FURTHER STUDIES OF THE HRR FIELD OF A MOVING CRACK,
AN EXPERIMENTAL ANALYSIS

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FURTHER STUDIES OF THE HRR FIELD OF A MOVING CRACK, AN EXPERIMENTAL ANALYSIS

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ABSTRACT

An improved moire interferometry was used to record simultaneously both the vertical and horizontal displacements associated with stable crack growth in an uniaxially loaded 5052-H32 aluminum, single edge notched specimens. For stable crack growth up to 2 mm, the vertical displacement field showed the dominance of the HRR field. HRR field was detected in the horizontal displacement only at the initial stage of loading. The far and near field J-integrals were path independent during this incremental crack extension. These results and previous results involving 2024-0 and 2024-T3 aluminum specimens indicate that J-characterization of a crack, is not valid for such ductile materials in this specimen configuration.

INTRODUCTION

One of the most popular fracture criterion in nonlinear fracture mechanics is Rice's [1968] J-integral concept for which large developmental efforts, experimental and theoretical, have been expended for the past fifteen years. For a nonlinear elastic solid, the J-integral is path independent and thus the experimentally more accessible far-field J-integral value can be used to characterize the crack tip field. Furthermore, Shih, Delorenzi and Andrews [1979] showed, through finite element analysis, that this path independency holds for small crack extension of about 5 mm in real engineering materials.
Physically, the J-integral value for a nonlinear elastic solid is the energy release rate which reduces to the familiar strain energy release rate for a linearly elastic solid. For a power hardening elastic-plastic material, HUTCHINSON [1968] and RICE and ROSENGREN [1968] showed that the extent of J-dominated crack tip region, which is commonly referred to as the HRR field, is characterized by this single parameter. MCMEEKING and PARKS [1979] and SHIH [1985] used finite element analysis to study the extent of the HRR fields in bend and tension fracture specimens. Both studies concluded that the HRR field is severely reduced in the tension specimens. The HRR field has been studied experimentally using the projection moire technique by CHIANG and HAREESH [1986]. This study showed that except for the very vicinity of the crack tip, the HRR field was a reasonable representation of the crack tip field. More recently, DADKHAH and KOBAYASHI [1989] showed through moire interferometry that the dominant displacement component, which is perpendicular to the crack, may conform with the corresponding HRR component but the displacement component parallel to the crack remained elastic and thus violated the premise of the HRR field.

The objective of this paper is to present further experimental evidence on the above and to discuss the significance of these and previously presented results.

EXPERIMENTAL PROCEDURE

The vertical and horizontal displacements in an uniaxially loaded 5052-H32 aluminum, single-edged notched plate were determined by an improved moire interferometry. Figure 1 shows the specimen configuration which was loaded in a special testing machine (HAWONG, KOBAYASHI, DADKHAH, KANG, RAMULU, [1987]). Figure 2a shows four results of uniaxial stress-strain tension tests in the vertical and horizontal directions for the 0.8 mm thick 5052-H32 aluminum alloy plate which were used in this study. The stress-strain relations for the vertical and horizontal directions for this plate were found to be within 5% of the experimental data shown in Figure 2a. Figure 2b shows the two parameters for the corresponding power hardening relation which was fitted to average of the four tests in Figure 2a.
The improved moire interferometry is based on the four beam arrangement [POST, 1987] with an additional beam splitter and a prism which records simultaneously the vertical and horizontal displacements in a single frame (DADKHAH, WANG and KOBAYASHI, [1988]). This method removes the approximation, which was necessary in previous studies (KANG, KOBAYASHI and POST, [1987]; KANG and KOBAYASHI, [1988]), in the J-estimation. It is conducive for high speed photographic recording of the transient moire fringes associated with a rapidly propagating crack. For this study, the moire fringes during stable crack growth were recorded by a motorized Nikon camera.

An AST Turboscan Digitizer and a Macintosh II computer were used to digitize the photographically recorded moire fringes. A software was developed to compute the two-dimensional strain components from the recorded displacement field. These strain components were used to compute the J-integral value along given rectangular contours, which encompass the crack tip, as shown in Figure 1 (DADKHAH, KOBAYASHI, WANG and GRAESSER, [1988]).

J-INTEGRAL EVALUATION

The evaluation of the J-Integral requires the strain components, the stress components and the strain energy density at each data along the contours (RICE, [1968]). KING and HERRMANN [1981] used measured strains along the two traction-free edges of a centrally cracked tensile specimen to determine the J-integral. Measured strains have also been used by (MACKENZIE, MCKELVIE and WALKER, [1986]; MULLER and GROSS, [1980]; KAWAHARA and BRANDON, [1982]; READ, [1983]) to determine J experimentally. Each procedure was somewhat limited in its applicability due to difficulties encountered in measuring the rotation terms, \( \partial u/\partial y \) and \( \partial v/\partial x \), simultaneously.

A major advantage which is gained by using the four-beam arrangement (POST, [1987]) is that both \( \partial u/\partial y \) and \( \partial v/\partial x \) can be quantified directly from the in-plane u- and v-displacement fields which are represented by the two moire fringe patterns. The J-measurement, which is derived for rectangular contours surrounding the crack tip, is
first divided into line integrals along the vertical and horizontal segments as shown in Figure 1. The integral value of J along the vertical segments is:

\[ J_V = \int_{V_1} \left[ W - \left( \sigma_{xx} \frac{\partial u}{\partial x} + \tau_{xy} \frac{\partial v}{\partial x} \right) \right] dy - \int_{V_2} \left[ W - \left( \sigma_{xx} \frac{\partial u}{\partial x} + \tau_{xy} \frac{\partial v}{\partial x} \right) \right] dy \]  

(1)

and along the horizontal segments, the value of J is:

\[ J_H = \int_{H_1} \left[ W - \left( \sigma_{xx} \frac{\partial u}{\partial x} + \tau_{xy} \frac{\partial v}{\partial x} \right) \right] dx + \int_{H_2} \left[ W - \left( \sigma_{xx} \frac{\partial u}{\partial x} + \tau_{xy} \frac{\partial v}{\partial x} \right) \right] dx \]  

(2)

Using the J₂-deformation theory for multiaxial states, the strains in the vicinity of a crack tip in a power hardening material is represented by

\[ \frac{\varepsilon_{ij}}{\varepsilon_0} = \frac{3}{2} \alpha (\sigma_0/\sigma_0)^{n-1} S_{ij} \]  

(3)

and

\[ \sigma_0 = \left( \frac{3}{2} S_{ij} S_{ij}^{1/2} \right) \]  

(4)

where \( S_{ij} \) is the deviatoric stress, and \( \varepsilon = \frac{\sigma_0}{E} \) is uniaxial strain with E as modulus of elasticity. \( \sigma_0 \) is the yield stress and \( \alpha \) and \( n \) are two the disposal parameters to fit the power law hardening stress-strain relation with the uniaxial tensile data.

For the state of generalized plane stress, the stress-strain relations for this power hardening material can be expressed in the following matrix forms;

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix} =
\begin{pmatrix}
(1+F) & (1+F/2) & 0 \\
(1+F/2) & (1+F) & 0 \\
0 & 0 & [2(1-v)/2+F/4]
\end{pmatrix}
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
2\varepsilon_{xy}
\end{pmatrix}
\]  

(5)
where \( E \) and \( \nu \) are the modulus of elasticity and Poison's ratio, respectively, and

\[
F = \alpha \sigma_0^{n-1}
\]

\[
\gamma = (1+\nu+1.5F) (1-\nu+0.5F)
\]

\[
\sigma_i^2 = \sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2
\]

Strain energy density \( W \) is defined as:

\[
W = \int_0^\varepsilon \sigma_{ij} d \varepsilon_{ij}
\]

\[
W = \frac{1}{E} \left\{ \frac{1+\nu}{3} \sigma_e^2 + \frac{1+2\nu}{6} \sigma_{kk}^2 + \frac{\alpha n}{n+1} \sigma_e^2 \left( \frac{\sigma_0}{\sigma_e} \right)^{-1} \right\}
\]  

(6)

where \( \sigma_0 \) is the yield stress.

The resultant J-integral value for this problem is given by

\[
J = J_v + J_H
\]

(7)

Accuracy of this procedure was assessed by evaluating equs. (1) and (2) along a 4.2 cm contour, which did not enclose the crack tip, using the moire fringe data. The resultant \( J = 49.0 \) (Pa-m), which theoretically should vanish, was 0.4 % of the minimum recorded J-value in this study.
Path independency of the J-integral during stable crack growth was evaluated using a series of rectangular paths encompassing the extending crack tip: as shown in Figure 1.

HRR FIELD

For a power hardening material, the $r^{-1}$ singularity in $W$ implies a $r^{-1/(n+1)}$ singularity in the stresses, a $r^{-n/(n+1)}$ singularity in the strains, and a $r^{1/(n+1)}$ variation in the displacements. The near-crack tip singularity fields can be written as

$$
\sigma_{ij} = \sigma_0 \left( \frac{J}{\alpha \sigma_0 \epsilon_0 n f} \right)^{1/(n+1)} \bar{\sigma}_{ij}(\theta, n) \tag{9}
$$

$$
\varepsilon_{ij} = \alpha \varepsilon_0 \left( \frac{J}{\alpha \sigma_0 \epsilon_0 n f} \right)^{n/(n+1)} \bar{\varepsilon}_{ij}(\theta, n) \tag{10}
$$

$$
u_i - u_i = \alpha \varepsilon_0 r \left( \frac{J}{\alpha \sigma_0 \epsilon_0 n f} \right)^{n/(n+1)} \bar{u}_i(\theta, n) \tag{11}
$$

The dimensionless $\theta$-variations of $\bar{\sigma}_{ij}(\theta, n)$, $\bar{\varepsilon}_{ij}(\theta, n)$ and $\bar{u}_i(\theta, n)$ and the normalizing constant $l_n$ depend on the fracture mode, on $n$ and on the state of plane strain or plane stress. These variations must be normalized in some manner and thus the maximum value of $\bar{\sigma}_e = 3/2 (S_{ij}S_{ij})^{1/2}$ was set to unity in this study. The contribution in $\bar{u}_i$ allows for a possible translation of the crack tip.

RESULTS

One 5052-H32 aluminum cruciform specimen was loaded uniaxially to generate a stable crack growth length of $\Delta a = 2$ mm at which point the experiment was terminated since
the moire fringes became too dense and indistinguishable. Figure 3 shows the load versus load-line displacement relation for this uniaxially loaded 5052-H32 aluminum cruciform specimen.

Figures 4a and 4b show typical moire fringes, i.e., the vertical displacement, v, and the horizontal displacement, u. Also shown are typical integration contours used for the J-integral evaluation. The J values obtained along these contours are shown in Table 1. Path independence, i.e., within a eight percent scatter in the J-values, is noted. The J-integral values which were computed (SHIH, GERMAN and KUMAR [1981]) by using the results of Figure 1, are also shown for comparison in Table 1. The specimen of the single edge notched specimen under uniaxial tension used in this computation was set to b = 85.7 mm.

Figure 5 and 6 show typical log-log plots of the v- and u-displacement fields obtained from the moire fringes at a crack extension of \( \Delta a = 1.95 \) mm. The load for these recording can be found from Fig. 1. Also shown are the log-log plots of the displacements versus radial distance, r, of the linear elastic fracture mechanics (LEFM) and HRR fields at a crack tip polar angle of \( \theta = 45^\circ \). The LEFM field was obtained by computing the equivalent stress intensity factor from the average J-value obtained from the contour integration. The HRR field was obtained by substituting this average J-value into equ. (11). The HRR field requires that the slope of the log-log plots be a constant of \( 1/(n + 1) \), which is 0.059 for the 5052-H32 aluminum used in this study. These plots indicate that the v-field conformed with the HRR field requirement at a radial distance approximately 7 mm away from the crack tip. The nonlinear zone as gleaned from Fig. 5a, (HUTCHINSON, [1983]) thus extends 7 mm from the crack tip along \( \theta = 45^\circ \). The u-field, on the other hand, conformed with the LEFM crack tip field beyond the 7mm radial distance and had a much smaller strain singularity inside of this 7 mm boundary. This nonlinear region in 5052-H32 aluminum alloy is much larger than 2024-0 and 2024-T3 aluminum alloys (DADKHAB and KOBAYASHI [1989]. A total of fourteen log-log plots of the u- and v-fields obtained in this study were evaluated to arrive at the conclusion that only the v-field exhibited the HRR field through the loading and stable crack growth process.
Figure 7 and 8 shows plots of the measured v- and u-displacements at a point of \( r = 1.2 \) and 5 mm, respectively (1.5 and 6.3 times the plate thickness) and at \( \theta = 45^\circ \), where HRR field was shown to extend the furthest (CHIANG and HAREESH, [1986]). With increasing applied load. Also shown are the corresponding u- and v-displacement fields for the LEFM and HRR fields at the same location. These results show that the region of \( r = 1.2 \) mm is entrenched in the nonlinear zone while the v-displacement of \( r = 5 \) mm follows the HRR field throughout the entire loading. The u-displacement field, on the other hand, fell away from the HRR after the initial loading.

Figure 9 shows the traditional \( J \)-resistance curve of this 5052-H32 specimen. Also shown are the approximate \( J \)-integral values which were reported by KANG and KOBAYASHI [1987], for the same material but for a much smaller conventional single edge notched specimen. These \( J \)-resistance curves differ with others in that crack extension occurs at a very low applied load without the significant blunting as reported by PARIS, TADA, ZAHOOR and ERNST [1979].

DISCUSSIONS

The results described in this paper are in complete agreement with those for the less ductile 2024-0 and 2024-T3 aluminum cruciform specimens by DADKHAH and KOBAYASHI [1989]. Figures 10 and 11 show typical variations of the measured v- and u-displacement fields respectively, in identical 2024-0 and 2024-T3 aluminum specimen together with the corresponding displacements for HRR and LEFM fields at a radial distance of \( r = 1.2 \) mm and polar angle of \( \theta = 45^\circ \) from the crack tip. Unlike the 5052-H32 aluminum specimen the v-displacement in the more strain hardening 2024-0 and 2024-T3 aluminum alloy followed the HRR field throughout the stable crack growth of \( \Delta a = 2 \) mm. The u-displacement field, on the other hand, exhibited the same trend of falling away from the HRR field after the quarter of the loading phase. These results are in agreement with the replotted experimental results (HU and LIU [1976]) of KE and LIU [1973] who used geometric moire method to determine the dominant strain component in 2024-0 aluminum double-edge-notched specimens.
It should be noted that this present discussion is limited to the analysis of HRR region of a moving crack in plane stress conditions. This and the above referenced previous results of DADKHAH and KOBAYASHI [1989] both showed that only the v-field exhibited the expected progression from the LEFM to the HRR crack tip fields with increasing load. Unlike 2024-0 and 2024-T3, the v-displacement in the more ductile 5052-H32 (n = 16) deviated from the corresponding HRR component at a higher loading where the HRR singularity field prevailed in the former two aluminum alloys. The u-displacement fields in both the 2024-0 and 5052-H32 specimens deviated from the corresponding HRR field at the very earlier stages of loading. Figure 2 shows that anisotropy in the rolled aluminum alloy sheet could hardly contribute to this deviation for the 5052-H32 specimen. Similar disclaimers are gleaned from the uniaxial stress-strain data of 2024-0 and 2024-T3, rolled aluminum sheets.

The above results are in disagreement with the requirements for the J-dominant region for plane strain condition by HUTCHINSON [1983] and SHIH [1985] where both the u- and v-displacement conformed with those of the HRR field. These results thus indicate that the desired HRR field does not exist in ductile materials and that J in this case is not a parameter which characterizes the crack tip singularity. J should thus be considered a contour integral, as defined by eqns. (1) and (2), and appears to be path independent for the small crack extension of approximately 2 mm considered in this study. The J-integral could thus be used as a far field parameter provided it is not used to characterize crack tip field.

Finally, one notes that the deformation theory of plasticity was used to compute the stresses for the contour integration in eqn. (3) through (6). Which are opened to experimental scrutiny. The path independency check and the vanishing J along a contour, which did not contain the crack tip, indirectly validate the use of deformation theory of plasticity in this study.

The large deviations, as shown in Table 1, between the J-values estimated by using SHIH, GERMAN and KUMAR [1981] formula requires further investigation. Since the cruciform specimen used in this study is substantially different in geometry with the
conventional single-edge notched specimen used by SHIH, GERMAN and KUMAR [1985], any conclusion regarding his formula is strictly speculative at this time.

CONCLUSIONS

1) HRR field existed in the v-field during the stable crack growth process in a region beyond the nonlinear zone at the crack tip.

2) LEFM field existed in the for u-field beyond the nonlinear zone.

3) J is path independent for a small crack extension of 2 mm in 5052-H32 aluminum alloy.

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REFERENCES


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Table 1. J-Integral Values Under Stable Crack Growth in 5052-H32 Aluminum Specimen, MD031188(1)

<table>
<thead>
<tr>
<th>Load in Y direction (N)</th>
<th>J (kPa-m)</th>
<th>Δa (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contour 1</td>
<td>Contour 2</td>
</tr>
<tr>
<td></td>
<td>b = 85.7 mm</td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>2370</td>
<td>4.8</td>
<td>5.2</td>
</tr>
<tr>
<td>3810</td>
<td>8.0</td>
<td>7.3</td>
</tr>
<tr>
<td>4400</td>
<td>12.9</td>
<td>11.6</td>
</tr>
<tr>
<td>52.50</td>
<td>18.6</td>
<td>17.1</td>
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<tr>
<td>5760</td>
<td>23.8</td>
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<td>40.4</td>
</tr>
<tr>
<td>7460</td>
<td>66.7</td>
<td>64.7</td>
</tr>
</tbody>
</table>

ASK/cm:bfp
Figure 1. Specimen Configuration and J-integral Paths
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Figure 11. \( v \)- and \( u \)-Displacements (\( r = 1.2 \text{ mm} \) and \( \theta = 45^\circ \)) Versus Average \( J \). 2024-T3 Aluminum MD031687
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