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An experimental and analytical study to test the effectiveness of a time-dependent inverse radiative transfer algorithm for estimating the single scattering properties of optically-thick media was investigated. The inversion algorithm requires measurements of the time resolved backscattered radiance following a pulsed illumination of an optically-thick atmosphere.

The project consisted of three components: a) further analytical development of the algorithm to numerically test the sensitivity of the estimated scattering properties to errors caused by its application in circumstances where it is less...
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The experiments were performed under companion grant DAAG29-84-K-0038, Determination of Optical Properties by Inversion of Multiple Scattering Data: Experimental Program, directed by R.A. Elliott of the Oregon Graduate Center. To model the atmosphere, most of the measurements were accomplished with 532 nm, 35 ps duration pulses from a frequency doubled, passively mode locked Nd:YAG laser and a 10 ps resolution streak camera, additional measurements were made with a synchroscan streak camera and 5 ps duration pulses from a dye laser synchronously pumped with an acousto-optically mode locked Argon ion laser. The target media were aqueous suspensions of nonabsorbing polystyrene latex spheres of diameters 91 and 482 nm, and weakly absorbing polymer Dynospheres of 1900 nm diameter.

Estimates of the single scatter albedo of weakly absorbing particles were obtained with a relative error of less than 1%. Estimates of the scattering asymmetry factor were found to be unreliable when particle diameters were much less than the wavelength of the scattered light, but relative errors of only a few percent were achieved when particle diameters were comparable to or larger than the wavelength.
DETERMINATION OF OPTICAL PROPERTIES BY INVERSION OF MULTIPLE SCATTERING DATA: ANALYTICAL PROGRAM

Final Report Prepared by

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Abstract

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The project consisted of three components: a) further analytical development of the algorithm to numerically test the sensitivity of the estimated scattering properties to errors caused by its application in circumstances where it is less accurate, or to inaccuracies in the experimental measurements, b) radiative transfer calculations to simulate the experiments, and c) application of the algorithm to results from experiments to test its viability in practice.

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DETERMINATION OF OPTICAL PROPERTIES BY INVERSION OF MULTIPLE SCATTERING DATA: ANALYTICAL PROGRAM

I. Introduction

The operation of optical systems in the low visibility atmosphere of a modern battlefield is a cause for concern. Laser ranging, tracking, target designation, and communication systems are all severely impacted by the aerosols and dust endemic to the battlefield environment. Laser pulse shapes can be grossly distorted and beam diameters and pulse durations increased by orders of magnitude by multiple scattering. Since smokes and artificial fogs can therefore be simple countermeasures to such laser systems, it is important to determine the optical properties of the battlefield atmosphere by remote measurement in order to possibly negate these countermeasures.

There has been considerable research on developing remote sensing methods, but most of this earlier work relies on the target being optically thin and neglects multiple scattering effects. The goal of this project was to further the development of an analytical inversion algorithm proposed by the principal investigator that was especially developed to provide estimates of the optical scattering properties from measurements of the multiply-scattered radiance emerging from an optically-thick atmosphere such as battlefield smoke.

The project consisted of three components: a) further analytical development of the algorithm to numerically test the sensitivity of the estimated scattering properties to errors caused by its application in circumstances where it is not strictly valid, or to inaccuracies in the experimental measurements, b) radiative transfer calculations to simulate the
experiments, and c) application of the algorithm to results from experiments to test its viability in practice.

II. Analytical Development of the Algorithm

The purpose of the inverse radiative transfer algorithm is to estimate the Legendre moments

\[ f_m = (2\pi)^{-1} f_0 \int_{4\pi} d\Omega P_m(\Omega' \cdot \Omega) p(\Omega' \cdot \Omega) \] (1)

of the function \( p(\Omega' \cdot \Omega) \), the scattering phase function, that depends upon the angle between incident and outgoing directions \( \Omega' \) and \( \Omega \). The coefficients \( f_0 \), the albedo of single scattering, and \( f_1/f_0 = g \), the mean cosine of the scattering angle (i.e., the scattering asymmetry factor), are the parameters that most directly influence the propagation of radiation in a medium, but the higher-order coefficients also have some influence.

The time-dependent inverse radiative transfer algorithm as originally proposed\(^3\) was based on the use of Fourier azimuthal moments \( B^m(\mu, t) \) of the radiance backscattered from a plane geometry semi-infinite medium. These moments are calculated from the surface backscattered radiance \( B(\mu, \phi, t) \) using

\[ B^m(\mu, t) = \pi(1 + \delta_m) \int_0^{2\pi} d\phi B(\mu, \phi, t) \cos m\phi, \] (2)

where \( \mu \) is the cosine of the polar angle measured from the outward surface normal and \( \phi \) is the azimuthal angle in the plane of the surface, as measured from the direction opposite to the incident illumination, and \( \delta_m \) is unity if \( m = n \) and zero otherwise. Long after the incident pulse centered about time \( t = 0 \), the Fourier backscatter moments asymptotically decay according to\(^1\)

\[ B^m(\mu, t) \cong C^m(\mu) t^{-\frac{3}{2}} \exp[-v(1 - f_m)t], \] (3)

where \( v \) is the inverse of the mean time between photon collisions with the scattering
centers. The objective of the inversion algorithm is to estimate $f_m$ from the values of $B^m(\mu, t)$.

To implement the algorithm, time resolved measurements of the backscattered radiance are required at a fixed polar angle $\theta = \cos^{-1} \mu$ and azimuthal angles $\phi_j$, $j = 1$ to $J$, following illumination of the surface of the scattering medium with a pulsed, monodirectional beam at a polar angle $\theta_0$.

The coefficients initially were estimated by differentiation of $B^m(\mu, t)$ in the asymptotic regime\(^3\) until later during the grant a least squares curve fitting procedure was developed for experimental data.\(^6\) With curve fitting the coefficients can be estimated by using the algorithm

$$v(1 - f_m) = \frac{< t > < \ln[t^{3/2}B^m(\mu, t)] > - < t \ln[t^{3/2}B^m(\mu, t)] >}{< t^2 > - < t >^2}, \quad (4)$$

where $< f(t) >$ is the average of $N$ values of $f(t)$ for $N$ time points $t_i$, $i = 1$ to $N$, if a linear logarithmic curve fitting is used. It was found preferable, however, to use an exact exponential curve fitting for which Eq. (4) has been shown to be a good approximation.\(^6\)

The above inversion algorithm is based on the assumptions of monochromatic radiation and requires that the backscattered time-dependent radiance be measured a few collision times after an incident short pulse. It is based on the assumption that the target thickness is essentially infinite, the pulse width is infinitesimally short, and the detector is perfectly timed and perfectly oriented with respect to the incident pulse and the target. It was shown during the grant that there is an inherent inaccuracy in the estimate of any $f_m$ of $O(t^{-1})$ for long times.\(^5\)

When originally proposed,\(^3\) the inversion algorithm had not been tested with simulated data obtained by radiative transfer calculations. The first numerical tests during the
grant showed that the albedo of single scattering and the scattering asymmetry factor, and perhaps the scattering parameter $f_2$, might be obtainable by use of the inversion algorithm at only a few azimuthal angles. The higher-order coefficients, $f_m$, $m > 2$, however, were predicted to be poorly estimated because they are even more sensitive to simulated experimental measurement errors than $f_0$, $f_1$, and $f_2$. This is because the corresponding azimuthal moment of the radiance, $B^m(\mu,t)$, is smaller and the onset of its asymptotic decay occurs at increasingly shorter times as $m$ increases.

The effects of the underlying assumptions in inversion algorithm (4) on the estimated values of $f_m$ also were investigated during the grant. It was shown that the errors in the coefficients arising from assuming that the target thickness was infinite (instead of just optically thick) were of $O(t^{-1})$, the same order as that inherent in the algorithm itself. Also, the errors in estimates of $f_m$ arising from pulse shapes that are short (instead of infinitesimal) duration are of $O(t^{-2})$, while those made by assuming that a perfectly oriented detector is perfectly timed are also of $O(t^{-2})$. Finally, it was shown that if the detector is not carefully oriented then the inversion algorithm will give a poor estimate of all coefficients except the albedo of single scattering.

A more detailed numerical study was then undertaken of the effects on the coefficients of a broadened incident pulse in time and of simulated systematic experimental errors. It was shown that excessive broadening of the initial pulse decreases the estimate of each $f_m$, with the magnitude of the relative error, $|\delta f_m/f_m|$, increasing with greater broadening and smaller values of $f_m$. Since the $f_m$ coefficients generally decrease with $m$, the related error in $f_m$ generally increases with increasing $m$. The study also showed that a directional misalignment during the measurements can cause an increase in the estimated coefficient, while applying the algorithm in a pre-asymptotic time regime can cause a decrease in the
estimated coefficients.

An additional analytical investigation was undertaken during the grant to learn more about estimating the expansion coefficients from the radiance deep within a weakly absorbing medium. While this configuration was not being simulated in the companion experimental program at the Oregon Graduate Center, it was felt that valuable information could be learned about the use of inverse algorithms in situations where the detector is not external to the target. It was shown that it is impractical to attempt to independently estimate $f_0$ and $f_1$, although the similarity parameter $[(1 - f_0)/(1 - f_1)]^{1/2}$ can be estimated.

III. Calculations to Simulate the Experiments

A direct radiative transfer program, AZDEP, was developed during the grant and then first used to predict the performance of the inverse algorithm. The program is a generalization to azimuthal-dependence of a multiple collision expansion method for a semi-infinite medium that was developed earlier. The program was later used to calculate the expected backscattered radiance for the experiments so that comparisons of theory and experiment could be made.

A second calculational task completed during the grant was to develop a program that would calculate the expansion coefficients $f_m$ for the suspensions of spherical particles that were used in the experiments. This was done by first making operational the Mie single scattering program MIEVO, the program was then extended to perform numerical integrations that enable calculation of the Legendre moments $f_m$. The final program, MIELEG, also was sent to the U.S. Army Chemical Research and Development Center in response to a request from Dr. David K. Cohoon.
IV. Application of the Algorithm to Experimental Data

A major portion of the program was the data analysis of the experimental results obtained under companion grant DAAG29-84-K-0038, *Determination of Optical Properties by Inversion of Multiple Scattering Data: Experimental Program*, directed by R.A. Elliott of the Oregon Graduate Center. To model the atmosphere, most of the measurements were accomplished with 532 nm, 35 ps duration pulses from a frequency doubled, passively mode locked Nd:YAG laser and a 10 ps resolution streak camera.

The target media were aqueous suspensions of nonabsorbing polystyrene latex spheres of diameters 91 and 482 nm, and weakly absorbing polymer *Dynospheres* of 1900 nm diameter. Typically 20 individual measurements of the backscattered radiance were recorded for each angular position. Measurements were made at three different polar angles, 15°, 21°, and 28°, and for at least 7 azimuthal angles; as many as 24 azimuthal angles were used in some cases.

The results for the 35 ps duration pulsed Nd:YAG laser system showed that it was possible, even in the presence of experimental noise, to estimate the single-scatter albedo with a relative error of < 1%. Estimates of the asymmetry factor were found to be less accurate (< 12%) and those for the $f_m$ for $m \geq 2$ to be impractical.

During the last 6 months of the experimental program some of the experiments were repeated with a synchroscan streak camera and 5 ps duration pulses produced at a repetition rate of 76 MHz from a dye laser synchronously pumped with an acousto-optically modelocked Argon ion laser. The high pulse repetition frequency and synchroscan streak camera made a great improvement in the signal-to-noise ratio of the data. Indeed, the estimates of the single scattering albedo and asymmetry factor were found to be improved because they were estimated with an error of < 1% when estimates could be made, but it
still was impractical to estimate the $f_m$ for $m \geq 2$.

References


5. Paper #7 of the List of Publications.


List of Publications


on p. 83 of the Technical Digest).


List of Participating Scientific Personnel

Norman J. McCormick, Professor and Principal Investigator (3.75 months support)

Tomasz Duracz, Research Assistant Professor (20 months support)

Hak Yi, graduate student (7 months support)

Richard Sanchez, senior investigator (15 days support)