
GRD	0.00016102	0.00003293	0.00000041	23.91	0.0002
APP	-0.00021812	0.00003293	0.00000076	43.87	0.0001

BOUNDS ON CONDITION NUMBER: 1, 4

STEP 3 VARIABLE GAP ENTERED R SQUARE = 0.91050618
C(P) = -4.93868570

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	0.00000129	0.00000043	999999.99	0.0001
ERROR	13	0.00000013	0.00000001		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
GRD	0.00016102	0.00002471	0.00000041	999999.99	0.0001
APP	-0.00021812	0.00002471	0.00000076	999999.99	0.0001
GAP	0.00008512	0.00002471	0.00000012	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1, 9

STEP 4 VARIABLE GAAP ENTERED R SQUARE = 0.93399783
C(P) = -4.00475917

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	0.00000133	0.00000033	999999.99	0.0001
ERROR	12	0.00000009	0.00000001		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
GRD	0.00016102	0.00002209	0.00000041	999999.99	0.0001
APP	-0.00021812	0.00002209	0.00000076	999999.99	0.0001
GAP	0.00008512	0.00002209	0.00000012	999999.99	0.0001
GAAP	-0.00004564	0.00002209	0.00000003	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1, 16

STEP 5 VARIABLE AAP ENTERED R SQUARE = 0.95435892
C(P) = -2.92876479

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	0.00000135	0.00000027	999999.99	0.0001
ERROR	11	0.00000006	0.00000001		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
GRD	0.00016102	0.00001918	0.00000041	999999.99	0.0001
APP	-0.00021812	0.00001918	0.00000076	999999.99	0.0001
GAP	0.00008512	0.00001918	0.00000012	999999.99	0.0001
AAP	0.00004249	0.00001918	0.00000003	999999.99	0.0001
GAAP	-0.00004564	0.00001918	0.00000003	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1, 25

STEP 6 VARIABLE GV ENTERED R SQUARE = 0.96461321
C(P) = -1.39411420

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	6	0.00000137	0.00000023	999999.99	0.0001
ERROR	10	0.00000005	0.00000001		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
GRD	0.00016102	0.00001772	0.00000041	999999.99	0.0001
APP	-0.00021812	0.00001772	0.00000076	999999.99	0.0001
GV	-0.00003016	0.00001772	0.00000001	999999.99	0.0001
GAP	0.00008512	0.00001772	0.00000012	999999.99	0.0001
AAP	0.00004249	0.00001772	0.00000003	999999.99	0.0001
GAAP	-0.00004564	0.00001772	0.00000003	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1, 36

STEP 7 VARIABLE VEL ENTERED R SQUARE = 0.97385661
C(P) = 0.18641173

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	7	0.00000138	0.00000020	999999.99	0.0001
ERROR	9	0.00000004	0.00000000		
TOTAL	16	0.00000142			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971425				
GRD	0.00016102	0.00001605	0.00000041	999999.99	0.0001
VEL	0.00002863	0.00001605	0.00000001	999999.99	0.0001
APP	-0.00021812	0.00001605	0.00000076	999999.99	0.0001
GV	-0.00003016	0.00001605	0.00000001	999999.99	0.0001
GAP	0.00008512	0.00001605	0.00000012	999999.99	0.0001
AAP	0.00004249	0.00001605	0.00000003	999999.99	0.0001
GAAP	-0.00004564	0.00001605	0.00000003	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1, 49

NO OTHER VARIABLES MET THE 0.1500 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE PS

STEP	VARIABLE ENTERED	VARIABLE REMOVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)
1	APP		1	0.5365	0.5365	8.03590
2	GRD		2	0.2924	0.8288	-3.23124
3	GAP		3	0.0817	0.9105	-4.93869
4	GAAP		4	0.0235	0.9340	-4.00476
5	AAP		5	0.0204	0.9544	-2.92876
6	GV		6	0.0103	0.9646	-1.39411
7	VEL		7	0.0092	0.9739	0.18641

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F	PROB>F
1	APP		17.3596	0.0008
2	GRD		23.9086	0.0002
3	GAP		11.8673	0.0044
4	GAAP		4.2711	0.0610
5	AAP		4.9072	0.0488
6	GV		2.8978	0.1195
7	VEL		3.1821	0.1081

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: PS

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	7	0.00000138	0.00000020	47.89
ERROR	9	0.00000004	0.00000000	PR > F
CORRECTED TOTAL	16	0.00000142		0.0001

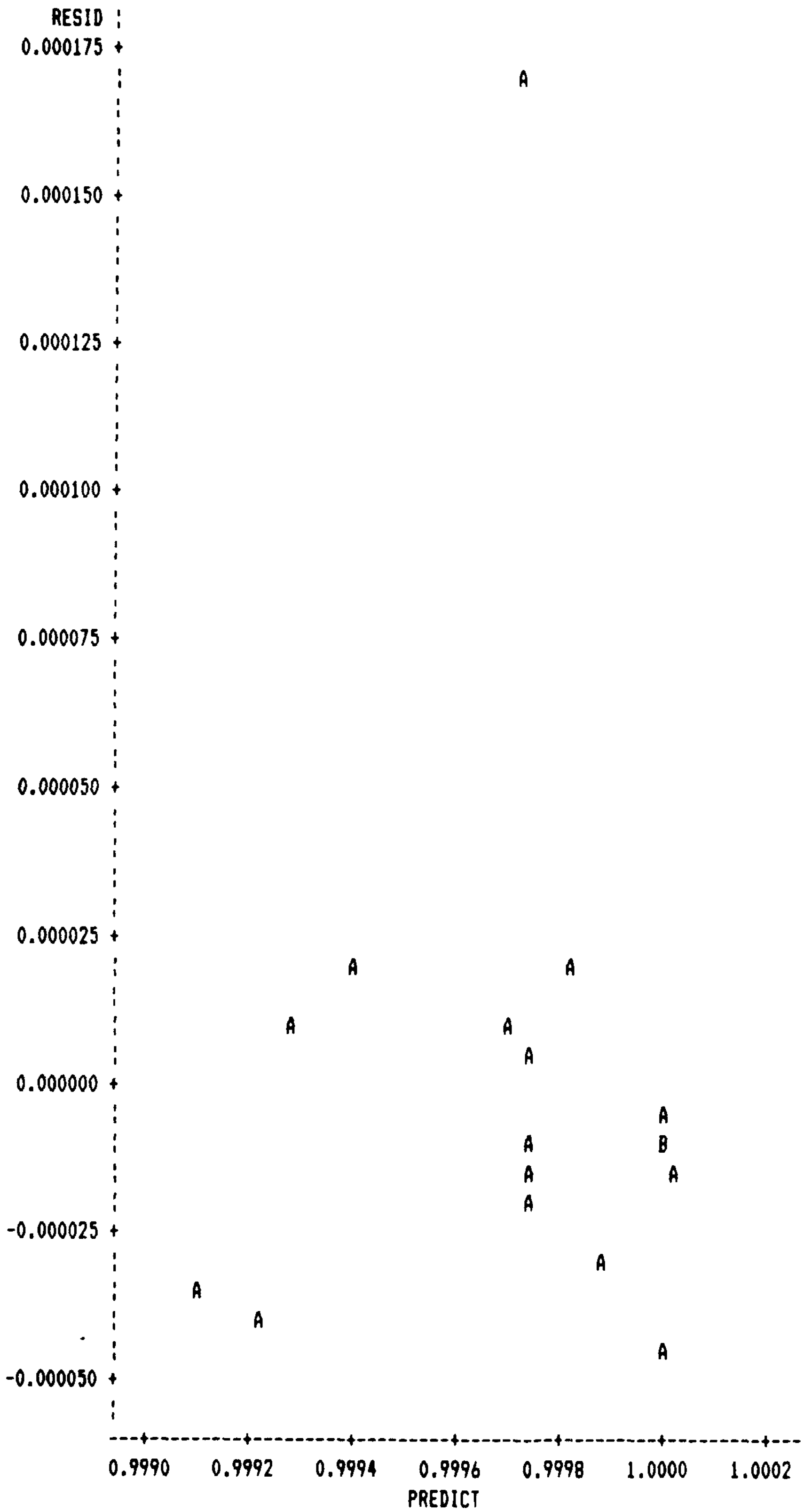
R-SQUARE	C.V.	ROOT MSE	PS MEAN
0.973857	0.0064	0.00006420	0.99971425

SOURCE	DF	TYPE I SS	F VALUE	PR > F
GRD	1	0.00000041	100.64	0.0001
VEL	1	0.00000001	3.18	0.1081
APP	1	0.00000076	184.68	0.0001
GV	1	0.00000001	3.53	0.0930
GAP	1	0.00000012	28.12	0.0005
AAP	1	0.00000003	7.01	0.0266
GAAP	1	0.00000003	8.09	0.0193

SOURCE	DF	TYPE III SS	F VALUE	PR > F
GRD	1	0.00000041	100.64	0.0001
VEL	1	0.00000001	3.18	0.1081
APP	1	0.00000076	184.68	0.0001
GV	1	0.00000001	3.53	0.0930
GAP	1	0.00000012	28.12	0.0005
AAP	1	0.00000003	7.01	0.0266
GAAP	1	0.00000003	8.09	0.0193

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	0.99971425	64203.12	0.0	0.00001557
GRD	0.00016102	10.03	0.0001	0.00001605
VEL	0.0000286	1.78	0.1081	0.00001605
APP	-0.00021812	-13.59	0.0001	0.00001605
GV	-0.0000302	-1.88	0.0930	0.00001605
GAP	0.0000851	5.30	0.0005	0.00001605
AAP	0.0000425	2.65	0.0266	0.00001605
GAAP	-0.0000456	-2.84	0.0193	0.00001605

PLOT OF RESID*PREDICT   LEGEND: A = 1 OBS, B = 2 OBS, ETC.



Part 2

Stepwise Regression of a Quadratic Model of P(s)

STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE PS

NOTE: SLENTRY AND SLSTAY HAVE BEEN SET TO .15 FOR THE STEPWISE TECHNIQUE.

STEP 1 VARIABLE APP ENTERED R SQUARE = 0.48007839
C(P) = 210.20514822

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	0.00000116	0.00000116	21.24	0.0001
ERROR	23	0.00000126	0.00000005		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971780				
APP	-0.00021998	0.00004773	0.00000116	21.24	0.0001

BOUNDS ON CONDITION NUMBER: 1, 1

STEP 2 VARIABLE GRD ENTERED R SQUARE = 0.79398411
C(P) = 72.61368480

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	0.00000192	0.00000096	42.39	0.0001
ERROR	22	0.00000050	0.00000002		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971780				
GRD	0.00017788	0.00003072	0.00000076	33.52	0.0001
APP	-0.00021998	0.00003072	0.00000116	51.27	0.0001

BOUNDS ON CONDITION NUMBER: 1, 4

STEP 3 VARIABLE GAP ENTERED R SQUARE = 0.84190317
C(P) = 53.30444762

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	0.00000204	0.00000068	37.28	0.0001
ERROR	21	0.00000038	0.00000002		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99971780				
GRD	0.00017788	0.00002755	0.00000076	41.70	0.0001
APP	-0.00021998	0.00002755	0.00000116	63.77	0.0001
GAP	0.00008512	0.00003374	0.00000012	6.37	0.0198

BOUNDS ON CONDITION NUMBER: 1, 9

STEP 4 VARIABLE APAP ENTERED R SQUARE = 0.88568369
C(P) = 35.83558739

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	0.00000214	0.00000054	38.74	0.0001
ERROR	20	0.00000028	0.00000001		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99978033				
GRD	0.00017788	0.00002400	0.00000076	54.92	0.0001
APP	-0.00021998	0.00002400	0.00000116	83.99	0.0001
GAP	0.00008512	0.00002940	0.00000012	8.38	0.0089
APAP	-0.00006514	0.00002354	0.00000011	7.66	0.0119

BOUNDS ON CONDITION NUMBER: 1, 16

STEP 5 VARIABLE GG ENTERED R SQUARE = 0.94769605
C(P) = 10.25916310

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	0.00000229	0.00000046	999999.99	0.0001
ERROR	19	0.00000013	0.00000001		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99987979				
GRD	0.00017788	0.00001666	0.00000076	999999.99	0.0001
APP	-0.00021998	0.00001666	0.00000116	999999.99	0.0001
GAP	0.00008512	0.00002040	0.00000012	999999.99	0.0001
GG	-0.00008081	0.00001703	0.00000015	999999.99	0.0001
APAP	-0.00008793	0.00001703	0.00000018	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 25.86429

STEP 6 VARIABLE GAAP ENTERED R SQUARE = 0.96147514
C(P) = 6.13170903

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	6	0.00000233	0.00000039	999999.99	0.0001
ERROR	18	0.00000009	0.00000001		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99987979				
GRD	0.00017788	0.00001469	0.00000076	999999.99	0.0001
APP	-0.00021998	0.00001469	0.00000116	999999.99	0.0001
GAP	0.00008512	0.00001799	0.00000012	999999.99	0.0001
GAAP	-0.00004564	0.00001799	0.00000003	999999.99	0.0001
GG	-0.00008081	0.00001501	0.00000015	999999.99	0.0001
APAP	-0.00008793	0.00001501	0.00000018	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 37.03714

STEP 7 VARIABLE AAP ENTERED R SQUARE = 0.97341799
C(P) = 2.82081610

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	7	0.00000235	0.00000034	999999.99	0.0001
ERROR	17	0.00000006	0.00000000		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99987979				
GRD	0.00017788	0.00001255	0.00000076	999999.99	0.0001
APP	-0.00021998	0.00001255	0.00000116	999999.99	0.0001
GAP	0.00008512	0.00001538	0.00000012	999999.99	0.0001
AAP	0.00004249	0.00001538	0.00000003	999999.99	0.0001
GAAP	-0.00004564	0.00001538	0.00000003	999999.99	0.0001
GG	-0.00008081	0.00001283	0.00000015	999999.99	0.0001
APAP	-0.00008793	0.00001283	0.00000018	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 50.21

STEP 8 VARIABLE VEL ENTERED R SQUARE = 0.98020425
C(P) = 1.80301605

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	8	0.00000237	0.00000030	999999.99	0.0001
ERROR	16	0.00000005	0.00000000		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99987979				
GRD	0.00017788	0.00001117	0.00000076	999999.99	0.0001
VEL	0.00002615	0.00001117	0.00000002	999999.99	0.0001
APP	-0.00021998	0.00001117	0.00000116	999999.99	0.0001
GAP	0.00008512	0.00001368	0.00000012	999999.99	0.0001
AAP	0.00004249	0.00001368	0.00000003	999999.99	0.0001
GAAP	-0.00004564	0.00001368	0.00000003	999999.99	0.0001
GG	-0.00008081	0.00001141	0.00000015	999999.99	0.0001
APAP	-0.00008793	0.00001141	0.00000018	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 65.38286

STEP 9 VARIABLE GV ENTERED R SQUARE = 0.98621893
C(P) = 1.12833440

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	9	0.00000239	0.00000027	999999.99	0.0001
ERROR	15	0.00000003	0.00000000		
TOTAL	24	0.00000242			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.99987979				
GRD	0.00017788	0.00000962	0.00000076	999999.99	0.0001
VEL	0.00002615	0.00000962	0.00000002	999999.99	0.0001
APP	-0.00021998	0.00000962	0.00000116	999999.99	0.0001
GV	-0.00003016	0.00001179	0.00000001	999999.99	0.0001
GAP	0.00008512	0.00001179	0.00000012	999999.99	0.0001
AAP	0.00004249	0.00001179	0.00000003	999999.99	0.0001
GAAP	-0.00004564	0.00001179	0.00000003	999999.99	0.0001
GG	-0.00008081	0.00000984	0.00000015	999999.99	0.0001
APAP	-0.00008793	0.00000984	0.00000018	999999.99	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 82.55571

NO OTHER VARIABLES MET THE 0.1500 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE PS

STEP	VARIABLE ENTERED	VARIABLE REMOVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)
1	APP		1	0.4801	0.4801	210.205
2	GRD		2	0.3139	0.7940	72.614
3	GAP		3	0.0479	0.8419	53.304
4	APAP		4	0.0438	0.8857	35.836
5	GG		5	0.0620	0.9477	10.259
6	GAAP		6	0.0138	0.9615	6.132
7	AAP		7	0.0119	0.9734	2.821
8	VEL		8	0.0068	0.9802	1.803
9	GV		9	0.0060	0.9862	1.128

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F	PROB>F
1	APP		21.2374	0.0001
2	GRD		33.5213	0.0001
3	GAP		6.3651	0.0198
4	APAP		7.6595	0.0119
5	GG		22.5267	0.0001
6	GAAP		6.4380	0.0206
7	AAP		7.6378	0.0133
8	VEL		5.4850	0.0324
9	GV		6.5467	0.0218

Part 3

Stepwise Regression and Residual Plot

of a Linear Model of E(h)

STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE HIT

NOTE: SLENTRY AND SLSTAY HAVE BEEN SET TO .15 FOR THE STEPWISE TECHNIQUE.

STEP 1 VARIABLE GRD ENTERED R SQUARE = 0.35343105
C(P) = 4.28927231

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	0.00039142	0.00039142	8.20	0.0118
ERROR	15	0.00071606	0.00004774		
TOTAL	16	0.00110748			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01779732				
GRD	-0.00494607	0.00172731	0.00039142	8.20	0.0118

BOUNDS ON CONDITION NUMBER: 1, 1

STEP 2 VARIABLE APP ENTERED R SQUARE = 0.67179576
C(P) = -2.22380924

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	0.00074400	0.00037200	14.33	0.0004
ERROR	14	0.00036348	0.00002596		
TOTAL	16	0.00110748			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01779732				
GRD	-0.00494607	0.00127384	0.00039142	15.08	0.0017
APP	0.00469429	0.00127384	0.00035258	13.58	0.0024

BOUNDS ON CONDITION NUMBER: 1, 4

STEP 3 VARIABLE VEL ENTERED R SQUARE = 0.80454538
C(P) = -3.77353787

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	0.00089102	0.00029701	17.84	0.0001
ERROR	13	0.00021646	0.00001665		
TOTAL	16	0.00110748			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01779732				
GRD	-0.00494607	0.00102014	0.00039142	23.51	0.0003
VEL	-0.00303127	0.00102014	0.00014702	8.83	0.0108
APP	0.00469429	0.00102014	0.00035258	21.17	0.0005

BOUNDS ON CONDITION NUMBER: 1, 9

STEP 4 VARIABLE ALT ENTERED R SQUARE = 0.88918950
C(P) = -4.03692419

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	0.00098476	0.00024619	24.07	0.0001
ERROR	12	0.00012272	0.00001023		
TOTAL	16	0.00110748			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01779732				
GRD	-0.00494607	0.00079948	0.00039142	38.27	0.0001
ALT	-0.00242051	0.00079948	0.00009374	9.17	0.0105
VEL	-0.00303127	0.00079948	0.00014702	14.38	0.0026
APP	0.00469429	0.00079948	0.00035258	34.48	0.0001

BOUNDS ON CONDITION NUMBER: 1, 16

STEP 5 VARIABLE GA ENTERED R SQUARE = 0.92173667
C(P) = -2.90723645

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	0.00102080	0.00020416	25.91	0.0001
ERROR	11	0.00008668	0.00000788		
TOTAL	16	0.00110748			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01779732				
GRD	-0.00494607	0.00070176	0.00039142	49.68	0.0001
ALT	-0.00242051	0.00070176	0.00009374	11.90	0.0054
VEL	-0.00303127	0.00070176	0.00014702	18.66	0.0012
APP	0.00469429	0.00070176	0.00035258	44.75	0.0001
GA	0.00150094	0.00070176	0.00003605	4.57	0.0557

BOUNDS ON CONDITION NUMBER: 1, 25

STEP 6   VARIABLE AAP ENTERED   R SQUARE = 0.94694457
C(P) = -1.58129633

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	6	0.00104872	0.00017479	29.75	0.0001
ERROR	10	0.00005876	0.00000588		
TOTAL	16	0.00110748			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01779732				
GRD	-0.00494607	0.00060600	0.00039142	66.62	0.0001
ALT	-0.00242051	0.00060600	0.00009374	15.95	0.0025
VEL	-0.00303127	0.00060600	0.00014702	25.02	0.0005
APP	0.00469429	0.00060600	0.00035258	60.01	0.0001
GA	0.00150094	0.00060600	0.00003605	6.13	0.0327
AAP	-0.00132092	0.00060600	0.00002792	4.75	0.0543

BOUNDS ON CONDITION NUMBER: 1,   36

NO OTHER VARIABLES MET THE 0.1500 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE HIT

STEP	VARIABLE ENTERED	VARIABLE REMOVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)
1	GRD		1	0.3534	0.3534	4.28927
2	APP		2	0.3184	0.6718	-2.22381
3	VEL		3	0.1327	0.8045	-3.77354
4	ALT		4	0.0846	0.8892	-4.03692
5	GA		5	0.0325	0.9217	-2.90724
6	AAP		6	0.0252	0.9469	-1.58130

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F	PROB>F
1	GRD		8.1994	0.0118
2	APP		13.5803	0.0024
3	VEL		8.8294	0.0108
4	ALT		9.1664	0.0105
5	GA		4.5745	0.0557
6	AAP		4.7512	0.0543

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: HIT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	0.00104872	0.00017479	29.75
ERROR	10	0.00005876	0.00000588	PR > F
CORRECTED TOTAL	16	0.00110748		0.0001

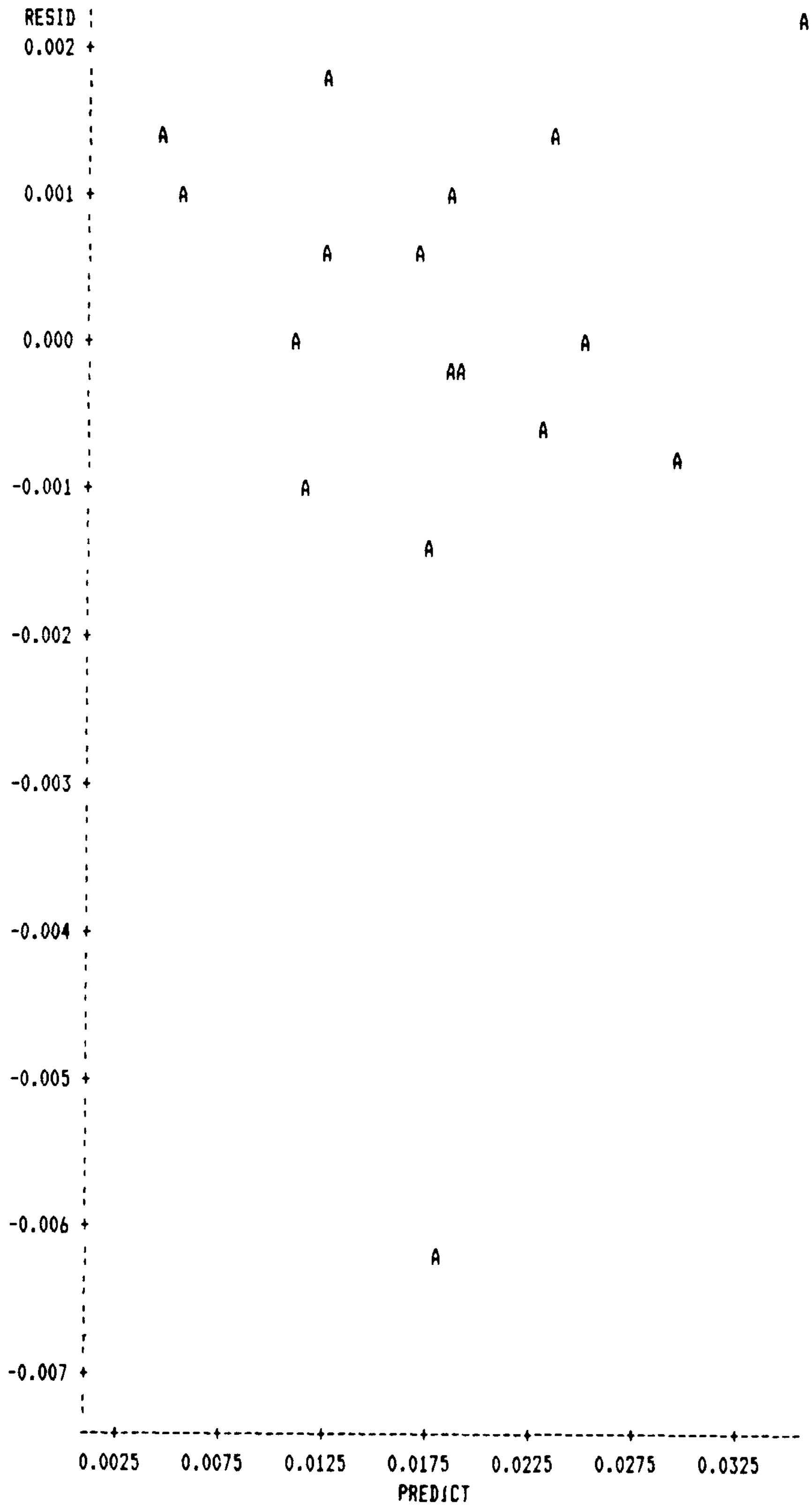
R-SQUARE	C.V.	ROOT MSE	HIT MEAN
0.946945	13.6200	0.00242400	0.01779732

SOURCE	DF	TYPE I SS	F VALUE	PR > F
GRD	1	0.00039142	66.62	0.0001
ALT	1	0.00009374	15.95	0.0025
VEL	1	0.00014702	25.02	0.0005
APP	1	0.00035258	60.01	0.0001
GA	1	0.00003605	6.13	0.0327
AAP	1	0.00002792	4.75	0.0543

SOURCE	DF	TYPE III SS	F VALUE	PR > F
GRD	1	0.00039142	66.62	0.0001
ALT	1	0.00009374	15.95	0.0025
VEL	1	0.00014702	25.02	0.0005
APP	1	0.00035258	60.01	0.0001
GA	1	0.00003605	6.13	0.0327
AAP	1	0.00002792	4.75	0.0543

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	0.01779732	30.27	0.0001	0.00058791
GRD	-0.00494607	-8.16	0.0001	0.00060600
ALT	-0.00242051	-3.99	0.0025	0.00060600
VEL	-0.00303127	-5.00	0.0005	0.00060600
APP	0.00469429	7.75	0.0001	0.00060600
GA	0.00150094	2.48	0.0327	0.00060600
AAP	-0.00132092	-2.18	0.0543	0.00060600

PLOT OF RESID*PREDICT   LEGEND: A = 1 OBS, B = 2 OBS, ETC.



Part 4

Stepwise Regression of a Quadratic Model of E(h)

STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE HIT

NOTE: SLENTRY AND SLSTAY HAVE BEEN SET TO .15 FOR THE STEPWISE TECHNIQUE.

STEP 1 VARIABLE APAP ENTERED R SQUARE = 0.39917676
C(P) = 36.30371119

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	0.00086585	0.00086585	15.28	0.0007
ERROR	23	0.00130324	0.00005666		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01285742				
APAP	0.00588978	0.00150670	0.00086585	15.28	0.0007

BOUNDS ON CONDITION NUMBER: 1, 1

STEP 2 VARIABLE GRD ENTERED R SQUARE = 0.62706985
C(P) = 16.56833401

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	0.00136017	0.00068008	18.50	0.0001
ERROR	22	0.00080892	0.00003677		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01285742				
GRD	-0.00453835	0.00123776	0.00049432	13.44	0.0014
APAP	0.00588978	0.00121372	0.00086585	23.55	0.0001

BOUNDS ON CONDITION NUMBER: 1, 4

STEP 3 VARIABLE APP ENTERED R SQUARE = 0.77330901
C(P) = 4.62072691

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	0.00167737	0.00055912	23.88	0.0001
ERROR	21	0.00049171	0.00002341		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01285742				
GRD	-0.00453835	0.00098773	0.00049432	21.11	0.0001
APP	0.00363550	0.00098773	0.00031721	13.55	0.0014
APAP	0.00588978	0.00096855	0.00086585	36.98	0.0001

BOUNDS ON CONDITION NUMBER: 1, 9

STEP 4 VARIABLE VEL ENTERED R SQUARE = 0.85655707
C(P) = -1.31908387

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	0.00185795	0.00046449	29.86	0.0001
ERROR	20	0.00031114	0.00001556		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01285742				
GRD	-0.00453835	0.00080511	0.00049432	31.77	0.0001
VEL	-0.00274296	0.00080511	0.00018057	11.61	0.0028
APP	0.00363550	0.00080511	0.00031721	20.39	0.0002
APAP	0.00588978	0.00078948	0.00086585	55.66	0.0001

BOUNDS ON CONDITION NUMBER: 1, 16

STEP 5 VARIABLE ALT ENTERED R SQUARE = 0.88602835
C(P) = -2.12991732

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	0.00192187	0.00038437	29.54	0.0001
ERROR	19	0.00024721	0.00001301		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01285742				
GRD	-0.00453835	0.00073630	0.00049432	37.99	0.0001
ALT	-0.00163205	0.00073630	0.00006393	4.91	0.0391
VEL	-0.00274296	0.00073630	0.00018057	13.88	0.0014
APP	0.00363550	0.00073630	0.00031721	24.38	0.0001
APAP	0.00588978	0.00072200	0.00086585	66.55	0.0001

BOUNDS ON CONDITION NUMBER: 1, 25

STEP 6 VARIABLE GG ENTERED R SQUARE = 0.90877812
C(P) = -2.29968373

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	6	0.00197122	0.00032854	29.89	0.0001
ERROR	18	0.00019787	0.00001099		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01105365				
GRD	-0.00453835	0.00067678	0.00049432	44.97	0.0001
ALT	-0.00163205	0.00067678	0.00006393	5.82	0.0268
VEL	-0.00274296	0.00067678	0.00018057	16.43	0.0007
APP	0.00363550	0.00067678	0.00031721	28.86	0.0001
GG	0.00146557	0.00069172	0.00004935	4.49	0.0483
APAP	0.00630314	0.00069172	0.00091276	83.03	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 37.03714

STEP 7 VARIABLE GA ENTERED R SQUARE = 0.92539586
C(P) = -1.88460606

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	7	0.00200726	0.00028675	30.12	0.0001
ERROR	17	0.00016182	0.00000952		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01105365				
GRD	-0.00453835	0.00062978	0.00049432	51.93	0.0001
ALT	-0.00163205	0.00062978	0.00006393	6.72	0.0190
VEL	-0.00274296	0.00062978	0.00018057	18.97	0.0004
APP	0.00363550	0.00062978	0.00031721	33.32	0.0001
GA	0.00150094	0.00077132	0.00003605	3.79	0.0684
GG	0.00146557	0.00064369	0.00004935	5.18	0.0360
APAP	0.00630314	0.00064369	0.00091276	95.89	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 50.21

STEP 8 VARIABLE AAP ENTERED R SQUARE = 0.93826636
C(P) = -1.11213394

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	8	0.00203518	0.00025440	30.40	0.0001
ERROR	16	0.00013391	0.00000837		
TOTAL	24	0.00216909			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.01105365				
GRD	-0.00453835	0.00059052	0.00049432	59.06	0.0001
ALT	-0.00163205	0.00059052	0.00006393	7.64	0.0138
VEL	-0.00274296	0.00059052	0.00018057	21.58	0.0003
APP	0.00363550	0.00059052	0.00031721	37.90	0.0001
GA	0.00150094	0.00072323	0.00003605	4.31	0.0544
AAP	-0.00132092	0.00072323	0.00002792	3.34	0.0865
GG	0.00146557	0.00060356	0.00004935	5.90	0.0273
APAP	0.00630314	0.00060356	0.00091276	109.06	0.0001

BOUNDS ON CONDITION NUMBER: 1.086429, 65.38286

NO OTHER VARIABLES MET THE 0.1500 SIGNIFICANCE LEVEL FOR ENTRY INTO THE MODEL.

SUMMARY OF STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE HIT

STEP	VARIABLE ENTERED	VARIABLE REMOVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)
1	APAP		1	0.3992	0.3992	36.3037
2	GRD		2	0.2279	0.6271	16.5683
3	APP		3	0.1462	0.7733	4.6207
4	VEL		4	0.0832	0.8566	-1.3191
5	ALT		5	0.0295	0.8860	-2.1299
6	GG		6	0.0227	0.9088	-2.2997
7	GA		7	0.0166	0.9254	-1.8846
8	AAP		8	0.0129	0.9383	-1.1121

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F	PROB>F
1	APAP		15.2808	0.0007
2	GRD		13.4439	0.0014
3	APP		13.5472	0.0014
4	VEL		11.6071	0.0028
5	ALT		4.9131	0.0391
6	GG		4.4890	0.0483
7	GA		3.7867	0.0684
8	AAP		3.3357	0.0865

Appendix G: Model Code

```
PREAMBLE
NORMALLY MODE IS UNDEFINED

''            ENTITIES AND VARIABLES DEALING WITH GRAPHICS
''           **********************************************

TEMPORARY ENTITIES INCLUDE PLOT, PLOT.BACK
GRAPHIC ENTITIES INCLUDE PLOT, PLOT.BACK

DEFINE COUNT AS A 1-DIM INTEGER ARRAY
DEFINE POINTS AS A 2-DIM REAL ARRAY
DEFINE HOLD AS A 3-DIM REAL ARRAY
DEFINE XMIN, XMAX, ZMAX AS REAL VARIABLES

DEFINE GRPH.TRAJ, FIELD.ID, DATIN AS TEXT VARIABLES
DEFINE FIN, PSE, PSA, EHE, EHA AS DOUBLE VARIABLES
DEFINE TRAJCAL, SURVCAL, GO.PROMPT, STOP.PROMPT, RESULTS,
       ENTER, INNAM, FILNAMS, VARYS, TGRPH, CALC, COMPLETE
       AS POINTER VARIABLES
DEFINE PSELE, PSAZI, EHELE, EHAZI AS 1-DIM DOUBLE ARRAYS

DISPLAY VARIABLES INCLUDE FIN, PSE, PSA, EHE, EHA


''                 VARIABLES FOR CALCULATING RESULTS
''                ***********************************


DEFINE OUT.FILE, IN.FILE, ARM.FILE, AIR.FILE AS TEXT VARIABLES

DEFINE NRUNS AS AN INTEGER VARIABLE
DEFINE MASS, AREA, VEL.I, STD.ELE, STD.AZI, ROF, GRD, ALT, OFR,
       VEL.AC, APP, APPROACH, T, RO, R.SPHERE AS REAL VARIABLES

DEFINE EXP.HITS, PROB.SURV AS DOUBLE VARIABLES

DEFINE COORDS AS A 4-DIM REAL ARRAY
DEFINE VELS, SLOPES AS 2-DIM REAL ARRAYS
DEFINE CD, PROB, PROB.KILL_H AS 1-DIM REAL ARRAYS

DEFINE DONE AS A 1-DIM INTEGER ARRAY
DEFINE AC_Z, Z_1, Z_2, X_1, X_2, Z.V_1, Z.V_2, X.V_1, X.V_2
       AS 1-DIM REAL ARRAYS

END
```

```
MAIN

DEFINE RUN AS AN INTEGER VARIABLE

RESERVE COORDS(*,*,*,*) AS 2 BY 2 BY 2 BY 31
RESERVE VELS(*,*), SLOPES(*,*) AS 2 BY 31
RESERVE CD(*), PROB.KILL_H(*) AS 101
RESERVE PROB(*) AS 16

RESERVE DONE(*), AC_Z(*), Z_1(*), Z_2(*), X_1(*), X_2(*),
        Z.V_1(*), Z.V_2(*), X.V_1(*), X.V_2(*) AS 2

RESERVE POINTS(*,*) AS 2 BY 127
RESERVE HOLD(*,*,*) AS 31 BY 2 BY 127
RESERVE COUNT(*) AS 31

RESERVE PSELE(*) AS 31
RESERVE PSAZI(*) AS 31
RESERVE EHELE(*) AS 31
RESERVE EHAZI(*) AS 31

CALL INIT.GRAPHICS          '' initialize graphic screens

CALL INTERACT               '' interactive inputs with graphic forms
CALL SET.DATA               '' read data from small arms and aircraft
                            '' files

FOR RUN = 1 TO NRUNS, DO

''  database run inputs
'' ************************************
   IF DATIN EQ "BASE"
      READ GRD, ALT, VEL.AC, APP
   ALWAYS
''*************************************


   CALL INIT.RUN                 '' initialize variables for each run
   CALL TRAJECT                  '' calcs trajectories and plane
                                 '' intersects

   IF GRPH.TRAJ EQ "YES"  CALL PLOT.TRAJ  ALWAYS

   CALL SURVIVAL.MAIN            '' main routine for P(s) and E(h)
   CALL PRINT.OUT GIVEN RUN      '' database output of results

LOOP

''  graphic output of results
'' ***************************************************
LET FIN = 0.
ERASE FIN
IF GRPH.TRAJ EQ "NO"  ERASE CALC  ALWAYS
```

```
IF DATIN EQ "BASE"
   CLOSE UNIT 10
OTHERWISE
   CALL RESULTS.OUT
ALWAYS

LET DTVAL.A(DFIELD.F("OUTFILE",COMPLETE)) = OUT.FILE
DISPLAY COMPLETE
LET FIELD.ID = ACCEPT.F(STOP.PROMPT,0)
ERASE COMPLETE
'' ****************************************************

CLOSE UNIT 3    ''  close output file

END
```

```
ROUTINE EXPECT.HITS GIVEN X1, Z1, X2, Z2, ALT.T, DIAM, DIRECT,
                          ELE.ID, AZI.ID
                 YIELDING HITS

DEFINE X1, Z1, X2, Z2, ALT.T, DIAM, T1, T2, I1, I2, INTERCEPT,
       DIST.BUL, DIST.AIR, X_SIGN AS REAL VARIABLES
DEFINE DIRECT, ELE.ID, AZI.ID AS INTEGER VARIABLES
DEFINE HITS AS A DOUBLE VARIABLE

'' angle of intercept
LET INTERCEPT = ARCTAN.F( (Z2-Z1), (X2-X1) )

'' z coords of points tangent to circle cut from aircraft sphere
LET T1 = ALT.T - DIAM * COS.F(INTERCEPT) /2
LET T2 = ALT.T + DIAM * COS.F(INTERCEPT) /2

'' select z coords with closest distance to each other
LET I1 = MAX.F( MIN.F(T1, T2), MIN.F(Z1, Z2) )
LET I2 = MIN.F( MAX.F(T1, T2), MAX.F(Z1, Z2) )

'' sign of the aircraft velocity vector
'' in relation to the bullet velocity
LET X_SIGN = (X2-X1)/ABS.F(X2-X1)

'' distance bullet stream travels during intersection
LET DIST.BUL = (I2 - I1) / SIN.F(INTERCEPT)

'' check for segment intersection
IF ABS.F(I1-ALT.T) LT DIAM/2  AND ABS.F(I2-ALT.T) LT DIAM/2

'' distance aircraft travels during intersection
   LET DIST.AIR = SQRT.F( (DIAM/2)**2 - (I1-ALT.T)**2 )
                + SQRT.F( (DIAM/2)**2 - (I2-ALT.T)**2 )
                + DIST.BUL * ABS.F( COS.F(INTERCEPT) )

'' number of hits for given trajectory segment times the
'' probability of a bullet being on the segement and the
'' rate of fire
   LET HITS = ABS.F(DIST.BUL/VELS(DIRECT,ELE.ID)
                 - X_SIGN*DIST.AIR/VEL.AC)
                 * ROF * PROB(ABS.F(16-ELE.ID)+1)
                 * PROB(ABS.F(16-AZI.ID)+1)

'' total hits
   LET EXP.HITS = EXP.HITS + HITS

'' hits by azimuth and elevation for graphic results only
   LET EHELE(ELE.ID) = EHELE(ELE.ID) + HITS
   LET EHAZI(AZI.ID) = EHAZI(AZI.ID) + HITS

ALWAYS

END
```

```
ROUTINE FILES_IN.CTRL GIVEN FIELD.ID, FORM.PTR  YIELDING STATUS

DEFINE FIELD.ID AS A TEXT VARIABLE
DEFINE FORM.PTR AS A POINTER VARIABLE
DEFINE I, STATUS AS AN INTEGER VARIABLE

'' status set to zero mean variables can be put in over
'' and over before accepting screen
LET STATUS=0

'' checking file names to see if they exist
SELECT CASE FIELD.ID

'' output database
CASE "OUT_FILE"

   LET OUT.FILE = DTVAL.A(DFIELD.F("OUT_FILE",FORM.PTR))
   OPEN UNIT 3 FOR OUTPUT, FILE NAME IS OUT.FILE, NOERROR
   USE UNIT 3 FOR OUTPUT
   IF ROPENERR.V NE 0

'' makes computer beep 5 times
      FOR I = 1 TO 5, DO
         WRITE 7 AS A 1, + USING 6
      LOOP

'' replace bad name with error message
      LET DTVAL.A(DFIELD.F("OUT_FILE",FORM.PTR))=
      "    ERROR -- NO SUCH FILE !!!!!"
   ALWAYS

'' echo back good name or error message
   DISPLAY DFIELD.F("OUT_FILE",FORM.PTR)
   CLOSE UNIT 3

'' small arms database
CASE "ARM_FILE"

   LET ARM.FILE = DTVAL.A(DFIELD.F("ARM_FILE",FORM.PTR))
   OPEN UNIT 2 FOR INPUT, FILE NAME IS ARM.FILE, NOERROR
   USE UNIT 2 FOR INPUT
   IF ROPENERR.V NE 0
      FOR I = 1 TO 5, DO
         WRITE 7 AS A 1, + USING 6
      LOOP
      LET DTVAL.A(DFIELD.F("ARM_FILE",FORM.PTR))=
      "    ERROR -- NO SUCH FILE !!!!!"
   ALWAYS
   DISPLAY DFIELD.F("ARM_FILE",FORM.PTR)
   CLOSE UNIT 2

'' aircraft database
CASE "AC_FILE"
```

```
   LET AIR.FILE = DTVAL.A(DFIELD.F("AC_FILE",FORM.PTR))
   OPEN UNIT 4 FOR INPUT, FILE NAME IS AIR.FILE, NOERROR
   USE UNIT 4 FOR INPUT
   IF ROPENERR.V NE 0
      FOR I = 1 TO 5, DO
         WRITE 7 AS A 1, + USING 6
      LOOP
      LET DTVAL.A(DFIELD.F("AC_FILE",FORM.PTR))=
      "     ERROR -- NO SUCH FILE !!!!!"
   ALWAYS
   DISPLAY DFIELD.F("AC_FILE",FORM.PTR)
   CLOSE UNIT 4

CASE "GO"
CASE "STOP"

DEFAULT
ENDSELECT

END
```