A lee wave model for deep-sea mudwave activity

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Abstract—Mudwaves, with wavelengths up to 6 km and heights up to 100 m, are commonly found in the deep sea where steady, sediment-laden currents are present; their internal structure suggests that they have migrated with time. Lee waves appear to be generated in the density gradient above the sinusoidal mudwave topography; the near-bottom flow field associated with the lee waves creates a cross-wave asymmetry in bottom current velocity. A model of bottom flow and sedimentation rate for a transverse mudwave shows that preferential deposition occurs on the upstream flanks and the bed forms migrate upstream. The flow conditions for such lee waves are common in the deep sea; therefore many mudwaves are probably active under present flow conditions. The model suggests that for a given wave, the ratio of downstream-upstream sedimentation rate varies primarily with flow velocity. Thus changes in this ratio, determined by, seismic or sampling techniques, might be used to determine past variations in flow velocity.

INTRODUCTION AND BACKGROUND

Deep-sea mudwaves are large-scale, quasi-sinusoidal bed forms found in many ocean basins where deep currents play an important role in sediment transport and deposition (Fig. 1). These waves, also known as migrating sediment waves, form in fine-grained sediments, and they often develop on the flanks of large sediment drifts (Fox et al., 1968; Hollister et al., 1974), along continental margins (Jacobi et al., 1975; Lonsdale, 1983), and near deep-sea channels (Damuth, 1979; Normark et al., 1980). The heights of these waves range from <10 to >100 m, and their spacings range from <1 to about 10 km. The waves can be followed within sub-bottom profiles, suggesting that the waves have been present for very long time intervals, at least 10's to 100's of thousands of years.

The relationship between mudwaves and bottom current flow has been poorly understood. Detailed surveys of mudwaves related to contour-current flow often indicate that the waves are typically oriented with their long axes at a low angle to the regional contours (Hollister et al., 1974; Jacobi et al., 1975). The waves often are found to migrate upslope, and, because the waves are not parallel to the bottom current direction (which is presumed parallel to the regional contours), upcurrent. In some instances, such as in the central Argentine Basin (Flood and Shor, 1988), the mudwave crests appear to be perpendicular to the bottom current, and the waves migrate upcurrent.

Studies of mudwaves on the Bahama Outer Ridge (Flood, 1978) suggested that internal waves (lee waves) are generated in the weakly stratified near-bottom flow as it passed over the mudwave (Fig. 2). Studies of the lee-wave phenomena by Miles (1968) and Huppert (1968), among others (see discussion in Turner, 1973), suggest that stratified flow over an isolated ridge will create strong lee waves when the quantity

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Fig. 1. Examples of migrating abyssal mudwaves from the Argentine Basin. Sub-bottom structure on these 3.5 kHz records suggest that the bed form has migrated by preferential deposition of sediments on one side of the wave, perhaps accompanied by erosion on the other side. Studies of mudwave character and bottom current flow directions suggest that the waves here migrate against the current, that is that preferential deposition occurs on the upstream side of the wave. The vertical exaggeration of these profiles is about 20:1. The waves in the lower profile appear to be perpendicular to the flow whereas those in the upper profile are at an angle to the flow (FLOOD and SHOR, 1988).

Fig. 2. Near-bottom towed temperature profile collected across a mudwave on the Bahama Outer Ridge (western North Atlantic) near 28°17'N, 74°23'W. (a) Temperature contoured along Deep-Tow fish track. (b) Temperature measured at Deep-Tow fish. During the time of this profile, the bottom current was 5 cm s\(^{-1}\) (3.9 cm s\(^{-1}\) perpendicular to the wave), and \(\kappa = Nh/U\) was 0.4 (from FLOOD, 1978).
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\[ \kappa = \frac{N h}{U} \] (where \( N \) is the stability, \( h \) is the obstacle height, and \( U \) is the free-stream velocity taken normal to the ridge) is near unity (Fig. 3). For flow conditions in a mudwave field on the Bahama Outer Ridge, \( \kappa \) values range from 0.4 to 1.5, suggesting that lee waves can be generated under present-day flows. Near-surface currents on the downstream flank of the ridge (where the streamlines are close together) are stronger than those on the upstream flank (where the streamlines are farther apart). Flood (1978) suggested that the flow pattern could lead to lower sedimentation rates on the downstream mudwave flank and higher sedimentation rates on the upstream side. Such a depositional pattern would result in upstream bed form migration.

Such a lee-wave model of mudwave dynamics can be quantified in the case where a sinusoidal mudwave is oriented perpendicular to the bottom current flow to predict variations in near-bottom velocity, bed shear stress, and sedimentation rate across a wave profile. Queney (1947, 1948) presented a model of stratified flow over a sinusoidal bed applied to the problem of air flow over mountain ranges. While more elaborate models of stratified flows over topography have been developed since then (e.g. Clark and Peltier, 1984), Queney’s solution is of particular use here since it deals with flow over sinusoidal topography. The model applies for small deformations of the flow induced by bottom topography, a condition met by mudwaves as the wave heights are almost always <2% of their wavelengths. Since the model applies for a flow which is perpendicular to the wave crest, it will not apply directly to mudwaves that occur at an angle to the bottom flow (e.g. the Bahama Outer Ridge); however, the model should apply to areas where the waves appear to be perpendicular to the bottom flow (e.g. the central Argentine Basin).

Fig. 3. Calculated lee wave streamlines created by a stratified flow over a semicircular ridge (Huppert, 1968). Note that the streamlines are more closely spaced on the downstream side of the ridge.
For a rotating, stratified atmosphere, the stability is

\[ N = \left( \frac{g}{\rho} \frac{\partial \rho}{\partial z} \right)^{1/2}, \]

where \( g \) is the gravitational acceleration, \( \rho \) is the density, and \( z \) is height.

Quenez (1948) presented streamlines for stratified flow over sinusoidal topography as a function of \( k \) (the wavenumber of the sinusoidal profile), \( k = 2\pi/L \), where \( L \) is the mudwave wavelength), \( k_s \) and \( k_f \), where \( k_s = N/U \) and \( k_f = fU \); \( f \) is the Coriolis parameter and \( U \) is the mean flow velocity of the stratified layer.

Solutions were presented for the following cases (Fig. 4):

Case 1, \( k_s < k \): the vertical variation in streamline displacement is exponential with no phase shift over the wave.

Case 2, \( k_f < k < k_s \): the vertical variation in streamline displacement is sinusoidal with a phase shift over the wave.

Case 3, \( k < k_f \): the vertical variation in streamline displacement is exponential with no phase shift over the wave.

For mudwaves in the deep sea, case 2 (lee-wave formation) leads to migrating sediment waves. For any given wavelength, lee waves form from \( U_{min} = f/k \) to \( U_{max} = N/k \). Case 1 applies where the flow is stronger than \( U_{max} \) and case 3 applies where the flow is weaker than \( U_{min} \). The flow patterns in cases 1 and 3 lead to preferential deposition in the trough.

Following Quenez (1948), the equations for the streamline displacement \( \zeta \) at a mean height \( z \) over a sinusoidal seabed topography \( b \) (for case 2) are given as follows.

\[ \zeta = U \left( B \left( \rho_0 / \rho \right)^{1/2} \cos(kx + k'z) + z \right), \]

where \( b = B \cos(kx) \),

\[ k' = k \left( \frac{k^2 - k_s^2}{k^2 - k_f^2} \right)^{1/2}. \]

Fig. 4. Schematic representations of streamlines for flow over sinusoidal topography in the three cases. A horizontal offset of flow lines occurs only in case 2. In cases 1 and 3 the greatest streamline spacing, and thus the lowest velocity and highest sedimentation rates, are in the trough (Quenez, 1947).
Here $U$ is the mean velocity, $B$ is the mudwave amplitude (one-half the wave height), $\rho$ is the mean density at height $z$, $\rho_0$ is the density at $z = 0$, and $x$ is the distance along the wave profile.

The velocity field along the mean flow direction can be determined by differentiating the streamline equation.

$$u = U(1 + Bk'(\rho_0/\rho)^{1/2} \sin(kx + k'z)).$$  \hspace{1cm} (3)

The velocity at the mudwave surface, $z = 0$, is

$$u = U(1 + B k' \sin(kx)).$$  \hspace{1cm} (4)

Since $k'$ is always positive, the near-surface velocity field reaches a maximum value at $1/4 L$ and a minimum value at $3/4 L$. The vertical gradient in $u$ is small near $z = 0$, and zero at $1/4 L$ and $3/4 L$, and the equation for $u$ at $z = 0$ is valid for $z$ up to several 10's of meters at $1/4 L$ and $3/4 L$.

The equation for $u$ near $z = 0$ can be used to estimate bed shear stress across the wave profile. The bed shear stress ($\tau$) can be estimated from $u$ by using a drag coefficient ($C_D$) and a quadratic stress law,

$$\tau = \rho \ C_D \ u^2.$$  \hspace{1cm} (5)

Drag coefficients have been determined for turbulent Ekman planetary boundary layers, where $u$ is the free-stream (geostrophic) velocity (e.g., Csanady, 1967; Weatherly and Wimbush, 1980), and for a logarithmic turbulent boundary layer, where $u$ is the velocity at 100 cm above the bed (e.g., Sternberg, 1968). Weatherly and Wimbush (1980) determined a $C_D$ value of 0.002 for a deep-sea turbulent Ekman boundary layer whose speed, bottom roughness ($z_0$), and stratification are similar to those observed near mudwaves on the Blake–Bahama Outer Ridge. Csanady (1967) suggested $C_D$ values of about 0.001 for atmospheric boundary layers for $U/fz_0 = 5 \times 10^5$, conditions similar to those of Weatherly and Wimbush (1980). Drag coefficients determined for logarithmic turbulent boundary layers range from about 0.002 to 0.004 (Stenberg, 1968). Here we assume a constant $C_D$ of 0.002 for the mudwave profile and take $u$ as the geostrophic velocity above the Ekman boundary layer. Weatherly and Martin (1978) suggested that the thickness of the Ekman boundary layer ($H$) can be represented as $H = 1.3 \ u_* / [f (1 + N^2 f^2)^{1/4}]$. With $u_* = 0.36 \ \text{cm} \ \text{s}^{-1}$ (corresponding to $\tau = 0.06 \ \text{Pa}$), $N = 5 \times 10^{-4} \ \text{s}^{-1}$ (a typical value in the deep sea) and at 45° latitude, $H$ is 20 m. The thickness of the logarithmic turbulent boundary layer is $0.10 - 0.15 \ H$, or 2-3 m (Businger and Arya, 1974). For simplicity, velocities calculated at $z = 0 \ \text{m}$ are considered to represent the geostrophic velocity above an Ekman boundary layer ($z = 20 \ \text{m}$). This approach is justified because velocities calculated at $z = 20 \ \text{m}$ are not significantly different from those calculated at $z = 0 \ \text{m}$. For the value of $C_D$ used here, erosion occurs when $u = 17.1 \ \text{cm} \ \text{s}^{-1}$. Erosion occurs at higher velocities for smaller values of $C_D$ (for example, erosion occurs at 24.6 cm s$^{-1}$ for $C_D = 0.001$).

McCave and Swift (1976) suggested that the rate of fine-grained sediment deposition ($R$, g cm$^{-2}$ s$^{-1}$) can be determined from the sediment concentration in the water ($C_w$, g cm$^{-3}$), the particle settling velocity ($W_s$, cm s$^{-1}$), the bed shear stress ($\tau$), and the maximum shear stress at which deposition will occur ($\tau_c$),

$$R = C_w W_s (1 - \tau/\tau_c) \ P.$$  \hspace{1cm} (6)
$P$ is the probability that a sediment particle will "stick" when deposited (taken as 1 here). Experimentally determined values of $\tau_e$ range from 0.04 to 0.08 Pa (McCave and Swift, 1976). Here we take an average value of 0.06 Pa. $R$ is multiplied by $3.16 \times 10^{10}$ (seconds in 1000 y) and divided by 1.5 (a typical near-surface sediment density) to convert from g cm$^{-2}$ s$^{-1}$ to geological units of cm y$^{-1}$ $\times$ 10$^{-3}$. A typical value of $C_w$ for regions of active sedimentation in the deep sea is $1 \times 10^{-7}$ g cm$^{-3}$ (100 $\mu$g l$^{-1}$; Ettreim et al., 1976; Biscaey and Ettreim, 1977). McCave (1985) suggests that much of the material in suspension at the HEBBLE site on the continental margin off Nova Scotia falls at a velocity of $5 \times 10^{-3}$ cm s$^{-1}$. As $\tau$ increases past $\tau_e$ sediment ceases to deposit.

We assume here that $\tau_e$, $C_D$, $C_w$ do not vary systematically across the mudwave profile. Near-bottom studies of mudwaves on the Bahama Outer Ridge suggest that bottom roughness is similar from one wave flank to another (Hollister et al., 1974; Flood, 1978), and there is presently little evidence to indicate that the concentration or character of surficial sediments or fine-grained suspended sediments varies systematically across a mudwave.

The maximum sedimentation rate occurs on the upstream wave flank ($3/4 L$) where the flow speed is the lowest, while the minimum sedimentation rate occurs on the downstream flank ($1/4 L$) where the flow speed is the highest. By forming the sedimentation rate ratio (SRR) between the minimum (downstream) sedimentation rate and the maximum (upstream) sedimentation rate, both $C_w$ and $W_s$ are eliminated from the equation.

$$SRR = \frac{\tau_e - \rho C_D U^2 (1 + B k')^2}{\tau_e - \rho C_D U^2 (1 - B k')^2}$$

As noted above, the near-bottom velocity is calculated at $z = 0$ to simplify the expression. If $C_D$ remains constant from one wave flank to another, the sedimentation rate ratio (SRR) will depend only on the mean flow velocity $U$ ($k'$ is also a function of $U$).

** USE OF THE MODEL FOR MUDWAVES IN THE ARGENTINE BASIN **

The model was used to calculate flow and sedimentation rates for a mudwave with a height of 36 m (amplitude of 18 m), a wavelength of 6000 m, and at a latitude of 45°. These parameters approximate those found in the field of apparently transverse mudwaves in the central Argentine Basin (Flood and Shor, 1988). The case 2 solution should be valid for a mean flow of 9.8–47.7 cm s$^{-1}$. At flow speeds slower than 9.5 cm s$^{-1}$, sediments should accumulate preferentially in the wave troughs. Streamlines calculated for a flow of 13 cm s$^{-1}$ show that lee waves are developed above the mudwave profile (Fig. 5) and $\kappa = 0.13$, suggesting that the interaction is weak. The near-bed flow speed (at the outer edge of the bottom boundary layer) varies from 14.3 cm s$^{-1}$ on the downstream wave flank to 11.7 cm s$^{-1}$ on the upstream flank, and bed shear stress varies from about 0.041 to 0.027 Pa (Fig. 6). The lowest calculated sedimentation rate is $3.3$ cm y$^{-1}$ $\times$ $10^{-3}$ on the downstream wave flank, while the highest rate is $5.7$ cm y$^{-1}$ $\times$ $10^{-3}$ on the upstream wave flank (Fig. 6). The sedimentation rates on the crest and in the trough are equal at $4.6$ cm y$^{-1}$ $\times$ $10^{-3}$.

The cross-wave near-bed velocity and sediment rate profiles calculated at several mean velocities (Fig. 7) show that the sedimentation rate decreases across the wave profile with
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Streamlines over Sinusoidal Topography (13 cm/sec)

> > > > > > FLOW DIRECTION > >

- 300 m
- 200 m
- 100 m

TOPOGRAPHY

Fig. 5. Calculated streamlines for flow over a mudwave with parameters typical of those in the central Argentine Basin (amplitude = 18 m, wavelength = 6000 m, latitude = 45°) at a mean velocity of 13 cm s⁻¹. The upstream shift of the streamlines with height above bottom suggests lee wave formation, and the near-bottom streamlines are most closely spaced on the downstream flank of the wave and most widely spaced on the upstream flank. This figure has a vertical exaggeration of 8.

> > > > > > FLOW DIRECTION > >

Mean flow speed 13 cm/sec

Fig. 6. Calculated near-bottom flow speed (c), bed shear stress (b), and sedimentation rate (a) for a flow of 13 cm s⁻¹ over sinusoidal bottom topography (d, mudwave parameters as in Fig. 5). The highest sedimentation rates occur on the upstream mudwave flank and the lowest rates occur on the downstream flank.

Increasing flow speed, but the highest sedimentation rate remains on the upstream wave flank. For a mean flow greater than 16 cm s⁻¹, the currents on the downstream flank are strong enough to erode, and the zone of erosion increases with mean flow speed until sediments can deposit only on the upstream flank. At higher velocities, erosion occurs across the wave profile. For these parameters the sedimentation rate ratio (SRR) will vary from about 0.65 at 10 cm s⁻¹ to 0 at 16 cm s⁻¹ when erosion begins to occur on the downstream wave flank (Fig. 8).

EFFECT OF VARYING WAVE PARAMETERS

The changes in depositional pattern produced by changing wave dimensions and latitude can be most readily observed by plotting SRR for waves of different dimensions...


Fig. 7. Cross-wave current speeds (a) and sedimentation rates (b) calculated for several mean flow velocities over sinusoidal topography (c. mudwave parameters as in Fig. 5). Sedimentation rates and SRRs decrease as flow speed increases. Erosion occurs on the downstream flank at a mean speed above about 16 cm s\(^{-1}\).

Fig. 8. Plots of calculated sedimentation rate ratio (SRR) vs mean flow speed for mudwaves with varying amplitudes, wavelengths and latitudes (\(C_D = 0.002\)). If \(C_D\) is taken constant at 0.001, the SRR vs velocity curves are similar to those shown in Fig. 8; however, SRR values of 0 occur when the current speed is about 24 cm s\(^{-1}\).

(Fig. 8). The minimum velocity necessary for wave migration (\(U_{\text{min}}\)) increases with latitude, suggesting that preferential deposition may occur in high-latitude wave troughs even at moderate flows. \(U_{\text{min}}\) also increases with wavelength, suggesting that mudwaves with longer wavelengths will become inactive at lower flow speeds. If \(U_{\text{max}}\) is greater than about 17 cm s\(^{-1}\) (for \(C_D = 0.002\)), then the downstream wave flanks can be erosional. For \(N = 5 \times 10^{-4} \text{s}^{-1}\), \(U_{\text{max}}\) is >17 cm s\(^{-1}\) when \(L\) is >2200 m. Thus preferential erosion on the downstream flank is more likely to occur on mudwaves with longer wavelengths.

At constant velocity, the SRR generally decreases with increasing wavelength (compare model calculations e, b and a) and with increasing wave amplitude (compare model calculations d, b and c). Thus mudwaves with longer wavelengths and greater heights tend to have asymmetric depositional patterns at low flow speeds (lower SRRs), whereas mudwaves with shorter wavelengths and smaller amplitudes tend to be more symmetric even at higher flow speeds (higher SRRs). Mudwaves at higher latitudes are somewhat
more asymmetric than those formed at lower latitudes (compare model calculations f and b).

For any given wave parameters, and if \( C_D \) is the same on both mudwave flanks and with time, the SRR is only a function of flow velocity. Since the SRR is also the ratio of downstream and upstream sediment thickness deposited during any time interval, past values of the SRR can be determined for existing sediment waves from well-located piston cores or high-resolution seismic profiles. Such information may allow us to calculate bottom water flow velocities for times in the past when the mudwaves were active.

The assumption of constant \( C_D \) across a mudwave is important to the success of this sedimentation model. For example, if \( C_D \) were allowed to vary across the wave, then \( C_D \) on the downstream flank would need to be only 70–80% of the value on the upstream flank in order to have equal shear stresses (and deposition rates) on the two wave flanks. However, since currents on the downstream wave flank are predicted to be stronger than those on the upstream wave flank, current-created roughness is probably more pronounced on the downstream flank, and thus \( C_D \) on the downstream flank may be greater than \( C_D \) on the upstream flank.

**DISCUSSION AND CONCLUSIONS**

This first-order analysis of sedimentation from a stratified flow over a sinusoidal topography suggests that preferential deposition and upstream migration are natural effects of such a flow. Preferential deposition occurs on the upstream mudwave flank because the flow pattern creates a weaker near-bottom flow there (Fig. 9). Less deposition and possibly erosion occur on the downstream flank where near-bottom flow is accelerated. The weak near-bottom density stratification of the flow is responsible for the development of such forms at the relatively low speeds of this environment. The present model is notable because internal waves develop on the density gradient that is commonly developed at the base of the water column, not on a discrete density step such as might exist between two water masses.

![Lee Wave Model of Mud Wave Dynamics](image)

**Fig. 9.** Summary model of mudwave growth process. The lee wave flow pattern leads to lower bed shear stress (higher sedimentation rates) on the upstream flank and higher shear stress (lower sedimentation rates) on the downstream flank.
The model simplifies the possible flow dynamics of mudwaves considerably. One important simplification is that only the cross-wave flow component has been considered. Preliminary investigations of the along-wave flow component (following Queney, 1947) suggest that such flows can be significant for low mean-flow velocity. However, the near-bed flow speed is \(<10\%\) greater at mean speeds \(>10-12 \text{ cm s}^{-1}\). This suggests that while there may be significant variations in local flow direction across the wave, the cross-wave speed variations, and the SRR plots, are similar to those presented here.

Many mudwaves are developed at an angle to the flow, and the present model may not be directly applicable to those waves. This angle may be the result of a long-term interaction between the bottom topography and the flow. Recent studies of mudwaves in the Argentine Basin (Flood and Shor, 1988) suggest that mudwave orientations can vary regionally, but that waves on south-facing slopes formed by eastward-flowing currents have similar orientations. Thus effects such as bottom slope or the earth’s rotation may be important in controlling wave orientation. Future models for mudwave dynamics will need to include cross-wave flow components and allow for topography that is developed at an angle to the flow.

We assume here that the same lee wave flow pattern exists for a long period of time, and thus that a single sedimentation regime applies for thousands of years. Variability in bottom current flow patterns may be due to tidal currents, eddies, benthic storms, benthic fronts, and long-term variations in bottom water flow patterns. All of these components can alter the relationship between the wave and the instantaneous current direction. Where current variability is small (mean flow with small tides and few benthic storms), a single sedimentation pattern may be representative of long periods. However, in regions where tidal components are significant, and especially where they dominate the mean flow, the simple model presented here may not apply. The passage of occasional "benthic storms" or other rare flow phenomena also will affect mudwave activity since short-lived high-velocity events can quickly erode sediments deposited over much longer periods of time. Long-term changes in bottom water flow patterns, velocity, and suspended sediment load and character also must affect wave development. We assume that bottom flow and sediment characteristics, including \(C_D\), \(\tau_c\), \(C_w\) and \(W_p\), are constant across a wave profile. The validity of such assumptions needs to be determined.

If the predictions of this or a more refined mudwave growth model are substantiated, then past SRRs and absolute accumulation rates may reflect past changes in mudwave activity growth which can be related to changes in flow rate or sediment input.

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