**Title:** Algorithms for Nonlinear equations

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**Date:** October 1, 1988

**Abstract:**

This project involved research in three areas: mathematical software, globally convergent homotopy methods, and hybrid quasi-Newton algorithms for large scale structural optimization. The homotopy research, centered mainly with low dimensional ferociously nonlinear problems, centered on proving convergence theorems, devising homotopy curve tracking algorithms and development of the mathematical software package Hompack. The structural optimization research concerned optimization algorithms for very large-sparse nonlinear problems, where maintaining sparsity is absolutely necessary and even matrix multiples are costly. Structural optimization and equilibrium configuration computation via quasi-Newton and homotopy techniques require an entirely different technology for quasi-Newton and homotopy algorithms, using realistic test problems from structural mechanics.
This project involved research in three areas: mathematical software, globally convergent homotopy methods, and hybrid quasi-Newton algorithms for large scale structural optimization. The homotopy research, concerned mainly with low dimensional ferociously nonlinear problems, centered on proving convergence theorems, devising homotopy curve tracking algorithms, and development of the mathematical software package HOMPACK. The structural optimization research concerned optimization algorithms for very large sparse nonlinear problems, where maintaining sparsity is absolutely necessary and even matrix multiplies are costly. Structural optimization and equilibrium configuration computation via quasi-Newton and homotopy techniques require an entirely different technology than for small dense problems. The goal of this research is to explore and develop sparse matrix technology for quasi-Newton and homotopy algorithms, using realistic test problems from structural mechanics.

Accomplishments under AFOSR. 85–0250.

First year: 9/1/85–8/30/86.

1) Steve Billups' MS thesis “An augmented Jacobian matrix algorithm for tracking homotopy zero curves”.

2) A fluid mechanics application “Slow viscous flow in a syringe”.


4) Final testing and documentation of the ODE based part of HOMPACK.

5) Final testing and documentation of the normal flow algorithm in HOMPACK.

6) Preliminary work on an augmented Jacobian matrix homotopy algorithm.

7) Preliminary work on sparse homotopy and hybrid algorithms.

Second year: 9/1/86–8/30/87.

1) Completion of theory for an augmented Jacobian matrix homotopy algorithm.

2) Final testing and documentation of the augmented Jacobian matrix algorithm in HOMPACK.

3) Preliminary work on theory and code for sparse versions of all the algorithms in HOMPACK.

5) Five fluid mechanics papers:
   - "Dissipative waves in fluids having both positive and negative nonlinearity",
   - "Magnetohydrodynamic flow and heat transfer about a rotating disk with suction and injection at the disk surface",
   - "Magnetohydrodynamic flow past a porous rotating disk in a circular magnetic field",
   - "Magnetohydrodynamic flow between a solid rotating disk and a porous stationary disk",
   - "Rotating magnetohydrodynamic flow in a circular magnetic field”.

6) A structural optimization paper: “Two point constraint approximation in structural optimization”.


9) Invited to lecture on homotopy methods at the 1986 ODE Conference (Albuquerque, NM), the ASCE Structures Congress (New Orleans, LA), and the AMS Regional Meeting (Logan, UT).

10) ACM Trans. Math. Software paper and algorithm “HOMPACK: A suite of codes for globally convergent homotopy algorithms”, representing the culmination of over 10 years of research and development. HOMPACK contains over 13,000 lines of code, is part of Argonne National Laboratory’s netlib software, and has been distributed to over 50 sites in the U.S., Mexico, Canada, England, Germany, Israel, Norway, Argentina, Australia, and Ireland.

Third year: 9/1/87-9/30/88.

1) Final testing and documentation of the sparse versions of the algorithms in HOMPACK, which are based on a new preconditioned conjugate gradient algorithm developed by Kamat and Watson.

2) Four papers on parallel homotopy methods:
   - “A globally convergent parallel algorithm for zeros of polynomial systems”,
   - “Message length effects for solving polynomial systems on a hypercube”,
   - “The granularity of parallel homotopy algorithms for polynomial systems of equations”,
   - “Granularity issues for solving polynomial systems via globally convergent algorithms on a hypercube”.

3) Two papers on polynomial systems, reporting significant advances in the algorithmic efficiency of solving polynomial systems (collaboration with A. Morgan from General Motors Research Laboratories and A. Sommese from the University of Notre Dame):
   - “The mathematical reduction of a system of equations for a heart dipole problem”,
   - “Finding all isolated solutions to polynomial systems using HOMPACK”.

4) Two mechanics papers (collaboration with C. Y. Wang from Michigan State University):
   - “Spinning of a liquid film from an accelerating disc”,

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• “Free rotation of a circular ring with an unbalanced mass”.

5) Two applications of sparse homotopy methods to large scale structural optimization:
   • “Tracing structural optima as a function of available resources by a homotopy method”,
   • “Design of laminated plates for maximum buckling load”.

6) A theoretical analysis of various homotopies for the linear complementarity problem (collab-
   oration with A. Poore from Colorado State University). The resulting paper “Continuous
   homotopies for the linear complementarity problem” will appear in *SIAM J. Matrix Anal.
   Appl.*

7) A theoretical analysis of quasi-Newton updates for underdetermined systems of equations
   (collaboration with H. Walker from Utah State University), “Least change secant update
   methods for underdetermined systems”.

8) Invited to lecture on parallel algorithms at the NATO Advanced Study Institute on Numerical
   Linear Algebra, Digital Signal Processing, and Parallel Algorithms (Leuven, Belgium), and on
   homotopy methods at the ORSA CSTS conference (Williamsburg, VA) and the ORSA/TIMS
   Joint National Meeting (Vancouver, British Columbia).