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RSRE MEMORANDUM No. 4211

ROYAL SIGNALS & RADAR ESTABLISHMENT

STATISTICAL DENSITY AND AMPLITUDE TAPERED ARRAYS

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PROCUREMENT EXECUTIVE,
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R S R E MALVERN,
WORCS.

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DATE: March 1988

SUMMARY

Statistical density and amplitude tapering are combined to improve the radiation characteristics of thinned, density tapered arrays by allowing changes in both element density and amplitude weight to better approximate the model amplitude distribution. Statistical density tapering (which uses only uniformly weighted elements) is extended so that each element is allocated one of several possible values of amplitude weight according to predefined selection probabilities. Increasing the number of possible values, or levels, for amplitude weight can reduce the sidelobe levels of an array. Several choices of levels and their probabilities can reduce sidelobes and satisfy the extension criteria. One of the lowest sidelobe levels is produced if: (a) the amplitude levels are equi-spaced in the allowable range between zero and unity, and (b) their selection probabilities are terms from the binominal distribution of the appropriate order. The properties of arrays designed using equi-spaced amplitude levels with binomially distributed selection probabilities are calculated and described as a function of the number of levels and aperture size. It was found firstly that density and amplitude tapering cannot reduce the sidelobe levels without an accompanying increase in the number of elements, and secondly that it cannot sufficiently lower the sidelobes of a thinned, density tapered array to equal the lowest obtainable from the comparable fully filled, amplitude tapered array. Thirdly however it was found that density and amplitude tapering can reduce the minimum aperture size and number of elements required to achieve a given sidelobe level.

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STATISTICAL DENSITY AND AMPLITUDE TAPERED ARRAYS

G J Ball

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1 INTRODUCTION

In this Memorandum statistical density[1] and amplitude tapering are combined to improve the radiation characteristics of density tapered arrays. Density tapered arrays are thinned arrays of uniformly weighted elements. These elements are placed in an aperture so that their density changes with position in proportion to a model amplitude distribution. This distribution is one known to give low sidelobes if applied to a continuous aperture. However, density tapered arrays only produce low sidelobes if they contain enough elements for the change in element density to sufficiently approximate the shape of the model amplitude distribution. Only larger aperture arrays can achieve this.

To improve the radiation characteristics of smaller aperture thinned arrays, and larger aperture arrays with small numbers of elements, statistical density tapering has been extended to arrays of elements with different amplitude weights. Changes in both element density and amplitude weight can then be used to better approximate the model amplitude distribution. The following sections describe firstly the general method used to extend statistical density tapering, secondly its particular application to produce arrays with low sidelobe levels, and thirdly the performance and properties of these arrays.

2 STATISTICAL DENSITY AND AMPLITUDE TAPERING

A fully filled array is one in which the elements are placed a half wavelength apart at the intersection points of a regular square grid. The procedure for designing a statistically density tapered array[1] may be described as one in which each element of a fully filled array is allocated an amplitude weight of either zero or unity according to predefined selection probabilities. As the zero weighted elements do not contribute to the radiation pattern they can be removed to thin the array.

The selection probability for the unity weight is the product of two quantities, one being the value of the normalised model amplitude distribution A at the element location, the other being the thinning factor q. The factor q is set to a single value between zero and unity for the whole array and controls the degree of thinning. Arrays with the unity value for q are termed "naturally" thinned. Greater thinning is achieved by setting q to a value less than unity. To allocate the weights a random number between zero and unity is generated for each element. If the random number for an element is less than the product qA it is allocated a unity weight, otherwise it is allocated a zero weight. Over many selections the average amplitude weight for a single element equals qA.

The above description may be extended to one in which an amplitude weight is allocated to each element from not two, but several, possible values. The selection probabilities for these possible values, or levels, must be assigned so that over many selections the average amplitude weight for a single element equals qA. That is, the following equation must be satisfied:

$$p_1 \ell_1 + p_2 \ell_2 \cdots\cdots + p_L \ell_L = qA \qquad (1)$$

where ℓ_i is the value of the amplitude level,
p_i its selection probability, and
L the total number of levels.

The probabilities p_i must be functions of the model amplitude distribution, so that Eqn (1) is satisfied as a function of position in the aperture. The sum of the probabilities p_i should equal unity. Hence:

$$p_1 + p_2 \cdots\cdots + p_L = 1.0 \qquad (2)$$

The probabilities and levels are constrained to have values in the following ranges:

$$0 \leq \ell_i \leq 1.0 \qquad (3)$$

$$0 \leq p_i \leq 1.0 \qquad (4)$$

Equations (1) to (4) state the basic criteria that must be satisfied for any extension.

To account for unity and zero values of qA there has to be at least one unity level with a selection probability of unity when qA is unity, and at least one zero level with a selection probability of unity when qA is zero. For example, when qA equals zero each term of the left hand side of Eqn (1) must also be zero, as each p_i and ℓ_i can only have a positive value. Therefore the probabilities of the non-zero levels must be zero, while that of the zero level must be unity. And when qA equals unity the left hand side of Eqns (1) and (2) can be set equal and rearranged to give the following equation:

$$p_1(1-\ell_1) + p_2(1-\ell_2) \; \ldots\ldots + p_L(1-\ell_L) = 0 \quad .$$

Each term on the left hand side of the equation must be zero, so that the probabilities of the non-unity levels must also be zero, while that of the unity level must be unity.

Amplitude weights are allocated to each element by an extension of the procedure used for density tapered arrays. Again, a random number is generated for each element of a fully filled array. Each amplitude level is then allocated a range equal to its probability. The range for the first level runs from zero to p_1, that for the second level from p_1 to $p_1 + p_2$ and so on. As the probabilities add up to unity, the total range also runs between zero and unity. The amplitude level selected is that whose range includes the random number.

Some of the general properties of statistical density and amplitude tapered arrays are derived in the remainder of this section. The radiation pattern of a density tapered array is the sum of two terms(1). The first term is proportional by a factor of q^2 to the pattern of an equally sized, fully filled, fully amplitude tapered array (ie one with elements having amplitude weights exactly equal to A). This latter array produces the lowest sidelobes obtainable from a fully filled array. The second term is the average value of the random sidelobes produced by a density tapered array, and equals:

$$\overline{S} = \frac{\overline{\sum_{m=1}^{M} F_m^2} - q^2 \sum_{m=1}^{M} A_m^2}{q^2 \left[\sum_{m=1}^{M} A_m \right]^2} \qquad (5)$$

where the summations are over the total number of elements in the aperture if the array were fully filled M, and A_m is the value of the normalised model amplitude distribution at location m. The parameter F_m equals the amplitude weight allocated to the element at location m. The bar denotes an average value. For a density tapered array each value of F_m is selected randomly and independently from location to location, so that the first term of the numerator of Eqn (5) may be expressed as:

$$\overline{\sum_{m=1}^{M} F_m^2} = \sum_{m=1}^{M} \overline{F_m^2} = \sum_{m=1}^{M} \overline{F_m} = \sum_{m=1}^{M} qA_m$$

Hence, Eqn (5) becomes:

$$\overline{S} = \frac{\sum_{m=1}^{M} qA_m(1 - qA_m)}{q^2 \left[\sum_{m=1}^{M} A_m\right]^2} \tag{6}$$

For density and amplitude tapered arrays possible values for F_m^2 are $\ell_1^2, \ell_2^2 \ldots\ldots \ell_L^2$. On average these levels are chosen a fraction $p_1, p_2 \ldots\ldots p_L$ respectively of the total of a large number of choices. Hence:

$$\overline{F_m^2} = p_1\ell_1^2 + p_2\ell_2^2 \ldots\ldots + p_2\ell_L^2 \quad .$$

and the average sidelobe level is given by:

$$\overline{S} = \frac{\sum_{m=1}^{M} \left[\sum_{i=1}^{L} p_i\ell_i^2\right] - q^2 \left[\sum_{m=1}^{M} A_m^2\right]}{q^2 \left[\sum_{m=1}^{M} A_m\right]^2} \tag{7}$$

The average number of elements N_i of each level in an array is given by the following equation:

$$\overline{N_i} = \sum_{m=1}^{M} p_i \tag{8}$$

as, at each possible element location, an element of weight ℓ_i will be chosen on average a fraction p_i of the total of a large number of choices. The total average number of elements in an array N is given by the sum of N_i for each non-zero level.

3 THE SELECTION OF AMPLITUDE LEVELS AND PROBABILITIES TO REDUCE SIDELOBES

3.1 NATURALLY THINNED ARRAYS

The amplitude levels and their selection probabilities that both satisfy Eqns (1) to (4), and reduce the average random sidelobe level of an array, were determined by comparing the properties of density and amplitude arrays with two and three levels. An array with two levels is one that is density tapered only. For an array with three levels, of which two must be unity and zero, Eqns (1), (2) and (7) become:

$$p_1(A) + p_2(A)\ell_2 = A \tag{9}$$

$$p_1(A) + p_2(A) + p_3(A) = 1.0 \tag{10}$$

$$\bar{S} = \frac{\sum_{m=1}^{M} \left[p_1(A) + p_2(A)\ell_2^2\right] - \sum_{m=1}^{M} A_m^2}{\left[\sum_{m=1}^{M} A_m\right]^2} \qquad (11)$$

where ℓ_1 is unity, ℓ_3 is zero, and the dependence of probability on A is now also stated. There are two unknown variables - ℓ_2 and one of the probabilities, for example p_1. The remaining two probabilities can be determined from Eqns (9) and (10).

No obvious choice of $p_i(A)$ and ℓ_i minimises the average sidelobe level $\bar{S}$. Therefore, $\bar{S}$ was calculated from Eqn (11) for various selection of $p_i(A)$ and ℓ_i. As an example these calculations were performed for an array with a circular aperture containing 812 possible element locations (ie 32 across a diameter). A 40 dB circular Taylor distribution was used as the model amplitude distribution. The results are shown in Table 1.

TABLE 1. Average Sidelobe Level of Two and Three Level Density and Amplitude Tapered Arrays for Different Choices of Level and Selection Probability

Array size: Circular aperture array containing 812 possible element locations.

Model amplitude distribution: 40 dB (n = 4) Taylor distribution.

Values for the two level (density tapered) array are shown in column (1).

COLUMN NUMBER		(1)	(2)	(3)	(4)	(5)	(6)	(7)
LEVELS	l_1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	l_2	0.0	0.5	0.25	0.75	0.50	0.67	0.75
	l_3	-	0.0	0.0	0.0	0.0	0.0	0.0
PROBABILITIES	$p_1(A)$	A	A^2	$A^{4/3}$	A^2	$A-A^2+A^3$	A^3	$(1.33).(A-A^4)$
	$p_2(A)$	1-A	$2A.(1-A)$	$4.(A-A^{4/3})$	$1.33.(A-A^2)$	$2.(A^2-A^3)$	$1.5.(A-A^3)$	$1.33.(A-A^4)$
	$p_3(A)$	-	$(1-A)^2$	$(1-4A-3A^{4/3})$	$(1-1.33A-0.33A^2)$	$(1-A-A^2+A^3)$	$(1-1.5A-0.5A^3)$	$(1-1/33A+0.33A^4)$
AVERAGE SIDELOBE LEVEL (calculated from eqn (7) in dB)		-28.6	-31.6	-30.4	-29.5	-29.5	-31.3	-30.8

Results for the same array but with density tapering only are also included for comparison. The average sidelobe levels of the three level arrays are all lower than that of the density tapered array.

The amplitude levels and selection probabilities of the density tapered array and the three level array with the lowest sidelobe level are shown in Columns (1) and (2) of Table 1 respectively. It can be seen that the selection probabilities of both arrays are terms of the following binomial distribution:

$$[A + (1-A)]^{L-1} \tag{12}$$

It can also be seen that the extension from the former to the latter type of array is achieved by increasing the order of the binomial distribution from 1 to 2, and introducing the third level so that the three levels are equi-spaced in an allowable range between zero and unity.

This extension may be extended further to arrays with several amplitude levels. Again the levels are equi-spaced in the allowable range so that the ith level is given by the following equation:

$$L_i = \frac{(L-i)}{(L-1)} \qquad i = 1,2.....L \tag{13}$$

Its selection probability is given by the relevant term from the binomial distribution of Eqn (12). The order of the distribution equals the number of non-zero weight levels. Hence the selection probability of the ith level is given by:

$$p_i(A) = \frac{(L-1)!}{(i-1)!(L-i)!} A^{L-i}(1-A)^{i-1} \tag{14}$$

Assigning the levels and probabilities in this way satisfies the criteria expressed in Eqns (1) to (4); as the levels and probabilities have values in the correct ranges, the probabilities add up to unity, and the average amplitude weight equals A. This last property can be derived either from the average value of the binomial distribution, or by direct substitution into Eqn (1). Over many choices the standard deviation of amplitude weight allocated to an element equals $A(1-A)/(L-1)^2$. It would be difficult to extend as simply some of the other levels and probabilities shown in Table 1.

The levels and probabilities of Eqns (13) and (14) give one of the best possible approximations to the model amplitude distribution, and hence one of the lowest sidelobe levels possible, because the binomial distribution gives those levels closest to A the highest selection probabilities, whatever the value of A, see Table 2.

TABLE 2. Values of levels and probabilities for a 17 level array

LEVEL NUMBER	VALUE OF LEVEL	PROBABILITY		
		TERM FROM BINOMIAL DISTRIBUTION	FOR A = 0.7	FOR A = 0.3
1	1.000	A^{16}	0.003	0.000
2	0.938	$16\ A^{15}(1-A)$	0.023	0.000
3	0.875	$120\ A^{14}(1-A)^2$	0.073	0.000
4	0.813	$560\ A^{13}(1-A)^3$	0.146	0.000
5	0.750	$1820\ A^{12}(1-A)^4$	0.204	0.000
6	0.688	$4368\ A^{11}(1-A)^5$	0.210	0.001
7	0.625	$8008\ A^{10}(1-A)^6$	0.165	0.006
8	0.563	$11440\ A^9\ (1-A)^7$	0.101	0.019
9	0.500	$12870\ A^8\ (1-A)^8$	0.049	0.049
10	0.438	$11440\ A^7\ (1-A)^9$	0.019	0.101
11	0.375	$8008\ A^6\ (1-A)^{10}$	0.006	0.165
12	0.313	$4368\ A^5\ (1-A)^{11}$	0.001	0.210
13	0.250	$1820\ A^4\ (1-A)^{12}$	0.000	0.204
14	0.188	$560\ A^3\ (1-A)^{13}$	0.000	0.146
15	0.125	$120\ A^2\ (1-A)^{14}$	0.000	0.073
16	0.063	$16\ A\ (1-A)^{15}$	0.000	0.023
17	0.000	$(1-A)$	0.000	0.003

3.2 ARRAYS WITH A THINNING FACTOR LESS THAN UNITY

The method described above may be modified to assign amplitude levels and selection probabilities to an array with a thinning parameter less than unity. The simplest modifications which also thin an array such that, as for a density tapered which array, the number of elements is proportional to q are:

(a) Reducing the selection probabilities for the naturally thinned array by the factor q. Hence:

$$p_i'(A) = q\ p_i(A) \tag{15}$$

where $p_i'(A)$ are the selection probabilities for the array with $q < 1.0$, and $p_i(A)$ are those given by Eqn (14). The amplitude levels defined by Eqn (13) remain unchanged.

(b) Introducing an extra zero level in addition to the one whose selection probability is determined from the binomial distribution. This extra level has a selection probability equal to 1−q. These extensions satisfy Eqns (1) to (4), with Eqns (1) and (2) becoming:

$$\sum_{i=1}^{L'} p'_i \ell'_i = q \sum_{i=1}^{L} p_i \ell_i = qA \tag{16}$$

and

$$\sum_{i=1}^{L'} p'_i = q \sum_{i=1}^{L} p_i + (1-q) = 1.0 \tag{17}$$

where L' is the number of levels for the array with $q < 1.0$, and equals L+1.

The average number of elements of each non-zero level becomes, from Eqn (8):

$$\overline{N_i} = \sum_{m=1}^{M} p'_i = q \sum_{m=1}^{M} p_i \tag{18}$$

showing that the total number of elements is proportional to q.

4 PROPERTIES AS A FUNCTION OF THE NUMBER OF AMPLITUDE LEVELS

Expressions for the average sidelobe level as a function of the number of levels may be derived by substituting into Eqn (7) for arrays with small numbers of amplitude levels. For a naturally thinned array the average sidelobe level is given by the following equation:

$$\bar{S} \approx \frac{1}{(L-1)} \left[\sum_{m=1}^{M} A_m (1-A_m) \right] \tag{19}$$

where the term in square brackets is the average sidelobe level of a density tapered array. The average sidelobe level for an array with $q < 1$ is given by:

$$\bar{S} \approx \frac{\frac{q}{L} \sum_{m=1}^{M} A_m \left[1 + (L-1)\, A_m\right] - q^2 \sum_{m=1}^{M} A_m^2}{q^2 \left[\sum_{m=1}^{M} A_m \right]^2} \tag{20}$$

Figure 1 shows how the values for the average sidelobe level $\bar{S}$ and the total number of elements N change as the number of levels is increased. Values for $\bar{S}$ and $\bar{N}$ were calculated both from Eqns as (8), (18), (19) and (20), and directly from arrays of point sources or their radiation patterns. The close agreement obtained between the different calculations demonstrates the validity of the equations. Results are shown for a naturally thinned array and for one with $q = 0.5$.

An array aperture containing 812 elements when fully filled, with 32 across the diameter, was used for all the calculations. The model amplitude distribution was a 40 dB ($\bar{n} = 4$) Taylor distribution. For direct calculations arrays were set up using values for amplitude levels and selection probabilities from Eqns (13) and (14). Each of the directly calculated values shown in Figure 2 is an average calculated from several arrays.

Figure 1 shows that an increase in the number of amplitude levels reduces the average sidelobe level, as required. It also shows that this reduction is accompanied by an unwanted increase in the number of elements in the array. As described by Eqn (19), the sidelobe level of a naturally thinned array is reduced at a constant rate of -3 dB for every factor of 2 (ie bit) increase in the number of non-zero amplitude levels. But, the array ultimately becomes fully filled. In contrast the array with $q = 0.5$ remains thinned as the number of amplitude levels is increased, but its average sidelobe level is not reduced at a constant rate. It tends to an asymptotic limit.

The effect on the radiation patterns is shown in Figure 2. An increase in the number of amplitude levels produces a much greater reduction in the sidelobes of the naturally thinned array. Sample element distributions for a density tapered array and density and amplitude tapered arrays with q equal to 1.0 and 0.5 are shown in Figure 3.

The average fill factor k for a naturally thinned array as a function of the number of amplitude weights is given in Table 3. The fill factor is the ratio of the actual number of elements in an array to the total number if it were fully filled. The fill factor of an array with $q < 1$ equals qk, ie that of the equivalent naturally thinned array reduced by the factor q. The values in Table 3 were determined from sample arrays with different numbers of elements when fully filled, and apply to all arrays large enough for statistical density tapering. One extra level increases the factor k of a naturally thinned array significantly, from 0.39 to 0.57. Sixteen extra levels increase it to 0.97, in which case the array is only slightly thinned.

TABLE 3. Average fill factor of a naturally thinned array as a function of the number of levels

Model amplitude distribution: 40 dB (n = 4) Taylor distribution

NUMBER OF LEVELS	FILL FACTOR
2	0.39
3	0.57
5	0.74
9	0.89
17	0.97

The changes in array properties are due to those levels with probabilities given by the binomial distribution. When the number of levels is infinite only the level exactly equal to A is selected from them, since the standard deviation of the levels is zero, while their average equals A. Only those levels with probabilities given by the binomial distribution are allocated to a naturally thinned array, so that each element is allocated with unity probability a level exactly equal to A at its location. Thus, as the number of amplitude levels is increased from two to infinity, a naturally thinned array changes from one that is density tapered to one that is both fully filled and fully amplitude tapered, with the result that its average sidelobe level falls to zero.

When the number of levels of an array with $q < 1$ is infinite each element is allocated either a level exactly equal to A, or the additional zero level. The selection probabilities of these levels are q and $1-q$ respectively. Thus, as the number of levels is increased from two to infinity, an array with $q < 1.0$ changes from one that is density tapered to one that is equivalent to a fully filled, fully amplitude tapered array from which on average a proportion $1-q$ of elements have been removed at random. As the array always remains thinned the element density cannot sufficiently approximate the model amplitude distribution, so that the average sidelobe level tends to an asymptotic limit. The value of this limit may derive from Eqn (20) to be:

$$\frac{1-q}{q} \frac{\sum_{m=1}^{M} A_m^2}{\left[\sum_{m=1}^{M} A_m\right]^2} \qquad (21)$$

Because of the asymptotic limit the radiation pattern of the thinned array is always the sum of two terms, so that its sidelobes are always higher than the lowest obtainable from an equivalent fully filled array.

The number of elements in an array may be kept constant by reducing q as the number of levels is increased. In this case, however, the sidelobe level also remains approximately constant. For example, the average sidelobe level of the sample array used in the earlier calculations changes from -28.6 dB to -25.6 dB if the number of elements is kept constant while the number of amplitude levels is increased from two to infinity.

5 REDUCTION OF APERTURE SIZE REQUIRED TO PRODUCE A GIVEN SIDELOBE LEVEL

The peak sidelobe level of a density tapered array is decreased as the aperture size is increased, so that a minimum aperture size is required to produce a given sidelobe level. Density and amplitude tapering can be combined to reduce this minimum aperture size, and hence also the minimum number of elements required. This is illustrated in Figure 4, which shows the peak sidelobe level of density and density and amplitude tapered arrays as a function of M, and hence also the aperture size (equal to $0.25\ M\lambda^2$) and the number of elements (equal to qkM). It can be seen that the minimum value of M to achieve, for example, a -40 dB sidelobe level is reduced by increasing the number of amplitude levels.

The peak sidelobe level was calculated by adding 10 dB to the average sidelobe level. This is a convenient and reasonably accurate approximation[2]. Graphs are shown for naturally thinned arrays with two and five levels, and for arrays with $q = 0.5$ with two and an infinite number of levels. The asymptotic limit was plotted for this last graph, to show the maximum improvement possible. For the naturally thinned arrays in Figure 4, Table 4 gives the minimum value for M, aperture size (expressed as the diameter of a circular aperture of area $0.25\ M\lambda^2$), and the number of elements required to achieve a -40 dB sidelobe level.

Figure 4 shows that the reduction possible in minimum aperture size is larger for naturally thinned arrays than for arrays with $q < 1.0$. For naturally thinned arrays three extra levels, for example, can halve the diameter of the minimum circular aperture from 205 to 100 wavelengths. The number of elements required is also approximately halved (see Table 4).

TABLE 4. Minimum circular aperture size and number of elements required to achieve a −40 dB peak sidelobe level

	DENSITY TAPERED ARRAY	5 LEVEL DENSITY AND AMPLITUDE TAPERED ARRAY
Diameter in wavelengths	205	100
Approximate number of element positions across a diameter	410	50
Total number of elements if array fully filled, M	132,000	32,000
Fill factor	0.39	0.74
Actual number of elements N	51,400	23,400

6 CONCLUSIONS

There are three main conclusions. Firstly that density and amplitude tapering cannot reduce the sidelobe levels of a density tapered array without an accompanying increase in the number of elements. Secondly that even analog amplitude tapering (ie using an infinite number of possible amplitude levels) cannot reduce the sidelobes of a density tapered array to equal those of an equivalent fully filled amplitude tapered array. Thirdly that density and amplitude tapering can significantly reduce the minimum aperture size and number of elements required by a density tapered array to achieve a given sidelobe level.

REFERENCES

1 M I Skolnik, J W Sherman and F C Ogg, IEEE Transactions on Antennas and Propagation, July 1964, pp 408–417.

2 "Principles of aperture and array system design", by B D Steinberg.

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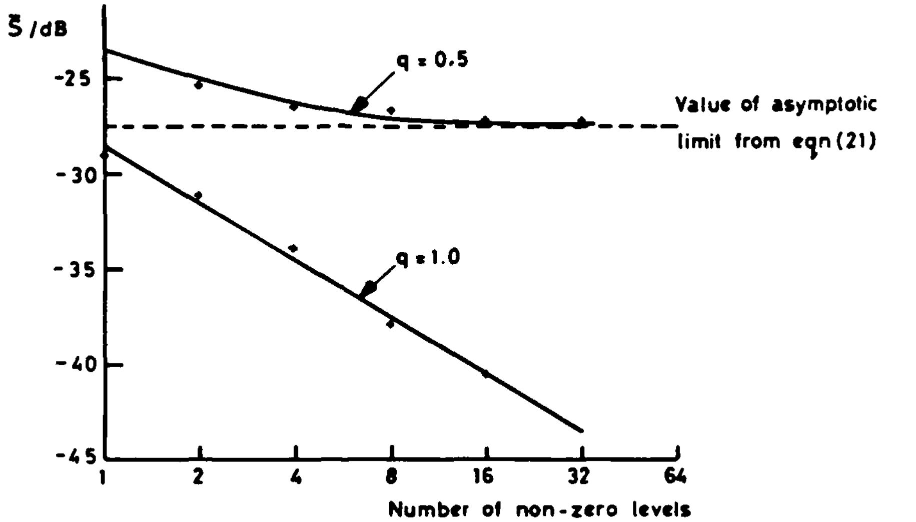
FIG.1. ARRAY PROPERTIES AS A FUNCTION OF THE NUMBER OF LEVELS

For a circular aperture array with 812 elements if fully filled, and having a 40dB (n=4) Taylor Model amplitude distribution.

Key :— Values predicted from eqns (8), (18), (19) or (20)

+ Values calculated directly from arrays.

(a) AVERAGE SIDELOBE LEVEL $\bar{S}$



(b) AVERAGE NUMBER OF ELEMENTS $\bar{N}$

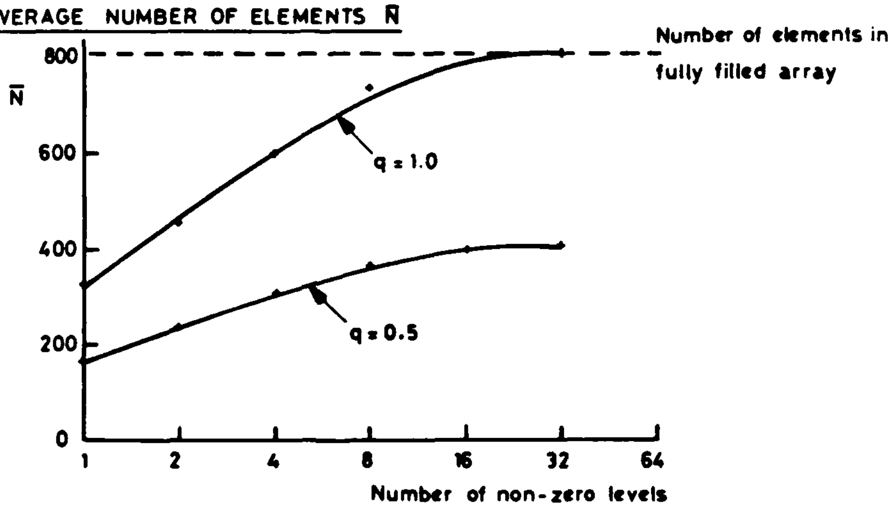


FIG. 2. IMPROVEMENT IN RADIATION PATTERN DUE TO EXTRA AMPLITUDE WEIGHTS

Aperture size: Circular aperture with 812 total possible element locations

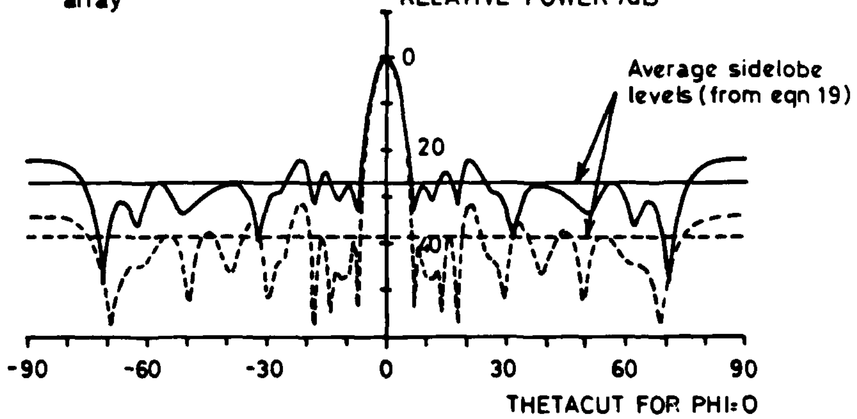
Model amplitude distribution: 40 dB (n = 4) Taylor distribution

Key:

——— Density tapered array

------ 17 level density and amplitude tapered array

a) Naturally thinned array



b) Array with q = 0.5

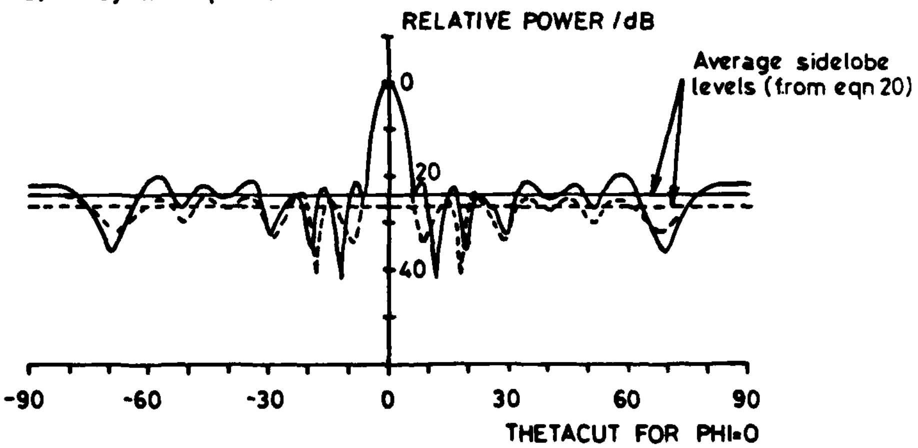


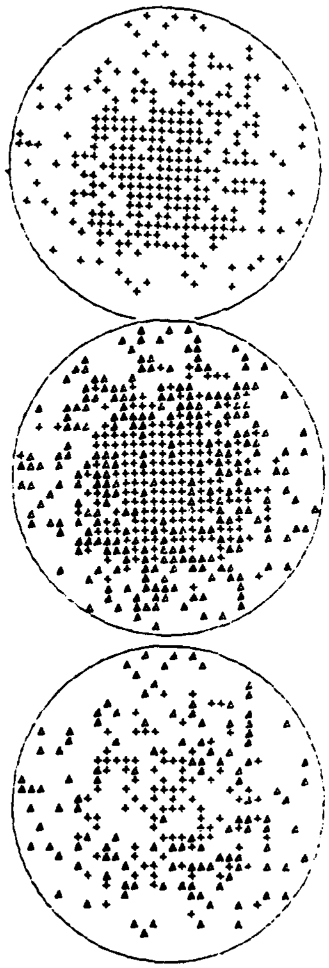
Figure 3. ELEMENT DISTRIBUTIONS

Aperture size: Circular aperture with 812 total possible element locations, 32 across a diameter

Model Amplitude distribution: 40dB Taylor

Key: + Element of weight equal to 1.0

△ Element of weight equal to 0.5



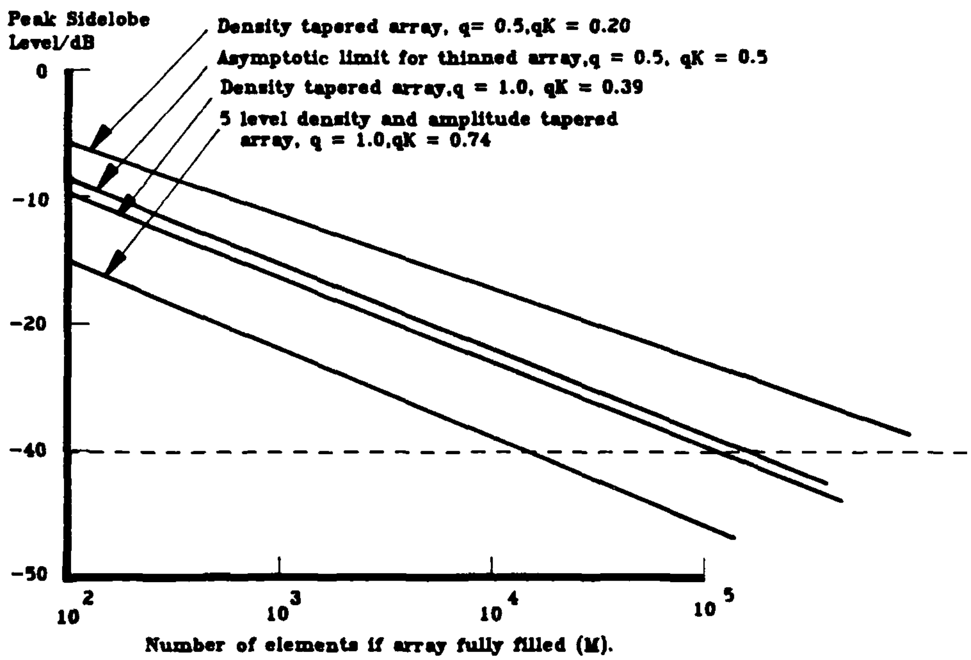
a) Density tapered array
q = 1.0

b) 3 level density and amplitude tapered array q = 1.0

c) 3 level density and amplitude tapered array q = 0.5

Figure 4. PEAK SIDELOBE LEVEL (S + 10dB) AS A FUNCTION OF APERTURE SIZE (M)

Model amplitude distribution: 40dB (n = 4) Taylor



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Abstract

See Overleaf:

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Abstract to Memorandum 4211

Statistical density and amplitude tapering are combined to improve the radiation characteristics of thinned, density tapered arrays by allowing changes in both element density and amplitude weight to better approximate the model amplitude distribution. Statistical density tapering (which uses only uniformly weighted elements) is extended so that each element is allocated one of several possible values of amplitude weight according to predefined selection probabilities. Increasing the number of possible values, or levels, for amplitude weight can reduce the sidelobe levels of an array. Several choices of levels and their probabilities can reduce sidelobes and satisfy the extension criteria. One of the lowest sidelobe levels is produced if: (a) the amplitude levels are equispaced in the allowable range between zero and unity, and (b) their selection probabilities are terms from the binominal distribution of the appropriate order. The properties of arrays designed using equispaced amplitude levels with binomially distributed selection probabilities are calculated and described as a function of the number of levels and aperture size. It was found firstly that density and amplitude tapering cannot reduce the sidelobe levels without an accompanying increase in the number of elements, and secondly that it cannot sufficiently lower the sidelobes of a thinned, density tapered array to equal the lowest obtainable from the comparable fully filled, amplitude tapered array. Thirdly however it was found that density and amplitude tapering can reduce the minimum aperture size and number of elements required to achieve a given sidelobe level.