



# Power Spectral Densities of Zonal and Meridional Winds in the Stratosphere

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## Abstract

In a previous publication [1], we described the power spectral densities (PSD's) of very high resolution (10 m) scalar winds in the stratosphere based on the triangulated, rocket-laid smoke trail method. The PSD's were obtained there by means of a fast fourier transform version of the Blackman-Tukey technique [2]. In the present paper we shall apply the same methods in order to obtain the PSD's of the individual velocity components. We found that the slopes and amplitudes of the spectra, plotted on log-log paper, were very similar to the results of the former analysis of scalar winds. In the case of the latter, we obtained the average slope of  $2.7 \pm 0.2$  and log-amplitude of  $3.24 \pm 0.2$ . The present results for the components did not differ in a statistically significant way from this. On the other hand, we found a statistical difference between the amplitudes of zonal and meridional component PSD's.

## 1. Introduction

In a previous publication [3] the power spectral densities (PSD's) were presented of 10 m resolution vertical profiles of horizontal wind speeds in the stratosphere. Figure 1 shows the five PSD's plotted on one set of axes. As can be seen, this figure shows the universality of both slope and amplitude of these PSD's very strikingly. The velocity fluctuations are here assumed to be due to atmospheric gravity waves. The universality shown in Fig. 1 is not surprising when one compares it to the findings made in the upper ocean. In that medium it is well known that the gravity waves have a universal spectral property described by the "G.M." or "Garrett-Munk Model" [4, 5]. The latter model was first associated with atmospheric velocity fluctuations by Dewan [6]. It was however, Van Zandt [7] who first proposed that the atmosphere displayed the universal aspects of this model and he has presented a significant body of evidence to demonstrate this.

At the suggestion of R. A. Vincent (private communication) we have, in the present paper, applied the same analysis techniques to the individual velocity components as were used in Ref. [3] in connection with the speeds analyzed there. The main question we address here is whether or not these velocity component PSD's differ significantly from the speed PSD's. As will be shown, the answer is that there is no statistically significant difference. On the other hand, there is evidence for a difference in spectral amplitudes between zonal and meridional components.

## 2. Experimental methods

Our data were obtained from rocket laid vertical smoke trails. These trails served as "fluid markers" and they were in the form of "dashes", the height ranges of which are given in Table 1. Time lapse photographs taken from ground based telescopic cameras were used to triangulate trail positions as

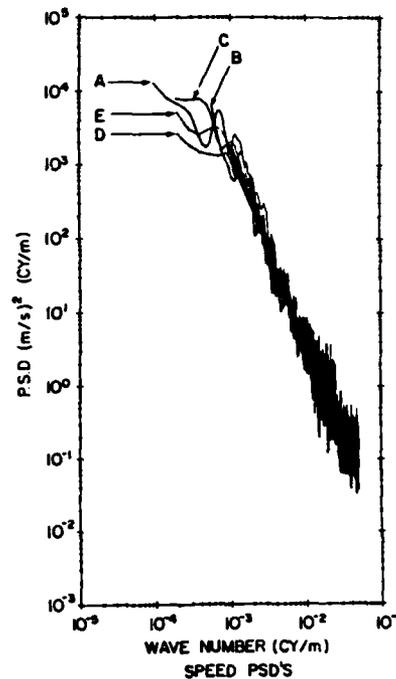


Fig. 1. Power spectral densities (PSD's) of speed (scalar wind) profiles. See Table 1 for labels.

Table I. Speed profiles (cubic detrend) PSD parameters based on straight line fit (from  $k = 10^{-3} \text{ m}^{-1}$  to  $k = 1/2$  Nyquist)

Profile	Height range (km)	PSD slope	PSD amplitude at $k = 10^{-3} \text{ c/m}$ units: $(\text{m}^2/\text{s}^2) (\text{c/m})$
A May 22	(1) 29.3-33.4 (2) 35.8-42.3	$25.9 \pm 0.08$	1558
B May 20	(1) 22.2-27.0 (2) 29.5-33.5	$2.83 \pm 0.05$	1970
C Apr 26	(1) 18.4-21.1 (2) 23.5-27.8 (3) 30-33.7	$2.74 \pm 0.03$	1886
D May 2	(1) 19.8-23.8 (2) 28.0-31.3 (3) 34.9-37.6	$2.93 \pm 0.03$	1889
E Sep 12	(1) 15.8-20.4 (2) 23.1-26.9 (3) 29.1-32.2 (4) 34.1-36.6 (5) 38.2-39.4	$2.46 \pm 0.02$	1388
AVERAGES:		$2.71 \pm 0.187$	$1.738 \pm 247$

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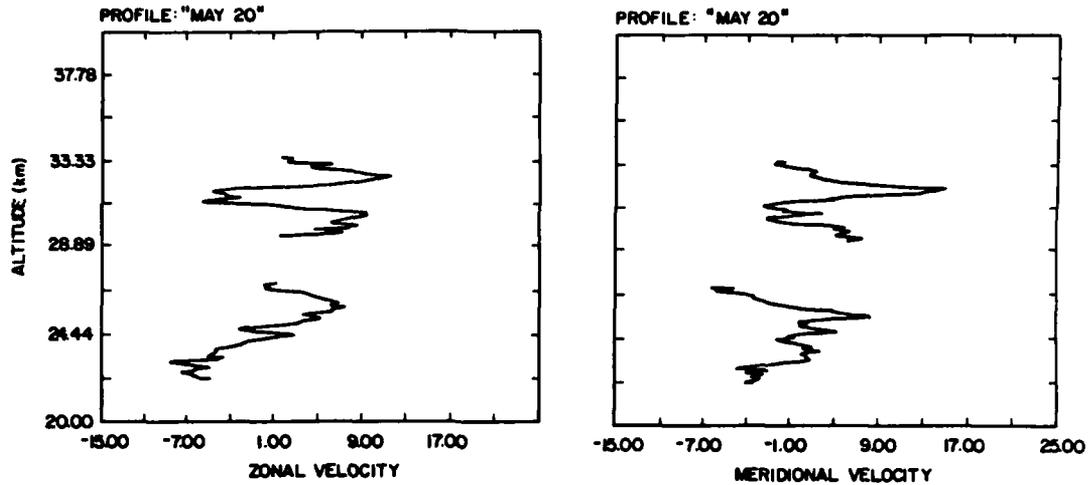


Fig. 2. Example of velocity component profiles. A, Zonal; B, Meridional. This is for the "May 20" profile.

a function of time and altitude. These in turn led to velocity profiles, an example of which is given in Fig. 2. Further details of the experimental methods were published in Ref. [3] and thus they will not be repeated here. In short, we obtained what is considered the "state of the art" (i.e., 10 m resolution) velocity profiles derived from such smoke trails.

### 3. Method of data analysis

The following is a description of the procedure that we used to spectrally analyze our data. It is identical to the one used for Ref. [3].

(1) The data were "prewhitened" (Ref. [8]) by "first differencing" the data as follows:

$$V_i = v_i - v_{i+1} \quad (1)$$

where  $v_i$  are the velocity component values,  $i$  is the data point number, and  $V_i$  represent the prewhitened data.

(2) The mean, trend, and rate of change of trend were then removed by subtracting out a fitted cubic polynomial.

(3) The autocorrelation was obtained from the  $n$  points by first adding zeros until there were  $M$  points such that  $M = 2\gamma > 2N$ . Then  $\gamma$  is an integer and this is done so that we can apply a Fast Fourier Transform (FFT). Next we used the following FFT technique to obtain the autocorrelation. First we computed

$$F \frac{k}{M\Delta Z} = \sum_{i=0}^{M-1} V_i \exp [2\pi j(ki)/M] \quad (2)$$

where  $\Delta Z$  is the altitude separation between points (10 m most of the time). The autocorrelation, which is  $A(Z)$ , is defined by

$$A(Z) = \left[ \frac{1}{N} \right] \sum_{j=0}^{N-1-|Z|} V_j V_{j+|Z|} \quad (3)$$

is obtained from (see Ref. [9])

$$A(\tau) = \left[ \frac{1}{N} \right] \left\{ \frac{1}{M} \sum_{i=0}^{M-1} \left| F \frac{k}{M\Delta Z} \right|^2 \exp [-2\pi j(kz/M)] \right\}. \quad (4)$$

(4) A "window" or "taper" is then applied (90%):

$$A_T(Z) = A(Z) \left\{ 1 - \frac{|Z|}{[(0.90)N + 1]} \right\} \quad (\text{zero outside}). \quad (5)$$

(5) The "whitened" ( $\text{PSD}_w$ ) is obtained from

$$\text{PSD}_w \left( \frac{k}{M\Delta Z} \right) = 2\Delta Z \sum_{Z=-|Z_{\max}|}^{Z=+|Z_{\max}|} A_T(Z) \exp [2\pi j(kz/M)] \quad (6)$$

$$1 \leq k \leq M/2.$$

(6) The "post darkened" PSD is, finally,

$$\text{PSD} \left( \frac{k}{M\Delta Z} \right) = \frac{\text{PSD}_w(k/M\Delta Z)}{[2 - 2 \cos (2\pi k/m)]} \quad (7)$$

### 4. Results

Figure 3 summarizes the PSD's of the meridional and zonal profiles. As can be seen, Figs 1 and 3 (A, B) resemble each other. Table II (A, B) displays the individual slopes and amplitudes of the least square fitted straight lines extending from wavenumber  $k = 10^{-3} \text{ m}^{-1}$  to  $k = 1/2$  Nyquist wavenumber, as well as to  $k^{\max} = (1/100 \text{ m})$  (compare Table I for the results for the speeds). Tables I and II also give the averages and standard deviations of the slopes and amplitudes.

A "t-test" [10] was applied to these averages to see if there was a significant difference between the (a) meridional and zonal component slopes and amplitudes (at  $k = 10^{-3} \text{ m}^{-1}$ ) and (b) the velocity component slopes and amplitudes vs. those of the speeds. Table III summarizes the results. To apply the t-test we used, as estimate of the error of the means,  $\sigma$ ,

$$\sigma^2 = \left[ \frac{(n_1 - 1)\sigma_1^2 + (n_2 - 1)\sigma_2^2}{n_1 + n_2 - 2} \right] \left[ \frac{1}{n_1} + \frac{1}{n_2} \right] \quad (8)$$

where  $\sigma_1$  and  $\sigma_2$  were calculated from the samples from

$$\sigma_i^2 = \frac{\sum V_i^2 - \frac{1}{n} \left( \sum V_i \right)^2}{n - 1}. \quad (9)$$

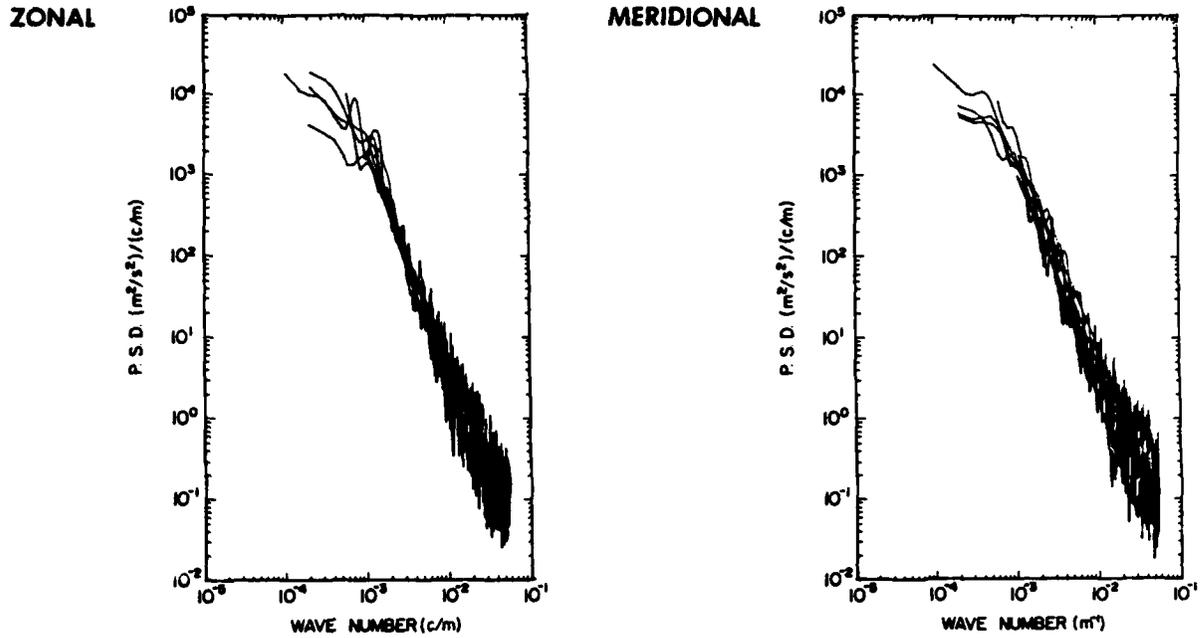


Fig. 3. Power spectral densities (PSD's) of Zonal (A), and meridional (B) velocity components. Table I shows labels, as in Fig. 1.

Table II.A. Velocity profiles (cubic detrend) PSD parameters based on straight line fit (from  $k = 10^{-3} m^{-1}$  to  $k = 1/2$  Nyquist)

Profile	Meridional		Zonal	
	PSD slope	PSD amplitude at $k = 10^{-3} m^{-1}$ units: $(m^2/s^2) (c/m)$	PSD slope	PSD amplitude at $k = 10^{-3} m^{-1}$ units: $(m^2/s^2) (c/m)$
May 22	2.59	1069	2.78	2402
May 20	2.57	995.4	2.99	2243
Apr 26	2.73	1951	2.65	1764
May 2	3.06	1618	2.93	1779
Sep 12	2.43	1642	2.53	1645
Averages	$2.68 \pm 0.24$	$1455 \pm 409$	$2.78 \pm 0.19$	$1967 \pm 334$
Combined averages: (both directions)	$2.73 \pm 0.21$	$1711 \pm 443$		

Table II.B. Velocity profiles (cubic detrend) PSD parameters based on straight line fit (from  $k = 10^{-3} m^{-1}$  to  $k = 1/100 m$ )

Profile	Meridional		Zonal	
	PSD slope	PSD amplitude at $k = 10^{-3} m^{-1}$ units: $(m^2/s^2) (c/m)$	PSD slope	PSD amplitude at $k = 10^{-3} m^{-1}$ units: $(m^2/s^2) (c/m)$
May 22	2.65	1141	2.94	2855
May 20	3.03	2028	3.22	3110
Apr 26	2.79	2072	3.00	2791
May	3.05	1623	3.52	4218
Sept 12	2.63	2183	2.88	2503
Averages	$2.83 \pm 0.202$	$1809 \pm 430$	$3.11 \pm 0.262$	$3095 \pm 664$
Combined averages	$2.97 \pm 0.266$	$2457 \pm 859$		

Table III. *t*-Test between means of PSD parameters ("significance" means  $\leq 5\%$  level of significance)

Type of profile	Slope		Amplitude	
	Value of <i>T</i>	Sig. level	Value of <i>T</i>	Sig. level
Meridional vs. zonal ( $\lambda_{max} = NYQ/2$ )	0.73 df = 8	50% Not sig.	2.17 df = 8	10% Not sig.
Meridional vs. Zonal ( $\lambda_{max} = 100m$ )	1.89 df = 8	10% Not sig.	3.64 df = 8	1% Signif.
Vector components vs. Speed ( $\lambda_{max} = NYQ/2$ )	0.180 df = 13	90% Not sig.	0.125 df = 13	90% Not sig.

Thus for *t* we have:

$$t = \frac{\sqrt{n_1 + n_2 - 2}(\bar{V}_1 - \bar{V}_2)}{\sqrt{(1/n_1) + (1/n_2)} \sqrt{\sum V_1^2 - n_1(\bar{V}_1)^2 + \sum V_2^2 - n_2(\bar{V}_2)^2}} \quad (10)$$

As can be shown from  $\sigma^2$ , and average variances, there are no significant differences between component and speed average  $\sigma^2$ . Note that, consistent with this finding, the PSD's (Table III) have no statistically different parameters from one another with one exception. The zonal and meridional spectral amplitudes do differ statistically, when we take the cut off wavelength as 100 m. This is an important result in the sense that universality does not apply to the component spectra but rather to the total spectra. Theoretically, this was to be expected.

6. Conclusion

High resolution vector wind profiles in the stratosphere have been spectrally analyzed and compared to speed profile spectra previously analyzed in Ref. [3]. The meridonal, zonal and speed PSD's were compared in regard to the slope and the

amplitude (at  $k = 10^{-3} m^{-1}$ ) and no significant differences were found between speeds and velocity components. There was however statistical evidence for an amplitude difference between zonal and meridional component PSD's. Our main conclusion is that there is no statistical difference between the power spectral densities of velocity components taken as a whole and the PSD's of speeds.

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