This is the final report for ARO research grant DAAG29-82-K-0077 for the period of March 1982 to September 1987. Research was conducted on several topics on image processing with special emphasis on mathematical modeling. Research topics include two-dimensional stochastic models for images, two-dimensional spectral factorization and spectral estimation, Radon transform theory for random fields, image data compression, image analysis and shape inspection based on contour processing using spline functions, pattern inspection based on mathematical morphology, Radon transform based image processing algorithms, VLSI architectures of image processing and computer vision algorithms, and so on. A brief summary of results obtained together with a list of publications and theses is included.
FINAL REPORT

DEVELOPMENT OF MATHEMATICAL MODELS
FOR APPLICATION
IN IMAGE PROCESSING

ARO GRANT DAAG29-82-K-0077
ARO PROPOSAL 18532-EL

March 1982 - September 1987

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Submitted to:
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U. S. Army Research Office
P. O. Box 1211
Research Triangle Park, NC 27709

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I. ACKNOWLEDGEMENT

The principal investigator, Professor Anil K. Jain and all participants of this research project wish to express their gratitude to the U. S. Army Research Office for its support of the research summarized in this final report. Special thanks are due to Dr. William A. Sander of ARO for his continuing interest in the technical aspects of the research throughout the duration of the project.
II. ABSTRACT

This is the final report for ARO research grant DAAG29-82-K-0077 for the period of March 1982 to September 1987. Research was conducted on several topics on image processing with special emphasis on mathematical modeling. Research topics include two-dimensional stochastic models for images, two-dimensional spectral factorization and spectral estimation, Radon transform theory for random fields, image data compression, image analysis and shape inspection based on contour processing using spline functions, pattern inspection based on mathematical morphology, Radon transform based image processing algorithms, VLSI architectures of image processing and computer vision algorithms, and so on. A brief summary of results obtained together with a list of publications and theses is included.
III. FINAL REPORT SUMMARY SHEET

A. ARO Proposal Number: 18532-EL


C. Title of Proposal: Development of Mathematical Models for Application in Image Processing

D. Contract or Grant Number: DAAG29-82-K-0077

E. Name of Institution: University of California, Davis

F. Authors of Report: Anil K. Jain

G. List of Manuscripts Submitted or Published under ARO Sponsorship during this Reporting Period, Including Journal References:


H. Scientific Personnel Supported by this Project and Degrees Awarded During this Reporting Period:

PERSONNEL SUPPORTED

Anil K. Jain - Principal Investigator

1. Visiting Scholars and Faculty:

J. L. C. Sanz - Adjunct Associate Professor (IBM)
S. Srinivasan - Visiting Associate Professor (Sabbatical)
Y. Yoshida - Visiting Associate Professor (Sabbatical)
F. Marvasti - Visiting Associate Professor (Sabbatical)
T. Ohta - Visiting Scholar
M. Buonocore - Assistant Professor of Radiology

2. Graduate Students:

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David Paglieroni
Carolyn Koenig
Ahmed Darwish
James Apffel
Hans Dohse
J. Bradley Van Tighem
S. Guliani
James Grishaw
M. Chin
Iskender Agi
Jonathan Brandt
Mathew Wagner
Stephen Azevedo
DEGREES AWARDED

PhD:


2. David Paglieroni, CONTROL POINT ALGORITHMS FOR CONTOUR PROCESSING AND SHAPE ANALYSIS, 1986.


M.S.:


14. I. Agi, AN EXPANDABLE VLSI PROCESSOR FOR CONTOUR TRACING (in progress).

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   James Apffel
   Hans Dohse
   J. Bradley Van Tighem
   S. Guliani
   James Grishaw
   M. Chin
   Iskender Agi
   Jonathan Brandt
   Mathew Wagner
   Stephen Azevedo

B. DEGREES AWARDED

PhD:

1. S. Ranganath, TWO-DIMENSIONAL SPECTRAL FACTORIZATION SPECTRAL
   ESTIMATION AND APPLICATIONS IN IMAGE PROCESSING, 1983.
2. David Paglieroni, CONTROL POINT ALGORITHMS FOR CONTOUR PROCESSING AND SHAPE ANALYSIS, 1986.


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VII. SUMMARY OF RESEARCH FINDINGS

During the entire period, research was conducted on several different topics including image modeling and representation, image restoration, 2-D spectral estimation and spectral factorization, image data compression, architectures of image processing algorithms, computer vision, Radon transform theory and applications, mathematical morphology for image processing and on. These projects are briefly summarized here.

A. TWO DIMENSIONAL STOCHASTIC MODELS FOR IMAGE PROCESSING (JAIN AND RANGANATH)

Three canonical representation, causal, semicausal, and noncausal were introduced for two-dimensional images. A new realization theory in the framework of 2-D prediction and spectral factorization was developed where it was shown that a well behaved 2-D power spectrum $S(\omega_1, \omega_2)$ can be factored as

$$S(\omega_1, \omega_2) = KH(\omega_1, \omega_2) H(-\omega_1, -\omega_2)$$

$$= \hat{S}(\omega_1, \omega_2)$$

such that $H$ is the frequency response of a finite order, stable, minimum variance causal, semicausal, or noncausal system and the spectral match between $S$ and $\hat{S}$ can be arbitrarily close. Moreover, the algorithm for realizing $H$ requires that only a finite number of linear equations be solved unlike prior methods of two dimensional spectral factorization which require solution of infinite number of equations to obtain finite order approximate realizations.

One practical significance of these models is that they spell different architectures for processing of 2-D data. For example, DPCM, Hybrid coding and Transform coding methods for image transmission arise from the causal, semicausal and noncausal models, respectively. Other important applications in image restoration, edge extraction, spectral estimation and texture modeling were reported. See Publications [1, 2, 7, 9, 29, 30, 32] and PhD thesis by S. Ranganath.
B. RADON TRANSFORM THEORY FOR RANDOM FIELDS (JAIN AND ANSARI)

The Radon Transform gives the mathematical formulation of the projections of a two (or higher) dimensional function. Simply stated, the Radon Transform $g(s, \theta)$ of a function $f(x, y)$ is its line integral along a line placed at a distance $s$ from the origin with an orientation $\theta$, i.e.,

$$g(s, \theta) = \iint f(x, y) \delta(s - x \cos \theta - y \sin \theta) \, dx \, dy$$

The mathematical theory associated with this transformation holds the key for computerized tomography (CT) which gives cross sectional images of a 3-D object from its 2-D projections.

A fundamental result associated with this theory is the Projection-Slice Theorem which states that the 1-D Fourier Transform, $G(\xi, \theta)$ of the projection $g(s, \theta)$ is equal to the central slice of the 2-D Fourier Transform $F(\xi \cos \theta, \xi \sin \theta)$ at angle $\theta$, i.e.,

$$G(\xi, \theta) = F(\xi \cos \theta, \xi \sin \theta)$$

where $(\xi, \theta)$ are the polar coordinates in the Fourier space. This result can be used to reduce the complexity of several 2-D and 3-D image processing algorithms. In our research, we introduced a Unitary Radon Transform [6, 29, 31, 32], which was shown to be the function $\tilde{g}(s, \theta)$ obtained by filtering $g(s, \theta)$ by a linear filter whose frequency response is $1|\xi|^{1/2}$. Via the use of the unitary Radon Transform, it was shown that for a stationary random field $f(x, y)$, its projections $g(s, \theta)$ and filtered projections $\tilde{g}(s, \theta)$ both are uncorrelated in $\theta$ and stationary in $s$. Moreover, the 1-D power spectrum density of $\tilde{g}(s, \theta)$ denoted by $S_{\tilde{g}}(\xi, \theta)$ is the central slice of the 2-D power spectrum of $f(x, y)$ denoted $S_f(\xi \cos \theta, \xi \sin \theta)$.

$$S_{\tilde{g}}(\xi, \theta) = S_f(\xi \cos \theta, \xi \sin \theta)$$

The result above is noteworthy because, contrary to earlier assumptions by various researchers, the power spectrum satisfies the projection theorem in the unitary Radon Transform domain whereas the amplitude spectrum satisfies it in the conventional Radon Transform domain. This result has a fundamental impact when processing is done in the
Radon space, as was demonstrated in [6] for reconstruction of CT images in the presence of detector noise. Thus, the Unitary Radon space is the correct projection space in which to apply stochastic estimation theory for derivation of new convolution kernels for 2-D image reconstruction from projections, 2-D pattern matching, and methods for estimating the power spectrum of 2-D objects from their unitary projections.

C. RECURSIVE BLOCK CODING - AN IMAGE DATA COMPRESSION METHODOLOGY (JAIN AND FARRELLE)

The limiting factor for image data compression using transform coding or VQ is the so-called block or tile effect. At sufficiently high rates, the loss of fidelity is a continuous function of the data rate. However, there comes a point when any further reduction in the rate produces the tile effect and this leads to a high discontinuity in the perceived fidelity and subjective results show that this artifact is ten times more objectionable than equal energy random noise. The cause of the problem is that each block of the image is coded independently, that is without reference to the neighboring blocks, and so the distortion introduced by the coding is discontinuous from block to block. The eye picks up the change in luminance at the edges of the blocks, especially in the smooth regions where no such change is expected. Furthermore, the effect is amplified since the regular block structure ensures that these problems areas are all aligned and the resulting image resembles a mosaic constructed from small tiles. In other words, we are sensitive not only to the amplitude of errors but also to their structure.

We have alleviated this problem when block coding an image by using a non-causal representation for the image which allows us to decompose it as the sum of two sources: a boundary response and a resulting residual image.

The reason why this two source decomposition reduces the tile effect is because if we share the boundary values, which are typically one pixel wide, between two neighboring blocks, then the boundary response which interpolates the boundary values will be continuous across the transition from one block to the next despite any coding distortion in the boundaries. See publications [2, 9, 18, 28, 29, 32] and the PhD thesis by Farrelle.
D. CONTOUR POINT ALGORITHMS FOR CONTOUR PROCESSING AND SHAPE ANALYSIS (JAIN AND PAGLIERONI)

Planar non-overlapping shape analysis is an important computer vision problem with applications in industrial parts inspection, character recognition, target identification, biological cell analysis, etc. Subtopics emphasized in this research include shape representation, measurement and classification. A unified approach composed of contour based algorithms for solving problems of each type is developed, where contour techniques are potentially efficient because they involve relatively small quantities of data.

Contour models are data compressed shape representations, where the model parameters are contour transforms. Polygon models are among the most popular. B-spline models are powerful and versatile generalizations of these parameters called control points. Algorithms for computing arbitrary shape moments, contour curvature, shape projections and related features directly from control points, independent of scale, have been developed. Use of these algorithms results in an useful contour based shape recognition scheme, with classification cost virtually independent of the number of prototypes. The scheme utilizes complete shape information, is scale-translation-rotation invariant and feasibly realizable. Error rates of less than 1 percent were achieved during single font character recognition at 500 dots per inch. Based on similar principles, a method of detecting positions of shapes flaws was also developed. See Publications [8, 26, 27] and the PhD thesis by Paglieroni.

E. MATHEMATICAL MORPHOLOGY TECHNIQUES FOR RULE BASED PATTERN INSPECTION (JAIN AND DARWISH)

The inspection problem is to find a procedure that can detect whether an item under test conforms with predefined specification standards within an acceptable margin of tolerance. This research concerns shape inspection of nonoverlapping flat objects imaged against a high contrast background.

Visual inspection provides a knowledge rich environment where the scene to be analyzed is known a priori to a high degree of confidence and products are, usually, tested against clear design criteria. In this research, the goal is to investigate the use of a priori knowledge about the scene to coordinate and control image segmentation, interpretation, and defect detection.
The approach is composed of two main steps. The first step consists of proper segmentation and labeling of individual regions in the image for subsequent interpretation. Low level morphology based techniques are used to perform segmentation, grouping and shape analysis. General as well as specific knowledge is used to improve the segmentation and interpretation decisions. Once every region in the image has been identified, the second step proceeds by testing different regions to ensure they meet the design requirements, which are formalized by a set of rules. Morphological techniques are also used to extract certain features from the previously processed image for rule verification purposes.

The rule based approach freed us from the precise alignment required for the well known image subtraction method. The approach is also an improvement over existing pure dimensional verification systems because it allows detection of missing and erroneous features as well as extraneous segments that might have normal looking features. See publications [12, 14, 23] and the PhD thesis by Darwish.

F. RADON TRANSFORM BASED IMAGE PROCESSING (JAIN, SANZ, BUONOCORE, HINKLE, DOHSE, AZEVEDO)

This project consisted of several subprojects as follows:

1. **P³E: A Parallel Pipeline Projection Engine (Jain, Sanz, Hinkle)**

   We developed a novel architecture that makes real-time projection-based image processing a reality. The design is founded on raster-mode processing, which is exploited in a powerful and flexible pipeline. This architecture, dubbed "P³E" (Parallel Pipeline Projection Engine), supports a large variety of image processing and image analysis applications. In this research, we were concerned with several image processing tasks, such as: discrete approximations of the Radon and inverse Radon transform, among other projection operators; CT reconstructions; 2-D convolutions; rotations and translations; etc. However, there is also an extensive list of key image analysis algorithms that are supported by P³E, thus making it a profound and versatile tool for projection-based computer vision. Recently, several image analysis operators were mapped onto this architecture to solve some important automated inspection problems. We have yet to apply P³E to many other unexplored image processing and image analysis tasks. Examples of these are: object
recognition, motion parameter computations, approximation of the Fourier transform on polar rasters, etc. See publications [11, 17, 22, 31].

2. **Object Classification and Registration by Radon Transform Based Invariants (Jain, Sanz, Yoshida, Dohse)**

Applications in machine vision systems and digital object recognition often require recognizing objects from a given class regardless of position and orientation in addition to determining the angle of rotation and the position of the input object.

The magnitude of the Fourier transform combined with a circular harmonics expansion is used to obtain several rotation and translation invariant representations. The Radon Transform is used as a tool to calculate the Fourier transform in polar coordinates. The reason for going into polar Fourier space is that the discrete implementation of the integrals required for the circular harmonics expansion is simplified when in polar space. In addition, at no extra computational expense, the location and rotation of an object can be found.

The recognition scheme has been tested on binary and gray level image sets. Favorable classification results were obtained for both sets. See [20] and the M.S. thesis by Dohse.

**G. MODEL-BASED COMPUTED TOMOGRAPHY FOR NONDESTRUCTIVE EVALUATION (JAIN, BUONOCORE, AZEVEDO)**

This research involves the use of model-based computer algorithms for solving the hollow-projections problem in computed tomography (CT) imaging. In particular, we are studying the algorithms best suited for nondestructive evaluation (NDE) of industrial objects where applicable mathematical models are available.

CT image reconstruction is a form of multidimensional signal processing that involves extracting or reconstructing a two-dimensional (2-D) planar density distribution through an object from measured data values that are inexact. The measurements are the detected X-ray photons that have been transmitted through a 2-D plane of the object at many different angles, and the extracted information is in the form of an image of the 2-D plane. Many standard methods of image reconstruction from projections have been proposed and implemented.
since the development of medical CT scanners, but there are still many unanswered questions about the quality and information content of the reconstructed images.

Furthermore, when there are missing or noisy measurements, artifacts can dominate the resultant reconstructed images. The hollow-projections problems is a case in point: some rays cast through the object are completely attenuated to zero by very dense materials. Our interest is in imaging the low-density regions surrounding the highly attenuating materials, not the dense materials themselves. Using standard reconstruction techniques, however, the low-density areas of interest will contain ghost artifacts caused by the missing data (zero-valued rays). Industrial tomography for NDE is especially plagued with this problem since the range of densities in an object can be far larger than the detector is capable of measuring. On the other hand, since the objects are designed and built by computer, we typically have very detailed and accurate expected models of the object profile from a computer-aided design (CAD) system. (This is in contrast to medical applications where every body is different.) Our premise is that any additional information that we can provide in the form of deterministic or stochastic mathematical models should help us to improve the imaging. Though some new techniques have been suggested with this premise in mind, they have not been generally adapted to the hollow-projection problems for NDE.

This project was recently started and will continue after the termination of this current grant.

H. AN EXPANDABLE VLSI PROCESSOR ARRAY APPROACH TO CONTOUR TRACING (HURST, JAIN, AGI)

Contour Tracing (or segmentation) is used in a variety of image-processing/computer-vision applications. In certain applications, contour tracing, implemented in software, takes too long, even for small images. Contour tracing in hardware can achieve much higher speeds especially if parallelism and other "speed up tricks" are used.

In the architecture proposed, input image needs to be thresholded and converted into binary pixels. Then the image is broken up into $2M$ by $M$ blocks, where each processor loads a single block. The contour tracer consists of a set of two processor arrays which work independently. As one array of processors traces out the contours contained in its portion of the image and broadcasts the contour information to the post-processor, the second set extracts the edges (or cracks) that are to be traced. These two arrays switch operations once
the loading of the on-board RAM is complete. A post-processor links all the partial contours into a set of continuous contours which can be used for further processing.

The compression ratios obtained by the processor array were no worse and sometimes better than the maximum possible by Run Length Coding (RLC) of the CCITT documents. The compression ratio can be further increased by the linking of the partial contours and run-length encoding the direction vectors. The tracing time was reduced significantly by keeping track of the first edge encountered on each line. See [24] and the M.S. thesis by Agi.

I. A RASTER SCAN ARCHITECTURE FOR BOUNDARY-BASED SEGMENTATION (JAIN, SANZ, APFFEL)

A novel hardware architecture for extracting region boundaries in two raster scan passes through a binary image has been developed. The first pass is a statistic gathering phase wherein data is collected regarding the size of each object contour. This information is used by the hardware to dynamically allocate available memory for storage of boundary codes. In the second raster pass, the same architecture constructs lists of Grid-Joint codes to represent the perimeter pixels of each object. These codes, referred to variously as "crack" codes or "raster-chain" codes, are later decoded by the hardware to reproduce the ordered sequence of coordinates surrounding each object. This list of coordinates is useful for a variety of shape recognition and manipulation algorithms which utilize boundary information.

The proposed VLSI architecture has been simulated in software. Measurements on the coding efficiency of the basic algorithm as well as estimates of the overall chip complexity have been explored. See [21] and the M.S. thesis by Apffel.

J. BLOCK PROCESSING OF IMAGES WITH SIMD MULTIPROCESSORS (JAIN, SRINIVASAN, VAN TIGHEM, CHIN)

Programmable Single Instruction Multiple Data (SIMD) stream chips can meet the high throughput requirements of many image processing applications. Because of the limited memory capacity of these devices, processing image data in blocks becomes a very practical and efficient way to utilize the full capacity of these chips and achieve real-time image processing at a reasonable cost. Specifically, the NCR-Geometric Arithmetic Parallel Processor (GAPP) was investigated and tested with various well-known enhancement algorithms. See [15] and the M.S. thesis by Van Tighem.
K. AN UNIFIED DCT/IDCT ARCHITECTURE FOR VLSI IMPLEMENTATION
(JAIN, CURRENT, SRINIVASAN, PARKHURST, FARRELLE, GULIANI, GRISHAW)

This project involved the design, layout, and simulation of a VLSI chip. This chip performs the discrete cosine transform (DCT) and inverse discrete cosine transform (IDCT). Two, three, and four-input nor gates, a register, a three-bit counter, a 15-bit adder as well as a fixed coefficient multiplier were designed and laid out. These layouts as well as the total chip layout were done using MAGIG, a CAD tool on the SUN workstations. Simulations, using SPICE and ESIM, were performed on component parts of the chip as well as the chip itself. The design and layout of the chip is finished while simulations on the chip are almost complete. See [13, 24] and the M.S. theses by Parkhurst and Grishaw.

L. SKELETONIZATION AND VECTOR CONVERSION OF DOCUMENT IMAGES (JAIN AND BRANDT)

This research examines the midline representation of an object. The midline, or medical axis, can be defined as the locus of points in an object which are equidistant to at least two boundary points. The midline is a contour that falls along the central axis of the shape. Using this contour, together with the object's thickness at each point along the contour, it is possible to reconstruct the object. It is possible to encode the midline into a vector representation so as to achieve efficient storage and reconstruction.

The following questions are being addressed:

A. How much compression can be achieved while retaining accurate reconstruction?

B. How are geometric transformations applied to the image using the vector representation?

C. What are the computational requirements for conversion and manipulation of this representation?

This is a continuing project. See the M.S. thesis by Brandt.