**Rapidly Convergent Algorithms for Nonsmooth Optimization**

The research supported by this grant has continued the development of efficient methods for solving optimization problems involving implicitly defined functions that are not everywhere differentiable.

A rapidly convergent algorithm for the single variable case where generalized derivatives are known is currently being extended to the n-variable case.

Also, a new fast method has been developed for the single variable case where only function values are available.
The research conducted under Grant Number AFOSR-83-0210 during the period 15 July 1987 to 14 July 1988 is partially documented in [7], [8], [9] and [10] and related to previous work in [1], [3], [4], [5] and [6].

Optimization problems with functions that are not everywhere differentiable often arise when certain techniques are applied to large or complicated nonlinear programming problems in order to convert them to a sequence of smaller or less complex problems. These transformation techniques include decomposition, nested dissection, relaxation, duality and/or exact $l_1$ penalty methods and often lead to functions to be minimized which are implicitly defined. Being able to solve such problems gives an analyst flexibility in modeling a problem for solution and the ability to exploit parallel processing in computation. Hence, it is important for practical applications to be able to solve such problems and it is the goal of this project to develop efficient solution methods.

Recent joint work with J-J. Strodiot (Namur, Belgium) has produced theoretically satisfying and practically useful ideas for solving single variable minimization problems using function, but not derivative, values. The work on a safeguarded bracketing technique and on quadratic approximation appears in [10] which has been accepted for publication in Mathematical Programming. Also, this effort has produced, what is probably, the first instance of a function-value-only method for nonsmooth functions with proved rapid convergence. The corresponding paper [9] is under revision in order to append some figures to illustrate various cases mentioned in the introduction and considered in the proofs that are, unfortunately, rather lengthy and complicated.
The above-mentioned work stems from a bracketing method using generalized derivatives at the bracket endpoints developed and tested in [4], [6] and [7]. A new result submitted for publication in [8] states that the next iterate is either superlinearly closer to the solution than both of the current bracket endpoints or the length of the next bracket is superlinearly shorter than that of the current bracket. This work was presented in October 1987 at the SIAM 35th Anniversary Meeting in Denver. A FORTRAN implementation of this method along with an application solved via nested optimization has appeared in [7]. A Masters degree student, Han Lim, is currently working on using this method in a nested manner to obtain maximum likelihood estimates of the three parameters in the Weibull probability density function [11].

Current research is concerned with developing a higher than first order method for n-variable implicitly-defined nonsmooth problems based upon ideas in [3] and [5]. A former research assistant, David Elwood, has written a FORTRAN code that can be used to test new ideas developed in the course of this research.

A Master's degree student, Allison Radcliffe, has written a computer code for smooth optimization which is a specialization of our nonsmooth method. It is intended to investigate the performance of a method that uses both BFGS and symmetric rank one updated Hessian matrix estimates. The desire is to obtain the advantages, and not the disadvantages, of both updates.
The two n-variable codes mentioned above use the general purpose quadratic programming subroutine QPSOL [7] to solve the search direction finding subproblems. For efficiency in future implementations it may be beneficial to develop a special purpose method based on the Ph.D. dissertation of A. Al-Saket [1]. Two papers based upon [1] are now being prepared for publication submission.
REFERENCES


