University of Colorado at Boulder

Surface and Interface Waves in a Sandwich Plate with Interface Soft Layers

S.K. Datta, A.H. Shah, T. Chakraborty, and R.L. Bratton

CUMER-87-5 December, 1987

DEPARTMENT OF MECHANICAL ENGINEERING

Surface and Interface Waves in a
Sandwich Plate with Interface Soft Layers

S.K. Datta, A.H. Shah, T. Chakraborty,
and R.L. Bratton

CUMER-87-5 December, 1987

The work reported here was supported by a grant from the Office of Naval Research (N00014-86-K-0280; Program Manager: Dr. Y. Rajapakse) and grants from the National Science Foundation (MSH-8609813, INT-8521422, INT-8610487). Partial support was also received from the National Science and Engineering Research Council of Canada and from the University of Colorado in the form of a Faculty Fellowship.

The paper was presented at the EUROMECH 226, Nonlinear and Other Non-classical Effects in Surface Acoustic Waves, Nottingham, England, September 2-5, 1987.

Surface and Interface Waves in a Sandwich Plate with Interface Soft Layers

S.K. Datta*, A.H. Shah†, T. Chakraborty*, and R.L. Bratton*

*Department of Mechanical Engineering and CIRES
University of Colorado, Boulder, CO 80309-0427, USA

†Department of Civil Engineering
University of Manitoba, Winnipeg, Canada R3T 2N2

1. INTRODUCTION

In the past, dynamic behavior of periodically laminated medium has been studied extensively. A review of the literature on exact and approximate analyses of this problem can be found in [1,2].

The corresponding problem of a plate of finite thickness having a large number of periodic anisotropic layers has not received much attention. In [2] a stiffness method was presented for studying harmonic wave propagation in a periodically laminated infinite medium. This method is also well suited for analyzing a finite thickness plate with many layers. In this approach each lamina is divided into several sublayers and the displacement distribution through the thickness of each sublayer is approximated by polynomial interpolation functions in such a way that displacements and tractions are continuous across the interfaces between adjacent sublayers. Details of the method can be found in [2,3]. Here we summarize the pertinent equations and present numerical results showing the effect of soft interface layers between reinforced laminae on the dispersion of guided waves.

The particular systems considered are a single ply (0°) and a cross-ply (0°/90°/0°) laminated plate. Each lamina is assumed to be reinforced by continuous aligned fibers, so that it can be modeled as a transversely isotropic medium with the symmetry axis aligned with the fiber direction. The geometry of a typical cross-ply laminate is shown in Fig. 1, which also shows thin interface layers between adjacent laminae. We have also considered the case when all the fibers are in the same direction (not shown). Although it is possible to derive an exact dispersion equation for propagation of harmonic waves in the plane of the plate, the equation is rather complicated when the number of layers is large. This equation simplifies for propagation in the 0° and 90° directions when one also considers small number of layers. In this paper we have considered a five layered plate: three reinforced layers of the same thickness and two interface isotropic soft equal thickness layers. Results are presented using both an analytical technique and the stiffness method.

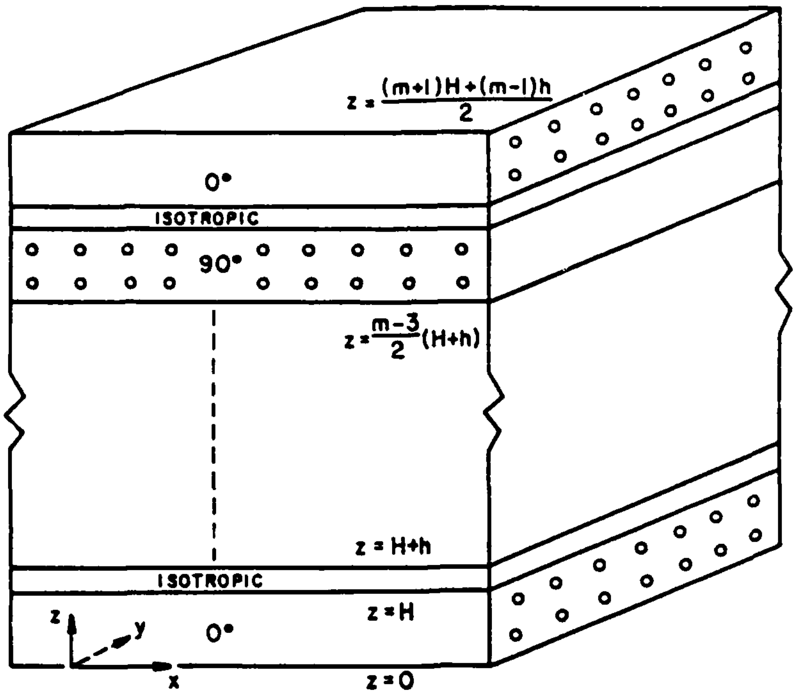

Fig. 1. Geometry of the laminated plate with interface layers. m is the total number of layers ($m \geq 5$).

2. GOVERNING EQUATIONS

We consider time harmonic wave propagation along the x or y axis. Because of symmetry the problem reduces to two uncoupled ones: plane strain in which the displacement components are u_x, o, u_z, and SH when the only nonzero displacement component is u_y. In this paper we consider only the plane strain case.

2.1 Stiffness Method

Consider the ith lamina bounded by $z=z_{i-1}$ and $z=z_i$. The stress strain relation in this lamina will be given by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} c_{11}^{(i)} & c_{13}^{(i)} & 0 \\ c_{13}^{(i)} & c_{33}^{(i)} & 0 \\ 0 & 0 & c_{55}^{(i)} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} \tag{1}$$

where σ_{ij} and ϵ_{ij} are the stress and strain components, respectively, and we have written $\gamma_{xz}=2\epsilon_{xz}$. Note that if the y axis is the axis of symmetry, then

$$c_{11}^{(i)} = c_{33}^{(i)} \text{ and } c_{55}^{(i)} = \frac{1}{2}\left(c_{11}^{(i)} - c_{13}^{(i)}\right)$$

Then the problem is equivalent to that of an isotropic one. In order to get good numerical results each lamina will be divided into several sublayers, p, say. Within the ith sublayer we will choose a local coordinate with the origin at the mid-plane and x_j, y_j, z_j, parallel to the global x,y,z axes, respectively. Let $2h_j$ be the thickness of this sublayer. Denoting $u^{(j)}$ to be the displacement at a point in the jth lamina we write

$$u_x^{(j)} = u_{j-1}f_1 + u_jf_2 + \left(\frac{1}{c_{55}^{(j)}}\chi_{j-1} - \frac{\partial w_{j-1}}{\partial x_j}\right)f_3 + \left(\frac{1}{c_{55}^{(j)}}\chi_j - \frac{\partial w_j}{\partial x_j}\right)f_4 \tag{2}$$

$$u_z^{(j)} = w_{j-1}f_1 + w_jf_2 + \left(\frac{1}{c_{33}^{(j)}}\sigma_{j-1} - \frac{c_{13}^{(j)}}{c_{33}^{(j)}}\frac{\partial w_{j-1}}{\partial x_j}\right)f_3 + \left(\frac{1}{c_{33}^{(j)}}\sigma_j - \frac{c_{13}^{(j)}}{c_{33}^{(j)}}\frac{\partial w_j}{\partial x_j}\right)f_4$$

where f_n (n=1,...,4) are cubic polynomials in the local coordinate z_j given by

$$f_1 = \frac{1}{4}(2-3\eta_j + \eta_j^3),\quad f_2 = \frac{1}{4}(2+3\eta_j - \eta_j^3)$$

$$f_3 = \frac{h_j}{4}(1-\eta_j - \eta_j^2 + \eta_j^3),\quad f_4 = \frac{h_j}{4}(-1-\eta_j + \eta_j^2 + \eta_j^3)$$

Here $\eta_j = z_j/h_j$ and u_j, w_j, χ_j, σ_j are the values of u_x, u_z, σ_{xz}, and σ_{zz} at the jth node. These nodal values of the displacement and traction components are functions of x_j (=x) and t. In this paper it will be assumed that the time dependence is of the form $e^{-i\omega t}$, ω being the circular frequency. The factor $e^{-i\omega t}$ will be dropped in the sequel.

The equations governing the nodal generalized coordinates $\{u_j, \chi_j, w_j, \sigma_j\}$ are obtained using Hamilton's principle. The details can be found in [3]. It is found that, if $\{\underset{\sim}{Q}\}$ is the vector of all the generalized coordinates, and if we assume for $\{\underset{\sim}{Q}\}$ the form

$$\{\underset{\sim}{Q}\} = \{\underset{\sim o}{Q}\}e^{ikx} \tag{3}$$

then $\{\underset{\sim o}{Q}\}$ satisfies the equation

$$(k^4[\underset{\sim 1}{K}] - ik^3[\underset{\sim 2}{K}] - k^2[\underset{\sim 3}{K}] + ik[\underset{\sim 4}{K}] + [\underset{\sim 5}{K}])\{\underset{\sim o}{Q}\} = 0 \tag{4}$$

Matrices $[\underset{\sim 1}{K}]$, etc., have been defined in [3]. $[\underset{\sim 3}{K}]$, $[\underset{\sim 4}{K}]$, and $[\underset{\sim 5}{K}]$ are linear in ω^2.

For nontrivial solution the determinant of the coefficient matrix must be zero. The solution to this equation provides the dispersion relation between k and ω.

2.2 Exact Solution

Consider wave propagation in one of the layers along one of the principal directions as described above. Then we may assume that

$$u_x^{(i)} = U_1(z_i)e^{ikx-i\omega t} \tag{5}$$

$$u_z^{(i)} = U_3(z_i)e^{ikx-i\omega t}$$

Further, if we let

$$\Omega_1^+ = A'e^{-is_1z_i} + A''e^{is_1z_i} \tag{6}$$

$$\Omega_1^- = A'e^{-is_1z_i} - A''e^{is_1z_i}$$

$$\Omega_2^+ = B'e^{-is_2z_1} + B''e^{is_2z_1}$$

$$\Omega_2^- = B'e^{-is_2z_1} - B''e^{is_2z_1}$$

then we may write

$$U_1 = V_1'\Omega_1^+ + V_1''\Omega_2^- \tag{7}$$

$$U_3 = V_3'\Omega_1^- + V_3''\Omega_2^+$$

Here the constants V_1', V_1'', V_3', and V_3'' may be taken as

$$V_1' = ik, \quad V_3' = \frac{k_2^2 - k^2\alpha - s_1^2}{is_1\delta} \tag{8}$$

$$V_1'' = -is_2, \quad V_3'' = -\frac{k_2^2 - k^2\alpha - s_2^2}{ik\delta} \quad .$$

s_1^2 and s_2^2 are given by

$$s_{1,2}^2 = \frac{1}{2\beta}[-(k^2\gamma - k_2^2(1+\beta)) \pm \{(k^2\gamma - k_2^2(1+\beta))^2 - 4\beta(k_2^2 - k^2)(k_2^2 - k^2\alpha)\}^{1/2}] \tag{9}$$

In the above we have defined

$$\alpha = c_{11}^{(i)}/c_{55}^{(i)}, \quad \beta = c_{33}^{(i)}/c_{55}^{(i)}, \quad k_2^2 = \rho\omega^2/c_{55}^{(i)}$$

$$\gamma = 1 + \alpha\beta - \delta^2, \quad \delta = 1 + c_{13}^{(i)}/c_{55}^{(i)} \, .$$

Note that if the y-axis is the symmetry axis then

$$c_{11}^{(i)} = c_{33}^{(i)}, \quad c_{13}^{(i)} = c_{11}^{(i)} - 2c_{55}^{(i)} \, .$$

In that case the solution is the same as that for an isotropic medium.

Using the solution given by equations (7) and (6) and applying the appropriate boundary conditions of continuity of tractions and displacements at the interfaces between the layers, and the traction-free boundary conditions at the free surfaces of the plate, one arrives at the dispersion equation governing k and ω. For the sake of brevity details will be omitted here.

3. NUMERICAL RESULTS AND DISCUSSION

As an application of the techniques described above, we consider a fiber-reinforced plate when the fibers are aligned with the x-direction (0°). Properties of the plate are given in Table 1.

Table 1. Properties of 0° and 90° laminae, and the interface layer. All the stiffnesses are in the units of 10^{11}N/m ^{2}.

	ρ(g/cm ^{3})	c_{11}	c_{22}	c_{12}	c_{44}	c_{55}
0° lamina	1.2	1.6073	0.1392	0.0644	0.0350	0.0707
Interface	1.8	0.0865	0.0865	0.0475	0.0195	0.0195
90° lamina	1.2	0.1392	1.6073	0.0644	0.0707	0.0350

Figure 2 shows the real and imaginary branches of the frequency spectrum for propagation in the x-direction. Solid lines in this figure are the results of the numerical method and the open circles and x's are the exact solutions. Note that the two agree very well. It is seen that as $k \rightarrow \infty$ the slopes of the first symmetric and antisymmetric modes tend to the ratio $V_R/\sqrt{c_{55}/\rho}$, where V_R is the Rayleigh wave velocity in the x-direction. Results by the stiffness method were obtained using 15 sublayers.

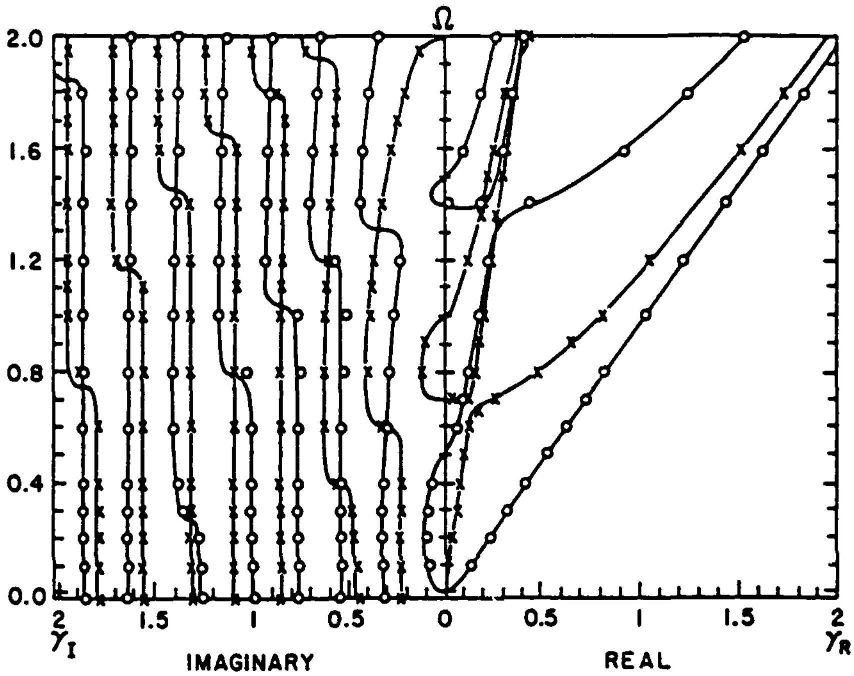

Fig. 2. Dispersion curves for the real and imaginary wave numbers. x: symmetric and o: antisymmetric modes. $\Omega = \frac{\omega H}{2\pi\sqrt{(c_{55}/\rho)}\,\sigma}$ and $\gamma = \frac{kH}{2\pi}$, H being the thickness of the plate.

To examine the effect of the interface soft layers we then consider a plate composed of three equal thickness unidirectional (0°) fiber-reinforced layers separated by two thin soft layers. Properties of each layer are as in Table 1. Figures 3a-3c show the dependence of the phase velocity in the x-direction on the frequency when there are no interface layers and when the ratio of the thicknesses of an interface layer and a lamina takes two values, 0.1 and 0.2. It is seen that the presence of the soft interface layers is to lower the cut-off frequencies and the phase velocities. Also it lowers the surface wave velocity. The results shown in Fig. 3 are obtained using the exact solution.

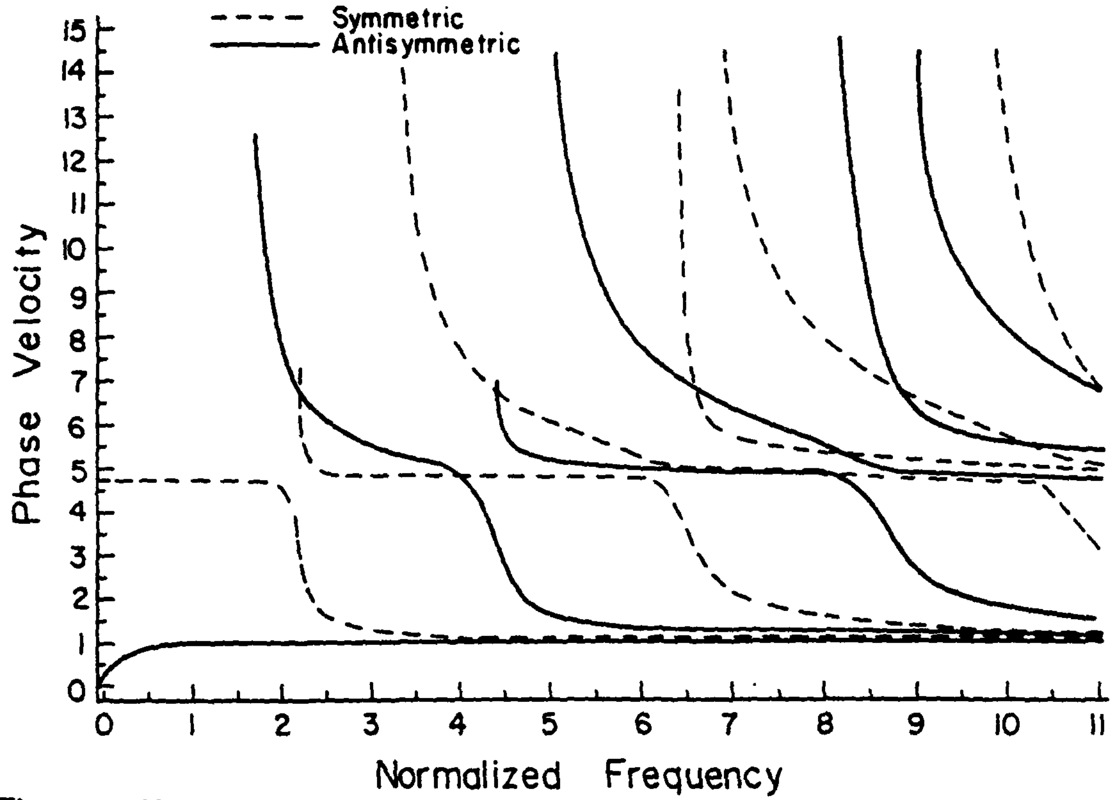

Fig. 3a. Normalized phase velocity ($c=\omega/k\sqrt{c_{55}/\rho}$) vs. normalized frequency ($\frac{1}{2}k_2H=\frac{1}{2}\omega H/\sqrt{c_{55}/\rho}$).

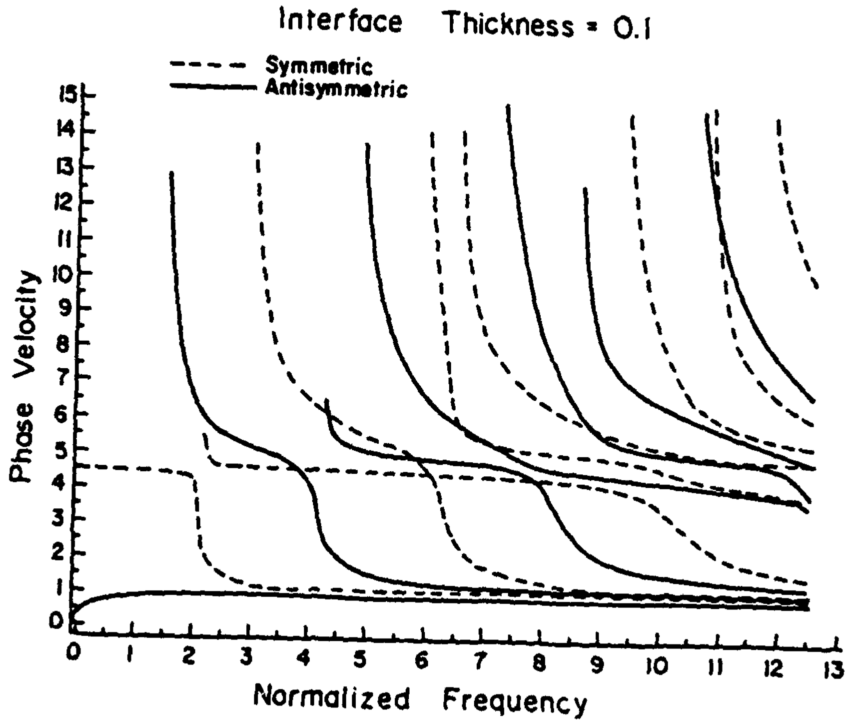

Fig. 3b. Variation of c with $\frac{1}{2}k_2H$ when the interface layer thickness ratio is 0.1.

p.6

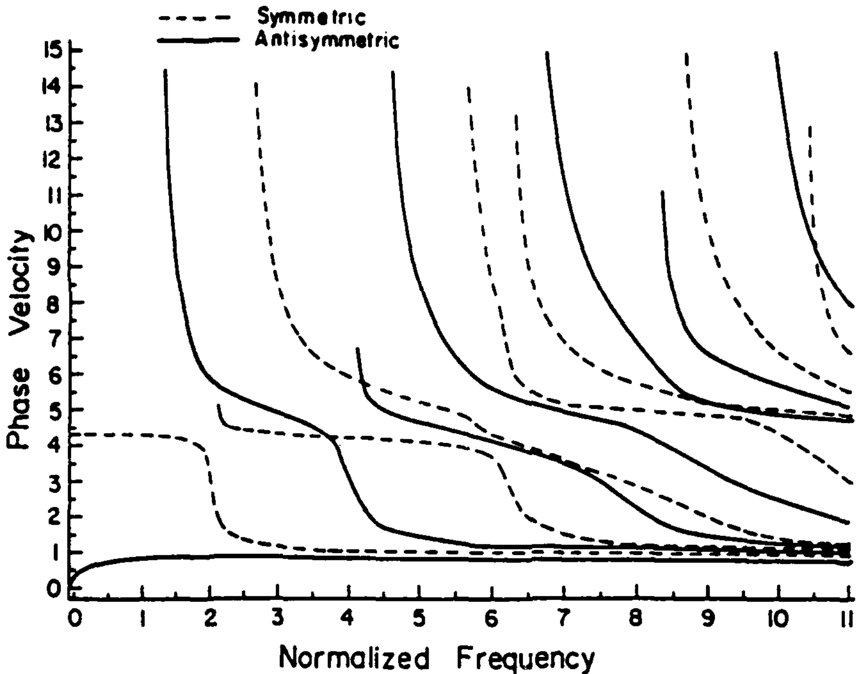

Fig. 3b. Variation of c with $\frac{1}{2}k_2H$ when the interface layer thickness ratio is 0.2.

Using the stiffness method we also studied the dispersion of guided waves in a 0°/90°/0° laminated plate with interface layers. Fig. 4 shows the frequency-wave number dependence when the interface layer thickness is 0.1. These curves are quite different from those shown in Fig. 2. To examine the behavior of the first three branches in more detail we show in Fig. 5 $\Omega = \frac{\omega h}{\pi\sqrt{(c_{55}/\rho)_{\text{isotropic}}}}$ vs. $\gamma = \frac{kh}{\pi}$. It is interesting to observe the slowing down of the waves as the wavelength becomes of the order of the interface layer thickness.

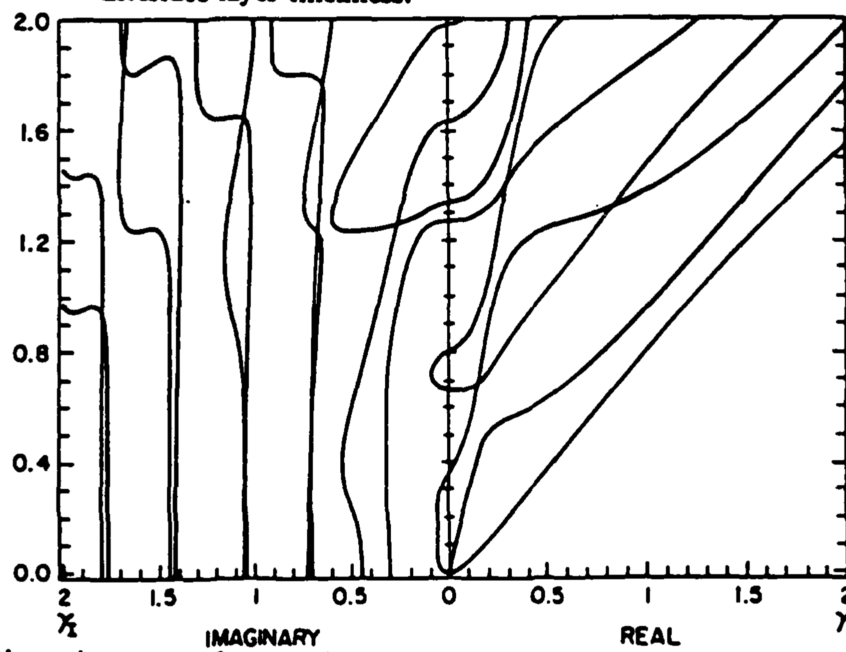

Fig. 4. Dispersion curves for a 0°/90°/0° plate with soft thin interface layers.

p.7

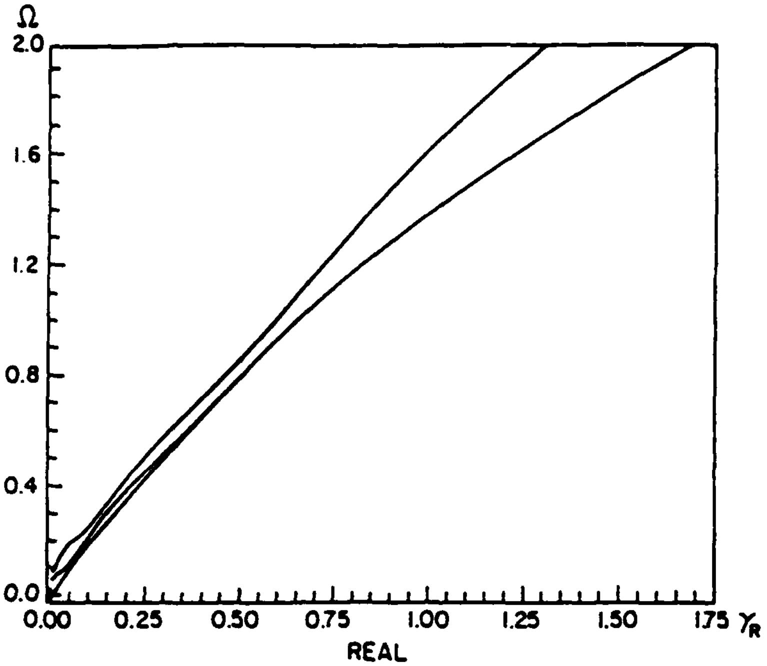

Fig. 5. First three real branches of the frequency spectrum for a 0°/90°/0° plate with soft thin interface layers.

4. CONCLUSION

Dispersion of guided waves in a fiber-reinforced laminated plate with or without soft interface layers between the laminae was ~~has been~~ studied ~~here~~ using an exact method of solution and an approximate stiffness method. It is shown that the approximate numerical method provides a solution that agrees very well with the exact solution. The advantage of the approximate method is that it is easily used for any number of laminae with arbitrary properties. The results for a laminated plate with soft interface layers show significant lowering of cut-off frequencies and phase velocities. Velocity of surface waves is found to decrease with increasing interface layer thickness.

laminates, Wave propagation, Periodic anisotropic

5. ACKNOWLEDGMENT

The work reported here has been supported by a grant from the Office of Naval Research (#N0014-86-K-0280; Program Manager: Dr. Y. Rajapakse) and grants (MSM-8609813, INT-8521422, and INT-8610487) from the National Science Foundation. Partial support was also received from the Natural Science and Engineering Research Council of Canada (A-7988). The first author gratefully acknowledges the support received from the University of Colorado in the form of a Faculty Fellowship for the academic year 1986-87. Parts of the work were done when he was visiting the University College, Galway, and the Chalmers University of Technology, Gothenburg.

6. REFERENCES

1. J.D. Achenbach: Arch. Mech. 28, 257 (1976).
2. A.H. Shah and S.K. Datta: Int. J. Solids Str. 18, 397 (1982).
3. S.K. Datta, A.H. Shah, R.L. Bratton, and T. Chakraborty: Wave Propagation in Laminated Composite Plates, Report CUMER87-4 (Department of Mechanical Engineering, University of Colorado, Boulder, 1987).