Final Report
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Stochastic Adaptive Control
and
Estimation Enhancement
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**New results were obtained in the following areas:** (a) stochastic dual control applied to a guidance problem where it enhances target discrimination; (b) stochastic adaptive control based on sensitivity functions for enhanced real-time system parameter identification; (c) state estimation in hybrid systems characterized by Markov and semi-Markov jumps with applications to target tracking and failure detection; (d) piecewise diffusion Markov processes.
The main results obtained and published during the period covered by this report, June 1984 - May 1988, are described below together with references given to the corresponding publication.

1. Dual Control Guidance for Simultaneous Identification and Interception


An adaptive dual-control guidance algorithm was developed for intercepting a moving target in the presence of an interfering target (decoy) in a stochastic environment. Two sequences of measurements are obtained at discrete points in time; however, it is not certain which sequence came from the target of interest and which from the decoy. Associated with each track, the interceptor also receives noisy, state-dependent feature measurements. The optimum control for the interceptor, which is given by the solution of the stochastic dynamic programming equation, is not numerically feasible to obtain. An approximate solution of this equation is obtained by evaluating the value of the future information gathering. This is done through the use of preposterior analysis: approximate prior probability densities are obtained and used to describe the future learning and control. In this way, the interceptor control is used for information gathering in order to reduce the future target and decoy inertial measurement errors and enhance the observable target/decoy feature differences for subsequent discrimination between the true target and the decoy. Simulation studies have shown the effectiveness of the scheme.
2. Dual Adaptive Control Based Upon Sensitivity Functions
(by J.A. Molusis, P. Mookerjee, Y. Bar-Shalom, in Proc. 23rd IEEE Conference on Decision & Control, Dec. 1984)

A new adaptive dual control solution was developed for the control of a class of multi-variable input-output systems. Both rapidly varying random parameters and constant but unknown parameters are included. The new controller modifies the cautious control design by numerator and denominator correction terms. This controller is shown to depend upon sensitivity functions of the expected future cost. A scalar example is presented to provide insight into the properties of the new dual controller. Monte-Carlo simulations are performed which show improvement over the cautious controller and the previous Linear Feedback Dual Controller.

3. Dual Control and Prevention of the Turn-Off Phenomenon in a Class of MIMO Systems
(by P. Mookerjee, Y. Bar-Shalom, and J.A. Molusis, in Proc. 24th IEEE Conference on Decision & Control, Dec. 1985)

The recently developed methodology of adaptive dual control based upon sensitivity functions was applied here to a multi-variable input-output model. The plant has constant but unknown parameters. It represents a simplified linear version of the relationship between the vibration output and the higher harmonic control input for a helicopter. The cautious and the new dual controller are examined. In many instances, the cautious controller is seen to turn off. The new dual controller modifies the cautious control design by numerator and denominator correction terms which depend upon the sensitivity functions of the expected future cost and
avoids the turn-off and burst phenomena. Monte-Carlo simulations and statistical tests of significance indicate the superiority of the dual controller over the cautious and the heuristic certainty equivalence controllers.

4. An Adaptive Dual Controller for a MIMO-ARMA System

An adaptive dual controller was developed for a multi-input multi-output dynamic system. The plant has constant but unknown parameters. The cautious and a new dual controller are examined. In many instances, the cautious controller is seen to turn off. The dual controller modifies the cautious control design by numerator and denominator correction terms which depend upon the sensitivity functions of the expected future cost and avoids the turn-off and slow convergence. Monte-Carlo comparisons based on parametric and nonparametric statistical analysis indicate the superiority of the dual controller over the cautious controller.

5. Sojourn Time Distribution in a Class of Semi-Markov Chains

The estimation and control of a plant with stochastic parameters subject to random input disturbances and measurement errors can be handled by adopting the usual techniques of state/parameter estimation and choosing an appropriate control policy, when the parameters are constant or slowly varying with time. Not all
practical processes can, however, be solved as above. There are numerous examples of systems which have discrete models that randomly vary with time and experience switchings between different models after a random sojourn time. In some situations the switching probabilities depend on the sojourn time. Such instances are commonly experienced in target tracking, manufacturing systems, power industry and also in the socio-economic environment. In such a system, knowledge of the sojourn time (in the form of a conditional distribution) is needed to infer the transition probabilities. In this paper we derive the conditional sojourn time distribution for systems with imperfect observations and changing structures of models.

6. State Estimation for Systems with Sojourn-Time-Dependent Markov Model-Switching


There are numerous examples of systems which have discrete models that randomly vary with time and experience switchings between different models after a random sojourn time. In some situations the switching probabilities depend on the sojourn time. Such a switching process, discussed in this paper, is a class of semi-Markov processes and is encountered in target tracking, systems subject to failures, and also in the socio-economic environment. In such a system, the transition probabilities depend on the sojourn time. It is shown in this paper how one can infer the transition probabilities via the evaluation of the conditional distribution of the sojourn time.
Subsequently, using the conditional sojourn time distribution for dynamic systems with imperfect observations and changing structures or models, a recursive state estimation for such systems is presented.

7. The Interacting Multiple Model Algorithm for Systems with Markovian Coefficients

A major problem in filtering for linear systems with Markovian switching coefficients is the problem of hypotheses management. For this dynamic multiple model situation the paper presents a novel approach to hypotheses merging. The novelty lies in an alternative timing of hypotheses merging. Applied to the problem of filtering in a linear system with Markovian coefficients it yields an elegant way to derive the IMM (Interacting Multiple Model) algorithm. After its derivation follows a further evaluation of the IMM algorithm and a comparison with other algorithms. This makes clear that the IMM performs very well at a relatively low computational load. These results imply a significant change to the picture of suboptimal Bayesian filtering for systems with Markovian coefficients.

8. From Piecewise Deterministic to Piecewise Diffusion Markov Processes
(by H.A.P. Blom, in Proc. IEEE Conf. on Decision and Control, Dec. 1988)

Piecewise Deterministic (PD) Markov Processes form a remarkable class of hybrid state processes because, in contrast to most other hybrid state processes, they include a jump reflecting boundary and
exclude diffusion. As such, they cover a wide variety of impulsively or singularly controlled non-diffusion processes. Because PD processes are defined in a pathwise way, they provide a framework to study the control of non-diffusion processes along the same lines as that of diffusions. An important generalization is to include diffusion in PD processes, but, as pointed out by Davis, combining diffusion with a jump reflecting boundary seems not possible within the present definition of PD processes. This paper presents PD processes as pathwise unique solutions of an IT stochastic differential equation (SDE), driven by a Poisson random measure. Since such an SDE permits the inclusion of diffusion, this approach leads to a large variety of piecewise diffusion Markov processes, represented by pathwise unique SDE solutions.


Systems with parameters that change in value due to component failures can be represented by a discrete set of models. For systems where the model switching process is Semi-Markov, a Semi-Markov based estimation scheme is required to obtain the optimal state estimate. The "sojourn-time-dependent Markov" (STDM) model switching is of the Semi-Markov type. Our recently developed STDM-based multiple-model recursive estimator computes conditional model probabilities which can be used to determine when a failure model is in effect. We show through an example that this estimator is better able to determine which model is in effect than the Markov-based multiple-model
algorithm and thus the STDM-based algorithm is a more reliable failure detection scheme. Our example also shows that the STDM-based algorithm gives a better state estimate, in terms of the rms error, than a Markov-based scheme.

10. Time-Reversion of a Hybrid State Stochastic Difference System with Jump-Linear Smoothing Application

The reversion in time of a stochastic difference equation in a hybrid space, with a Markovian solution, is presented. The reversion is obtained by a martingale approach, which previously led to reverse time forms for stochastic equations with Gauss-Markov or diffusion solutions. The reverse time equations follow from a particular non-canonical martingale decomposition, while the reverse time equations for Gauss-Markov and diffusion solutions followed from the canonical martingale decomposition. The need for this non-canonical decomposition stems from the hybrid state space situation. Moreover, the non-Gaussian discrete time situation leads to reverse time equations that incorporate a Bayesian estimation step. The latter step is carried out for linear systems with Markovian switching coefficients, and the result is shown to provide the solution to the problem of fixed-interval smoothing. For an application of this smoothing approach to a trajectory with sudden maneuvers, simulation results are given to illustrate the practical use of the reverse time equations obtained.