(4)

MASSACHUSETTS INSTITUTE OF TECHNOLOGY VLSI PUBLICATIONS

VLSI Memo No. 88-494
November 1988

Stability Criterion for Lateral Inhibition and Related Networks that is Robust in the Presence of Integrated Circuit Parasitics

David L. Standley and John L. Wyatt, Jr.

Abstract

In the analog VLSI implementation of neural systems, it is sometimes convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. A serious problem of unwanted spontaneous oscillation often arises with these circuits and renders them unusable in practice. This paper reports a design approach that guarantees such a system will be stable, even though the values of designed elements in the resistive grid may be imprecise and the location and values of parasitic elements may be unknown. The method is based on a mathematical analysis using Tellegen's theorem and the Popov criterion. The criteria are *local* in the sense that no overall analysis of the interconnected system is required for their use, *empirical* in the sense that they involve only measurable frequency response data on the individual cells, and *robust* in the sense that they are not affected by unmodelled parasitic resistances and capacitances in the interconnect network.

Microsystems Research Center Room 39-321
Massachusetts Institute of Technology
Cambridge Massachusetts 02139
Telephone (617) 253-8138

Acknowledgements

To appear in *IEEE Transactions on Circuits and Systems*, Special Issue on Neural Networks, 1989. This research was supported in part by the Defense Advanced Research Projects Agency under contract number N00014-87-K-0825 and the National Science Foundation under contract MIP-8814612.

Author Information

Standley: Electrical Engineering and Computer Science, MIT, Room 36-863, Cambridge, MA 02139. (617) 253-2631.

Wyatt: Electrical Engineering and Computer Science, MIT, Room 36-864, Cambridge, MA 02139. (617) 253-6718.

To appear in IEEE Transactions on Circuits and Systems, Special Issue on Neural Networks, 1989.

Stability Criterion for Lateral Inhibition and Related Networks that is Robust in the Presence of Integrated Circuit Parasitics

D.L. Standley and J.L. Wyatt, Jr.

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

In the analog VLSI implementation of neural systems, it is sometimes convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. A serious problem of unwanted spontaneous oscillation often arises with these circuits and renders them unusable in practice. This paper reports a design approach that guarantees such a system will be stable, even though the values of designed elements in the resistive grid may be imprecise and the location and values of parasitic elements may be unknown. The method is based on a mathematical analysis using Tellegen's theorem and the Popov criterion. The criteria are *local* in the sense that no overall analysis of the interconnected system is required for their use, *empirical* in the sense that they involve only measurable frequency response data on the individual cells, and *robust* in the sense that they are not affected by unmodelled parasitic resistances and capacitances in the interconnect network.

I. Introduction

The term "lateral inhibition" first arose in neurophysiology to describe a common form of neural circuitry in which the output of each neuron in some population is used to inhibit the response of each of its neighbors. Perhaps the best understood example is the horizontal cell layer in the vertebrate retina, in which lateral inhibition simultaneously enhances intensity edges and acts as an automatic gain control to extend the dynamic range of the retina as a whole [1]. The principle has been used in the design of artificial neural system algorithms by Kohonen [2] and others and in the electronic design of neural chips by Mahowald and Mead [3,4].

In the VLSI implementation of neural systems, it is convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. Linear resistors fabricated in, e.g., polysilicon, could yield a very compact realization, and nonlinear resistive grids, made from MOS transistors, have been found useful for image segmentation [4,5]. Networks of this type can be divided into two classes: feedback systems and feedforward-only systems. In the feedforward case one set of amplifiers imposes signal voltages or currents on the grid and another set reads out the resulting response for subsequent processing, while the same amplifiers both "write to" the grid and "read from" it in a feedback arrangement. Feedforward networks of this type are inherently stable, but feedback networks need not be.

A practical example is one of Mahowald and Mead's retina chips [3] that achieves edge enhancement by means of lateral inhibition through a resistive grid and temporal sharpening by comparing the present input with a capacitively filtered version. Figure 1 shows a single cell in a continuous-time version of this chip, and Fig. 2 illustrates the network of interconnected cells. Note that the voltage on the capacitor in any given cell is affected both by the local light intensity incident on that cell and by the capacitor voltages on neighboring cells of identical design. Each cell drives its neighbors, which drive both their distant neighbors and the original cell in turn. Thus the necessary ingredients for instability — active elements and signal feedback — are both present in this system. Experiment has shown that the individual cells in this system are open-circuit stable and remain stable when the output of transamp # 2 is connected to a voltage source through a resistor, but the interconnected system oscillates so badly that the original design is scarcely usable[1] in practice [6]. Such oscillations can readily occur in any resistive grid circuit with active elements and feedback, even when each individual cell is quite stable. Analysis of the conditions of instability by conventional methods appears hopeless, since the number of simultaneously active feedback loops is enormous.

This paper reports a practical design approach that rigorously guarantees these and related systems will be stable under certain conditions. The

[1]The later design reported in [3] avoids this problem altogether, at a small cost in performance, by redesigning the circuits to passively sense the grid voltage in a "feedforward" style as described above.

work begins with the naïve observation that the system would be stable if we could design each individual cell so that, although internally active, it acts like a passive system as seen from the resistive grid. The design goal in that case would be that each cell's output impedance should be a *positive-real* (i.e., passive) [7-9] function. This is sometimes possible in practice: we will show that the original network in Fig. 1 satisfies this condition in the absence of certain parasitic elements. Furthermore, it is a condition one can verify experimentally by frequency-response measurements.

It is obvious that a collection of cells that appear passive at their terminals will form a stable system when interconnected through a passive medium such as a resistive grid, and that the stability of such a system is *robust* to perturbations by passive parasitic elements in the network. The contribution of this paper is to go beyond that observation to provide i) a demonstration that the passivity or positive-real condition is much stronger than we actually need and that weaker conditions, more easily achieved in practice, suffice to guarantee robust stability of the linear network model, and ii) an extension of the analysis to the *nonlinear* domain that furthermore rules out sustained *large-signal* oscillations under certain conditions.

Note that these results do not apply directly to networks created by interconnecting neuron-like elements, as conventionally described in the literature on artificial neural systems. The "neurons" in, e.g., a Hopfield network [10] are *unilateral 2-port elements* in which the input and output are both voltage signals. The input voltage uniquely and instantaneously determines the output voltage of such a neural model, but the output can only affect the input via the resistive grid. In contrast, the cells in our system are *1-port electrical elements* (temporarily ignoring the optical input channel) in which the port voltage and port current are the two relevant signals, and each signal affects the other through the cell's internal dynamics (modelled as a Thévenin equivalent impedance) as well as through the grid's response.

II. The Linear Theory

This work was motivated by the following linear analysis of a model for the circuit in Fig. 1. For an initial approximation to the output admittance of the cell we use the elementary model shown in Fig. 3 for the amplifiers and simplify the circuit topology within a single cell (without loss of information relevant for stability) as shown in Fig. 4.

Straightforward calculations show that the output admittance is

$$Y(s) = [g_{m_2} + R_{o_2}^{-1} + sC_{o_2}] + \frac{g_{m_1} g_{m_2} R_{o_1}}{(1 + sR_{o_1} C_{o_1})}. \quad (1)$$

This is a positive-real admittance that could be realized by a network of the form shown in Fig. 5, where

$$R_1 = (g_{m_2} + R_{o_2}^{-1})^{-1}, R_2 = (g_{m_1} g_{m_2} R_{o_1})^{-1}, \text{ and } L = C_{o_1} / g_{m_1} g_{m_2}. \quad (2)$$

Of course this model is oversimplified, since the circuit *does* oscillate. Transistor parasitics and layout parasitics cause the output admittance of the individual active cells to deviate from the form given in eqs. (1) and (2), and any very accurate model will necessarily be quite high order. The following theorem shows how far one can relax the positive-real condition and still guarantee that the entire network is robustly stable. Though we have been unable to find such a theorem in the literature, the concepts used in the proof are well-established and this result is not surprising.

Terminology

The terms *open right-half plane* and *closed right-half plane* refer to the set of all complex numbers $s = \sigma + j\omega$ with $\sigma > 0$ and $\sigma \geq 0$, respectively, and the term *closed second quadrant* refers to the set of complex numbers with $\sigma \leq 0$ and $\omega \geq 0$. A *natural frequency* of a linear network is a complex frequency s_o such that, when all independent sources are set to zero and all branch impedances and admittances are evaluated at s_o, there exists a nonzero solution for the complex branch voltages $\{V_k\}$ and currents $\{I_k\}$ [11]. For the purposes of this section, a lumped linear network is said to be *stable* if a) it has no natural frequencies in the closed right-half plane except perhaps at the origin, and b) any natural frequency at the origin results only in network solutions that are constant as functions of time. (The latter condition rules out unstable transient solutions that grow polynomially in time resulting from a repeated natural frequency at the origin.)

Theorem 1

Consider a linear network of arbitrary topology, consisting of any number of positive 2-terminal resistors and capacitors and of N lumped linear impedances $Z_n(s), n = 1, 2, ..., N$, that are open- and short-circuit stable in isolation, i.e., that have no poles or zeroes in the closed right half plane. Then the network is *stable* if at each frequency $\omega \geq 0$ there exists a phase angle $\theta(\omega)$ such that $0 \geq \theta(\omega) \geq -90^\circ$ and $|\angle Z_n(j\omega) - \theta(j\omega)| < 90^\circ, n = 1, 2, ..., N$.

An equivalent statement of this last condition is that the Nyquist plot of each cell's output impedance for $\omega \geq 0$ never intersects the closed 2nd quadrant, and that no two cells' output impedance phase angles can ever differ by as much as 180°. If all the active cells are designed identically and fabricated on the same chip, their phase angles should track fairly closely in practice and thus this second condition is a natural one.

The theorem is intuitively reasonable. The assumptions guarantee that the cells cannot resonate with one another at any purely sinusoidal frequency $s = j\omega$ since their phase angles can never differ by as much as 180°, and they can never resonate with the resistors and capacitors since there is no $\omega \geq 0$ at which both $Re\{Z_n(j\omega)\} \leq 0$ and $Im\{Z_n(j\omega)\} \geq 0$ for some $n, 1 \leq n \leq N$. Figure 6 illustrates these ideas. The proof, which appears in the Appendix, formalizes this argument using conservation of complex power, extends it to rule out natural frequencies in the right-half plane as well, and shows why instabilities resulting from a repeated natural frequency at the origin cannot occur. Note that this formulation of the theorem does not guarantee a unique equilibrium, since capacitor-only cut-sets are not ruled out. In the absence of such cut-sets the equilibrium will be unique because the d.c. cell conductances $Y_n(0)$ are strictly positive.

III. Stability Result for Networks with Nonlinear Resistors and Capacitors

The previous results for linear networks can afford some limited insight into the behavior of nonlinear networks. First the nonlinear equations are linearized about an equilibrium point and Theorem 1 is applied to the linear model. If the linearized model is asymptotically stable, then the equilibrium point of the original nonlinear network is ***locally stable***, i.e., the network will return to that equilibrium point *if* the initial condition is sufficiently near it. But the result in this section, in contrast, applies to the full nonlinear circuit model and allows one to conclude that it cannot blow up or oscillate ***even if*** the initial state is ***arbitrarily far from*** the equilibrium point.

Terminology

We say that an impedance $Z(s)$ satisfies the ***Popov criterion*** if $(1+\tau s)Z(s)$ is ***positive real*** [7,8,9] for some $\tau \geq 0$. (Note that this formulation of the Popov criterion differs slightly from that given in standard references [8,9].)

Theorem 2

Consider a network consisting of possibly nonlinear resistors and capacitors, along with two-terminal cells each consisting of a linear impedance $Z_n(s)$ in series with a d.c. Thévenin equivalent voltage source of value v_n^{th}, for $n = 1, 2, ..., N$. Assume the network has an equilibrium point (i.e., a d.c. solution) characterized by a voltage v_k^0 and current i_k^0 for each branch k, and that for any initial condition at $t = 0$, there exists a unique, continuously differentiable solution for the branch voltage and current waveforms defined for all $t \geq 0$. Assume

i) the resistor curves are continuously differentiable functions $v_k = f_k(i_k)$, with the derivative $f_k'(i_k) \geq R_{min} > 0$ for all k and i_k,

ii) the capacitors are characterized by continuous functions $i_k = C_k(v_k)\dot{v}_k$ where $0 < C_k(v_k) \leq C_{max}$ for all k and v_k, and

iii) the impedances $Z_n(s)$ all satisfy the Popov criterion for some common value of $\tau \geq 0$. Then the network is stable in the sense that, for any initial condition,

$$\int_0^\infty \sum_{\text{all resistors}} (i_k(t) - i_k^0)^2 dt + \tau \int_0^\infty \sum_{\text{all capacitors}} i_k^2(t) dt < \infty. \tag{3}$$

The proof, which relies on interesting dualities between resistor co-content and capacitor energy and between resistor power and a form of capacitor reactive power, is in the Appendix. Note that Thm. 2, as stated, does not require the equilibrium point to be unique, since any cell impedance may have a (nonrepeated) pole at the origin and cut-sets composed entirely of capacitors and/or such cells are not ruled out. In the absence of such cut-sets the equilibrium will in fact be unique for each value of the Thévenin equivalent voltage sources.

The Popov criterion condition on the cell impedances can be weakened, given some simple topology and modeling restrictions. In many circuits, such as the regular array in Fig. 2, each cell has one terminal connected only to a fixed or limited number of grid resistors, never to the terminal of another cell. This, along with a limit on the incremental conductance of the resistors, can be used to effectively add linear resistors to the terminals of each cell impedance, allowing the user to shift the Nyquist plots to the right before applying the Popov criterion. Figure 7 illustrates this process in stages, and Thm. 3 below states the improvement.

Theorem 3

Consider a network satisfying the following conditions in addition to the assumptions of Theorem 2:

i) the resistor curves have derivative $f_k'(i_k) \geq R_{min} + \epsilon, R_{min} > 0$ for some $\epsilon > 0$, for all k and i_k,

ii) each cell (which includes the series voltage source v_n^{th}) has at least one terminal connected to zero capacitors, zero cells, and to no more than M resistors, and

iii) the impedances $Z_n(s)$ are such that the corresponding expressions $Z_n(s) + (R_{min}/2M)$ all satisfy the Popov criterion for some common value of $\tau \geq 0$. Then the network is stable in the sense that, for any initial condition,

$$\int_0^\infty \sum_{\text{all resistors}} (i_k(t) - i_k^0)^2 dt + \tau \int_0^\infty \sum_{\text{all capacitors}} i_k^2(t) dt < \infty. \qquad (4)$$

The proof is in the Appendix.

IV. Concluding Remarks

The design criteria established by these theorems are simple and practical, though at present their validity is restricted to linear models of the cells. The results are consistent, in an interesting way, with the spirit of neural networks. Attention is focussed on the global collective behavior of the system. Neither the stability criteria nor the methods of proof rely on assigning any specific functional role to any particular component. This feature is characteristic of Tellegen's theorem and most, if not all of, the results that follow from it [12].

There are several areas of further work to be pursued, one of which is an analysis of the cell that includes amplifier clipping effects. Others include the synthesis of a compensator for the cell, an extension of the nonlinear result to include impedance multipliers other than the Popov operator, a waveform bounding analysis of the network which would guarantee adequate convergence after an allotted settling time, and an input-output stability theorem.

V. Appendix

Proof of Theorem 1

Let s_o denote a natural frequency of the network and $\{V_k\}$, $\{I_k\}$ denote any complex network solution at s_o. By Tellegen's theorem [12], or conservation of complex power, we have

$$\sum_k V_k I_k^* = 0, \tag{5}$$

i.e., for $s_o \neq$ any pole of $Z_n, n = 1, ..., N$ and $s_o \neq 0$,

$$\underset{\text{resistors}}{\sum |I_k|^2 R_k} + \underset{\text{capacitors}}{\sum |I_k|^2 (s_o C_k)^{-1}} + \underset{\text{cell impedances}}{\sum |I_n|^2 Z_n(s_o)} = 0 \tag{6}$$

and for $s_o \neq$ any zero of $Z_n, n = 1, ..., N$,

$$\underset{\text{resistors}}{\sum |V_k|^2 R_k^{-1}} + \underset{\text{capacitors}}{\sum |V_k|^2 s_o^* C_k} + \underset{\text{cell admittances}}{\sum |V_k|^2 Y_n^*(s_o)} = 0 \tag{7}$$

where the superscript $*$ denotes the complex conjugate operation.

i) There are no natural frequencies at $s_o = j\omega \neq 0$.

For each $\omega > 0$ all the cell impedance values lie strictly below and to the right of the half-space boundary in Fig. 6. The capacitance impedances $\{(j\omega C_k)^{-1}\}$ and the resistor impedances $\{R_k\}$ also lie below and to the right of this line. Thus no positive linear combination of these impedances can vanish as required by eq. (6). A similar argument holds for $\omega < 0$.

ii) There are no instabilities resulting from a repeated natural frequency at the origin.

The assumptions that the cell impedances have no $j\omega$-axis zeroes and that their Nyquist plots for $\omega \geq 0$ never intersect the closed 2nd quadrant imply that $Y_n^*(0) > 0, n = 1, ..., N$. Thus eq. (7) requires that all the voltages across resistor branches and cell output branches must vanish in any complex network solution at $s_o = 0$. Thus only the capacitor voltages can be nonzero and the network solution will be unaltered if all non-capacitor branches are replaced by short circuits. But every solution to a network comprised only of positive, linear 2-terminal capacitors is constant in time and hence stable.

iii) There are no natural frequencies in the open right-half plane.

Assume the contrary, i.e., that there exists such a network with a natural frequency $\hat{s}_o$ with $Re\{\hat{s}_o\} > 0$. Alter each element in the network (except resistors) as follows. For each cell having a $Z_n(s)$ of relative degree less than zero, add a series resistance R; for all other cells and for capacitors, add a parallel conductance G to each. Call each resulting pair a "composite element", and choose $R = G = \lambda \geq 0$. For λ sufficiently large all natural frequencies must lie in the open left-half plane since every branch element is strictly passive for λ sufficiently large. Since the natural frequencies are continuous functions of λ [13] and $Re\{\hat{s}_o\} > 0$ for $\lambda = 0$, there exists some $\acute{\lambda} > 0$ for which some natural frequency $\acute{s}_o$ lies on the imaginary axis. But this is ruled out by the proof in part i) unless $\acute{s}_o = 0$, and the argument in part ii) rules out $\acute{s}_o = 0$, since any network solution at $\acute{s}_o = 0$ consists of zero branch voltages except for capacitor branches, and for $\lambda > 0$ each capacitor has a conductance G in parallel with it. Since the voltage across

G is zero in such a network solution, all branch voltages (and thus all branch currents) in that solution must be zero, which is a contradiction because a natural frequency at $\acute{s}_o$ would imply the existence of a nonzero solution.

Proof of Theorem 2

First consider the special case in which all branch voltages and currents can be zero in equilibrium. By Tellegen's theorem, for any set of initial conditions and any time $T > 0$,

$$\int_0^T \sum_{\text{resistors}} (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt \; +$$
$$\int_0^T \sum_{\text{capacitors}} (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt \; +$$
$$\int_0^T \sum_{\text{cell impedances}} (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt = 0, \tag{8}$$

where $\dot{v}$ denotes dv/dt. The *resistor co-content* $\phi_k(v)$ is, for all v in the range of $f_k(i)$,

$$\phi_k(v) = \int_0^v g_k(v')dv' \geq 0, \tag{9}$$

where g_k is the inverse of the function f_k; note that g_k exists because f_k is monotonically increasing. Then for resistors,

$$\int_0^T (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt =$$
$$\int_0^T v_k(t) i_k(t) dt \; +$$
$$\tau[\phi_k(v_k(T)) - \phi_k(v_k(0))]$$

(10a)

and using the inequality in (9), along with $\tau \geq 0$, gives

$$\int_0^T (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt \geq$$

$$\int_0^T v_k(t) i_k(t) dt - \tau \phi_k(v_k(0)) \tag{10b}$$

for all resistor branches. Similarly the *capacitor energy* $E_k(q)$, as a function of the charge q, is

$$E_k(q) = \int_0^q v_k(q') dq' \geq 0. \tag{11}$$

Then for capacitors,

$$\int_0^T (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt =$$

$$\tau \int_0^T \dot{v}_k(t) i_k(t) dt +$$

$$[E_k(q_k(T)) - E_k(q_k(0))], \tag{12a}$$

and using the inequality in (11) gives

$$\int_0^T (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt \geq$$

$$\tau \int_0^T \dot{v}_k(t) i_k(t) dt - E_k(q_k(0)) \tag{12b}$$

for all capacitor branches. And for cells, the assumption that $(1+\tau s)Z_n(s)$ is positive-real (by the Popov condition) implies that

$$\int_0^T (v_n(t) + \tau \dot{v}_n(t)) i_n(t) dt \geq -E_n(0), \tag{13}$$

where $E_n(0) \geq 0$ is the "initial energy in the Popov-multiplied cell impedance" at $t = 0$, a function of the initial condition only. Using (10b), (12b), and (13) in (8) gives

$$\begin{aligned} &\int_0^T \sum_{\text{resistors}} v_k(t) i_k(t) dt + \\ &\tau \int_0^T \sum_{\text{capacitors}} \dot{v}_k(t) i_k(t) dt \leq \\ &\sum_{\text{resistors}} \tau \phi_k(v_k(0)) + \sum_{\text{capacitors}} E_k(q_k(0)) + \\ &\sum_{\text{cell impedances}} E_n(0). \end{aligned} \tag{14}$$

But the right side of (14) depends on the initial conditions only, so that

$$\begin{aligned} &\int_0^\infty \sum_{\text{resistors}} v_k(t) i_k(t) dt + \\ &\tau \int_0^\infty \sum_{\text{capacitors}} \dot{v}_k(t) i_k(t) < \infty. \end{aligned} \tag{15}$$

For resistors, the inequality condition on f_k', the zero equilibrium branch voltage and current assumption, and $R_{min} > 0$ yields $i_k^2 \leq R_{min}^{-1} v_k i_k$. Also, for capacitors $i_k^2 = C_k^2(v_k)\dot{v}_k^2 \leq C_{max} C_k(v_k) \dot{v}_k^2$ and $C_{max} \geq 0$ so that $i_k^2 \leq C_{max} \dot{v}_k i_k$. The resultant inequalities for resistors and capacitors, along with $R_{min} > 0$ and $C_{max} \geq 0$, in (15) yield the result (3) for the special case of $i_k^0 = 0$ for all branches:

$$\int_0^\infty \sum_{\text{resistors}} i_k^2(t) dt + \tau \int_0^\infty \sum_{\text{capacitors}} i_k^2(t) dt < \infty. \tag{16}$$

To prove the result for the general case of a nonzero equilibrium solution, a constant additive transformation on the branch voltage and current waveforms will result in a network satisfying the special case just proven. Note that capacitor branch currents vanish at equilibrium.

Proof of Theorem 3

As in the proof of Thm. 2, we specifically address only the case in which all the Thévenin equivalent cell voltages are zero and thus all the branch voltages and currents are zero at equilibrium. By condition ii), each impedance $Z_n(s)$ has a terminal connected to m_n resistors and to no other elements, where $0 \leq m_n \leq M$. If $m_n = 0$, then the cell is open-circuited; it does not affect the rest of the circuit. If $m_n \geq 1$, then $Z_n(s)$ is connected to m_n linear resistors of value $R_{min}/2$ as explained in Fig. 7. Let i_j be the branch currents entering the resistors at the dashed boundary shown in Fig. 7d, $j = 1, 2, ..., m_n$, and let v_j be the corresponding voltages referenced to the terminal of the cell shown as ground. Also let v_z and i_z be the voltage across and current through $Z_n(s)$, respectively, in the direction sense such that

$$i_z(t) = \sum_{j=1}^{m_n} i_j(t). \tag{17}$$

Then

$$v_j(t) = v_z(t) + (\frac{R_{min}}{2})i_j(t), \tag{18}$$

$j = 1, 2, ..., m_n$. Using (17) and (18) gives

$$\int_0^T \sum_{j=1}^{m_n}(v_j(t) + \tau\dot{v}_j(t))i_j(t)dt = \int_0^T (v_z(t) + \tau\dot{v}_z(t))i_z(t)dt +$$

$$\int_0^T \sum_{j=1}^{m_n}(\frac{R_{min}}{2})(i_j(t) + \tau\dot{i}_j(t))i_j(t)dt. \tag{19}$$

Using (17), (19) can be written as

$$\int_0^T \sum_{j=1}^{m_n}(v_j(t) + \tau\dot{v}_j(t))i_j(t)dt =$$

$$\int_0^T [v_z(t) + \tau\dot{v}_z(t) + (\frac{R_{min}}{2m_n})(i_z(t) + \tau\dot{i}_z(t))]i_z(t)dt +$$

$$\int_0^T \sum_{j=1}^{m_n}(\frac{R_{min}}{2})(i_j(t) - m_n^{-1}i_z(t))i_j(t)dt +$$

$$\int_0^T \sum_{j=1}^{m_n}(\frac{R_{min}\tau}{2})(\dot{i}_j(t) - m_n^{-1}\dot{i}_z(t))i_j(t)dt. \tag{20}$$

Since $m_n^{-1}i_z(t)$ is the mean of $i_j(t)$ over all j, and $R_{min} > 0$, the integrand in the second term on the right side of (20) is nonnegative (as is easily shown by a standard quadratic form), e.g.

$$\sum_{j=1}^{m_n}(\frac{R_{min}}{2})(i_j(t) - m_n^{-1}i_z(t))i_j(t) \geq 0. \tag{21}$$

Also, (17) and (21) can be used to show that

$$\int_0^T \sum_{j=1}^{m_n} 2(\dot{i}_j(t) - m_n^{-1}\dot{i}_z(t))i_j(t)dt =$$

$$\sum_{j=1}^{m_n}(i_j(T) - m_n^{-1}i_z(T))i_j(T) -$$

$$\sum_{j=1}^{m_n}(i_j(0) - m_n^{-1}i_z(0))i_j(0) \geq K, \tag{22}$$

where K depends on the initial conditions only. Condition iii) in the statement of the theorem and $m_n \leq M$ mean that the first integral on the right side of (20) is bounded below by $-E_n(0)$, where $E_n(0)$ is the "initial energy in the impedance $[Z_n(s) + \frac{R_{min}}{2M}](1 + \tau s)$." This, with (20), (21), and (22), show that

$$\int_0^T \sum_{j=1}^{m_n}(v_j(t) + \tau\dot{v}_j(t))i_j(t)dt \geq K_n(0) \tag{23}$$

where $K_n(0)$ depends only on the initial conditions at $t = 0$, for $n = 1, 2, ..., N$. The resistor inequality $f_k'(i_k) \geq R_{min} + \epsilon$ shows that the nonlinear resistors in Fig. 7c have $f_k'(i_k) \geq \epsilon/2$. Thus the circuit satisfies the conditions of Thm. 2, except for the form of the cell impedances; in this regard, equation (23), summed over all the cells, is used in place of (13) for the proof.

ACKNOWLEDGEMENT

We sincerely thank Professor Carver Mead of Cal Tech for encouraging this work, which was supported by Defense Advanced Research Projects Agency (DARPA) Contract No. N00014-87-K-0825 and National Science Foundation (NSF) Contract No. MIP-8814612. We also thank Mr. George Syrmos and Professor Robert Newcomb of the University of Maryland at College Park for helpful comments.

REFERENCES

1. F.S. Werblin, "The Control of Sensitivity in the Retina," *Scientific American*, vol. 228, no. 1, Jan. 1973, pp. 70–79.

2. T. Kohonen, **Self-Organization and Associative Memory**, (vol. 8 in the Springer Series in Information Sciences), Springer Verlag, New York, 1984.

3. C.A. Mead and M.A. Mahowald, "A Silicon Model of Early Visual Processing," *Neural Networks*, vol. 1, no. 1, pp. 91–97, Jan. 1988.

4. C.A. Mead, **Analog VLSI and Neural Systems**, Addison-Wesley, to appear in 1988.

5. J. Hutchinson, C. Koch, J. Luo and C. Mead, "Computing Motion Using Analog and Binary Resistive Networks," *Computer*, March 1988.

6. M. Mahowald and C.A. Mead, personal communication.

7. B.D.O. Anderson, and S. Vongpanitlerd, **Network Analysis and Synthesis – A Modern Systems Theory Appraoch**, Prentice-Hall, Englewood Cliffs, NJ, 1973.

8. M. Vidyasagar, **Nonlinear Systems Analysis**, Prentice-Hall, Englewood Cliffs, NJ, 1970, pp. 211–217.

9. C. Desoer and M. Vidyasagar, **Feedback Systems: Input-Output Properties**, Academic Press, New York, 1975.

10. J.J. Hopfield, "Neurons with Graded Response have Collective Computational Properties Like Those of Two-state Neurons," *Proc. Nat'l. Acad. Sci.*, U.S.A., vol. 81, May 1984, pp. 3088–3092.

11. L.O. Chua, C.A. Desoer and E.S. Kuh, **Linear and Nonlinear Circuits**, McGraw-Hill, 1987, sect. 4.3.

12. P. Penfield, Jr., R. Spence, and S. Duinker, **Tellegen's Theorem and Electrical Networks**, MIT Press, Cambridge, MA, 1970.

13. M. Marden, **Geometry of Polynomials**, American Mathematical Society, Providence, R.I., no. 3, 1985, pp. 4–5.

Figure Captions

Figure 1: The cell in dotted lines, a photoreceptor and signal processor circuit using two MOS wide-range amplifiers, realizes spatial lateral inhibition by communicating with similar cells through a resistive grid. Bias adjustments ensure that the output impedance of amp #1 is much lower than that of amp #2.

Figure 2: Interconnection of cells through a hexagonal resistive grid. Ground connections are internal to each cell. Cells are drawn as 2-terminal elements with the power supply and signal output lines suppressed to emphasize that each is modelled as a black box characterized by its driving-point impedance seen from the grid terminal. The grid resistors will be nonlinear by design in many such circuits.

Figure 3: Elementary model for an MOS amplifier.

Figure 4: Simplified network topology for the circuit in Fig. 1. The capacitor that appears explicitly in Fig. 1 has been absorbed into the output capacitance of transamp #2.

Figure 5: Passive network with driving-point impedance identical to that of the cell model in Fig. 3 with the transamp modelled as in Fig. 4.

Figure 6: Illustration for the proof of Theorem 1. For each $\omega \geq 0$ the cell impedances $Z_n(j\omega)$ must all lie below some half-space boundary lying in the closed first and third quadrants. The boundary can vary with ω.

Figure 7: a) Nonlinear resistor connected to cells at nodes a and e. The dashed lines emphasize that each cell is also connected to other resistors, not shown. b) The resistor in a) can be split into a series string of two identical resistor pairs, where each pair consists of a linear resistor of value $R_{min}/2$ and a nonlinear resistor. If R_{min} is chosen to be less than the smallest value of $f'(i)$, then the resulting nonlinear resistors between nodes b, c, and d will still have strictly monotone increasing $v-i$ curves. c) The model for parasitics associated with the resistor in a) can include capacitors between any of the nodes b, c, d, and ground; it can not include any connected to a or e. d) Each cell impedance can be thought of as a multiterminal linear

element, shown in the dashed box, as seen from the external circuit.

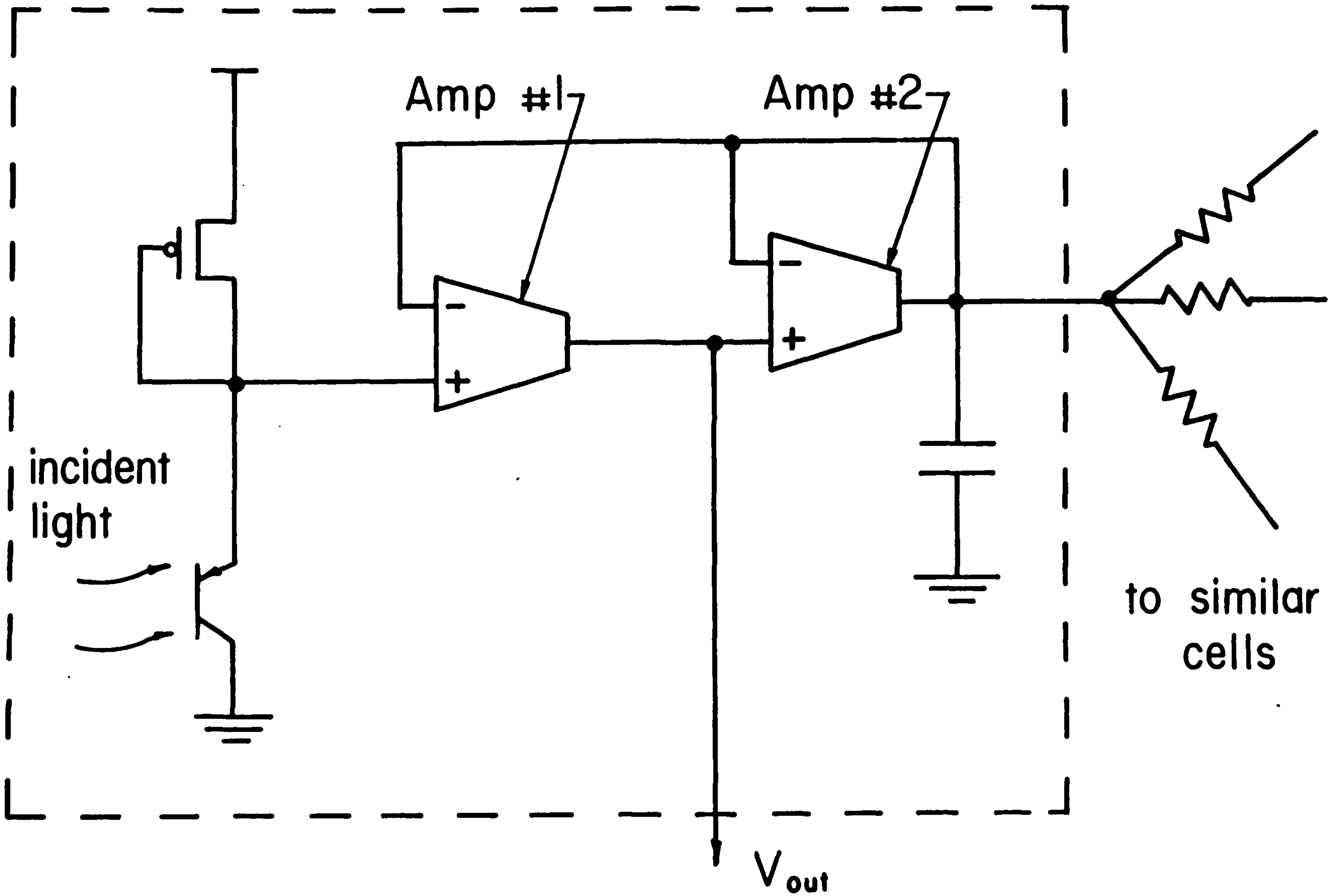


FIGURE 1

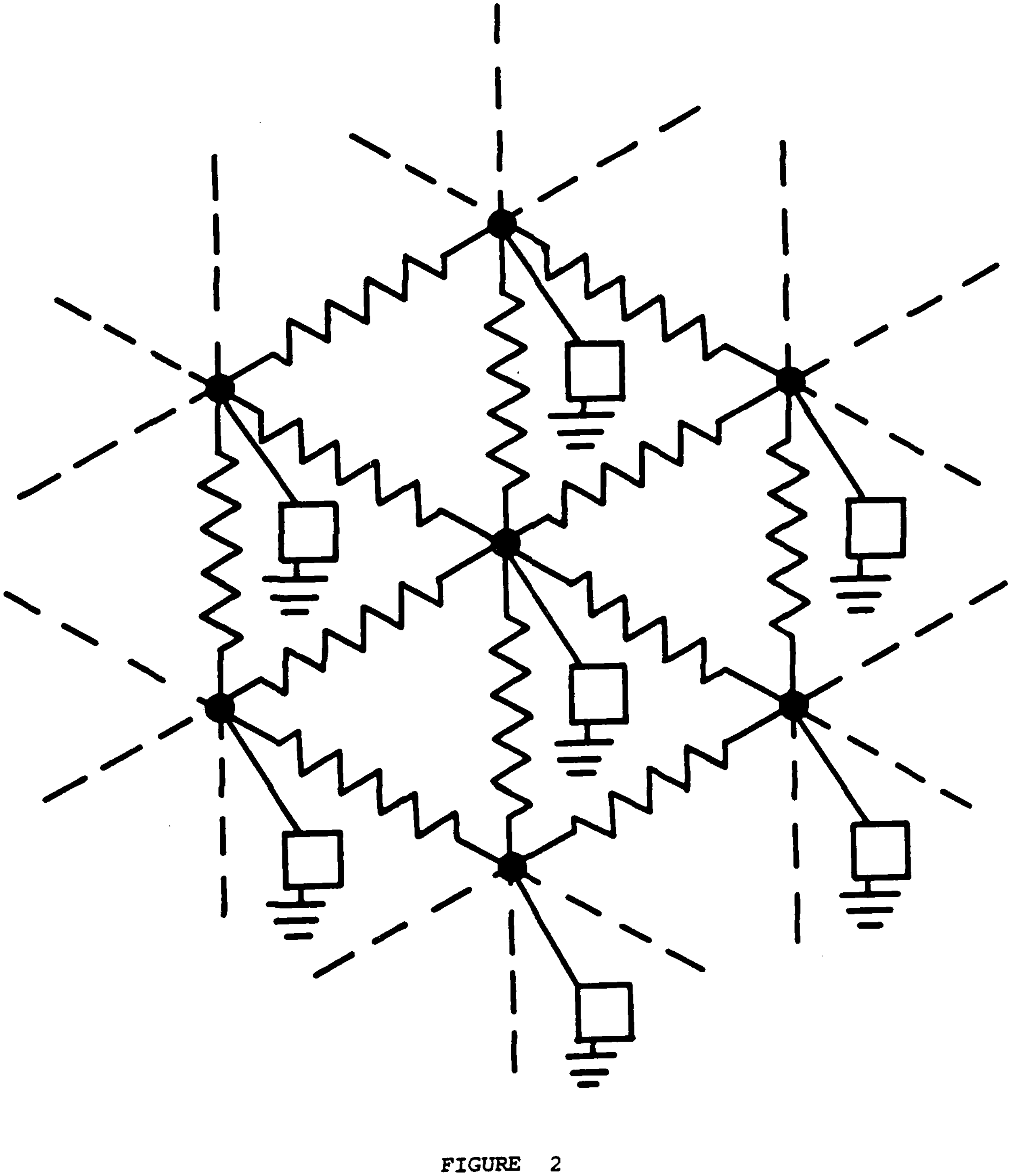


FIGURE 2

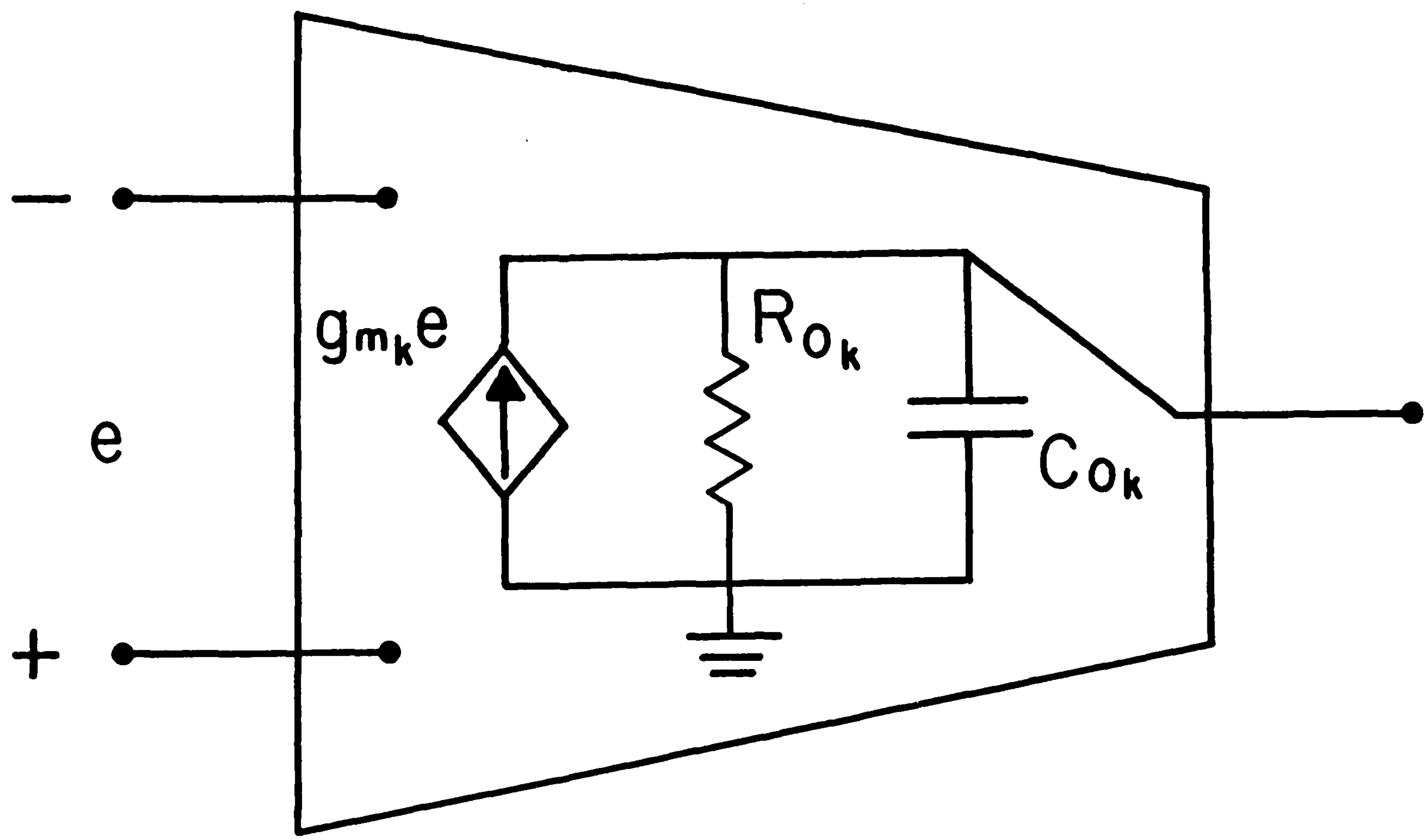


FIGURE 3

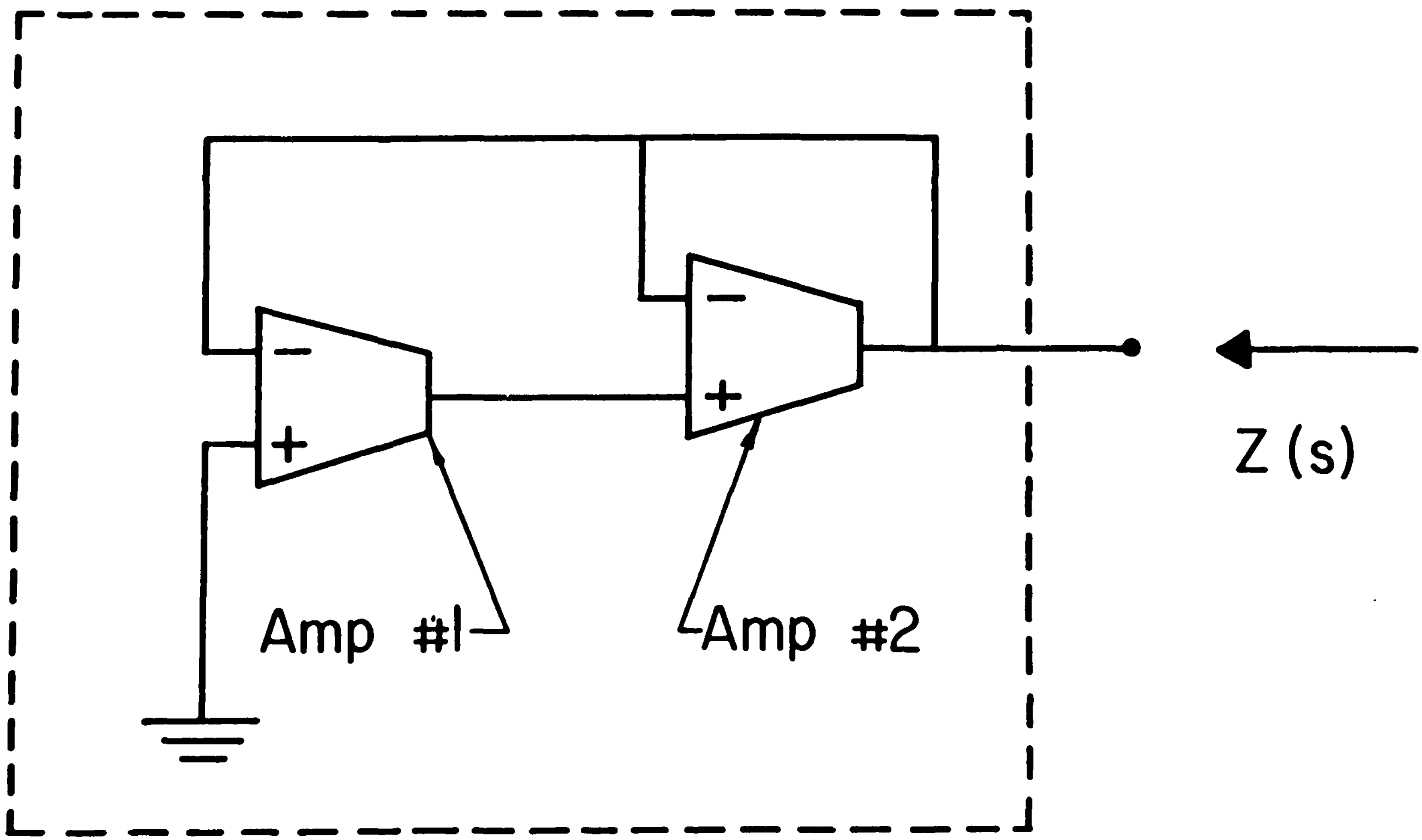


FIGURE 4

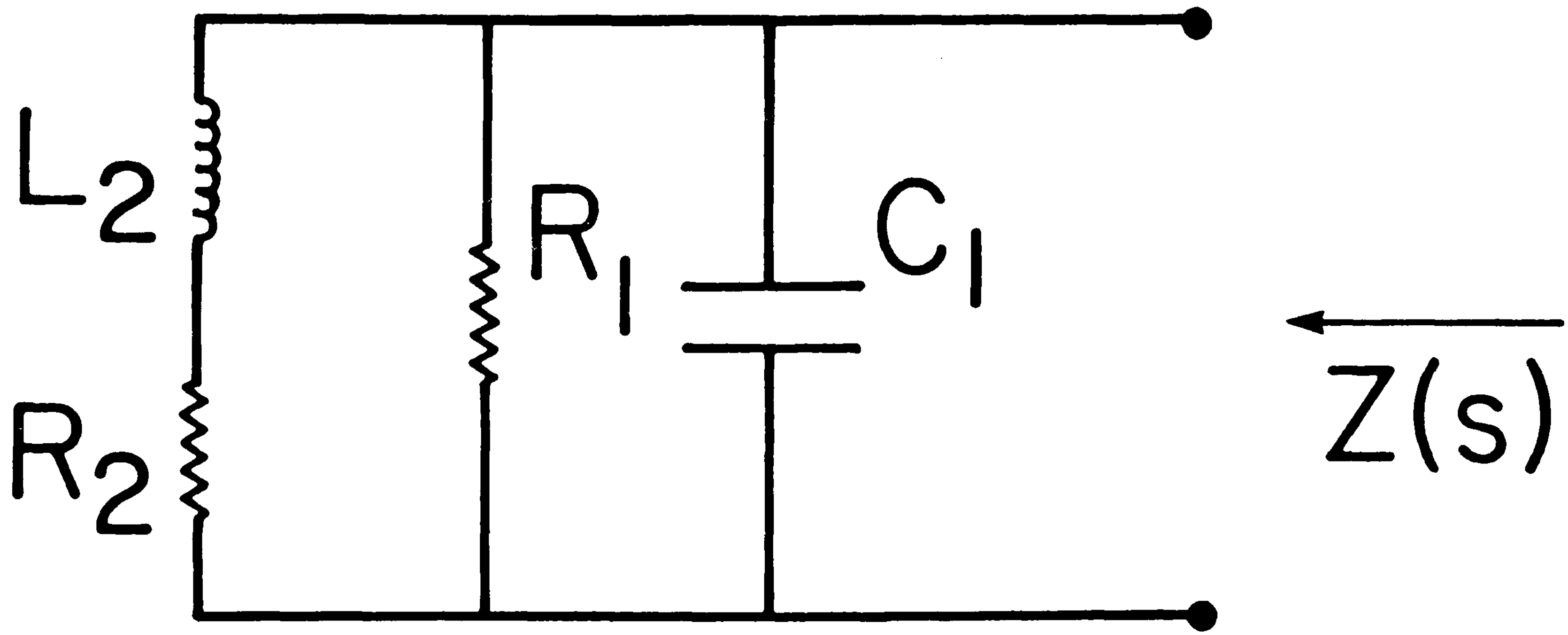
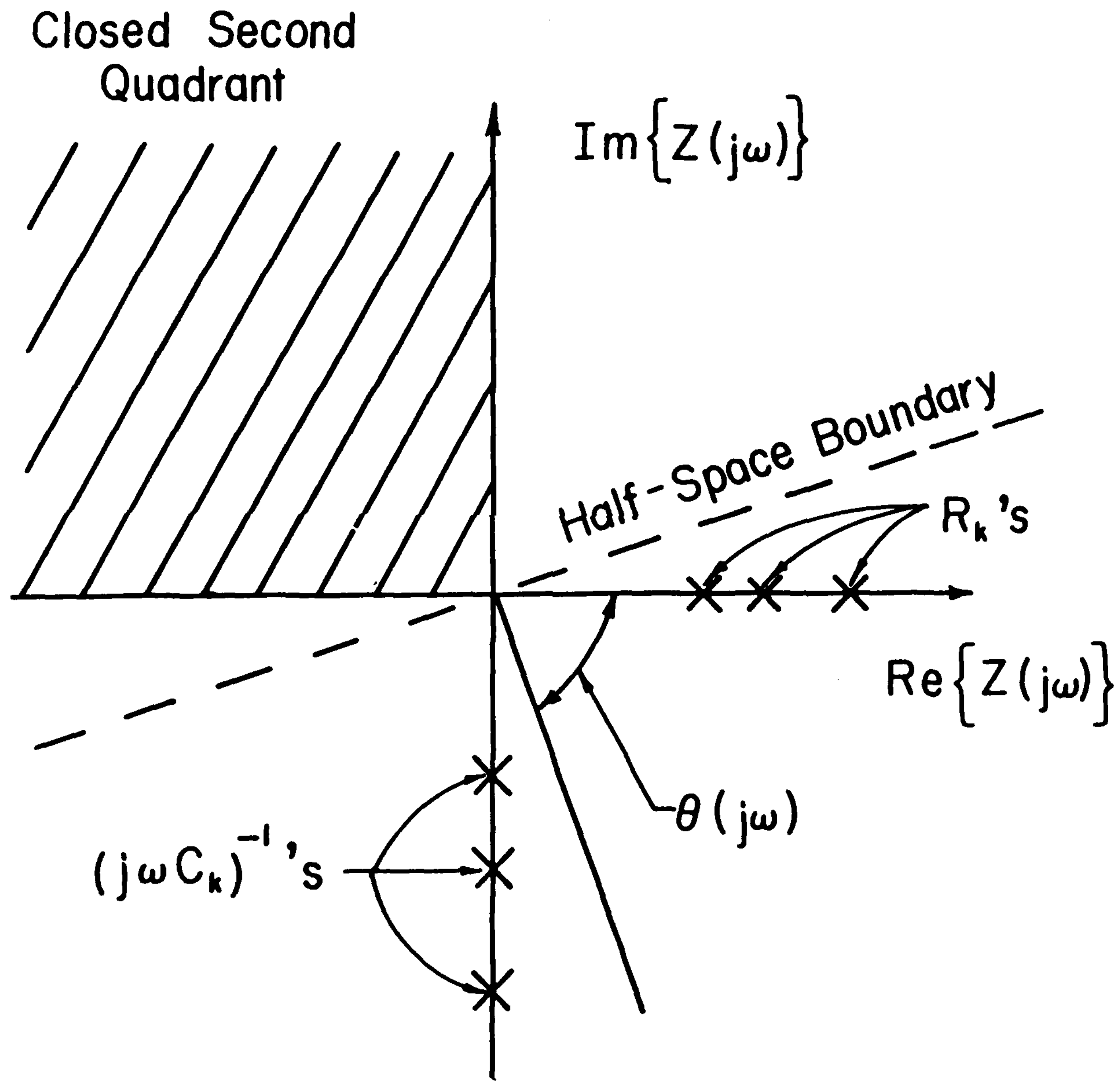


FIGURE 5



Closed Second Quadrant
Im{Z(jω)}
Half-Space Boundary
R_k's
Re{Z(jω)}
θ(jω)
(jωC_k)^-1's

FIGURE 6

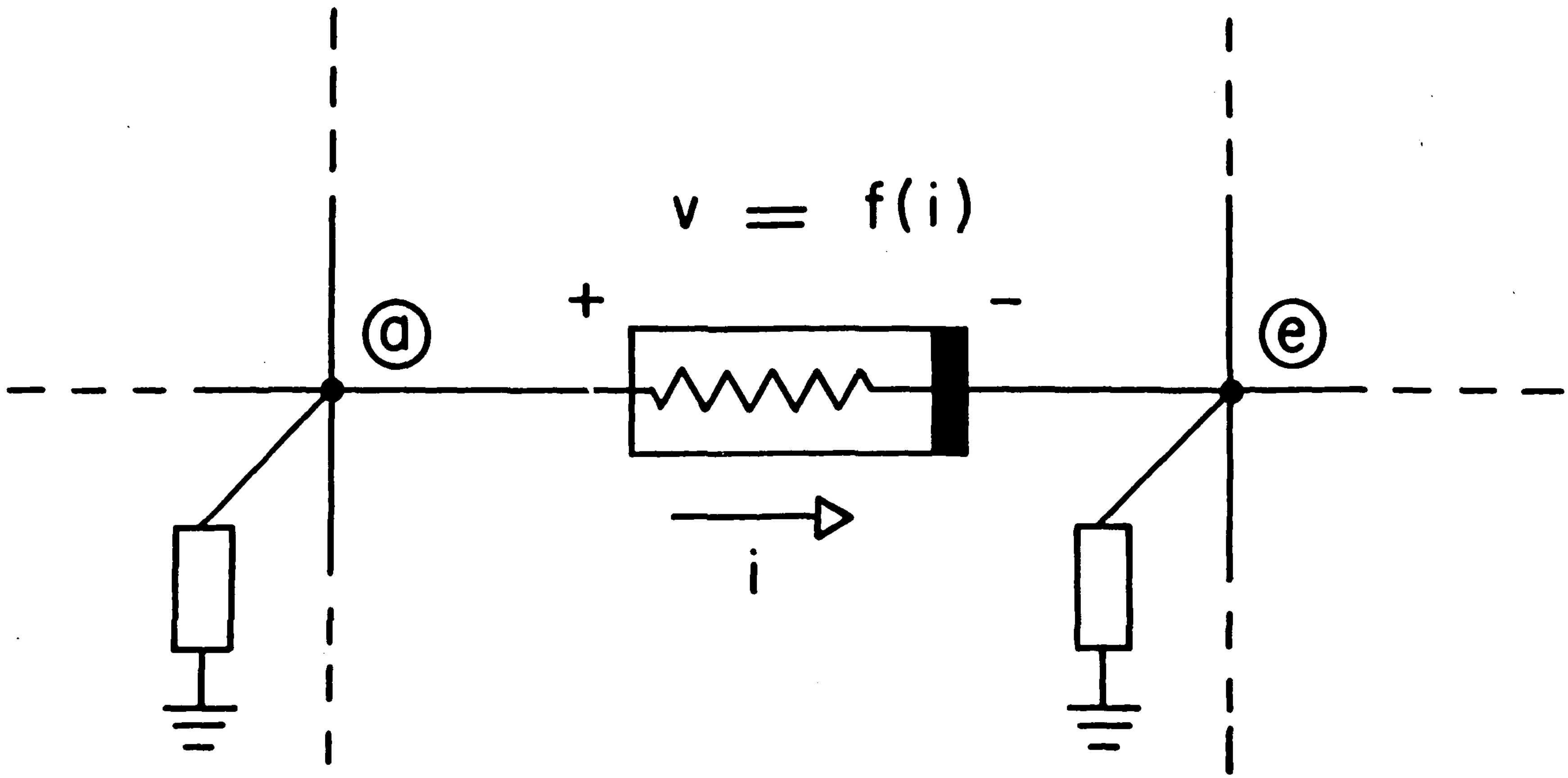


FIGURE 7a

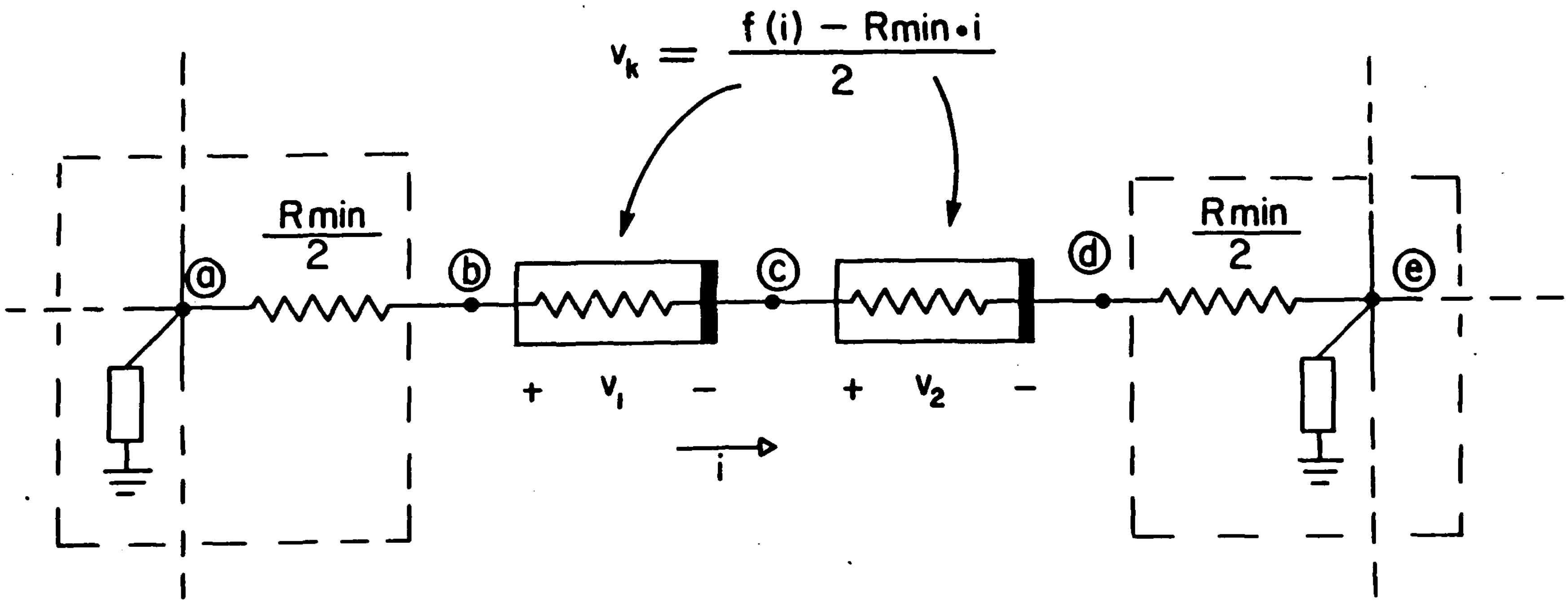


FIGURE 7b

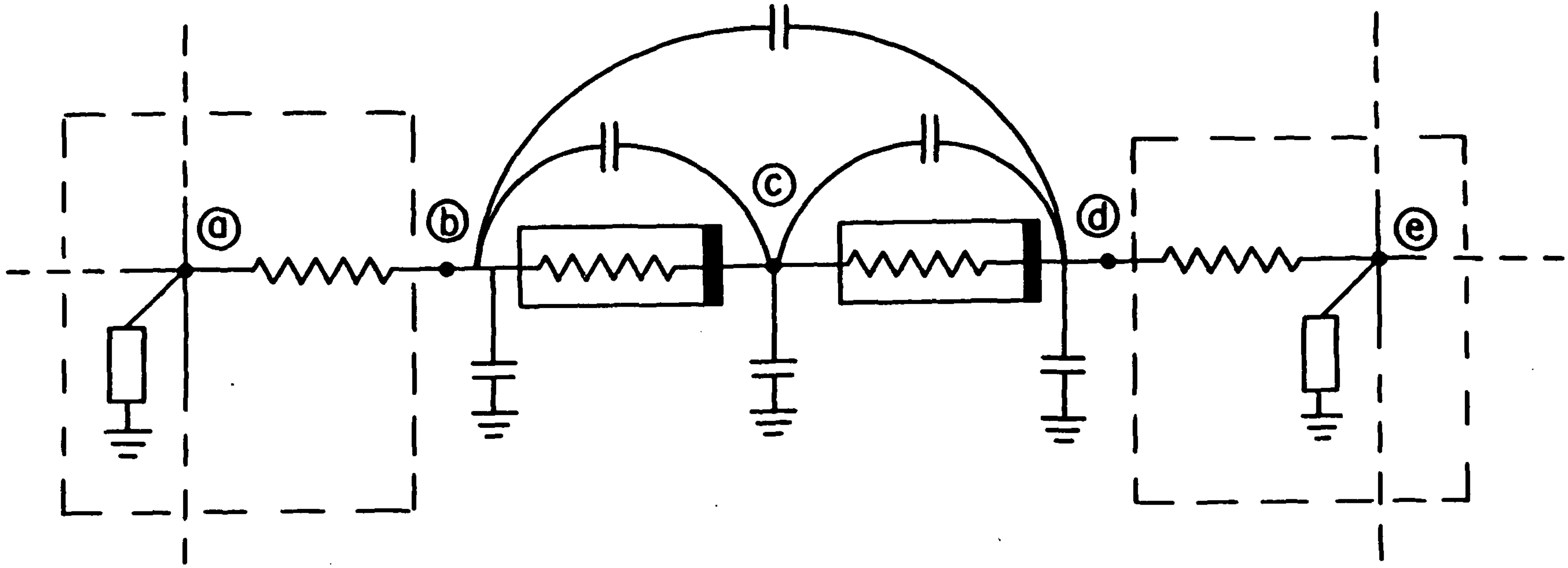


FIGURE 7c

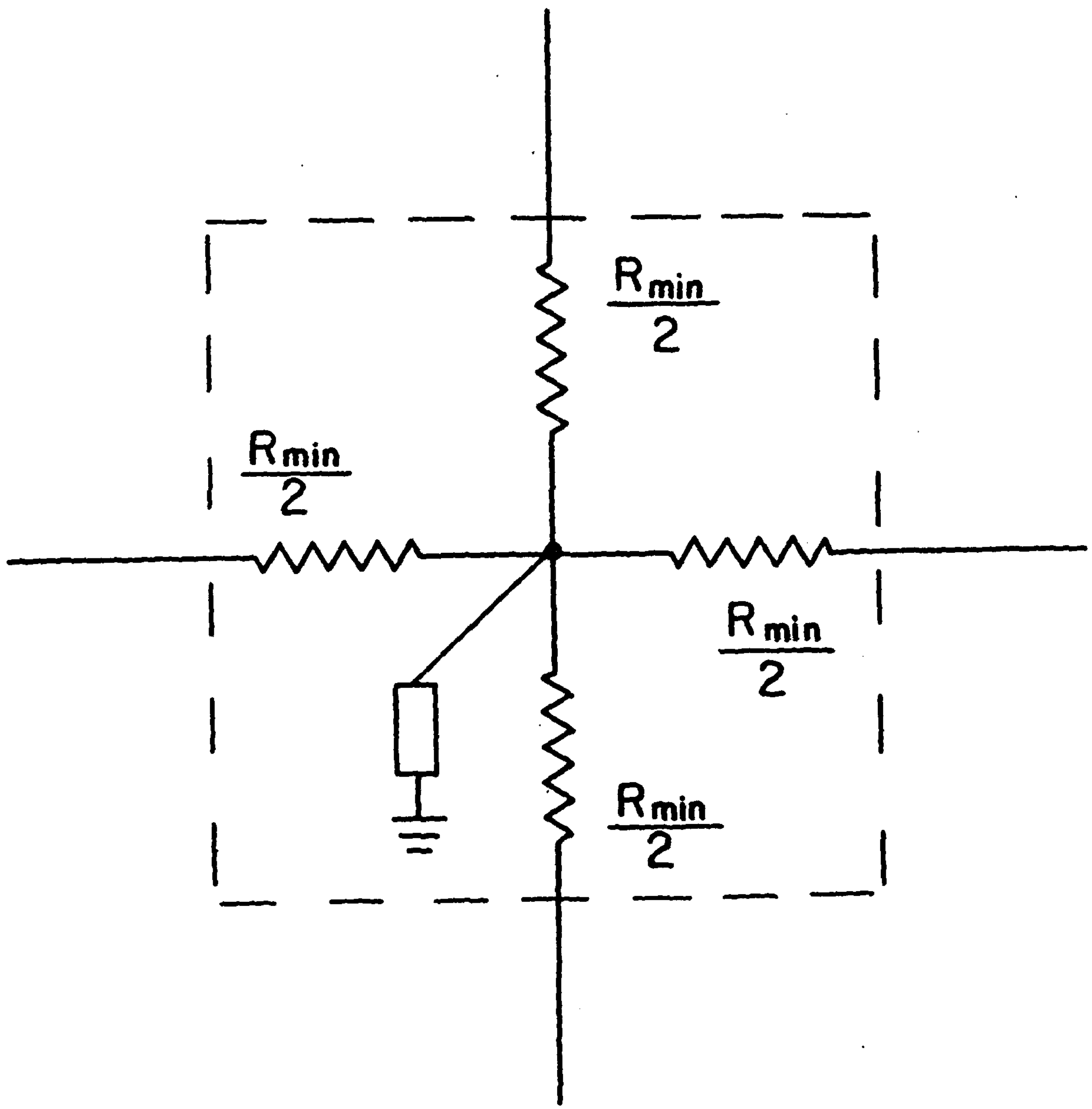


FIGURE 7d