Detection Performance of Normalizer for a Multi-Pulse Signal Subject to Partially-Correlated Fading with Chi-Squared Statistics

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Preface

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The false alarm and detection probabilities for a multipulse signal subject to partially correlated fading, in the presence of Gaussian noise of unknown level, are derived in closed form. The number \( N \) of signal pulses, as well as the number \( L \) of noise-only pulses used to estimate the noise background power level, are arbitrary. The power fading is characterized by a chi-squared distribution with \( 2m \) degrees of freedom and a normalized set of covariance coefficients \( \{ \rho_{kl} \} \), all of which can be selected arbitrarily, in order to match an experimental realization or an actual measured situation. The performance capability of this processor depends additionally on the received signal-to-noise ratio. This study covers the case of a nonconstant threshold; comparisons of this normalizer with earlier results (for \( \zeta = \infty \)) enable a quantitative evaluation of the losses incurred by...
lack of knowledge of the noise level. The important capability of constant false alarm rate is achieved by this normalizer.
DETECTION PERFORMANCE OF NORMALIZER FOR A MULTI-PULSE SIGNAL
SUBJECT TO PARTIALLY-CORRELATED FADING WITH
CHI-SQUARED STATISTICS

BY

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The following 5 pages give the text of the oral presentation at the JASA meeting.

The succeeding 8 pages constitute the poster presentation. The particular 5 pages that were employed for the oral presentation are labeled with VG1, VG2, VG3, VG4, VG5, at the top center.
The underwater propagation of acoustic signals is very adversely affected by the presence of deep fades in time and/or frequency. In an effort to combat fading, diversity in the transmitted signal is often employed, by emitting energy distributed over time and frequency. However, the fading of nearby pulses can be highly dependent, thereby thwarting some of the gains to be expected from diversity combination. This paper outlines the evaluation of the performance of a diversity-combining system in the presence of partially-correlated fading and additive noise of unknown level, in terms of the false alarm and detection probabilities.
VU-GRAPH 2

The transmitted signal consists of \( K \) CW tone bursts, each of duration \( T_1 \) seconds, distributed in a known pattern over time and frequency. The \( k \)-th received signal pulse undergoes an amplitude-scaling \( r_k \) and a phase-shift \( \theta_k \), both of which are constant over an individual pulse duration of \( T_1 \) seconds and bandwidth \( 1/T_1 \) Hertz. That is, we are considering slow non-selective fading.

The additive noise is stationary over the total observation time and has a flat spectrum over the total bandwidth utilized. However, since the noise level is unknown, a group of \( L \) noise-only bins, located in time-frequency where there is no signal, is used to extract an estimate of the noise level, \( N_0 \).
In order to be quantitative about the fading statistics and dependencies, we define the power-scaling variate $q_k = r_k^2$ the square of the amplitude-scaling variate. The first-order probability density of $q$ is taken to be chi-squared with $2m$ degrees of freedom. Thus $m = 1$ corresponds to exponential power-fading or Rayleigh amplitude-fading. The parameter $a$ is a measure of the average strength of the fading. In particular, the average power scaling equals $m$ times $a$. Particularly deep fading is modeled by small values of parameter $m$, which need not be integer. In fact, some recent measurements yielded fading which was characterized by $m$ near $1/2$.

The statistical dependencies between fading of separate signal pulses is modeled by allowing arbitrary covariance coefficients between any two power scaling variates. The exact statistics and dependencies of phase shifts $\theta_k$ are not relevant, due to the particular form of receiver processing adopted.
The potential-signal channels are matched-filtered and square-law envelope-detected at the instants of peak signal output, if signal is present. The sum of these $K$ squared envelopes is then compared with a similar sum of $L$ envelope-squared outputs of filters which are known to operate in a region of noise-only. If the ratio $\psi$ exceeds a threshold, a signal is declared present.

For signal absent, the value of output $\psi$ on any trial is independent of the absolute unknown noise level, $N_0$. Hence, this processor has constant false alarm rate capability; that is, we can realize a specified false alarm probability without knowledge of noise level $N_0$. 
The detailed analysis of performance of this system is too lengthy to be presented here. All we can do is to present a sample which illustrates the type of results that are currently available. This plot gives the receiver operating characteristics on normal probability paper, with input signal-to-noise ratio as a parameter. This particular case is for four signal pulses subject to Rayleigh amplitude fading with a covariance coefficient of 0.5, and utilizing 16 noise-only filters. Many additional examples and a general program are available in NUSC Technical Report 8133, from which this paper was drawn.
DETECTION PERFORMANCE OF NORMALIZER FOR A MULTI-PULSE SIGNAL SUBJECT TO PARTIALLY CORRELATED FADING WITH CHI-SQUARED STATISTICS

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PRESENTED AT
ACOUSTICAL SOCIETY OF AMERICA CONFERENCE
SHERATON-WAIIIKI HOTEL
HONOLULU, HAWAII

15 NOVEMBER 1988
TIME-FREQUENCY OCCUPANCY

FREQUENCY

\[ \begin{array}{ccc}
\{ & \text{NOISE-ONLY} & \text{BINS} \\
& r_k, \theta_k & \\
\{ & \text{POTENTIAL-SIGNAL} & \text{BINS} \\
\end{array} \]

\[ \begin{array}{c}
\text{TIME} \\
\end{array} \]

BINS: T, SECONDS DURATION
1/T, HERTZ BANDWIDTH \{ C W TONE BURSTS

RECEIVED SIGNAL (IF PRESENT):
AMPLITUDE SCALING \( r_k \) \{ OF k-TH SIGNAL PULSE
PHASE SHIFT \( \theta_k \) 

ADDITIVE NOISE:
STATIONARY OVER TOTAL OBSERVATION TIME.
FLAT SPECTRUM OVER TOTAL BANDWIDTH UTILIZED.
UNKNOWN LEVEL.
SIGNAL FADING STATISTICS

\( r_k, \theta_k: \) CONSTANT OVER EACH INDIVIDUAL PULSE DURATION \( t_i \) (SLOW FADING). ARBITRARY DEPENDENCIES BETWEEN PULSES.

DEFINE \( q_k = r_k^2 \) = POWER SCALING OF \( k \)-TH RECEIVED SIGNAL PULSE.

FIRST-ORDER PDF OF \( q_k \) IS CHI-SQUARED WITH \( 2m \) DEGREES OF FREEDOM:

\[
P_q(u) = \frac{u^{m-1} \exp(-u/a)}{\Gamma(m)} a^m \quad \text{for } u > 0
\]

AVERAGE POWER SCALING \( \bar{q} = ma \) FOR ALL \( k \).

\( m = 1 \) CORRESPONDS TO RAYLEIGH AMPLITUDE-FADING OR EXPONENTIAL POWER-FADING.

\( m \) NEED NOT BE AN INTEGER.
STATISTICS (CONTINUED):

ARBITRARY CORRELATION COEFFICIENTS BETWEEN POWER SCALINGS:

\[ \rho_{kj} = \frac{q_k q_j - \bar{q}_k \bar{q}_j}{\sigma_k^2} \]

PHASEhifts (\(\theta_k\)) HAVE ARBITRARY STATISTICS AND DEPENDENCIES.

ADDITIVE NOISE SPECTRAL LEVEL, \(N_0\) WATTS/Hz, IS UNKNOWN.
RECEIVER PROCESSING

TIME AND FREQUENCY SYNCHRONIZATION PRESUMED.
(DELAY AND DOPPLER COMPENSATED)

FILTERING CAN BE ACCOMPLISHED VIA FFTS.

DECISION VARIABLE $\gamma = \frac{X(k)}{X_0(l)} \geq \text{THRESHOLD.}$
PROBLEM DEFINITION

RELEVANT PARAMETERS:

\( K = \) NUMBER OF SIGNAL PULSES
\( L = \) NUMBER OF NOISE PULSES
\( m = \) SIGNAL FADEING PARAMETER (NON-INTEGER); 2m DEGREES OF FREEDOM
\( \beta_{ij} = \) POWER-FADEING COVARIANCE COEFFICIENT
\( \overline{E}_i = \) AVERAGE RECEIVED SIGNAL ENERGY PER PULSE
\( N_o = \) SINGLE-SIDED RECEIVED NOISE SPECTRAL LEVEL

WANT PROB(\( \gamma \) > THRESHOLD | SIGNAL PRESENT).
CAN THEN DETERMINE \( P_f \) AND \( P_d \).

\( L = \infty \) CORRESPONDS TO KNOWN NOISE LEVEL.

FOR SIGNAL ABSENT, \( \gamma \) IS INDEPENDENT OF ACTUAL (UNKNOWN)
NOISE LEVEL \( N_o \); CFAR PROCESSOR.
CAN ACHIEVE SPECIFIED \( P_f \) WITHOUT KNOWLEDGE OF \( N_o \).
ANALYTIC APPROACH

HOLD \( \{r_k\} \) AND \( \{\theta_k\} \) FIXED.

THEN CONDITIONAL CHARACTERISTIC FUNCTION (CF) OF SUM \( \gamma(k) \) DEPENDS ONLY ON

\[
S = \sum_{k=1}^{K} r_k^2 = \sum_{k=1}^{K} q_k.
\]

APPROXIMATE \( S \) BY A CHI-SQUARED VARIATE WITH EXACT SAME MEAN AND VARIANCE.

AVERAGE THE CONDITIONAL CF TO GET THE UNCOND. CF OF \( \gamma(k) \).

FOURIER TRANSFORM TO GET PDF OF \( \gamma(k) \).

COMPUTE \( \text{PROB}( \gamma(k) > t \times \gamma_0(l) ) \) IN TERMS OF A FINITE SERIES OF TERMINATING GAUSSIAN-HYPERGEOMETRIC FUNCTIONS.
Figure 25. ROC for K=4, m=1, ρ=.5, L=16
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