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SBIR Phase I Final Report

Digital Spectroscopy of Piezoelectric Crystalline Media

DAAL01-86-C-0031

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SUMMARY

This Final Report for the Department of the Army under Contract No., DAAL01-86-C-0031 describes the work undertaken for a DOD SBIR Phase I award, "Digital Spectroscopy of Piezoelectric Crystalline Media". The report describes the development and application of a new method for measuring the anisotropic permittivities of piezoelectric materials which we call Digital Refractometry (or Digital Spectroscopy). We show that Fresnel's equations for anisotropic media reduce to very simple forms for an incident angle of 45° and we transform these equations to the Stokes parameters and the Mueller matrix. Fresnel's equations for anisotropic media have been cast in a very useful form for experimental measurement by Wooten. And, one form of Wooten's equations allows the ordinary and extraordinary refractive indices to be uniquely and separately determined. The experimental configuration for measuring the ordinary and extraordinary refractive indices is then described. In our experiments a dual beam approach was used to overcome the laser intensity fluctuations. With the equipment at hand the ordinary and extraordinary refractive indices were measured using a HeNe laser and at room temperature the measured values were found to be 1.54290 ± 0.00008 and 1.55213 ± 0.00008 , respectively, in very good agreement with published data. We also briefly describe two other variants of Digital Refractometry which we found during the course of this six month program.

1. Introduction.

During the past twenty years there has been an increasing need for the development of rapid and automated methods for measuring the refractive index and extinction coefficient of optical materials. Among these materials are the class of piezoelectric materials. In an attempt to satisfy this requirement of increased speed, accuracy and automation Measurement Concepts, Inc. has developed a new technique which we call Digital Refractometry. Not only can this technique be used to measure the refractive indices of piezoelectric crystals such as quartz (uniaxial) but it is equally applicable to the measurement of the refractive index of glass, metals and semiconductors.

In July, 1986 Measurement Concepts, Inc. was awarded a Phase I DoD SBIR contract (DAAL01-86-C-0031, "Digital Spectroscopy of Piezoelectric Crystalline Media"). In this Final Report we discuss the theory and application of Dual-Beam Digital Refractometry as it relates to the measurement of permittivities (or refractive indices) piezoelectric crystals. In our experiments we measured, specifically, the refractive indices (ordinary and extraordinary) of quartz at HeNe wavelength (0.6328 microns) and at room temperature.

Piezoelectric investigations usually involve the determination of the elastic and dielectric constants of crystals. The electrical state of a medium is known when two vector quantities such as the electric field and the electric

displacement are known. Similarly, the elastic state is known when two second-order tensor quantities, stress and strain, are specified. Piezoelectricity is concerned with the interaction between the electrical and elastic behavior of a crystal and therefore with relations involving the two electrical and two elastic variables. The electrical variables are the electric field (\vec{E}) and the electric displacement (\vec{D}). The electric displacement is chosen as the second electrical variable in preference to other possible variables (e.g., polarization) as being more useful from an engineering and experimental point of view. The components of the electric field and electric displacement are designated by E_i and D_i , respectively. The subscript i takes the values of 1,2,3 and denotes the axis along which the component is directed.

Crystals are commonly classified into seven systems: triclinic (the least symmetrical), monoclinic, orthorhombic, tetragonal, hexagonal, trigonal, and isometric. Some authorities, however, treat trigonal crystals as a division of the hexagonal system. The seven systems in turn are divided into point-groups (classes) according to their symmetry with respect to a point. There are thirty-two such classes, of which twelve have too high a degree of symmetry to show piezoelectric properties. Thus, twenty classes can be piezoelectric. Every system contains at least one piezoelectric class.

In this Final Report we are only interested in determining the permittivities of uniaxial, piezoelectric crystals. The displacement, \vec{D} , and the electric field, \vec{E} , are related by a second-order permittivity tensor. In theory, measurement of \vec{D} and \vec{E} can determine the elements of the permittivity tensor. In practice this is quite difficult to do, however. Another method which can be used involves the reflection of light. It is well known that Fresnel's equations for isotropic media can be used to determine the refractive index and the extinction coefficient of optical materials. Also, the fact has been known for a very long time that Fresnel's equations reduce to simple forms for two conditions, namely, incident light which is normal to the surface and incident light at the Brewster angle.

There is another angle, much less known, which leads to a very simple form for Fresnel's equations. This is at an incident angle of 45° . Over the past several years Measurement Concepts, Inc. has shown that the use of this angle and high resolution digital voltmeters can lead to very accurate values for the refractive index and the extinction coefficients of optical materials. Analysis shows that the measurement of the refractive index to m significant figures requires a digital voltmeter with $(m+1)\frac{1}{2}$ digit resolution. Digital voltmeters with resolutions of five and six digits have appeared within this decade which now permit this kind of measurement. We have named this simplification in Fresnel's equations at 45° and the measurement of the refractive index with high resolution digital voltmeters

Digital Refractometry. We are currently applying for a patent on this method.

At first sight it would appear that the modification of Fresnel's equation for reflection from anisotropic media is well known. An investigation of the literature shows that this is not the case, however. In fact, surprisingly, only recently have Fresnel's equations for reflection and transmission in anisotropic media been published. Because piezoelectric crystals are anisotropic, e.g., quartz, Digital Refractometry can be a very useful method for measuring the permittivities. This method, which is optical, is especially valuable because the permittivities can be determined while a varying stress or electric field is being applied to the crystal.

During the measurement of quartz which began in earnest in early December, 1986 and lasted through all of January, 1987 we discovered two additional methods for measuring the refractive index. This occurred because we had set up the dual-beam configuration using a Rochon prism polarizer; the output of the polarizer consists of two orthogonally linearly polarized beams. While adjusting the optical polarizers the plane of polarization of the incident beam was rotated. The fact was noted that the intensity of one of the beams emerging from the Rochon prism increased while, simultaneously, the intensity of the other beam decreased. The question was asked if the refractive index could be obtained for the condition that the intensities of the two beam were equal. Analysis showed that a very simple relation

existed between the angle of the plane of polarization and the refractive index. We now call this method Equi-intensity Digital Refractometry. Several weeks later while aligning the reflected beam with the Rochon prism removed, a polarizer in a rotating mount was placed between the reflected beam and the detector. Normally, the polarizer is rotated through 90° and the intensities at these two extreme angles recorded. However, while passing from 0° to 90° with our new Ardel rotating mounts we observed a null intensity. We had not ever noticed this null before because our rotating mounts were accurate only to 0.1 . The Ardel mounts, on the other hand, are accurate to 5" of arc. An analysis was made of this configuration and, indeed, a null phenomenon is expected. Again, a very simple relation exists between the angle where the null intensity occurs and the refractive index. We call this measurement method Null Digital Refractometry. Thus, at time three measurement methods exist (the first one which we discovered we call the Minimum-Maximum intensity method and is the original proposed measurement method.) We are continuing to develop these methods and we shall report on them in the future. We are now writing up a large definitive paper on the entire subject of Digital Refractometry to be submitted in the near future. We plan to describe not only the measurement methods and the foundations of Digital Refractometry but the entire subject of the measurement of the refractive index of optical materials, e.g., the minimum-deviation prism method, the Brewster angle null method, the Pulfrich method, the Abbe method, etc.

2. Wooten's Equations for Reflection from Uniaxial Crystals.

In order to apply Digital Refractometry to uniaxial as well as piezoelectric crystals it is necessary to know the appropriate form of Fresnel's equations. While Fresnel's equations for reflection are well known for isotropic media the same cannot be said for Fresnel's equations for reflection from uniaxial (and uniaxial absorbing) crystals. In fact, only in 1984 was a paper written by Wooten in which equations for uniaxial crystals suitable for experimental verification were presented (1).

Wooten pointed out that information on certain aspects of reflection, refraction and absorption of light by anisotropic materials are available. A general analysis of optical properties of such crystals does not appear to have been published. Moreover, formulas for the incident and reflected rays are in a form which are very inconvenient for the experimentalist. Wooten, therefore, presented equations based on a much earlier work by Mosteller and Wooten that are likely to be used by experimentalists for measuring the optical properties of uniaxial crystals (2). These equations are the basis for applying the concepts of Digital Refractometry to the measurement of uniaxial (piezoelectric) crystals. Our ultimate objective is to transform these equations to a Mueller matrix and to determine the intensities which then lead to the refractive indices. We now discuss the three experimental configurations described by Wooten as they relate to Digital Refractometry.

The first case is reflection from the basal plane as depicted in Fig.1. For this case the incident and reflected waves can be decomposed into two linearly polarized waves: (1) a wave with $\vec{E} = E_y \hat{y}$ perpendicular to the axis of symmetry (the optic axis); (2) a wave $\vec{E} = E_{xz} \hat{u}$ in the plane defined by the direction of wave propagation and the optic axis. The first is the ordinary wave and the second is the extraordinary wave, respectively.

The ratio of the reflected to the incident wave amplitudes for the ordinary wave is given by

$$\frac{E_y^r}{E_y^i} = \frac{\cos \theta - (n_{\perp}^2 - \sin^2 \theta)^{1/2}}{\cos \theta + (n_{\perp}^2 - \sin^2 \theta)^{1/2}}, \quad (2.1)$$

where n_{\perp} is the ordinary index of refraction (n_o) for electromagnetic waves polarized with the \vec{E} vector normal to the optic axis. The corresponding equations for the ordinary waves are

$$\frac{E_{xz}^r}{E_{xz}^i} = \frac{n_{\perp} n_{\parallel} \cos \theta - (n_{\parallel}^2 - \sin^2 \theta)^{1/2}}{n_{\perp} n_{\parallel} \cos \theta + (n_{\parallel}^2 - \sin^2 \theta)^{1/2}}, \quad (2.2)$$

where n_{\parallel} is the refractive index for electromagnetic waves polarized with the \vec{E} vector parallel to the optics axis, i.e., $\vec{E} = E_z \hat{z}$. The quantity n_{\parallel} is the extraordinary refractive index (n_e). Two special cases of these equations are of interest. The first is for normal incidence so $\theta = 0$. From Eq.(2.1) and (2.2) we then have

$$\frac{E_y^r}{E_y^i} = \frac{1 - n_{\perp}}{1 + n_{\parallel}}, \quad (2.3a)$$

and

$$\frac{E_Y^r}{E_Y^i} = - \left(\frac{1 - n_{\perp}}{1 + n_{\perp}} \right) . \quad (2.3b)$$

Thus, at normal incidence the magnitude of the ratios are identical. They differ in that there is a 180° phase shift between the reflected components, Eq.(2.3b). This result is very useful because it shows that the normal reflection method can be used to determine the direction of the optical axis. Thus, in this configuration the orthogonal intensities are equal and show that the beam is propagating along the optical axis.

For Digital Refractometry the incident angle of interest is 45° . Thus, from Eqs.(2.1) and (2.2) we find

$$\frac{E_Y^r}{E_Y^i} = - \frac{(2n_{\perp}^2 - 1)^{1/2} - 1}{(2n_{\perp}^2 - 1)^{1/2} + 1} , \quad (2.4a)$$

and

$$\frac{E_{x_2}^r}{E_{x_2}^i} = \frac{n_{\perp} n_{\parallel} - (2n_{\parallel}^2 - 1)^{1/2}}{n_{\perp} n_{\parallel} + (2n_{\parallel}^2 - 1)^{1/2}} . \quad (2.4b)$$

We see that reflection of an optical beam from a basal plane leads to a unique expression for n_{\perp} , Eq.(2.4a), and an equation for both n_{\perp} and n_{\parallel} , Eq.(2.4b). Thus, we must measure n_{\perp} first, Eq.(2.4a), and then substitute this result into Eq.(2.4b) and solve for n_{\parallel} . This was the approach proposed in our Phase I SBIR proposal. It is still valid.

During the course of this Phase I we have read Wooten's paper further and considered his remaining two cases. The second

case is shown in Fig.2. Wooten has shown that the following equations are valid for the configuration shown in Fig.2,

$$\frac{E_z^r}{E_z^i} = \frac{\cos\theta - (n_{11}^2 - \sin^2\theta)^{1/2}}{\cos\theta + (n_{11}^2 - \sin^2\theta)^{1/2}}, \quad (2.5a)$$

and

$$\frac{E_{xy}^r}{E_{xy}^i} = \frac{n_{\perp}^2 \cos\theta - (n_{\perp}^2 - \sin^2\theta)^{1/2}}{n_{\perp}^2 \cos\theta + (n_{\perp}^2 - \sin^2\theta)^{1/2}}. \quad (2.5b)$$

We see that in this set of equations n_{11} and n_{\perp} are not coupled. It is again of interest to examine the condition of normal incidence. Thus, we have,

$$\frac{E_z^r}{E_z^i} = - \left(\frac{n_{11} - 1}{n_{11} + 1} \right), \quad (2.6a)$$

and

$$\frac{E_{xy}^r}{E_{xy}^i} = \left(\frac{n_{\perp} - 1}{n_{\perp} + 1} \right). \quad (2.6b)$$

In this configuration the amplitudes of the reflected field are unequal and there is, again, a 180° phase shift, Eq.(2.6a). Consequently, the unequal reflected intensities would show that the beam is not propagating along the optic axis. However, unique expressions for both n_{11} and n_{\perp} can be obtained.

For Digital Refractometry we again consider the incident angle at 45° . We then find from Eqs.(2.5) that

$$\frac{E_z^r}{E_z^i} = - \frac{(2n_{11}^2 - 1)^{1/2} - 1}{(2n_{11}^2 - 1)^{1/2} + 1}, \quad (2.7a)$$

and

$$\frac{E_{xy}^r}{E_{xy}^i} = - \frac{(2n_{\perp}^2 - 1)^{1/2} - n_{\perp}^2}{(2n_{\perp}^2 - 1)^{1/2} + n_{\perp}^2}. \quad (2.7b)$$

Eqs.(2.7a) and Eq.(2.7b) are the fundamental pair of equations which we use for the application of Digital Refractometry to uniaxial (piezoelectric) crystals.

Finally, the third case discussed by Wooten leads to expressions in which $n_{||}$ and n_{\perp} are again coupled. Because Eqs.(2.7a) and (2.7b) express $n_{||}$ and n_{\perp} uniquely we chose to use the experimental configuration, Fig.2., corresponding to these equations. In order to carry out the measurement of the refractive indices Eqs.(2.7a) and (2.7b) are transformed to their corresponding Mueller matrices.

3. Mueller Matrix for Uniaxial (Piezoelectric) Crystals.

In order to describe the experimental configuration for Dual-Beam Digital Refractometry we transform Fresnel's equations to the Stokes parameters and the Mueller matrix formalism. This enables us to describe the experiment in terms of the measured intensities rather than the unobservable amplitudes.

From Eq. (2.7) the reflected field amplitudes are given by

$$E_z^r = a E_z^i, \quad (3.1a)$$

and

$$E_{xy}^r = b E_{xy}^i, \quad (3.1b)$$

where

$$a = - \frac{(2n_{\parallel}^2 - 1)^{1/2} - 1}{(2n_{\parallel}^2 - 1)^{1/2} + 1}, \quad (3.1c)$$

$$b = - \frac{(2n_{\perp}^2 - 1)^{1/2} - n_{\perp}^2}{(2n_{\perp}^2 - 1)^{1/2} + n_{\perp}^2}, \quad (3.1d)$$

The incident Stokes parameters are defined by

$$S_0^i = \cos \theta (E_{xy}^i E_{xy}^{i*} + E_z^i E_z^{i*}), \quad (3.2a)$$

$$S_1^i = \cos \theta (E_{xy}^i E_{xy}^{i*} - E_z^i E_z^{i*}), \quad (3.2b)$$

$$S_2^i = \cos \theta (E_{xy}^i E_z^{i*} + E_z^i E_{xy}^{i*}), \quad (3.2c)$$

$$S_3^i = j \cos \theta (E_{xy}^i E_z^{i*} - E_z^i E_{xy}^{i*}), \quad (3.2d)$$

where θ = the incident angle = 45° . Similarly, the reflected Stokes parameters are given by

$$S_0^r = \cos \theta (E_{xy}^r E_{xy}^{r*} + E_z^l E_z^{l*}), \quad (3.3a)$$

$$S_1^r = \cos \theta (E_{xy}^r E_{xy}^{r*} - E_z^i E_z^{i*}), \quad (3.3b)$$

$$S_2^r = \cos \theta (E_{xy}^r E_z^{r*} + E_z^r E_{xy}^{r*}), \quad (3.3c)$$

$$S_3^r = j \cos \theta (E_{xy}^r E_z^{r*} - E_z^r E_{xy}^{r*}). \quad (3.3d)$$

Straightforward substitution of Eq.(3.1a) into Eq.(3.2) and using Eq.(3.3) yields

$$\begin{pmatrix} S_0^r \\ S_1^r \\ S_2^r \\ S_3^r \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a^2 + b^2 & a^2 - b^2 & 0 & 0 \\ a^2 - b^2 & a^2 + b^2 & 0 & 0 \\ 0 & 0 & 2ab & 0 \\ 0 & 0 & 0 & 2ab \end{pmatrix} \begin{pmatrix} S_0^l \\ S_1^i \\ S_2^l \\ S_3^i \end{pmatrix}, \quad (3.4)$$

where the 4x4 matrix is the Mueller matrix of a polarizer and a and b are, repeating Eqs.(3.1c) and (3.1d),

$$a = - \frac{(2n_{||}^2 - 1)^{1/2} - 1}{(2n_{||}^2 - 1)^{1/2} + 1}, \quad (3.1c)$$

$$b = \frac{(2n_{\perp}^2 - 1)^{1/2} - n_{\perp}^2}{(2n_{\perp}^2 - 1)^{1/2} + n_{\perp}^2}. \quad (3.1d)$$

4. Experimental Configuration for Dual-Beam Digital Refractometry.

In order to determine a and b and, ultimately, $n_{||}$ and n_{\perp} the experimental configuration to be used is shown in Fig.3. In the experiment the incident beam on the sample crystal is linearly polarized at $+45^{\circ}$. The Stokes vector of the incident beam is $I_0(1, 0, 1, 0)$. The Stokes vector of the reflected beam is then

$$\begin{pmatrix} S_{0r} \\ S_{1r} \\ S_{2r} \\ S_{3r} \end{pmatrix} = \frac{I_0}{2} \begin{pmatrix} a^2 + b^2 \\ a^2 - b^2 \\ 2ab \\ 0 \end{pmatrix}. \quad (4.1)$$

In the experimental configuration for Phase II we have developed a method for measuring the refractive indices (ordinary and extraordinary) whereby there is no mechanical motion of the polarizers. This is accomplished by using either a Rochon or Wollaston polarizing prism. These types of prisms split the incident beam into two beams which are orthogonally linearly polarized. The prism(s) are characterized by the following pair of Mueller matrices,

$$M_{||} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M_{\perp} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (4.2)$$

The output beams are described by matrix multiplying Eq.(4.1) with each of the matrices in Eq.(4.2). The result is

$$\frac{I_0}{2} a^2 \{ 1, 1, 0, 0 \}_{||}, \quad \frac{I_0}{2} b^2 \{ 1, -1, 0, 0 \}_{\perp}. \quad (4.3)$$

The intensities on the detector are then given by the first Stokes parameter,

$$I_{\parallel} = \frac{I_0}{2} a^2, \quad I_{\perp} = \frac{I_0}{2} b^2. \quad (4.4)$$

In Digital Refractometry the source fluctuations can be cancelled out by measuring both polarization channels simultaneously. Thus, taking the ratio of the equations in Eq.(4.4) yields

$$\frac{I_{\parallel}}{I_{\perp}} = \frac{a^2}{b^2}. \quad (4.5)$$

Similar to the case of isotropic media for uniaxial (anisotropic) media the intensity is also cancelled out. But, we now have two unknowns, a and b , in a single equation. For the isotropic media, a and b are related by the expression $a^2 = b^4$ and so only a single unknown appears in Eq.(4.5). For anisotropic media we must modify the experimental configuration to determine n_{\parallel} and n_{\perp} . This is accomplished by using a reference intensity path and then measuring n_{\parallel} and n_{\perp} sequentially. In Fig.3 the reference path is shown. We now describe the measurement of the n_{\parallel} and n_{\perp} using only the intensities and reflection coefficients. We consider the "sample" path first. The laser intensity incident on the first beamsplitter is I_0 . The coefficients of the reflection and transmission of the beamsplitters and mirrors are represented by T_B , R_B and R_M . The reflection coefficient for the

sample is represented for either polarization as $R_s(\parallel, \perp)$. Then the intensity on the detector due to the sample path is

$$I_s = R_B R_s T_B I_0 \quad (4.6)$$

Similarly, the intensity on the detector due to the reference path is

$$I_R = T_B R_M R_B I_0 \quad (4.7)$$

We now take the ratio of Eq.(4.7) and Eq.(4.8) and find

$$\frac{I_s}{I_R} = \frac{R_s}{R_M} \quad (4.8)$$

Thus, the source fluctuations cancel out. However, the coefficient must be known and we must measure the factor

$$F = \frac{1}{R_M} \quad (4.9)$$

Thus, we now need only evaluate R_M , the reflection coefficient of an isotropic medium. This is easily accomplished by placing the mirror in the sample mount and by using Digital Refractometry for isotropic media, that is, taking the ratio of the orthogonal intensities emerging from the Rochon (or Wollaston) prisms. Previous analyses have shown that the refractive index is given by³

$$n = \frac{[1 + \sqrt{1 - \xi^2}]^{1/2}}{\xi}, \quad (4.10)$$

where

$$f = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}}, \quad (4.11)$$

are the intensities of the orthogonal polarizations. The reflection coefficient, R_M , is then found from the equation

$$R_M = \frac{n^2 (n^2 - \sqrt{2n^2 - 1})}{(n^2 + \sqrt{2n^2 - 1})^2}. \quad (4.12)$$

During the course of the Phase II program we shall make a matrix analysis of the proposed experimental configuration. For the sake of brevity and, hopefully, clarity we have only described the intensity (scalar) formulation here.

The beamsplitters and mirror have now been completely characterized. The quantities a and b are now determined separately. In order to do this we use the same configuration as shown in Fig.3. with the uniaxial crystal mounted according to Wooten's configuration 2. The experimental details for the reflected beam path are shown in Fig.4. The incident beam is again linearly polarized at 45° . It is then reflected from the sample and passes through the Rochon polarizing prism. Both beams are chopped at the same frequency. In order to measure either a or b , that is, $n_{||}$ or n_{\perp} , one of the beams is blocked from reaching the detector. The beam is incident on the collimating lens, passes through the beamsplitter along with the reference beam and is then focused onto the detector. The sample beam is chopped at one frequency, f_S , and the reference beam at

another frequency, f_R . The output current of the silicon detector consists of $i_R + i_S$. This signal is then processed by two lock-in amplifiers whose outputs are transmitted to a pair of 5 1/2 digital voltmeters. The data from the voltmeters are read over a GPIB interface to a PC computer. From the foregoing equations the refractive index is then determined. After this the entire process is repeated for the other orthogonal beam by blocking the previously measured beam from the detector.

In the actual measurements a quartz sample was used in the configuration corresponding to Fig.2. In order to obtain data accurate to five significant figures several new items were purchased. The first was an amplitude stable HeNe laser, the SP117A Spectra-Physics Laser. This laser arrived in early December and its intensity (amplitude) stability was immediately tested. After leaving the laser on for a twenty-four hour period the intensity fluctuations were found to be ± 0.00002 which was stable indeed. In fact, this new laser is nearly fifty times more stable than the HeNe laser we were previously using which was the Spectra-Physics 120S. An extremely important feature of the SP117A laser is that it is cylindrical in shape. This allowed us to purchase a new laser mount from the Newport Corp. into which the laser could be mounted. This mount is controlled by micrometers. With this new mount and laser we were finally able to assure ourselves that we were indeed parallel to the optical plane and exactly aligned. This was done by placing the sample at 45° and checked with the Gaertner auto-collimator. The detector was replaced by a micrometer controlled mounted mirror. The laser

beam was transmitted to the sample and to the mirror and then reflected back into the cavity. The mirror was adjusted until the retroreflected beam was exactly on line with the incident beam. This behavior could be seen by watching the stability lamp on the laser power supply. Only when the retroreflected beam is precisely on line and out of phase with the incident beam was the lamp observed to fluctuate. In this way we were completely confident that we were at the angles required to implement the equations of Digital Refractometry.

The other major purchase was an ultra-resolution mount from the Newport Corporation. This mount is also micrometer controlled and its precised motion could be observed with our Gaertner MC551 autocollimator. With this mount we were able to obtain a 45° angle to with $1''$ of arc, a resolution we were unable to obtain with our previous mounts (which could resolve only to $20''$ of arc.) The accuracy and resolution of the mount were checked by inserting a polished sample of BK7 into the mount. The refractive index was measured to be $n=1.51513 \pm 0.00003$, in excellent agreement with published data. Thus, we were very confident that we would be able to obtain the required accuracy for the measurement of quartz.

Because the quartz sample was in the form of a rectangle a special platform in the form of a clamp was machined. On the surfaces in contact with the quartz crystal very thin pieces of teflon were attached to the inner surfaces of the clamp. The crystal was gently tightened to hold it in place. The position of

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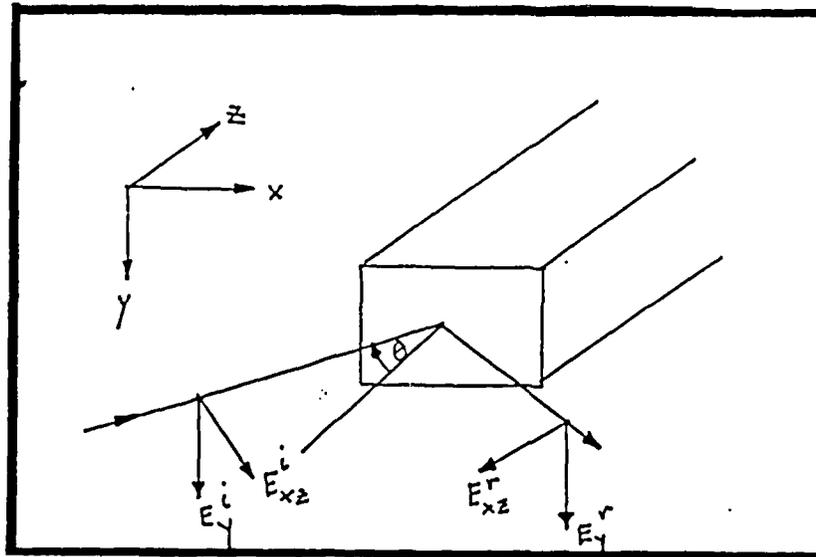


Fig. 1. Reflection from the basal plane (the x-y plane) of a uniaxial (anisotropic) crystal.

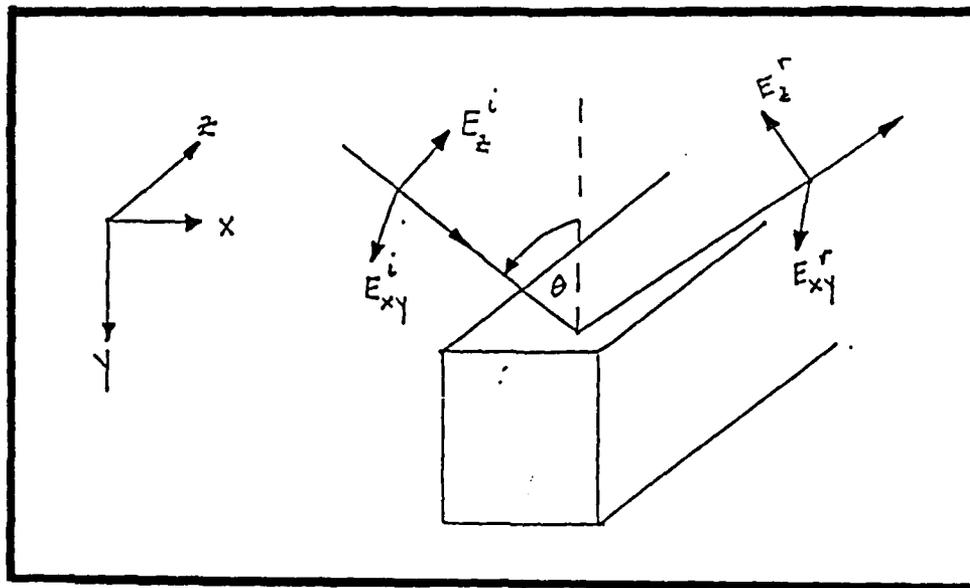


Fig. 2. Reflectance from a plane parallel to the optics axis with the plane of incidence parallel to the basal (the x - y plane). The optic axis is along the z axis.

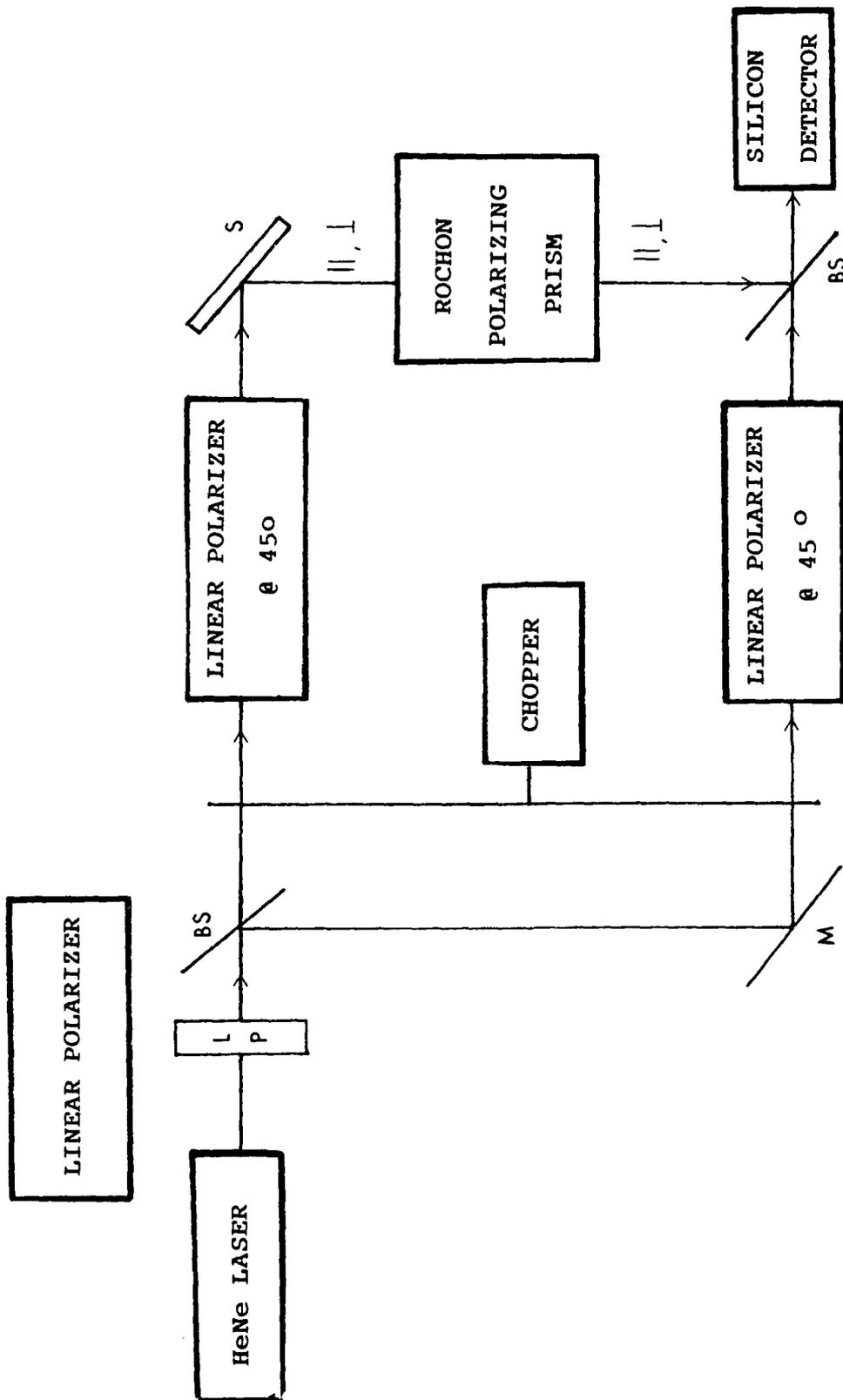


Fig. 3. Dual-Beam configuration for Digital Refractometry.

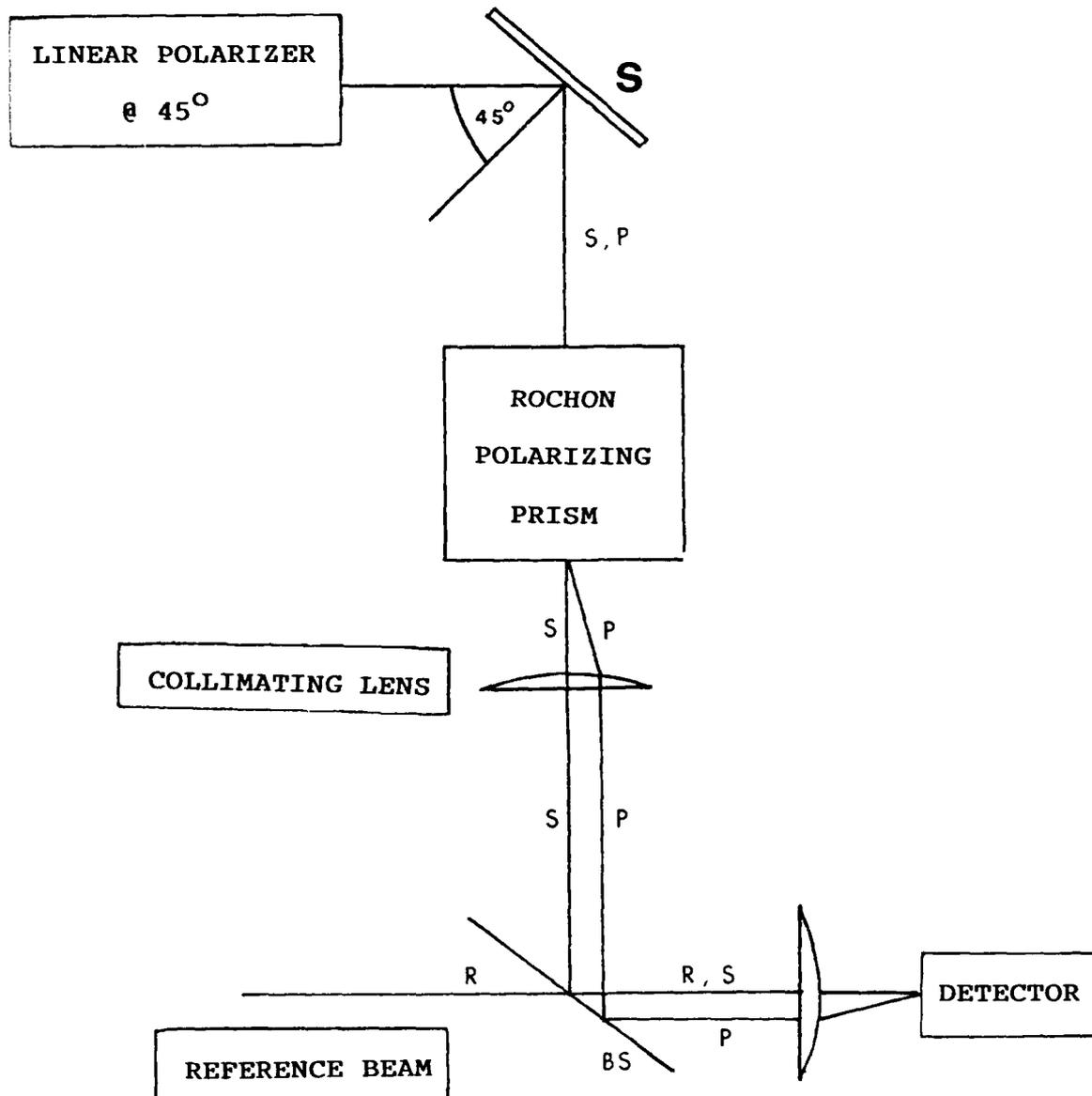


Fig. 4. Orthogonal polarizations of a Rochon Prism.