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Description of the Derivation of the Collision Risk Model Used in the Vertical Separation Simulation Risk Model

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16. Abstract

This report presents a brief description of the derivation of the collision risk equations for the use on the vertical separation Midair Collision Simulation Risk Model. It also describes the estimation of the Collision Risk Model parameters for the current 2000-foot standard and the proposed 1000-foot planned vertical separation standard.

The model itself consist of specialized computer programs and systematic procedures that realistically and economically simulate aircraft flight-planned movements in the National Airspace System (NAS). These aircraft movements are based on flight plans and tracking data transmitted to Central Flow Control Facility (CFCF) from all the 20 centers that make up the NAS.

The task is to find the frequency, $N_a^\wedge$, with which a pair of aircraft flying at and above flight level (FL)290 would, by flight-planned intent, be proximate (near each other) in the NAS. The purpose of this mathematical model is to make a quantitative judgment about the safety of the proposed 1000-foot vertical separation, and provide an estimate of the risk of midair collision due to the loss of 1000-foot planned vertical separation.

As the result of this first phase of the study, it is recommended that the model be enhanced to do the following: (1) step climbing, and (2) point-to-point navigation.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

INTRODUCTION

In February 1982, the Federal Aviation Administration (FAA) publicly announced its intention to study the feasibility of reducing the vertical separation standard from 2000 to 1000 feet at and above flight level (FL)290. Since then, the FAA Technical Center vertical separation project has conducted 10 separate field data collections, to evaluate the measurement methodology required to empirically estimate vertical separation performance. To support this analysis, it is necessary to realistically simulate the flight-planned aircraft movements in the National Airspace System (NAS), and record the resulting proximities at and above FL290 which would take place if the proposed 1000-foot vertical separation standard was implemented.

For the purpose of this work, a proximity is defined as the event where a pair of aircraft in level flight and on adjacent flight levels are within 5 nautical miles horizontally and 1000 feet vertically. This simulation model is needed to make quantitative judgments about the safety of 1000-foot vertical separation to determine two very important values: (1) an estimate of the risk of midair collision due to the loss of 1000-foot planned vertical separation, and (2) a target level of safety which is acceptable to decision makers to allow implementing a 1000-foot vertical separation standard.

This model consists of specialized computer programs and systematic procedures that realistically simulate aircraft movements under varying separation standard environments. These aircraft movements are based on flight plan and tracking data received from the Central Flow Control Facility (CFCF) which are used as input to the model.

The key to this study is the FAA Technical Center modeling and analysis of the risk of midair collision. One important element which will affect the risk is the likelihood that pairs of aircraft will simultaneously be on adjacent flight levels and in horizontal proximity, thus giving rise to the potential for midair collision if vertical position keeping is inadequate.

Evaluation of this exposure-to-risk is a complex task because the behavior of the air traffic control system with a vertical separation standard of 1000 feet above FL290 must be studied before the system exists. Quantitative investigation of this issue was the motivation for developing the Midair Collision Simulation Risk Model, utilizing Ronald Hershkowitz's technical report, Collision Risk Model For North Atlantic Region.

The derivation of the collision risk equations depends on a clear understanding of four key concepts:

1. The tracking systems.
2. The proximity shell.
3. The separation vector.
4. The collision slab.

THE TRACKING SYSTEMS

The route structure of the air traffic control system may consist of parallel or intersecting tracks. If parallel, the track system is used where each aircraft is clear to fly down tubes, nominally centered at specific vertical and lateral positions. The separation distance between the center lines of these usable tracks is chosen in order to maintain safety standards. For those aircraft entering a particular track, the allowable times are set in accordance with the along-track safety requirements as shown in figure 1.

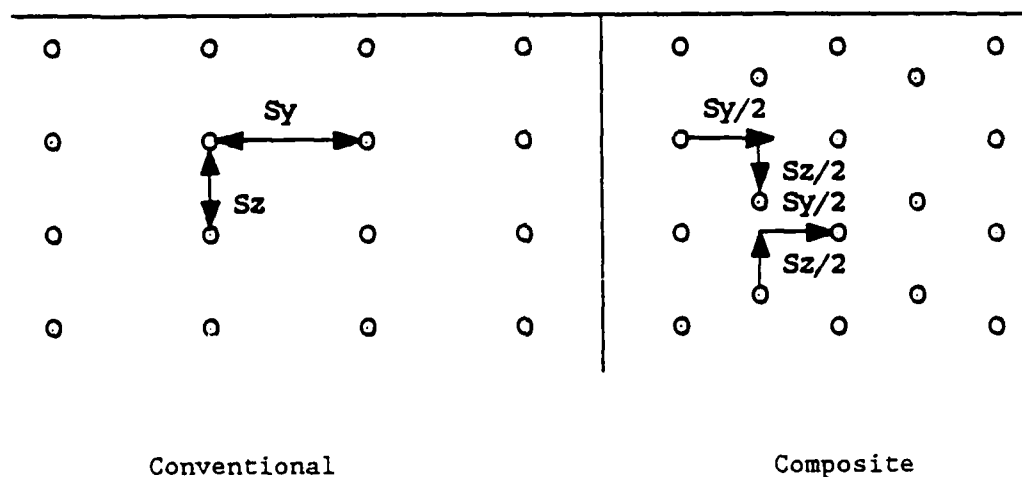Conventional   Composite

FIGURE 1.   TRACKING SYSTEMS

In the conventional, or rectangular system, each flightpath has two vertical and two lateral neighbor paths. The composite system path is bordered by four diagonal paths, in addition to the two vertical and two lateral found in the conventional system. In other words, the composite system is two rectangular systems offset from one another by half a standard separation in each dimension. If the route system contains crossing routes, the additional risk of collision (due to the loss of vertical separation at the intersections) must be taken into account. Under the assumptions of a parallel track system and no air traffic control (ATC) loop errors, midair collision can occur only because of imperfections in navigating and piloting or flying errors on the part of one or both members of an aircraft pair.

One way to approach the estimation of the collision risk is to sum up the individual risks of collisions to all aircraft due to flying errors in order to give the expected number of accidents (aircraft involved in a collision) in the time period of interest. Figure 2 illustrates the general setting of an aircraft pair potentially exposed to the risk of collision.
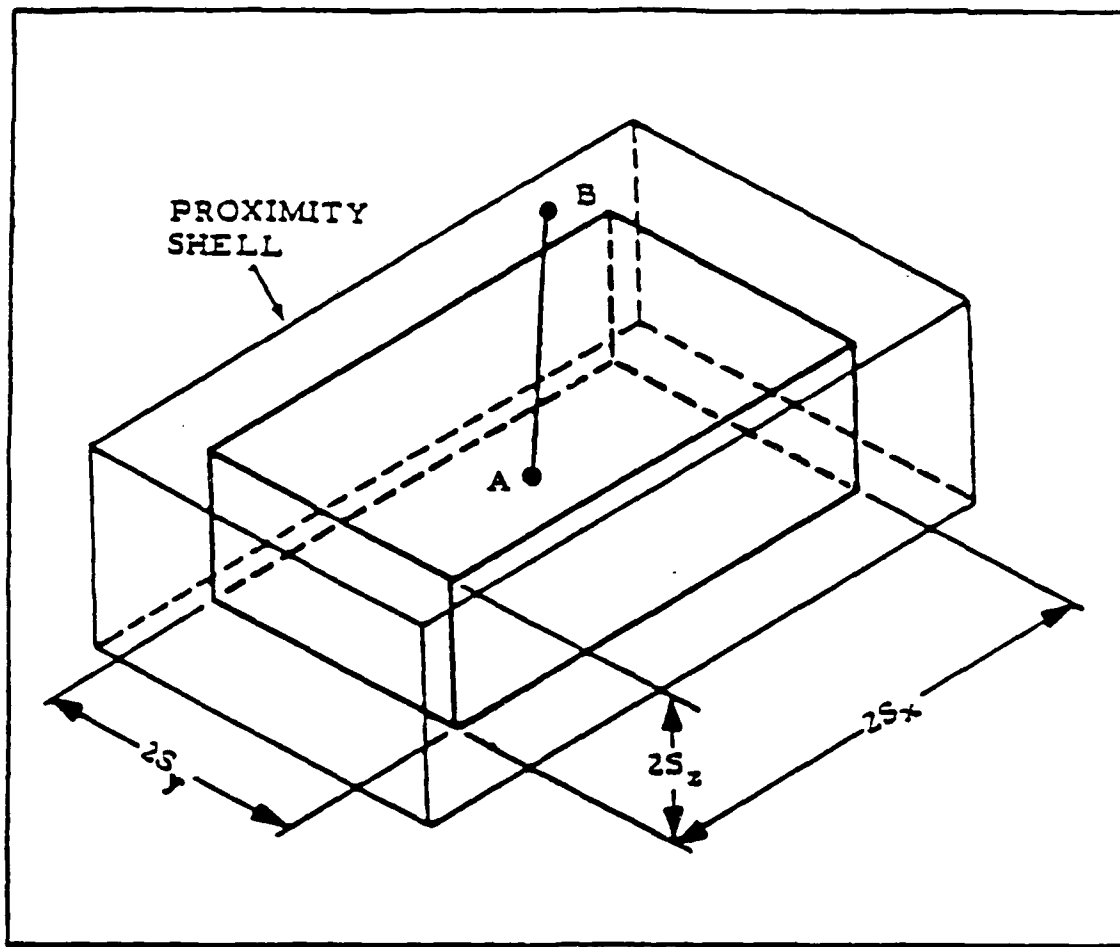

FIGURE 2. EXPOSURE TO RISK

The intended or planned position of an aircraft is shown at A. Surrounding A is an inner box of length $2S_x$, width $2S_y$, and height $2S_z$, with A located at a distance S_x from either end of the box, S_y from either side, and S_z from the top and bottom. In fact, this box is the representation of the imposition of ATC separation standards upon the aircraft located at A. The flight-planned position of a second aircraft is shown at B. A significant risk of collision between A and B will arise only when the planned position of B is on or very near to one of the faces of the inner box at A, and the risk will fall off very rapidly as the distance of B from the

inner box increases. The second, outer box about A is called the Proximity Shell of A. For positions B outside this shell, the risk of collision between the two aircraft can be considered negligible, while for position B, within the proximity shell, two aircraft are said to be proximate and exposed to the risk of collision. Therefore, it is necessary to take into account (1) the expected number of times each aircraft has another aircraft in its proximity shell, (2) the expected length of time this other aircraft will remain in the shell, and (3) the path of this aircraft through the shell.

THE SEPARATION VECTORS

Figure 3 shows the intended or planned position of two aircraft at A and B and the true positions at A' and B' resulting from flying errors.
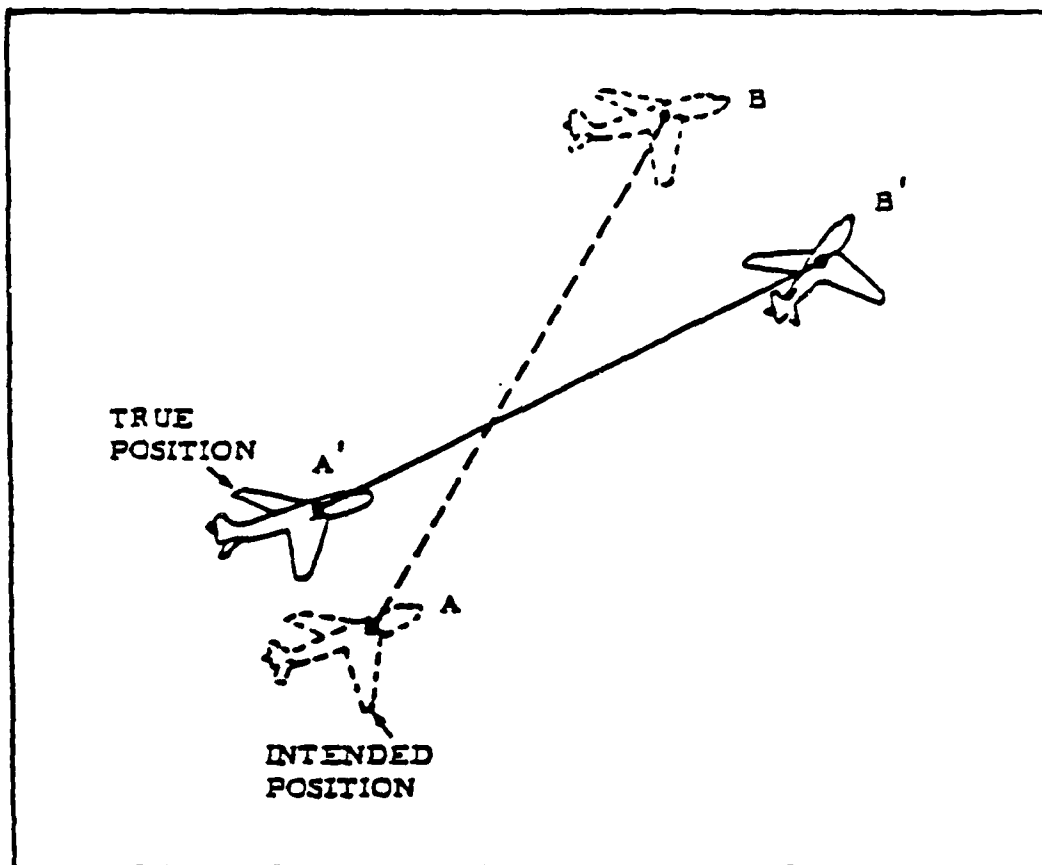

FIGURE 3.   SEPARATION VECTORS

The collision risk between an aircraft pair is the chance that the time varying separation vector A'B' shrinks small enough such that the aircraft collide. This chance depends upon the intended separation vector AB and the flying errors committed by both aircraft. Both the intended position vector AB and the true position A'B' are time dependent.

4

The simplest way to represent the positions of the two aircraft under study is in terms of components of motion (which are assumed to be independent of one another) along the cartesian coordinates (X,Y,Z) corresponding to the along-track, cross-track, and the vertical position.

Assuming that each aircraft in the track system is the same size, and representing each aircraft as a box with sides λ_x, λ_y, and λ_z which represent the aircraft's outer metallic dimensions. The collision process can be represented as the motion of one aircraft relative to the other, as shown in figure 4 below.

FIGURE 4.   COLLISION SLAB
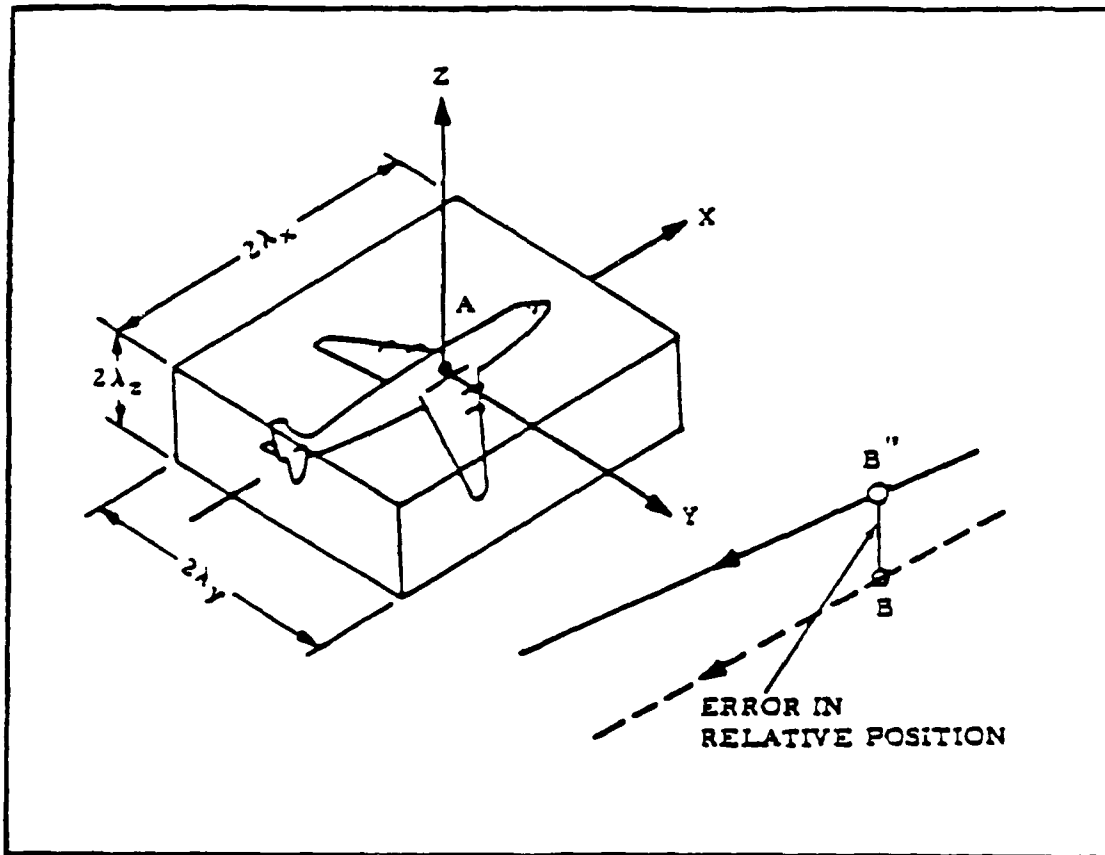
The intended positions of the two aircraft are shown as A and B in the proximity shell figure 2, while the change in separation due to the combined flying errors of the two aircraft is shown by the vector BB". The box of length $2\lambda_x$, width $2\lambda_y$ and height $2\lambda_z$ with centroid of the aircraft at position A midway between any two opposing faces is called the Collision Slab.

Not all pairs of aircraft pose threats to each other. Only those aircraft nominally close enough to be drawn, with some threshold probability, to within the collision slab dimensions as a result of flying errors, are considered as potential hazards to one another. A significant risk will occur if and only if one aircraft has an intended flightpath which enters, or is close to, the proximity shell of another. The risk is assumed to fall off rapidly outside the shell.

Taking the motion of one aircraft relative to the other, the collision process may be looked on as a particle bombarding a slab, or it may be viewed mathematically as the entry of a particle, B, representing the second aircraft, into the collision slab. Therefore, the collision rate is given by the expected number of times the particle enters the slab through (1) the sides, (2) the ends, and (3) the top and bottom.

The collision rate for an aircraft pair is developed for a period of time short enough that the intended separation vector AB may be considered to be essentially constant. The calculation of the collision risk in each dimension is done in a three-step process:

1. The collision rate (CR) for a proximate aircraft pair is determined.

2. The average time, (T), in which two aircraft are proximate during the time a flight is determined.

The product of these two quantities is the collision risk during the flight period:

Collision Risk $= (CR) * (T)$

3. Determine the expected number of accidents during the time period of interest, normally 10 million track system flying hours, N_a.

It is first necessary to divide T by H (total hours), that would give the average number of flying hours over which the proximity was calculated, which gives the average collision rate per flying hour. This result is then multiplied by two in order to determine the average accident level per hour of flight (since each collision is counted as two accidents): then multiplying by 10 million hours to give the desired quantity, the frequency:

$N_a = 2(10**7) * (CR) * (T/H)$.

where,

H - represents the total number of flying hours being considered.

THE FREQUENCY

The frequency at which particle B enters the slab through either end is obtained by multiplying the probabilities that particle B's Y and Z coordinates are within the distances λ_y and λ_z of the center of the slab, P_yP_z, by N_x the frequency at which its x-coordinates lie within λ_x of the center. For the constant intended separation $(_{AB})$, this is written as:

$$(N_xP_yP_z)(_{AB})$$

The flying errors of either proximate aircraft are assumed to be dimensionally independent. Similarly, the frequency of B entering the slab through top or bottom is:

$$(N_zP_xP_y)(_{AB})$$

and through either side:

$$(N_yP_xP_z)(_{AB}),$$
where,

P_x - is the probability that B's x-coordinate is within λ_x of the center of the collision slab; i.e., the proportion of time the aircraft spends in this condition.

P_y - is the probability that the cross-track separation is less than λ_y; i.e., the proportion of time the aircraft is in this condition.

P_z - is the probability that the vertical separation is less than λ_z; i.e., the proportion of time the aircraft spends in this condition.

N_x - is the frequency with which B enters the slab through the ends; i.e., the expected frequency with which the along-track separation shrinks to less than λ_x.

N_y - is the frequency with which B enters the slab through the sides.

N_z - is the frequency with which B enters the slab through the top and bottom.

Therefore, the expected collision rate, $CR(_{AB})$, for an aircraft pair during a time period sufficiently short that the intended separation vector, $_{AB}$, may be assumed to be constant is given by:

$$CR(_{AB}) = (N_xP_yP_z)(_{AB}) + (N_yP_xP_z)(_{AB}) + (N_zP_xP_y)(_{AB})$$

Now, this equation must be evaluated for all possible intended separation vectors, AB, planned for the aircraft pair during a flying period and the individual contributions properly summed to give an overall collision rate for the desired time period.

7

AN EXAMPLE OF HOW THE COLLISION RISK MODEL IS USED TO ESTIMATE THE RISK

Consider a pair of aircraft at adjacent flight levels, flying in the opposite directions, a typical way in which the air traffic control system is used, as shown in figure 5 below.
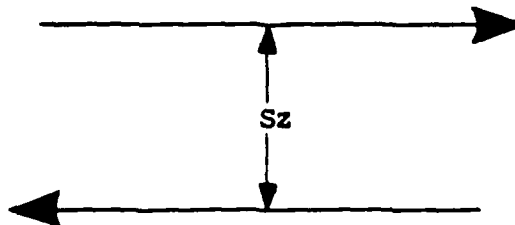

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FIGURE 5.   ADJACENT FLIGHT LEVELS AND OPPOSITE DIRECTIONS

What is the risk of midair collision due to the loss of vertical separation S_z?

$$CR_{az} = N_x P_y P_z + N_y P_x P_z + N_z P_x P_y$$

$$= P_x P_y P_z (1/t_x + 1/t_y + 1/t_z)$$

For a pair of aircraft planned to pass in opposite directions, the average duration of the event is t_x, in other words, the average time spent by a particle in the collision slab is simply the total distance it travelled while traversing the slab divided by its average speed.

therefore;

$$t_x \ll t_y, \ t_z$$

so,

$$1/t_x \gg 1/t_y, \ 1/t_z$$

$$CR_{az} = P_x P_y P_z * 1/t_x = N_x P_y P_z$$

i.e., CR_{az} is the product of overlap times properties in y and z coordinates multiplied by the frequency with which they pass. The task is to find an estimate for:

P_y - using radar data collection for a pair of aircraft assigned to the same route, $(P_y(0))$.

P_z - using radar data collection for a pair of aircraft assigned to adjacent flight levels, $(P_z(S_z))$.

N_x - frequency which is the count of passing per system flying hour for a pair of aircraft planned to be at adjacent flight levels on the same route; i.e., the same ground track, either on a jet route or point-to-point navigation.

SIMULATION OF THE PROPOSED 1000-FOOT VERTICAL SEPARATION, AND THE CURRENT 2000-FOOT VERTICAL SEPARATION STANDARD

To characterize the demand on the NAS, the model utilizes flight plan data provided by the CFCF. Mainly, the flight plan message (FZ), the boundary crossing message (UZ), and the tracking message (TZ). This information covers one day's actual flight-planned demand by all users of the system. The model will identify flight plans which include en route operations at and above FL290. The model also separates and stores the relevant demand information from this reduced group of users. Such information for a flight includes origin and destination points, route of flight, requested flight levels, times of boundary crossing, aircraft identity and type, true airspeed and planned groundspeed for the various flights.

This very important and unique model is capable of simulating variable separation. Mainly, the current 2000-foot vertical separation standard at and above FL290. More importantly, the proposed 1000-foot vertical separation standard at and above FL290. This mathematical model is needed to make quantitative judgment about the safety of 1000-foot vertical separation to determine two very important values: (1) an estimate of the risk of midair collision due to the loss of 1000-foot planned vertical separation, and (2) a target level of safety which is acceptable to decision makers and the world aviation community at large to implement a 1000-foot vertical separation standard at and above FL290. The model consist of the following functions.

NETWORK STRUCTURE.

The jet route structure of NAS is a set of paths in the horizontal plane through which aircraft can fly from one point to another. A route is composed of great circle segments connecting the locations of very high frequency (VHF) omnidirectional radio range (VOR) and other navigational aids, and is defined as a series of these segments. The purpose of this function is to provide a file system which is used to relate geographical positions and aircraft information of all points and fields that exist on each of the flight plans.

This function performs the task of processing the AIRWAY and navigational aid (NAVAID) data tapes received from the National Data Center (NDC) and the standard instrument departure (SIDS) and Standard Terminal Arrival Route System (STARS) tape files received from the National Oceanic and Atmospheric Administration (NOAA). The result of this important process is the Network Structure data base and a series of indices that enable quick access of the data base. As a part of this function, the sequential access Ascent and Decent rate files is converted to a relatively direct access file. This conversion allows for a quick access of aircraft flight data in the Input Demand Profiles process.

INPUT DEMAND PROFILE.

The purpose of this function is to reduce and prepare the input data to establish the location and disposition of an aircraft flying at and above FL290 in a given geographical area at fixed-time intervals. The input data for this function are received from CFCF and composed of variable length records ordered by receipt time. The purpose of the data reduction is to: (a) identify each flight segment that contains the total track history and eliminate all records associated with any segment that does not have activity at and above FL290, or (b) has insufficient data to reconstruct the flightpath of that segment. In order to determine the geographical position of each aircraft, a flight history for each aircraft is assembled. Using this information, both time and geographic reference points are established. The exact location of each aircraft is then calculated at fixed-time intervals. The end products of this process are a sorted Simulation File to be used in the next process, and the sorted System Load Airway file and the System Load Point-to-Point navigation file to be used in the System Load process.

NETWORK SIMULATION.

The Network Simulation performs the task of processing the sorted Simulation File produced as the end product of the Input Demand Profile. The purpose of this function is to count and report the number of the proximities that occur in a given time period for the chosen airspace. Using the Input Demand Profile as input, the distance between aircraft is measured at each time iteration. Each proximity is accounted for and reported. The resulting simulation is similar to that of a radar sweep, with aircraft location being identified at precise time intervals.

SYSTEM LOAD.

This function will report the system load or flight activities on each of the jet routes being used and identify point-to-point navigation in the geographical area being simulated. The inputs to this process are the sorted System Load Airway file and the System Load Point-to-Point file created as part of the Input Demand Profile process.

The end product is a set of reports that lists the system routes and identifies the Traffic Loads file. This file consist of records that identify activities on the elementary segments of the jet route being simulated. Each of these records contains the jet route number, the jet route segment number which was provided by the Network Structure data base, the fixes that delineate the position of the identified jet route, a time of arrival at each of the fixes, and the altitude of each fix.

If the model is executing a 1000-foot vertical separation standard, it will establish new cruising altitude for the flight segment using the following method as showing in tables 1, 2, 3, and 4.

TABLE 1.   EAST TO WEST ROUTES

Current Flight Levels	Proposed Flight Levels
310	300, 320
350	340, 360
390	380, 400
430	420, 440
470	460, 480

TABLE 2.   ROUTES REDISTRIBUTION (EAST TO WEST)

First Redistribution	Second Redistribution
50% of FL310 to FL300	50% of FL310 to FL320
50% of FL350 to FL340	50% of FL350 to FL360
50% of FL390 to FL380	50% of FL390 to FL400
50% of FL430 to FL420	50% of FL430 to FL440
50% of FL470 to FL460	50% of FL470 to FL480

TABLE 3.   WEST TO EAST ROUTES

Current Flight Levels	Proposed Flight Levels
290	290, 310
330	330, 350
370	370, 390
410	410, 430
450	450, 470

11

TABLE 4.   ROUTES REDISTRIBUTION (WEST TO EAST)

First Redistribution	Second Redistribution
50% remain at FL290	50% of FL290 to FL310
50% remain at FL330	50% of FL330 to FL350
50% remain at FL370	50% of FL370 to FL390
50% remain at FL410	50% of FL410 to FL430
50% remain at FL450	50% of FL450 to FL470

The simulation process is a three-dimensional grid containing many cells arranged side by side with two flight levels of the airspace being simulated. Each one of the cells represents a window approximately 5 square nautical miles latitude by longitude and one flight level. The dimensions of 120° latitude by 95° longitude by two flight levels occurrences approximately represents the area of a given center. The first dimension represents 5 minutes of latitude, the second dimension represents 9 minutes of longitude and the third dimension represents one of the two flight levels being processed.

The coordinates of the simulation file record are converted to the logical coordinates, then stored in the first and second locations of the coordinates tables. The logical coordinates are calculated by obtaining the integer part of lat or long minus the lower boundary of the lat or long divided by the logical constant of 5 for the latitude and 9 for the longitude, symbolically as follows:

    logical lat = lat - (lower boundary lat) / 5

    logical long = long - (lower boundary long) / 9

By converting the coordinates of the stored simulation file record into the logical coordinates of the grid, the aircraft's relative position can be established, and any aircraft within the horizontal proximity will be contained in either cell or a cell adjacent to it. For the vertical proximity to be established, a cell on the lower level of the grid need only be checked against the cell directly above it and those adjacent to it. Instead of loading the large three-dimensional grid described above in memory, the logical coordinates are represented in a three-dimensional table (150,8,2) in which all aircraft on the two adjacent flight levels are listed. Each table appearance include the logical coordinates calculated from the simulation file coordinates, and its subscripted location corresponds to the aircraft data being represented.

The first table dimension subscripted at 150 refers to the aircraft that is being simulated at a particular time interval. Defining this array to be 150, then the maximum number of aircraft on one flight level that can be simulated for a given time interval is limited to 150. If there is a higher demand on the system, then this array has to be made larger in order for the model to execute.

The second table dimension subscripted at 8 refers to the aircraft's corresponding flight information the Simulation File record, including latitude, longitude, altitude, ground speed, a pointer to the flight plan file and a pointer to the detailed portion of the flight route being used. The third dimension subscripted at 2 represents one of the two adjacent flight levels being processed.

The model divides the simulation process into two subfunctions. The first identifies and categorizes each proximity, by writing a .ata record to the proximity history file for each occurrence. The second subfunction processes the proximity history file produced by the first subfunction into two reports, consisting of the four proximities categories, as well as the coaltitude and along-track reporting requirements. The first report lists the proximities by aircraft identity (ACID) pair and Time (duration) and classifies them by type. The other lists the proximities by Time and ACID pairs, to cross-reference multiple proximities at the same time.

In other words, the simulation model looks at a window as showing in figure 6. Where Lon1, Lon2, Lat1 and Lat2 represent the geographical area being simulated for a 4-hour period. These geographical locations and the time must be supplied by the user to the model in order for it to execute.

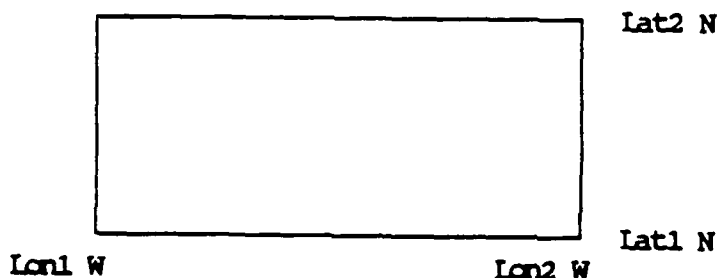Lat2 N

Lat1 N

Lon1 W   Lon2 W

FIGURE 6.   WINDOW

then, gives a snapshot account of the occurring proximities every 15-second interval with a true north heading. The model also keeps a record and stores the following:

    1. Number of proximities and duration, t_{xy}, in the window.

    2. Total flying time, H, in the window.

then calculates the probability of the horizontal overlap:

$$P_{xy} = t_{xy} \ / \ H.$$

In the case of the proposed 1000-foot planned vertical separation standard, the collision rate, $(C.R.)_{az}$, equations are given by:

$$(C.R.)_{az} = N_{xy} * P_z(1000) + N_z(1000) * P_{xy}$$

where,

N_{xy} - is the frequency with which an aircraft's separation from another becomes less than the horizontal overlap distance.

P_z - is the proportion of time that a typical aircraft pair with 1000-foot planned vertical separation spends in vertical overlap.

N_z - is the frequency with which an aircraft pair with planned 1000-foot vertical separation becomes less than the vertical overlap distance.

P_{xy} - is the proportion of time that a typical aircraft spends in horizontal overlap.

The frequency term, N_{xy}, can be expressed as the ratio of P_{xy} and the corresponding average durations of overlap in the horizontal dimensions, t_{xy}. Thus,

$$N_{xy} = P_{xy} / t_{xy}$$

similarly

$$N_z(1000) = P_z(1000) / t_z$$

then:

$$(C.R.)_{az} = P_{xy} * P_z(1000) * (1/t_{xy} + 1/t_z)$$

RECOMMENDATIONS

The Federal Aviation Administration (FAA) Technical Center recommends continuing the development of the Midair Collision Simulation Risk Model by enhancing the existing model to enable the FAA to realistically and economically study the probability of the vertical and the horizontal overlap resulting from changes occurring to the National Airspace System (NAS) navigational and the air traffic control system. The enhancements are the following:

1. The existing software and procedures are to be converted from the VM/Jobshop International Business Machines (IBM) 4341 computer system to execute on the FAA Technical Center's General Purpose Main Frame IBM 4381 computer system.

2. The model should be modified to do step climbing, to realistically simulate the NAS en route flight activities. Under normal operating procedures, aircraft request and receive permission from ATC to climb to higher available altitudes as their gross weight decreases due to the heavy fuel consumption at the beginning of the flight. Certain types of aircraft are designed to operate more efficiently at higher altitudes.

3. The model should also be modified to do point-to-point navigation because it is predicted that this type of navigation will be heavily used in the future for economic reasons. When the model is executing in this mode, it will be using an approximation of a great circle distance to connect the aircraft's calculated high-altitude entry and exit points to the system. The simulated aircraft flightpaths will not follow the existing network.

BIBLIOGRAPHY

1. Busch, Allen C., Colamosca, Brian, and Vander Veer, John R., Collision Risk and Economic Benefit Analysis of Composite Separation for the Central Pacific Track System, Final Report, June 1977.

2. Hershkowitz, Ronald, Collision Risk Model for North Atlantic Region, Technical Report, May 1971.

3. Reich, P.G., Analysis of Long-Range Air Traffic Systems: Separation Standards-I, Journal of the Institute of Navigation, Volume 19, No. 1, January 1966.

4. Reich, P.G., Analysis of Long-Range Air Traffic Systems: Separation Standards-II, Journal of the Institute of Navigation, Volume 19, No. 2, April 1966.

5. Reich, P.G., Analysis of Long-Range Air Traffic Systems: Separation Standards-III, Journal of the Institute of Navigation, Volume 19, No. 3, July 1966.